Cosmological large-scale structure

Lecture 6

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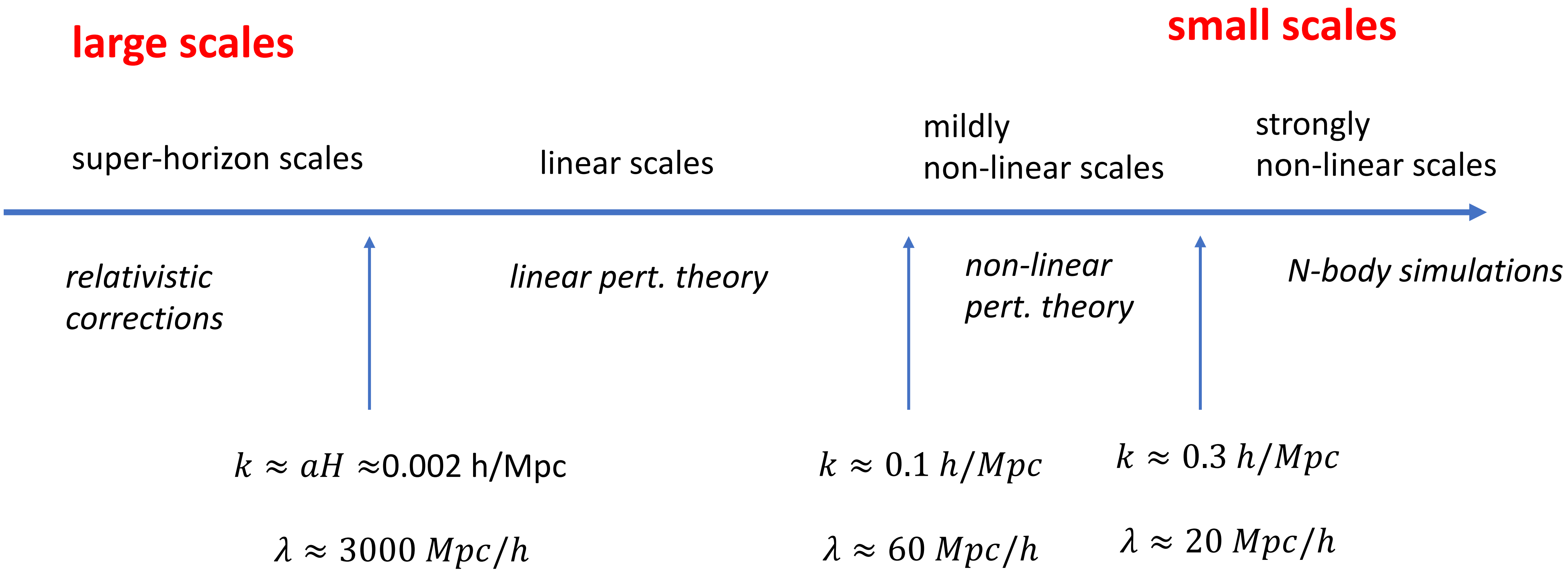
The structure of the large scale structure



Recap

Non-linear equations for pressureless fluid and EdS

$$\dot{\delta} = -\nabla(1+\delta)\mathbf{v}$$

$$\dot{\theta} + H\theta + \frac{3}{2}a^2H^2\delta = -\nabla(\mathbf{v}\cdot\nabla\mathbf{v})$$

expansion

$$\dot{\delta} = \varepsilon\delta^{(1)} + \varepsilon^2\delta^{(2)} + \ldots$$

$$\dot{\theta} = \varepsilon\theta^{(1)} + \varepsilon^2\theta^{(2)} + \ldots$$

final expressions for EdS

$$\theta^{(2)} = -G^2 f \int \delta_1\delta_2 G_2(\mathbf{k}_1,\mathbf{k}_2)\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}$$

$$\delta^{(2)} = G^2 \int \delta_1\delta_2 F_2(\mathbf{k}_1,\mathbf{k}_2)\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}$$

$$F_2 = \frac{5}{7} + \frac{\mathbf{k}_1\mathbf{k}_2}{2k_1k_2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right) + \frac{2}{7}\left(\frac{\mathbf{k}_1\mathbf{k}_2}{k_1k_2}\right)^2$$

$$G_2 = \frac{3}{7} + \frac{\mathbf{k}_1\mathbf{k}_2}{2k_1k_2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right) + \frac{4}{7}\left(\frac{\mathbf{k}_1\mathbf{k}_2}{k_1k_2}\right)^2$$

