Cosmological large-scale structure

Lecture 3

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The structure of the large scale structure

large scales | **small scales**

super-horizon scales | linear scales | mildly non-linear scales | strongly non-linear scales

relativistic corrections | *linear pert. theory* | *non-linear pert. theory* | *N-body simulations*

$k \approx aH \approx$ 0.002 h/Mpc

$\lambda \approx 3000\ Mpc/h$

$k \approx 0.1\ h/Mpc$

$\lambda \approx 60\ Mpc/h$

$k \approx 0.3\ h/Mpc$

$\lambda \approx 20\ Mpc/h$

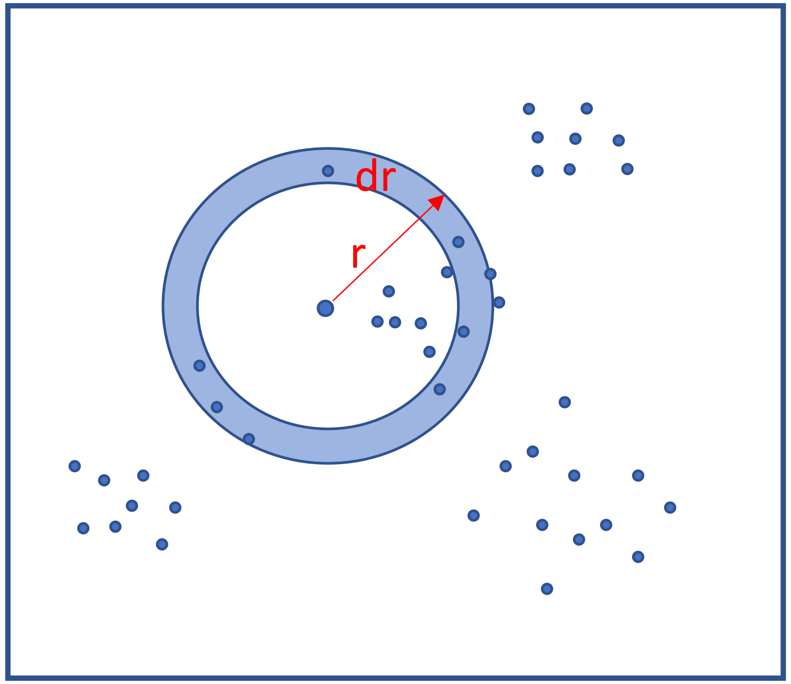
Recap

correlation function

$$\xi(r_{ab}) = \frac{dN_{ab}}{\rho_0^2 dV_a dV_b} - 1 = \frac{\langle n_a n_b \rangle}{\rho_0^2 dV_a dV_b} - 1 = \langle (\delta_a + 1)(\delta_b + 1) \rangle - 1 = \langle \delta(r_a) \delta(r_b) \rangle$$

$$\xi = \frac{DD}{DR} - 1$$

practical implementation



Recap

definition $$P(\mathbf{k}) = \int \xi(r) e^{-i\mathbf{kr}} dV$$

conversely $$\xi(\mathbf{r}) = (2\pi)^{-3} \int P(k) e^{i\mathbf{kr}} d^3k$$

Fourier conjugates

$$V\langle \delta_k \delta^*_{k'} \rangle = \frac{1}{V} \int \xi(r) e^{i(k-k')y + ikr} dV_r dV_y = \frac{(2\pi)^3}{V} P(k) \delta_D(k-k')$$

The correlation function is the variance of $\delta(x)$, the power spectrum is the variance of δ_k

Power spectrum
for a finite-size set
of particles

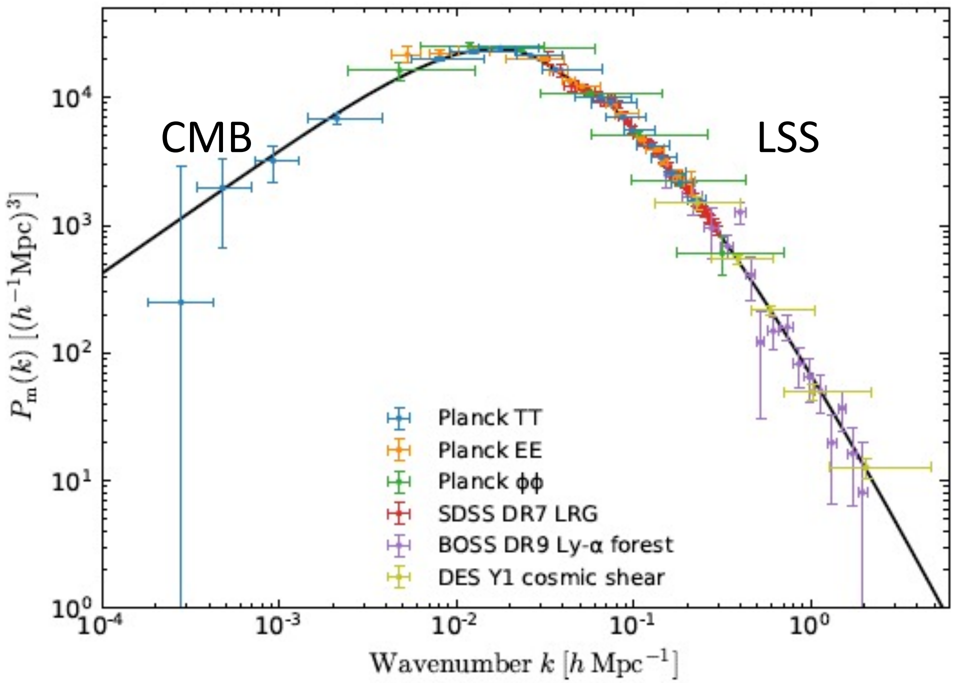
$$P(k) = \frac{V}{N^2} \sum_{ij} w_i w_j \langle e^{ik(x_i - x_j)} \rangle - V W_k^2$$

$$\sigma_R^2 = (2\pi^2)^{-1} \int P(k) W_R^2(k) k^2 dk$$

Roadmap for today

- The theoretical power spectrum
- Linear bias
- Redshift distortion
- Fingers-of-God effect
- Baryon Acoustic Oscillations
- Alcock-Paczynski effect
- A glimpse of non-linearity