Recap

General growth

$$\delta^{(2)} = G_{2A}(a)A(\mathbf{k}) + G_{2B}(a)B(\mathbf{k})$$

$$G''_{2A} + FG'_{2A} - SG_{2A} = \frac{7}{5}G^2(2f^2 + f' + Ff) = \frac{7}{5}G^2(f^2 + S)$$

$$G''_{2B} + FG'_{2B} - SG_{2B} = \frac{7}{2}G^2 f^2$$

Real space-redshift space mapping

$$\mathbf{s} = \mathbf{r}\left[1 + \frac{u(r)}{r}\right]$$

RSD

$$\delta_s(\mathbf{k}) = \int d^3r[\delta(\mathbf{r}) - \frac{du}{dr}]\{e^{i\mathbf{kr}} + \sum_{n=1} \frac{(k\mu)^n}{n!} \int \frac{d^3q_1}{(2\pi)^3}\frac{\mu_1}{q_1}\theta(\mathbf{q}_1) \int \frac{d^3q_2}{(2\pi)^3}\frac{\mu_2}{q_2}\theta(\mathbf{q}_2) .. \int \frac{d^3q_n}{(2\pi)^3}\frac{\mu_n}{q_n}\theta(\mathbf{q}_n) e^{i(\mathbf{k} - \sum_i^n \mathbf{q}_i)\mathbf{r}}\}$$

Roadmap for today

- Redshift distortion
- General non-linear bias
- Kernels with bias and RSD
- Power spectrum corrections

Bias and RSD

$$\delta_s(\mathbf{k}) = \int d^3r[\delta(\mathbf{r}) - \frac{du}{dr}]\{e^{i\mathbf{kr}} + \sum_{n=1} \frac{(k\mu)^n}{n!} \int \frac{d^3q_1}{(2\pi)^3}\frac{\mu_1}{q_1}\theta(\mathbf{q}_1) \int \frac{d^3q_2}{(2\pi)^3}\frac{\mu_2}{q_2}\theta(\mathbf{q}_2).. \int \frac{d^3q_n}{(2\pi)^3}\frac{\mu_n}{q_n}\theta(\mathbf{q}_n)e^{i(\mathbf{k}-\sum_i^n \mathbf{q}_i)\mathbf{r}}\}$$

real space

$$\delta_g(\mathbf{r}) = b_1\delta(\mathbf{r}) + \frac{1}{2}b_2\delta(\mathbf{r})^2 + ...$$

Fourier space

$$\delta_g(\mathbf{k}) = b_1\delta^{(1)}(\mathbf{k}) + \frac{1}{2}b_2 \int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\delta_D(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)\delta^{(1)}(\mathbf{q}_1)\delta^{(1)}(\mathbf{q}_2) + ...$$

first-order term

$$\delta_g^{(1)}(\mathbf{k}) = \int d^3r[b_1\delta(\mathbf{r}) - \frac{du}{dr}]e^{i\mathbf{kr}} = \int d^3r b_1\delta(\mathbf{r})e^{i\mathbf{kr}} - \int d^3r\frac{du}{dr}e^{i\mathbf{kr}}$$

Bias and RSD

$$\delta_g^{(1)}(\mathbf{k}) = \int d^3r[b_1\delta(\mathbf{r}) - \frac{du}{dr}]e^{i\mathbf{kr}} = \int d^3r b_1\delta(\mathbf{r})e^{i\mathbf{kr}} - \int d^3r\frac{du}{dr}e^{i\mathbf{kr}}$$

from linear theory

$$\mathbf{v}(\mathbf{k}) = i\mathcal{H}\delta_k f\frac{\mathbf{k}}{k^2}$$

projection along LOS

$$u(r) = \frac{\mathbf{r}}{r}\cdot\frac{\mathbf{v}}{\mathcal{H}}(\mathbf{r}) = if\int\frac{d^3k'}{(2\pi)^3}\delta(\mathbf{k}')\frac{\mathbf{k'r}}{k'^2r}e^{-i\mathbf{k}'\cdot\mathbf{r}}$$