The observed power spectrum



ESA and the
Planck collaboration

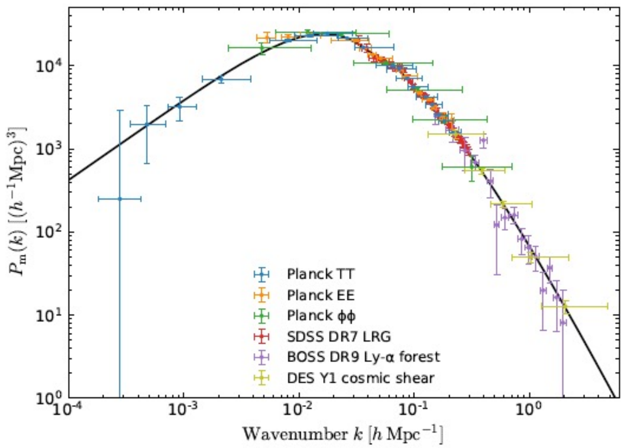
Large scales
~1000 Mpc

Small scales
~10 Mpc

The observed power spectrum

$P(k)_{initial} = Ak^{n_s}$ Initial (inflationary) Power spectrum n_s=slope

$P(k)_{today} = Ak^{n_s}T^2(k; cosmology)$ T=transfer function



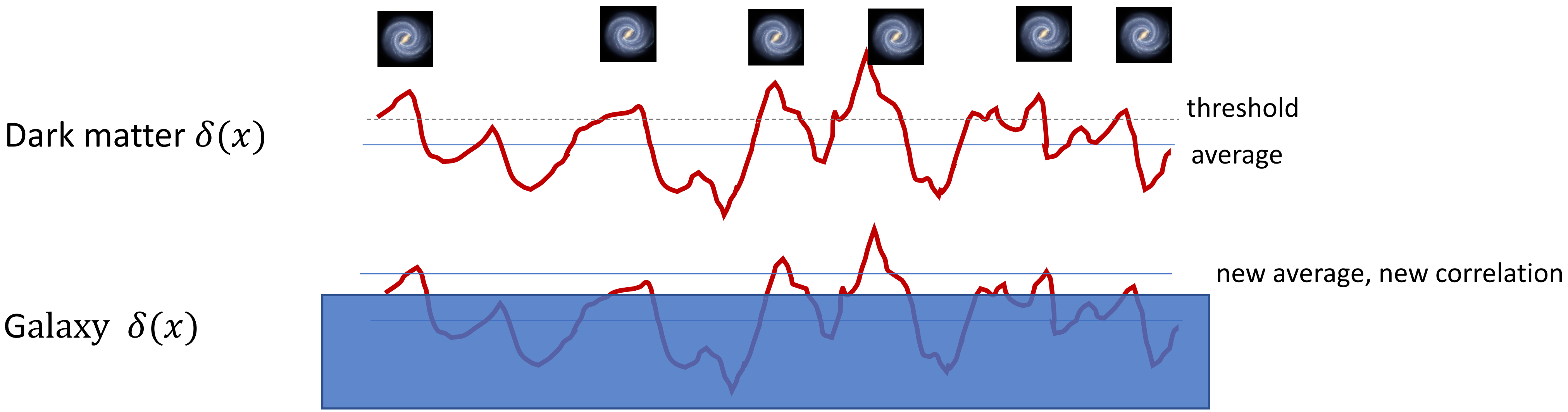
The exact form of T is obtained by solving the coupled linear perturbation equations for all dark matter, baryons, radiation, neutrinos…

A simplified form for LCDM:

$$
\begin{aligned}
P(k) &= Ak^{n_s}T(k)^2 \\
T(k) &= \left[1 + \left[ak + (bk)^{1.5} + (ck)^2\right]^{\nu}\right]^{-1/\nu} \\
(a, b, c) &= (6.4, 3.0, 1.7)\,\Gamma^{-1} Mpc/h, \nu = 1.13 \\
\Gamma &= \Omega_{nr} h
\end{aligned}
$$

Linear bias

In the simplest model of bias, galaxies form only above some threshold



The higher the peak, the more galaxies form

$$\delta_g = b\delta_m \qquad b \approx 1$$

Linear bias

$$\delta_g = b\delta_m \Rightarrow P_g = b^2 P_m$$

This deterministic bias is very simplified but seems to work!

In general, the bias:

- Can depend on time and scale
- Can be stochastic
- Can depend on galaxy type, mass, and environment

Bias for galaxy color in SDSS

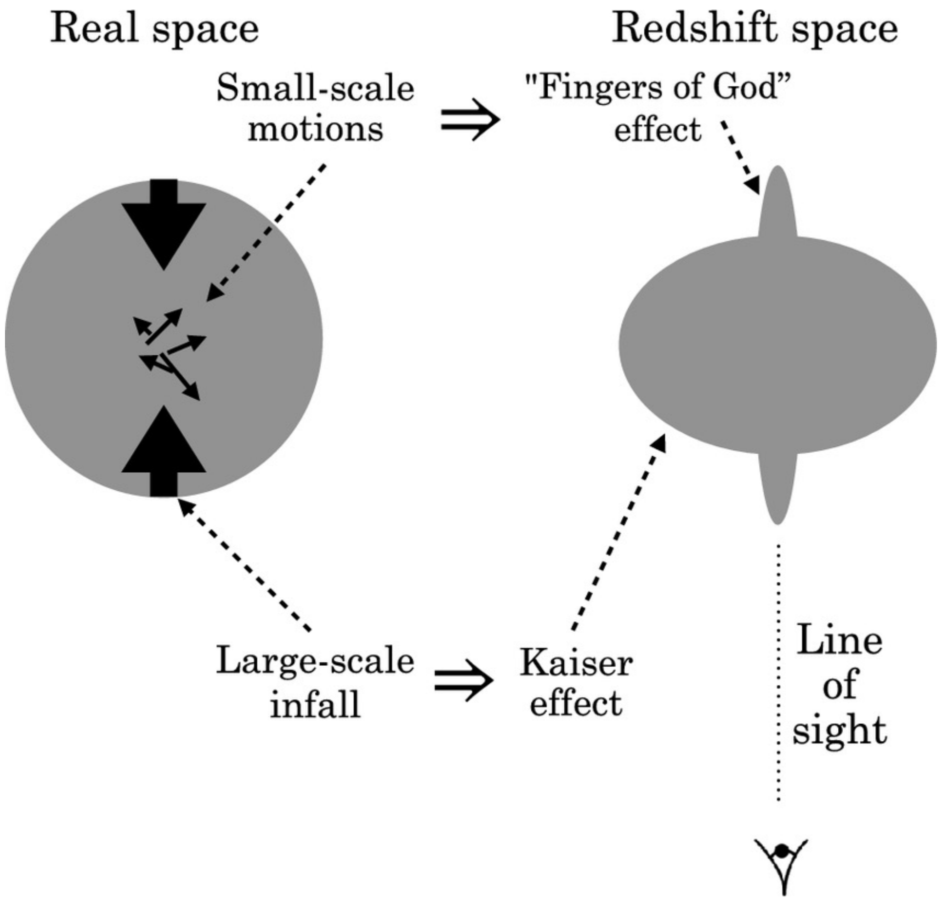
bin	Q-model b_{lin}	Q	P-model b_{lin}	P
red 1	1.40 ± 0.02	9.45 ± 0.70	1.35 ± 0.02	512 ± 48
red 2	1.22 ± 0.03	9.24 ± 0.83	1.17 ± 0.03	375 ± 40
red 3	1.17 ± 0.03	9.44 ± 0.94	1.13 ± 0.03	351 ± 42
red 4	1.20 ± 0.04	7.72 ± 0.93	1.16 ± 0.04	277 ± 46
red 5	1.25 ± 0.05	7.26 ± 1.03	1.20 ± 0.05	272 ± 51
red 6	1.44 ± 0.08	8.03 ± 1.45	1.39 ± 0.07	419 ± 80
blue 1	1.09 ± 0.03	13.74 ± 1.29	1.04 ± 0.03	541 ± 48
blue 2	0.96 ± 0.03	9.89 ± 1.34	0.92 ± 0.03	274 ± 40
blue 3	0.94 ± 0.04	7.74 ± 1.43	0.90 ± 0.04	177 ± 43
blue 4	0.89 ± 0.05	7.44 ± 1.74	0.86 ± 0.05	151 ± 46
blue 5	0.91 ± 0.07	4.64 ± 1.78	0.87 ± 0.06	66 ± 51
blue 6	0.92 ± 0.14	2.99 ± 2.97	0.87 ± 0.13	17 ± 80

James G. Cresswell and Will J. Percival, MNRAS 2008

Mapping real space to redshift space

$$r_{obs} = z/H_0 = (z_c + z_p)/H_0$$

Two effects at different scales:
squeezing and elongation



Huterer 2023

Mapping real space to redshift space

Peculiar velocity along LOS

$$\mathbf{v}_p = \mathbf{v} \cdot \frac{\mathbf{r}}{r}$$

Measured Hubble law

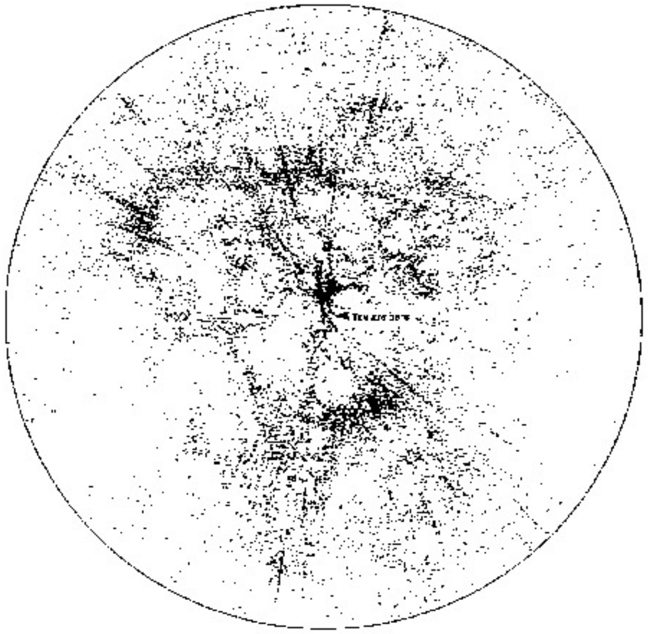
$$r_{obs} = \frac{v_{cosm} + v_p}{H}$$

Distance (u=v/H : velocity in units of 1/H)

$$s = r + u(r) - u(0)$$

3D version:
From real space **r** to redshift space **s**

$$\mathbf{s} = \mathbf{r}\left[1 + \frac{u(r) - u(0)}{r}\right]$$



Redshift space distortion

From real space **r** to redshift space **s**

$$\mathbf{s} = \mathbf{r}\left[1 + \frac{u(r) - u(0)}{r}\right]$$

Number of particle must remain the same!