$$\frac{du}{dr} = -f\int\delta(\mathbf{k}')e^{-i\mathbf{k}'\cdot\mathbf{r}}\left(\frac{\mathbf{k'r}}{k'r}\right)^2\frac{d^3k'}{(2\pi)^3} = -f\int\delta(\mathbf{k}')e^{-i\mathbf{k}'\cdot\mathbf{r}}\mu^2\frac{d^3k'}{(2\pi)^3}$$

$$\int d^3r\frac{du}{dr}e^{i\mathbf{kr}} = -f\int d^3r\frac{d^3k}{(2\pi)^3}\delta(\mathbf{k}')e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}}\mu^2 = -f\int\frac{d^3k}{(2\pi)^3}\delta_D(\mathbf{k}-\mathbf{k}')\delta(\mathbf{k}')\mu^2 = -f\mu^2\delta(\mathbf{k}) = \mu^2\theta(\mathbf{k})$$

Linear Kaiser effect
(no FoG !)

$$\delta_g^{(1)}(\mathbf{k}) = \int d^3r[b_1\delta(\mathbf{r}) - \frac{du}{dr}]e^{i\mathbf{kr}} = \int d^3r b_1\delta(\mathbf{r})e^{i\mathbf{kr}} - \int d^3r\frac{du}{dr}e^{i\mathbf{kr}}$$
$$= \delta^{(1)}(\mathbf{k})(b_1 + f\mu^2)$$

Bias and RSD: second order

$$\delta_s(\mathbf{k}) = \int d^3r[\delta(\mathbf{r}) - \frac{du}{dr}]\{e^{i\mathbf{kr}} + \sum_{n=1} \frac{(k\mu)^n}{n!} \int \frac{d^3q_1}{(2\pi)^3} \frac{\mu_1}{q_1}\theta(\mathbf{q}_1) \int \frac{d^3q_2}{(2\pi)^3} \frac{\mu_2}{q_2}\theta(\mathbf{q}_2).. \int \frac{d^3q_n}{(2\pi)^3} \frac{\mu_n}{q_n}\theta(\mathbf{q}_n)e^{i(\mathbf{k}-\sum_i^n \mathbf{q}_i)\mathbf{r}}\}$$

$$\delta_g(\mathbf{k}) = b_1\delta^{(1)}(\mathbf{k}) + \frac{1}{2}b_2 \int \frac{d^3q_1}{(2\pi)^3} \frac{d^3q_2}{(2\pi)^3} \delta_D(\mathbf{k} - \mathbf{q}_1 - \mathbf{q}_2)\delta^{(1)}(\mathbf{q}_1)\delta^{(1)}(\mathbf{q}_2) + \ldots$$

Take the first NL term *n=1*:

$$\begin{aligned}\delta_g(\mathbf{k}) &= \int d^3r[b_1\delta(\mathbf{r}) - \frac{du}{dr}]e^{i(\mathbf{k}-\mathbf{q}_1)\mathbf{r}} k\mu \frac{d^3q_1}{(2\pi)^3} \frac{\mu_1}{q_1}\theta(\mathbf{q}_1) \\ &= \int d^3r \int \frac{d^3q_0}{(2\pi)^3}[b_1\delta(\mathbf{q}_0) - \theta(\mathbf{q}_0)\mu_0^2]e^{i(\mathbf{k}-\mathbf{q}_0-\mathbf{q}_1)\mathbf{r}} k\mu \frac{d^3q_1}{(2\pi)^3} \frac{\mu_1}{q_1}\theta(\mathbf{q}_1) \\ &= \int \frac{d^3q_0}{(2\pi)^3} \frac{d^3q_1}{(2\pi)^3}[b_1\delta(\mathbf{q}_0) - \theta(\mathbf{q}_0)\mu_0^2]\delta_D(\mathbf{k} - \mathbf{q}_0 - \mathbf{q}_1)k\mu\frac{\mu_1}{q_1}\theta(\mathbf{q}_1)\end{aligned}$$

Apply the same to every term

$$\delta_g(\mathbf{k}) = \sum_{n=1} \int \frac{d^3q_1}{(2\pi)^3} \int \frac{d^3q_2}{(2\pi)^3} \cdots \int \frac{d^3q_n}{(2\pi)^3}[\delta(\mathbf{q}_1) - \theta(\mathbf{q}_1)\mu_1^2]\delta_D(\mathbf{k} - \sum_{i=1}^n \mathbf{q}_i)\frac{(k\mu)^{n-1}}{(n-1)!} \frac{\mu_2}{q_2}\theta(\mathbf{q}_2)\frac{\mu_3}{q_3}\theta(\mathbf{q}_3)..\frac{\mu_n}{q_n}\theta(\mathbf{q}_n)$$

Bias and RSD: second order

$$\delta_g(\mathbf{k}) = \sum_{n=1} \int \frac{d^3q_1}{(2\pi)^3} \int \frac{d^3q_2}{(2\pi)^3} \cdots \int \frac{d^3q_n}{(2\pi)^3} [\delta(\mathbf{q_1}) - \theta(\mathbf{q_1})\mu_1^2] \delta_D(\mathbf{k} - \sum_{i=1}^{n} \mathbf{q}_i) \frac{(k\mu)^{n-1}}{(n-1)!} \frac{\mu_2}{q_2} \theta(\mathbf{q_2}) \frac{\mu_3}{q_3} \theta(\mathbf{q_3}) .. \frac{\mu_n}{q_n} \theta(\mathbf{q_n})$$

Expand the fields

$$\delta = \varepsilon\delta^{(1)} + \varepsilon^2\delta^{(2)} + ...$$
$$\theta = \varepsilon\theta^{(1)} + \varepsilon^2\theta^{(2)} + ...$$

$$\theta^{(2)} = -G^2 f \int \delta_1\delta_2 G_2(\mathbf{k}_1, \mathbf{k}_2) \delta_D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3}$$
$$\delta^{(2)} = G^2 \int \delta_1\delta_2 F_2(\mathbf{k}_1, \mathbf{k}_2) \delta_D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3}$$

And expand the bias

$$\delta_g(\mathbf{k}) = b_1\delta^{(1)}(\mathbf{k}) + \frac{1}{2}b_2 \int \frac{d^3q_1}{(2\pi)^3} \frac{d^3q_2}{(2\pi)^3} \delta_D(\mathbf{k} - \mathbf{q}_1 - \mathbf{q}_2) \delta^{(1)}(\mathbf{q}_1) \delta^{(1)}(\mathbf{q}_2) + ...$$

Use all this and select terms at the same order in ε

Bias and RSD: second order

$$\delta_g(\mathbf{k}) = \sum_{n=1} \int \frac{d^3q_1}{(2\pi)^3} \int \frac{d^3q_2}{(2\pi)^3} \cdots \int \frac{d^3q_n}{(2\pi)^3} [\delta(\mathbf{q_1}) - \theta(\mathbf{q_1})\mu_1^2] \delta_D(\mathbf{k} - \sum_{i=1}^{n} \mathbf{q}_i) \frac{(k\mu)^{n-1}}{(n-1)!} \frac{\mu_2}{q_2}\theta(\mathbf{q_2}) \frac{\mu_3}{q_3}\theta(\mathbf{q_3}) .. \frac{\mu_n}{q_n}\theta(\mathbf{q_n})$$

$$\delta_g^{(2)}(\mathbf{k}) = b_1\delta^{(2)}(\mathbf{q}_1) + \frac{b_2}{2} \int \frac{d^3q_1}{(2\pi)^3} \frac{d^3q_2}{(2\pi)^3} \delta_D(\mathbf{k} - \mathbf{q}_1 - \mathbf{q}_2) \left[\delta^{(1)}(\mathbf{q}_1)\delta^{(1)}(\mathbf{q}_2) - \theta^{(2)}(\mathbf{q}_1)\mu^2\right] \quad (5.4.23)$$

$$+ \int \frac{d^3q_1}{(2\pi)^3} \frac{d^3q_2}{(2\pi)^3} [b_1\delta^{(1)}(\mathbf{q}_1) - \theta^{(1)}(\mathbf{q}_1)\mu_1^2] \delta_D(\mathbf{k} - \mathbf{q}_1 - \mathbf{q}_2) k\mu \frac{\mu_2}{q_2} \theta^{(1)}(\mathbf{q}_2) \quad (5.4.24)$$