$$n(r)dV_r = n(s)dV_s$$

$$dV_s = s^2 ds d\cos\theta d\phi = r^2\left(1 + \frac{\Delta u(r)}{r}\right)^2 |J| dr d\cos\theta d\phi = \left(1 + \frac{\Delta u(r)}{r}\right)^2 |J| dV_r$$

$$s = r + u(r) - u(0) \quad \Rightarrow \quad |J| = |\frac{\partial s}{\partial r}| = 1 + \frac{du}{dr}$$

Redshift space distortion

$$dV_s = s^2 ds\, d\cos\theta\, d\phi = r^2 \left(1 + \frac{\Delta u(r)}{r}\right)^2 |J| dr\, d\cos\theta\, d\phi = \left(1 + \frac{\Delta u(r)}{r}\right)^2 |J| dV_r$$

Relation between density contrast in real and redshift space

$$\delta_s = \frac{n(s) dV_s}{n_0 dV_s} - 1 = \frac{n(r) dV_r}{n_0 dV_r \left(1 + \frac{\Delta \mathbf{u}(r)}{r}\right)^2 |J|} - 1$$

Simplify to first order

$$\begin{aligned} \delta_s &= \frac{n(r)}{n_0}\left(1 - 2\frac{\Delta u(r)}{r} - \frac{du}{dr}\right) - 1 \\ &= \left[\frac{n(r)}{n_0} - 1\right] - \frac{n(r)}{n_0}\left[2\frac{\Delta u(r)}{r} + \frac{du}{dr}\right] \\ &= \delta_r - \cancel{2\frac{\Delta u(r)}{r}} - \frac{du}{dr} \end{aligned}$$

Valid for any tracer!

Redshift space distortion

$$\boxed{\delta_s = \delta_r - \frac{du}{dr}}$$

Back to linear pert theory:

$$a\frac{d\delta}{dt} = -ik^i v_i \qquad \dot{v}^i = -\mathcal{H}v^i - a^2 ik^i\psi$$

Assuming irrotational fluid
v is parallel to ***k***

$$v^i = iHa\delta f\frac{k^i}{k^2}$$

derive this equation!

In real space:

$$\mathbf{v}(x) = i\mathcal{H}f\frac{V}{(2\pi)^3}\int \delta_k \frac{\mathbf{k}}{k^2}e^{ikr}d^3k$$

Redshift space distortion

In real space: $$\mathbf{v}(x) = i\mathcal{H}f\frac{V}{(2\pi)^3}\int \delta_k \frac{\mathbf{k}}{k^2}e^{ikr}d^3k$$

Units of 1/H and along the LOS

$$u(r) = \mathcal{H}^{-1}\frac{\mathbf{r}}{r}\cdot\mathbf{v} = if\int \delta_k e^{i\mathbf{k}\cdot\mathbf{r}}\frac{\mathbf{kr}}{k^2 r}d^3k^*$$

$$\frac{du}{dr} = -f\int \delta_k e^{i\mathbf{k}\cdot\mathbf{r}}\left(\frac{\mathbf{kr}}{kr}\right)^2 d^3k^*$$

using $$\frac{d}{dr}e^{i\mathbf{k}\cdot\mathbf{r}} = i\frac{\mathbf{k}\cdot\mathbf{r}}{r}e^{i\mathbf{kr}}$$

And since $$\delta_s = \delta_r - \frac{du}{dr}$$

we get $$\delta_s = \delta_r - \frac{du}{dr} = \delta_r + f\int \delta_k e^{i\mathbf{kr}}\left(\frac{\mathbf{kr}}{kr}\right)^2 d^3k^* = \delta_r + f\int \delta_k e^{i\mathbf{kr}}\mu^2 d^3k^*$$

$$\mathbf{k}\cdot\mathbf{r}/(kr) = \mu$$

Redshift space distortion

$$\delta_s = \delta_r + f \int \delta_k e^{i\mathbf{kr}} \mu^2 d^3 k^*$$

Apply to any tracer (eg galaxies)

Obtained from continuity,
so apply to the total mass fluctuations

So we can write: $$\delta_{g,s} = \delta_{g,r} + \frac{f}{b} \int \delta_{k,g} e^{i\boldsymbol{kr}} \mu^2 d^3 k = \delta_{g,r} + \beta \int \delta_{k,g} e^{i\boldsymbol{kr}} \mu^2 d^3 k$$

Crucial simplification: $\mu = const$ $$\delta_{g,s} = \delta_{g,r} + \beta\mu^2 \int \delta_{k,g} e^{i\boldsymbol{kr}} d^3 k = \delta_{g,r}(1 + \beta\mu^2)$$

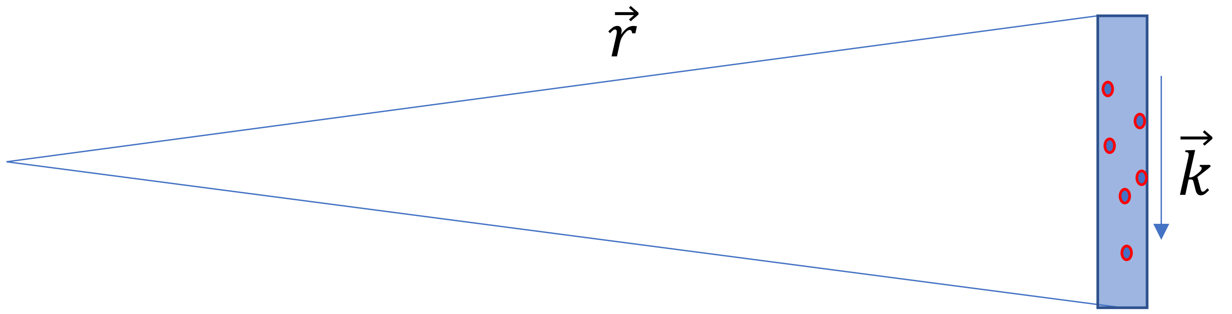
(same in Fourier space!)

Redshift space distortion

$$\delta_{g,s} = \delta_{g,r} + \beta\mu^2 \int \delta_{k,g} e^{i\mathbf{kr}} d^3k = \delta_{g,r}(1 + \beta\mu^2)$$

RSD
Redshift space distortion

Crucial simplification: $\mu = const$
Flat sky/distant observer approximation



$$\mathbf{k} \cdot \mathbf{r}/(kr) = \mu$$

almost constant

Linear power spectrum

All together

the spectrum is no longer isotropic!

$$P_g(k,\mu,z) = b^2 G^2 P_m(k, z=0)(1+\beta\mu^2)^2$$

bias

growth

Matter spectrum shape

RSD

If we average it over angles we get

$$P_s(k) = P_r(k)(1 + 2\beta\langle\mu^2\rangle + \beta^2\langle\mu^4\rangle)$$

where

$$\langle\mu^2\rangle = \frac{1}{2}\int_{-1}^{1} \cos^2\theta' d\cos\theta' = 1/3$$

$$\langle\mu^4\rangle = \frac{1}{2}\int \cos^4\theta' d\cos\theta' = 1/5$$

Finally we obtain for the μ-averaged spectrum

$$P_s(k) = P_r(k)(1 + 2\beta/3 + \beta^2/5)$$

Averaging over angles:

RSD measures $\beta = f/b$

Linear power spectrum

$$P_g(k,\mu,z) = b^2 G^2 P_m(k, z=0)(1+\beta\mu^2)^2$$

bias

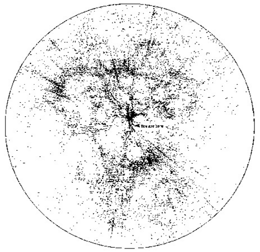
growth

Matter spectrum shape

RSD

Final touch

$$P_g(k,\mu,z) = b^2 G^2 P_m(k, z=0)(1+\beta\mu^2)^2 e^{-k^2\mu^2\sigma_v^2}$$



Fingers-of-God damping

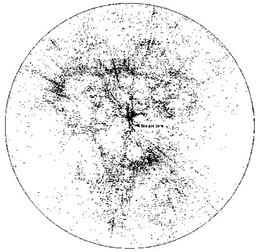
Fingers-of-God

how do we get this?

$$\boxed{P_g(k,\mu,z) = b^2 G^2 P_m(k, z=0)(1+\beta\mu^2)^2 e^{-k^2\mu^2\sigma_v^2}}$$

smoothed corr funct

$$\xi(s,\mu) = \frac{1}{\sqrt{2\pi}\,\sigma_v} \int \xi(\mathbf{r}) e^{-\frac{1}{2}\frac{(r-s)^2\,\mu^2}{\sigma_v^2}} dr$$



Two theorems:

The FT of a convolution of two functions
is the product of the individual FT of the functions

The FT of a Gaussian is another Gaussian

$$P_{\mathrm{FoG}}(k,\mu) = P(k) e^{-k^2\,\mu^2\,\sigma_v^2}$$

Recap

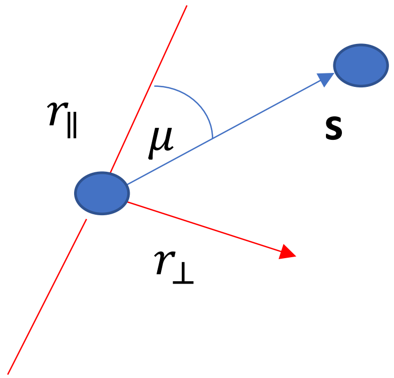
theoretical power spectrum $$P(k)_{today} = Ak^{n_s}T^2(k; cosmology)$$

bias $$P_g = b^2 P_m$$

RSD $$\delta_{g,s} = \delta_{g,r}(1 + \beta\mu^2)$$

(almost) final form $$P_g(k, \mu, z) = b^2 G^2 P_m(k, z = 0)(1 + \beta\mu^2)^2 e^{-k^2\mu^2\sigma_v^2}$$

RSD on the correlation function



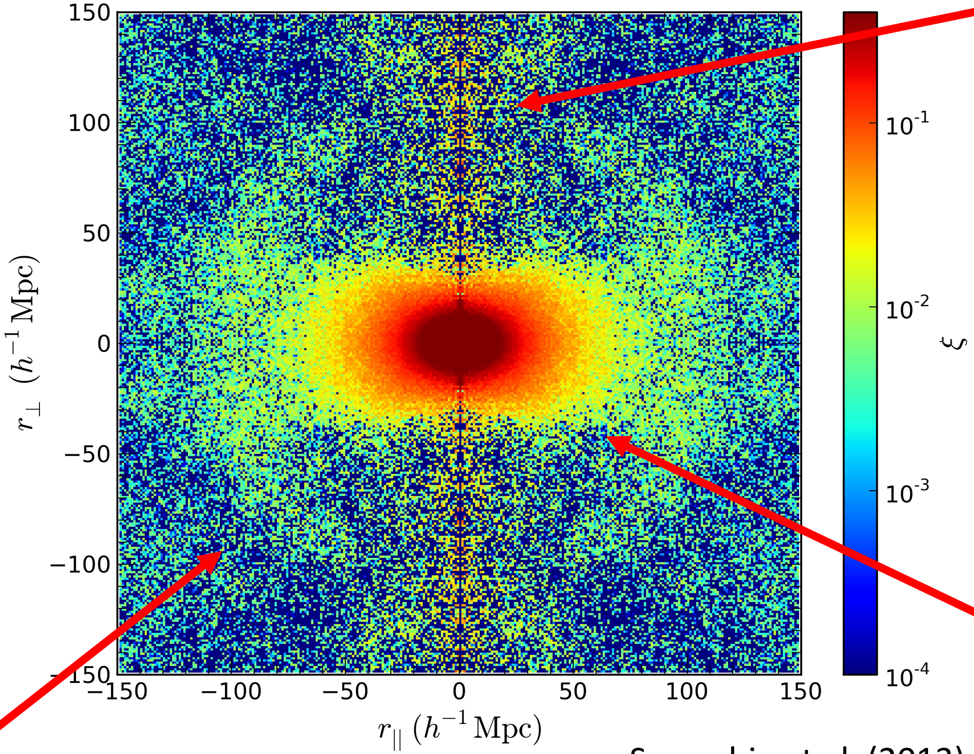
LoS

Averaging over angles:

RSD measures $\beta = f/b$

$$\xi(s,\mu) = \xi(r_{\perp}, r_{\parallel})$$

FoG



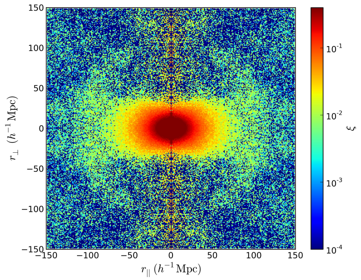
Linear RSD

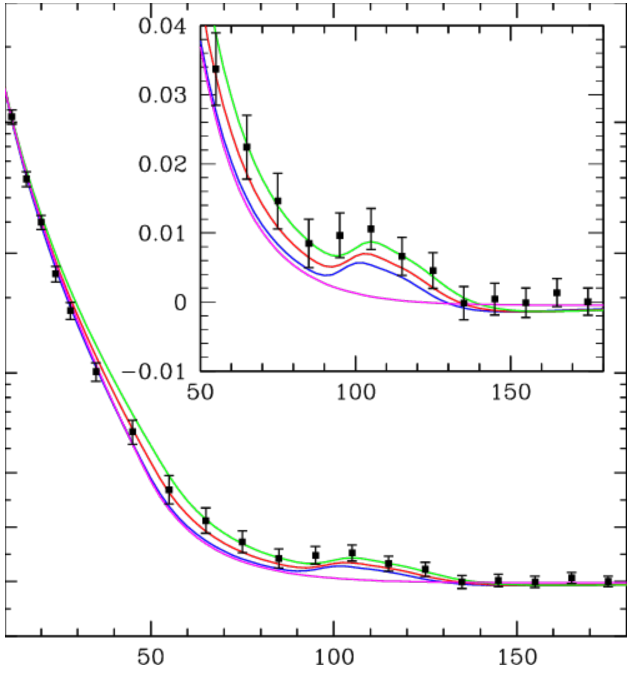
Samushia et al. (2013)

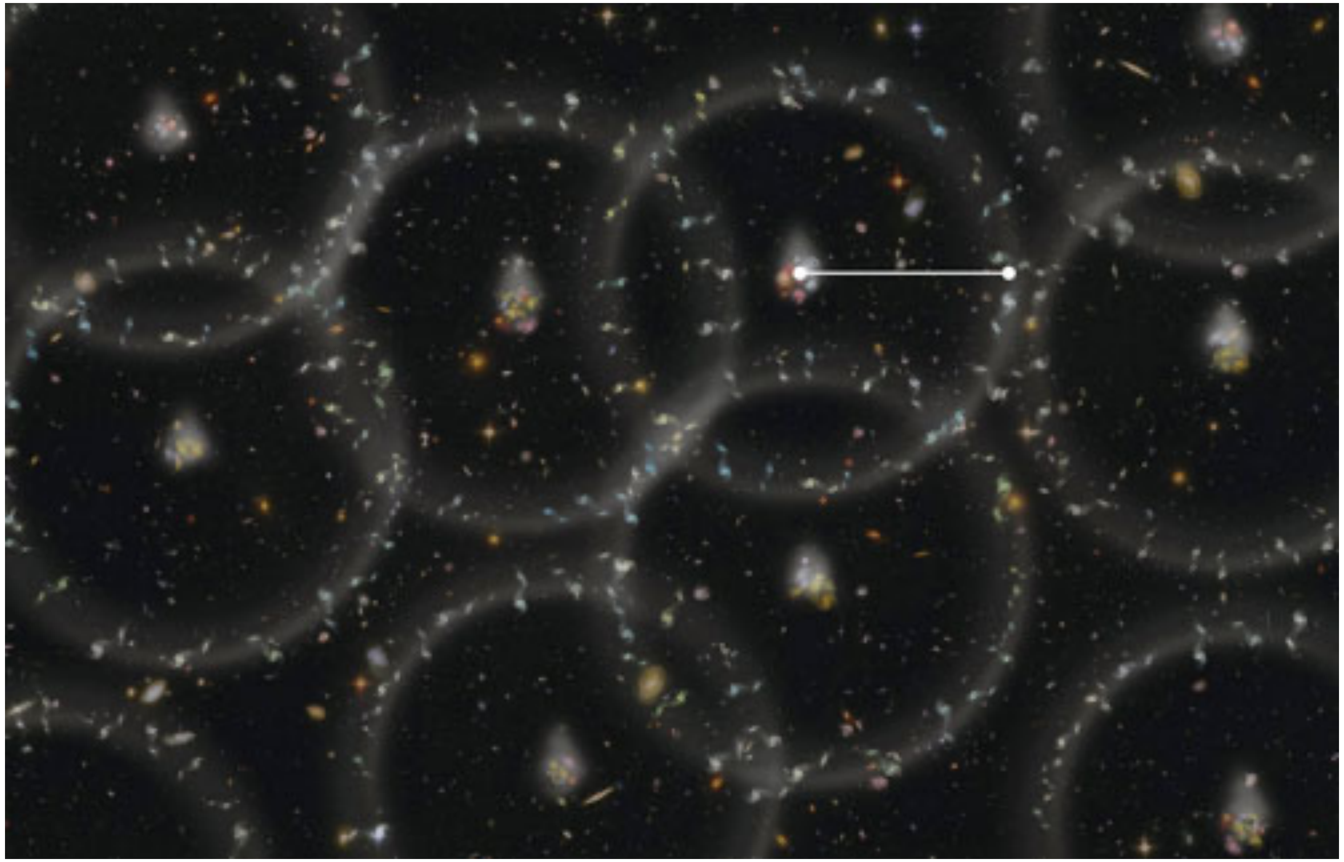
…and what is this ring??

Baryon Acoustic Oscillations (BAO)

excess of clustering at separation around 140 Mpc

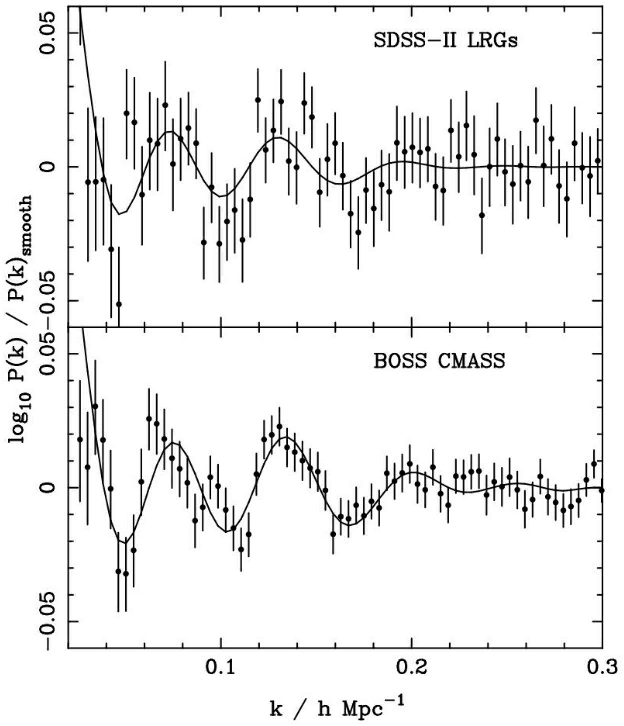




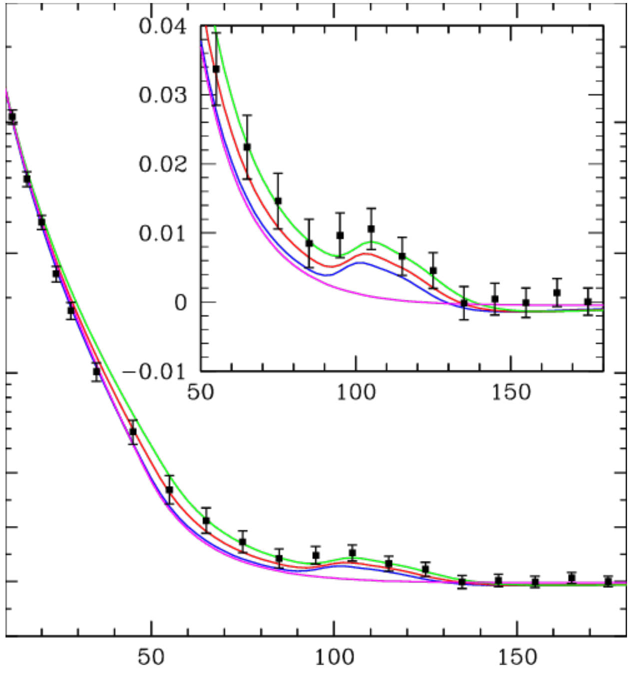


BOSS collaboration

Baryon Acoustic Oscillations (BAO)