$$= \int \frac{d^3q_1}{(2\pi)^3} \frac{d^3q_2}{(2\pi)^3} \delta^{(1)}(\mathbf{q}_1)\delta^{(1)}(\mathbf{q}_2) [b_1F_2 + G_2 f\mu^2 + \frac{b_2}{2} + fk\mu b_1 \frac{\mu_2}{q_2} + f^2\mu_1^2\mu k \frac{\mu_2}{q_2}] \delta_D(\mathbf{k} - \mathbf{q}_1 - \mathbf{q}_2) \quad (5.4.25)$$

$$= \int \frac{d^3q_1}{(2\pi)^3} \frac{d^3q_2}{(2\pi)^3} \delta^{(1)}(\mathbf{q}_1)\delta^{(1)}(\mathbf{q}_2) [b_1F_2 + G_2 f\mu^2 + \frac{b_2}{2} + fk\mu[\frac{\mu_2}{q_2}(b_1 + f\mu_1^2)] \delta_D(\mathbf{k} - \mathbf{q}_1 - \mathbf{q}_2) \quad (5.4.26)$$

$$= \int \frac{d^3q_1}{(2\pi)^3} \frac{d^3q_2}{(2\pi)^3} \delta^{(1)}(\mathbf{q}_1)\delta^{(1)}(\mathbf{q}_2) [b_1F_2 + G_2 f\mu^2 + \frac{b_2}{2} + \frac{fk\mu}{2}[\frac{\mu_1}{q_1}(b_1 + f\mu_2^2) + \frac{\mu_2}{q_2}(b_1 + f\mu_1^2)] \delta_D(\mathbf{k} - \mathbf{q}_1 - \mathbf{q}_2) \quad (5.4.27)$$

$$= \int \frac{d^3q_1}{(2\pi)^3} \frac{d^3q_2}{(2\pi)^3} \delta^{(1)}(\mathbf{q}_1)\delta^{(1)}(\mathbf{q}_2) Z_2(\mathbf{q}_1, \mathbf{q}_2) \delta_D(\mathbf{k} - \mathbf{q}_1 - \mathbf{q}_2) \quad (5.4.28)$$

A new bias+RSD kernel !

$$Z_2(\mathbf{q}_1, \mathbf{q}_2) = b_1F_2 + G_2 f\mu^2 + \frac{fk\mu}{2}[\frac{\mu_1}{q_1}(b_1 + f\mu_2^2) + \frac{\mu_2}{q_2}(b_1 + f\mu_1^2)] + \frac{b_2}{2}$$

However: bias is more complicated!

We don't know much about sources of bias. It could depend on nearby gravitational sources

Tidal tensor

$$K_{ij} = (\partial_i \partial_j \Phi) - \frac{1}{3}\delta_{ij}\Delta\Phi$$

Tidal scalar

$$\mathcal{G}_2 = K_{ij}K^{ij} = (\partial_i \partial_j \Phi)^2 + \frac{1}{9}\delta_{ij}\delta^{ij}(\Delta\Phi)^2 - \frac{2}{3}(\partial_i \partial_j \Phi)\delta^{ij}(\Delta\Phi)$$
$$= (\partial_i \partial_j \Phi)^2 - \frac{1}{3}(\Delta\Phi)^2$$

Tidal scalar in Fourier space…

$$K_{ij}(\mathbf{k}) = (\frac{k_i k_j}{k^2} - \frac{1}{3}\delta_{ij})\delta(\mathbf{k})$$

…and another tidal scalar from velocity Potential…

$$\Gamma_3 = \mathcal{G}_2(\Phi_g) - \mathcal{G}_2(\Phi_v)$$

More general bias

$$\delta_g = b_1\delta + \frac{b_2^2}{2}\delta^2 + b_{\mathcal{G}}\mathcal{G}_2 + b_{\Gamma}\Gamma_3 + \ldots$$

However: bias is more complicated!