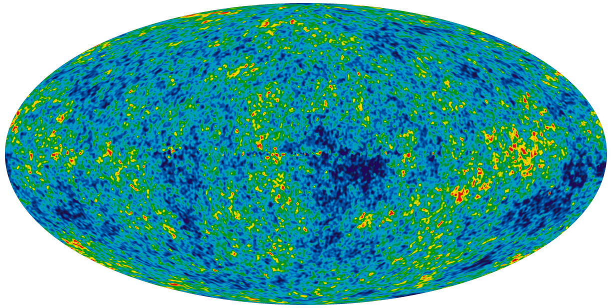


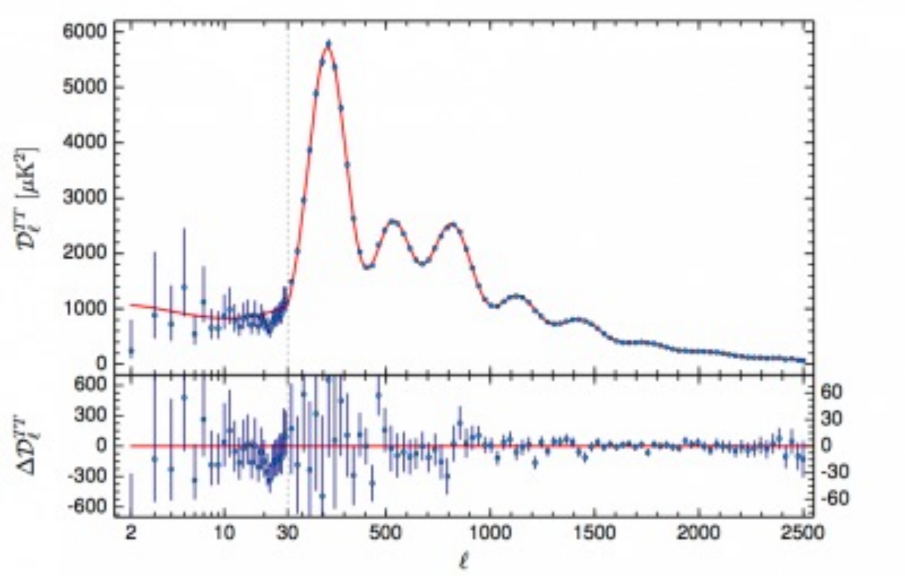
Power spectrum



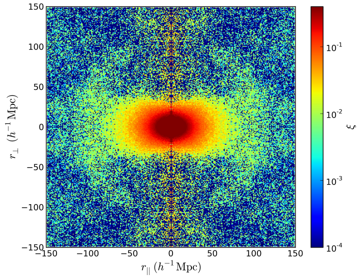
Correlation function

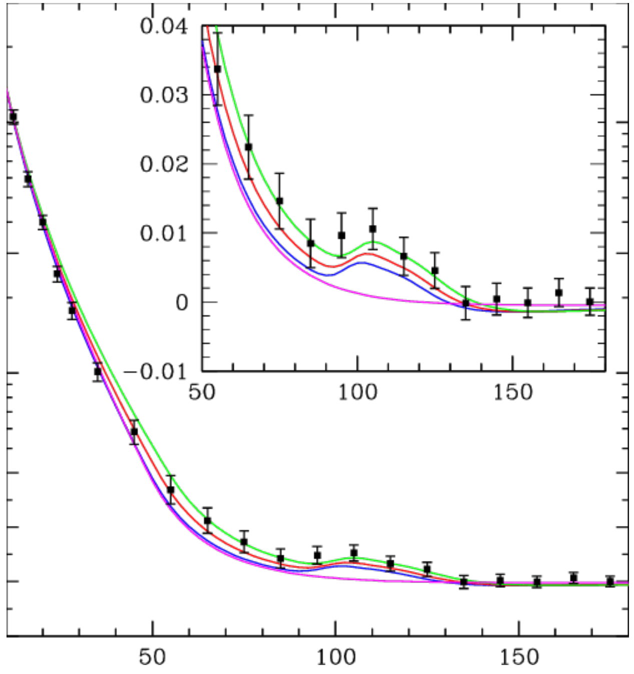
Baryon Acoustic Oscillations (BAO)





CMB
z=1000

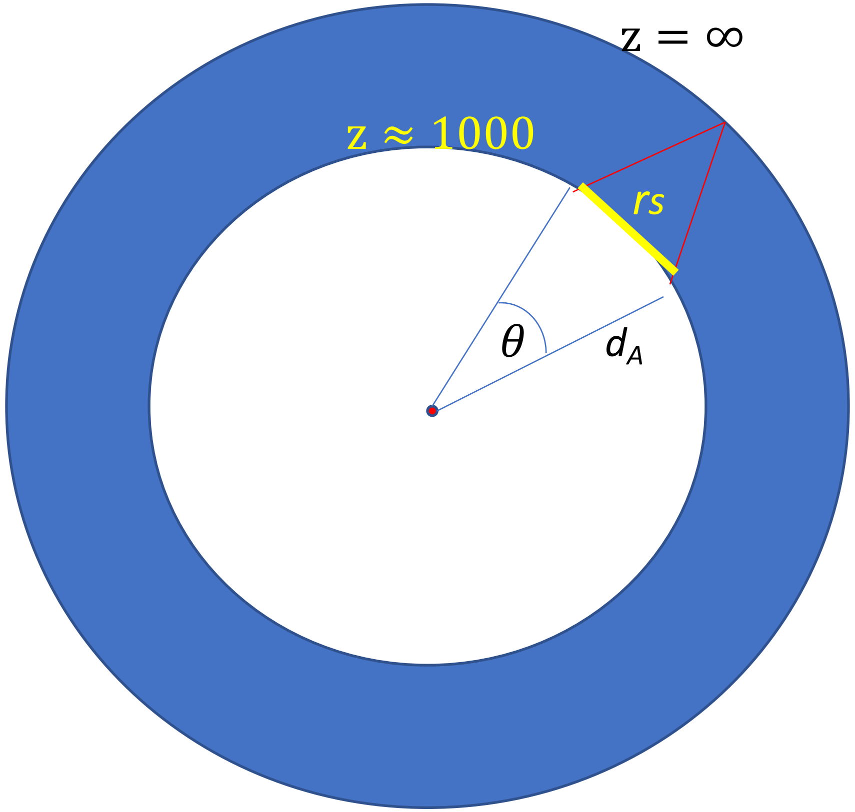




LSS
z=0

Baryon Acoustic Oscillations (BAO)

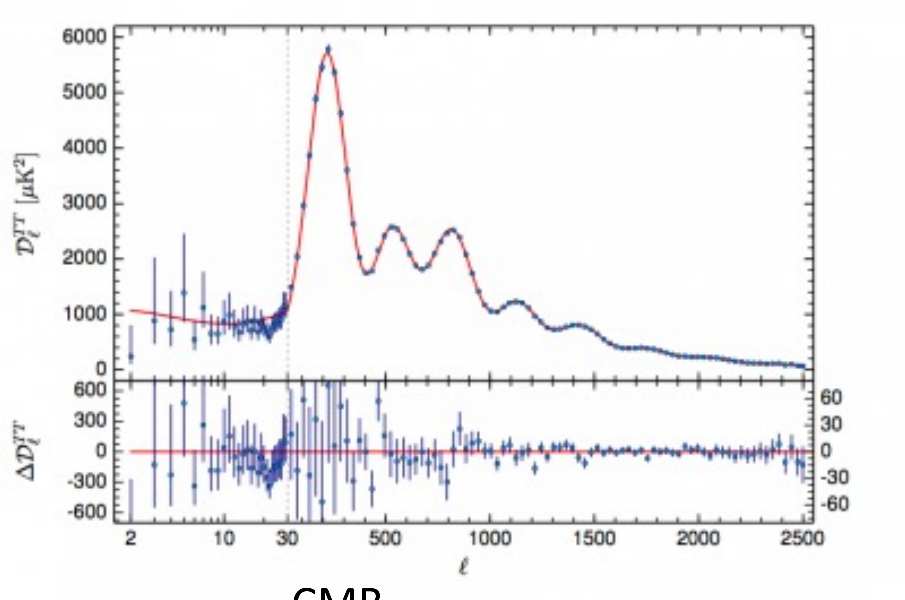
perturbations in the coupled baryon-photon plasma
from Big Bang to decoupling



$$\theta_A \equiv \frac{r_s(z_{\rm dec})}{d_A^{(c)}(z_{\rm dec})},$$

Baryon Acoustic Oscillations (BAO)

perturbations in the coupled baryon-photon plasma
from Big Bang to decoupling



CMB
z=1000

$$\theta_A \equiv \frac{r_s(z_{\rm dec})}{d_A^{(c)}(z_{\rm dec})},$$

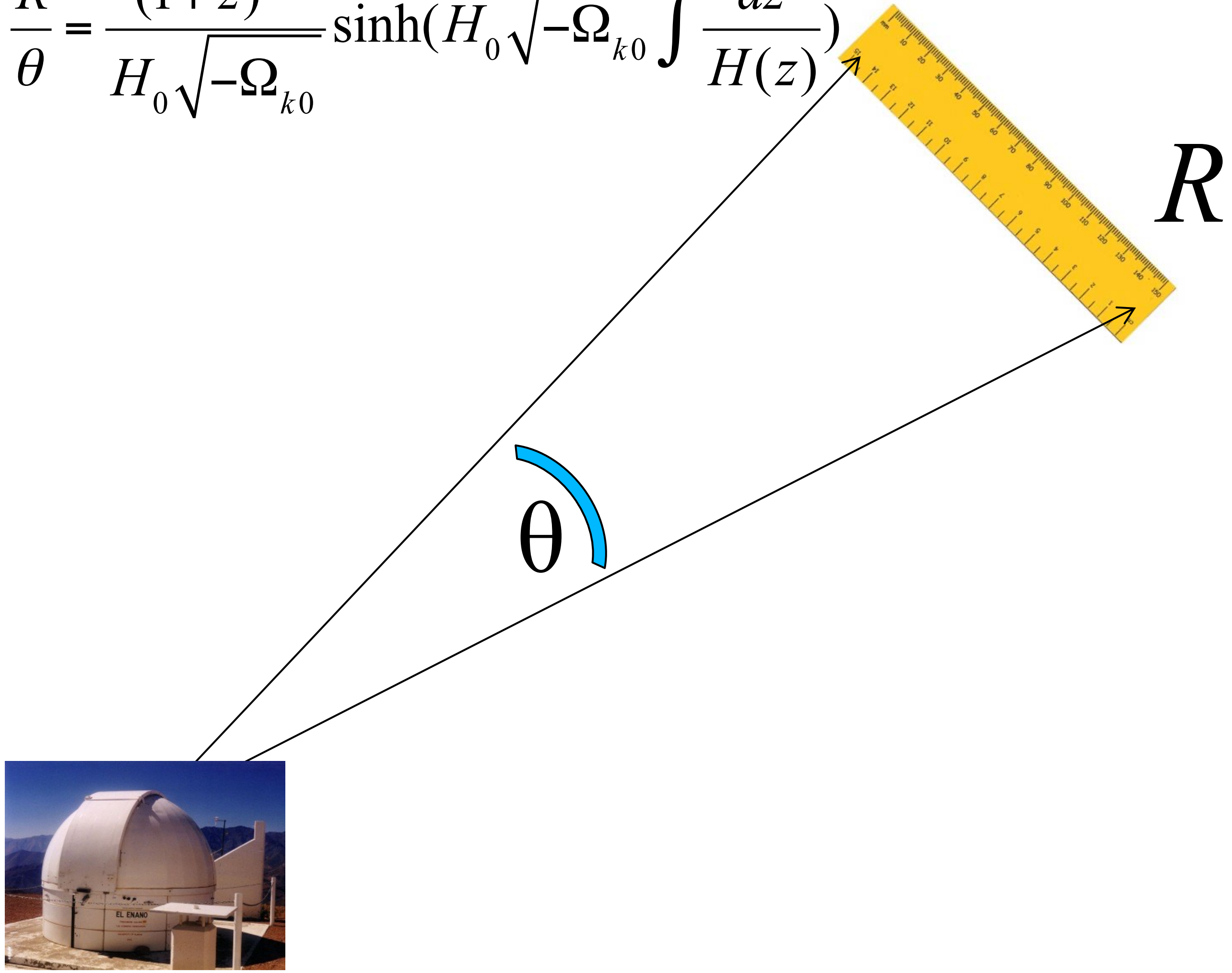
$$r_s(z_{\rm dec}) = \frac{c}{\sqrt{3a_0 H_0}} \int_{z_{\rm dec}}^{\infty} \frac{\mathrm{d}z}{\sqrt{1+R_s}\,E(z)}.$$

Independent of
late-time cosmology!

$$R_s = \frac{3\omega_b}{4\omega_\gamma}\frac{1}{1+z},$$

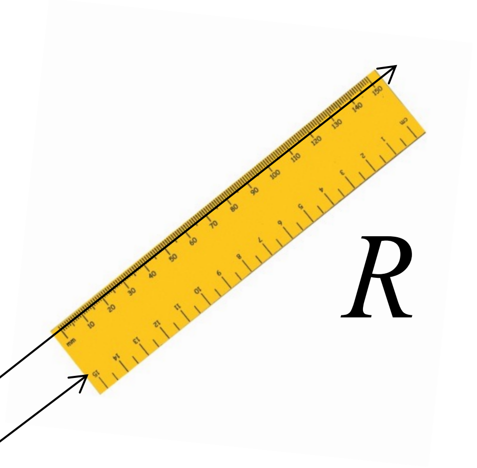
BAO as standard rod

$$D(z)=\frac{R}{\theta}=\frac{(1+z)^{-1}}{H_0\sqrt{-\Omega_{k0}}}\sinh\left(H_0\sqrt{-\Omega_{k0}}\int\frac{dz}{H(z)}\right)$$



BAO as standard rod

$$H(z) = \frac{dz}{R}$$



Alcock-Paczyński effect

$$D(z) = \frac{\lambda_\perp}{\theta}$$

$$H(z) = \frac{dz}{\lambda_\parallel}$$

$$\theta = \frac{\lambda_\perp}{D}$$

$$dz = \lambda_\parallel H$$

we measure angles and redshifts, not distances!

when we produce a *P(k)* or $\xi(r)$ we convert angles/redshifts
into wavevectors *k* (or separations *r*) with a reference cosmology
(i.e. expansion rate *H(z)*)

for any other cosmology…

Alcock-Paczyński effect

from these relations…

$$D(z) = \frac{\lambda_\perp}{\theta}$$
$$H(z) = \frac{dz}{\lambda_\parallel}$$

…we see that these combinations are the same in every cosmology

$$\theta = \frac{\lambda_\perp}{D}$$
$$dz = \lambda_\parallel H$$

…therefore for any two cosmologies

$$\frac{\lambda_\perp}{D}\Big|_1 = \frac{\lambda_\perp}{D}\Big|_2$$
$$\lambda_\parallel H|_1 = \lambda_\parallel H|_2$$

…and therefore

$$Dk_\perp|_1 = Dk_\perp|_2$$
$$\frac{H}{k_\parallel}\Big|_1 = \frac{H}{k_\parallel}\Big|_2$$

Alcock-Paczyński effect

$$\boxed{\begin{aligned} Dk_\perp|_1 &= Dk_\perp|_2 \\ \frac{H}{k_\parallel}\Big|_1 &= \frac{H}{k_\parallel}\Big|_2 \end{aligned}}$$

$$k_\perp = k_{r\perp} D_r / D\,.$$

"r" is a reference cosmology

$$k_\parallel = k_{r\parallel} H / H_r\,.$$

We need modulus and direction cosine:

$$k = (k_\parallel^2 + k_\perp^2)^{1/2} = \alpha k_r\,,$$

$$\mu = \frac{k_\parallel}{(k_\parallel^2 + k_\perp^2)^{1/2}} = \frac{H\mu_r}{H_r\alpha}\,,$$

where, putting $h = H/H_r$ and $d = D/D_r$

$$\alpha = \frac{\sqrt{\mu_r^2(h^2d^2 - 1) + 1)}}{d}\,.$$

Alcock-Paczyński effect

$$k = (k_\parallel^2 + k_\perp^2)^{1/2} = \alpha k_r \, ,$$

$$\mu = \frac{k_\parallel}{(k_\parallel^2 + k_\perp^2)^{1/2}} = \frac{H \mu_r}{H_r \alpha} \, ,$$

where, putting $h = H/H_r$ and $d = D/D_r$

$$\alpha = \frac{\sqrt{\mu_r^2(h^2 d^2 - 1) + 1)}}{d} \, .$$

Final linear spectrum with bias, growth, RSD, FoG, AP

$$P_g(k, \mu, z) = b^2 G^2 P_m(\alpha k_r)(1 + \beta \mu_r^2 \frac{h^2}{\alpha^2})^2 e^{-k^2 \mu^2 \sigma_v^2}$$

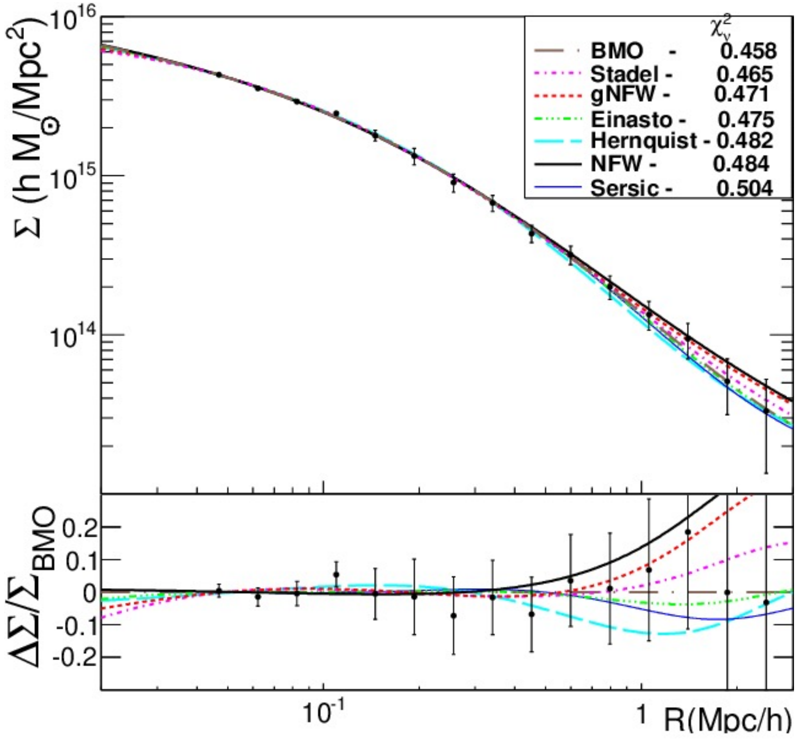
observations

theory

Quiz time

1. How can we quantify the amount of correlations?
2. Can we measure the peculiar velocity?
3. In the final expression for P(k), where are the cosmological parameters?
4. Is the BAO really a standard rod?
5. Is the k-range of the linear regime wider or smaller at high z?

First glimpse on non-linearity



Leandro Beraldo e Silva and Marcos Lima and Laerte Sodre 2013

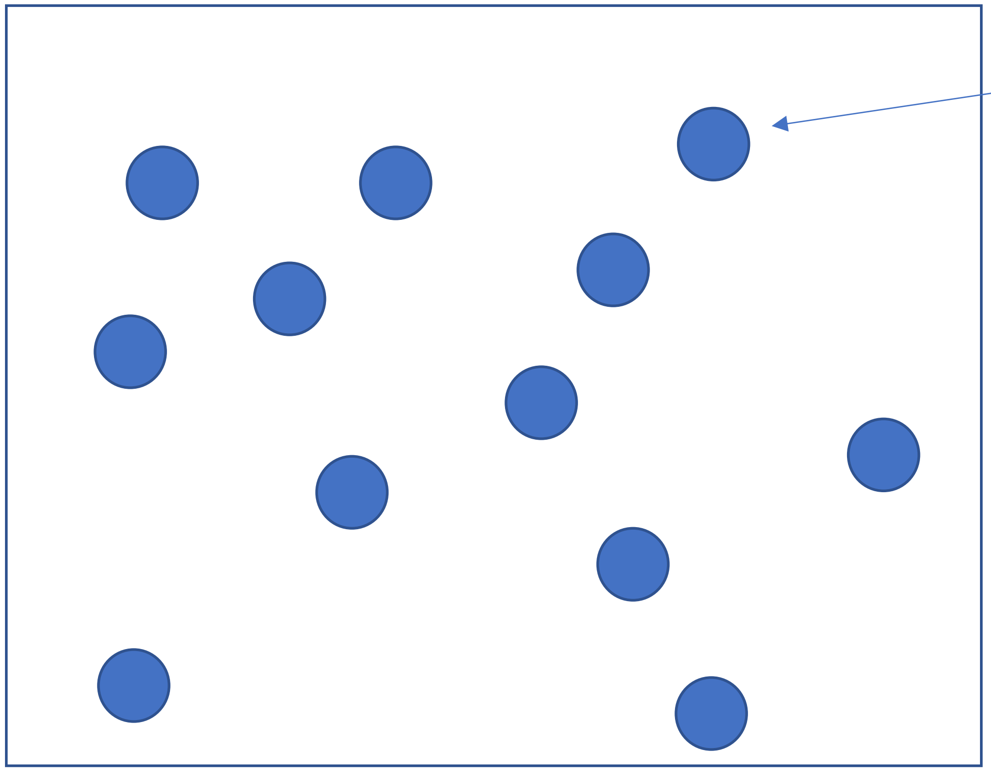
Navarro-Frenk-White profile for DM halos

$$\rho_{NFW} = \frac{\rho_0}{\frac{r}{r_s}(1+\frac{r}{r_s})^2}$$

First glimpse on non-linearity

Navarro-Frenk-White profile

$$\rho_{NFW} = \frac{\rho_0}{\frac{r}{r_s}(1+\frac{r}{r_s})^2}$$



Simple model:
NFW halos of DM
randomly distributed on top
of the linear spectrum

First glimpse on non-linearity

$$\rho_{NFW} = \frac{\rho_0}{\frac{r}{r_s}(1+\frac{r}{r_s})^2}$$

$$\begin{aligned}\delta_1 &= \frac{4\pi}{V}\int_0^R \left(\frac{\rho_{NFW}}{\bar{\rho}} - 1\right)\frac{\sin(kr)}{(kr)} r^2 dr \\ &= \frac{1}{V}[\frac{4\pi}{\bar{\rho}}\int_0^R \rho_{NFW}\frac{\sin(kr)}{(kr)}r^2 dr - \frac{4\pi}{\bar{\rho}}\int_0^R \frac{\sin(kr)}{(kr)}r^2 dr] \\ &= \frac{1}{V}(W_{NFW} - W_{TH})\end{aligned}$$

$$\bar{\rho} = \frac{4\pi}{V}\int_0^R \rho_{NFW} r^2 dr$$

Density contrast in Fourier space at small scales (<10 Mpc)

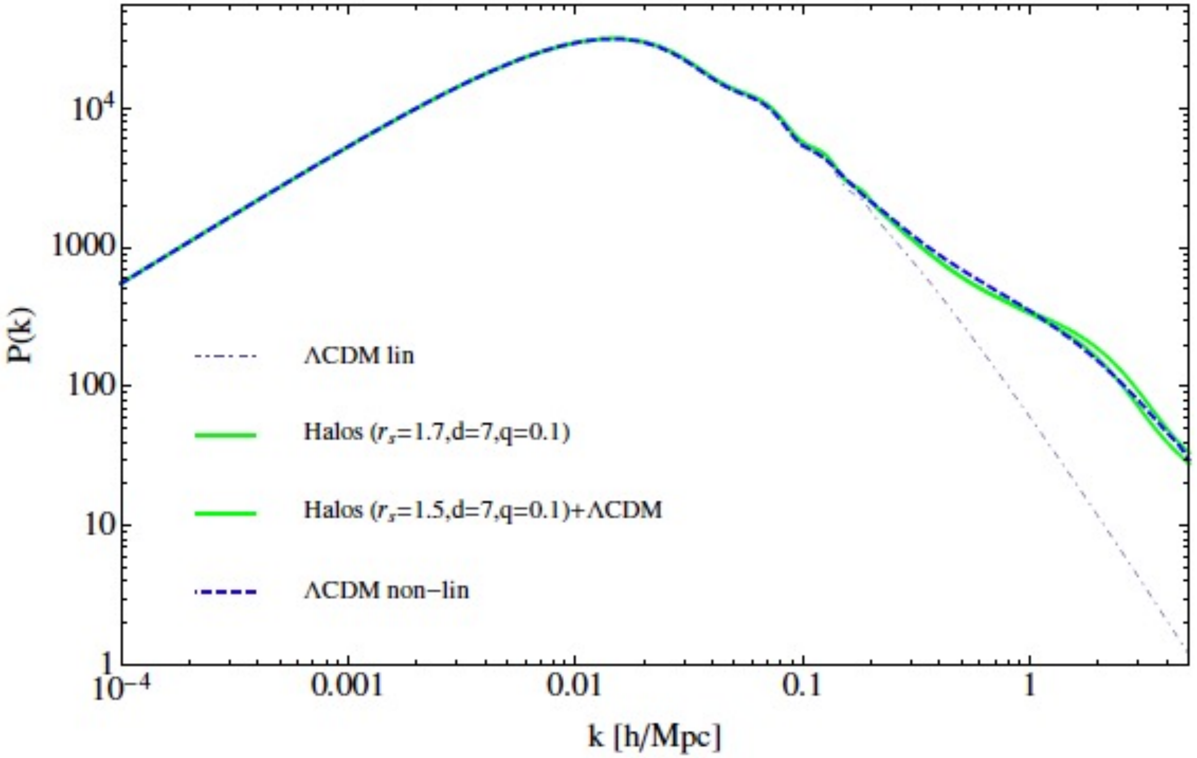
Spectrum of N uncorrelated haloes

$$P_h(n, R, r_s) = NV\delta_1^2 = n(W_{NFW} - W_{TH})^2$$

$$P_{NL} = P_{LIN} + P_h$$

First glimpse on non-linearity

$$P_{NL} = P_{LIN} + P_h$$



The structure of the large scale structure

large scales

small scales

super-horizon scales

linear scales

mildly
non-linear scales

strongly
non-linear scales

relativistic corrections

linear pert. theory

non-linear pert. theory

N-body simulations

$k \approx aH \approx$0.002 h/Mpc

$\lambda \approx 3000\ Mpc/h$

$k \approx 0.1\ h/Mpc$

$\lambda \approx 60\ Mpc/h$

$k \approx 0.3\ h/Mpc$

$\lambda \approx 20\ Mpc/h$