More general bias

$$\delta_g = b_1\delta + \frac{b_2^2}{2}\delta^2 + b_G\mathcal{G}_2 + b_\Gamma\Gamma_3 + \dots$$

More general kernels!

$$Z_2(\mathbf{q}_a, \mathbf{q}_b) = b_1\{F_2(\mathbf{q}_a, \mathbf{q}_b) + \beta\mu^2 G_2(\mathbf{q}_a, \mathbf{q}_b) + \frac{\beta b\mu k}{2}\left[\frac{\mu_{az}}{q_a}(1+\beta\mu_{bz}^2) + \frac{\mu_{bz}}{q_b}(1+\beta\mu_{az}^2)\right]\} + \frac{b_2}{2} + b_G S_1(\mathbf{q}_a, \mathbf{q}_b)$$

$$Z_3(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) = b_1\{F_3(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) + \beta\mu^2 G_3(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) + \beta\mu k b[F_2(\mathbf{q}_1, \mathbf{q}_2) + \beta\mu_{12z}^2 G_2(\mathbf{q}_1, \mathbf{q}_2)]\frac{\mu_{3z}}{q_3}$$
$$+ \beta\mu k b(1+\beta\mu_{1z}^2)\frac{\mu_{23z}}{q_{23}}G_2(\mathbf{q}_2, \mathbf{q}_3) + \frac{(\beta\mu k)^2}{2}b_1^2(1+\beta\mu_{1z}^2)\frac{\mu_{2z}}{q_2}\frac{\mu_{3z}}{q_3}\}$$
$$+ 2b_G S_1(\mathbf{q}_1, \mathbf{q}_2 + \mathbf{q}_3)F_2(\mathbf{q}_2, \mathbf{q}_3) + b_G b_1 \beta\mu k\frac{\mu_{1z}}{q_1}S_1(\mathbf{q}_2, \mathbf{q}_3)$$
$$+ 2b_\Gamma S_1(\mathbf{q}_1, \mathbf{q}_1 + \mathbf{q}_3)(F_2(\mathbf{q}_2, \mathbf{q}_3) - G_2(\mathbf{q}_2, \mathbf{q}_3))$$

Recap

Non-linear equations for pressureless fluid and EdS

$$\dot{\delta} = -\nabla(1+\delta)\mathbf{v}$$

$$\dot{\theta} + H\theta + \frac{3}{2}a^2H^2\delta = -\nabla(\mathbf{v}\cdot\nabla\mathbf{v})$$

expansion

$$\dot{\delta} = \varepsilon\delta^{(1)} + \varepsilon^2\delta^{(2)} + ...$$

$$\dot{\theta} = \varepsilon\theta^{(1)} + \varepsilon^2\theta^{(2)} + ...$$

final expressions for EdS

$$\theta^{(2)} = -G^2 f \int \delta_1\delta_2 G_2(\mathbf{k}_1,\mathbf{k}_2)\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}$$

$$\delta^{(2)} = G^2 \int \delta_1\delta_2 F_2(\mathbf{k}_1,\mathbf{k}_2)\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}$$

$$F_2 = \frac{5}{7} + \frac{\mathbf{k}_1\mathbf{k}_2}{2k_1k_2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right) + \frac{2}{7}\left(\frac{\mathbf{k}_1\mathbf{k}_2}{k_1k_2}\right)^2$$

$$G_2 = \frac{3}{7} + \frac{\mathbf{k}_1\mathbf{k}_2}{2k_1k_2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right) + \frac{4}{7}\left(\frac{\mathbf{k}_1\mathbf{k}_2}{k_1k_2}\right)^2$$

Recap

General growth

$$\delta^{(2)} = G_{2A}(a)A(\mathbf{k}) + G_{2B}(a)B(\mathbf{k})$$

$$G''_{2A} + FG'_{2A} - SG_{2A} = \frac{7}{5}G^2(2f^2 + f' + Ff) = \frac{7}{5}G^2(f^2 + S)$$

$$G''_{2B} + FG'_{2B} - SG_{2B} = \frac{7}{2}G^2 f^2$$

Real space-redshift space mapping

$$\mathbf{s} = \mathbf{r}\left[1 + \frac{u(r)}{r}\right]$$

RSD

$$\delta_s(\mathbf{k}) = \int d^3r[\delta(\mathbf{r}) - \frac{du}{dr}]\{e^{i\mathbf{kr}} + \sum_{n=1} \frac{(k\mu)^n}{n!} \int \frac{d^3q_1}{(2\pi)^3}\frac{\mu_1}{q_1}\theta(\mathbf{q}_1) \int \frac{d^3q_2}{(2\pi)^3}\frac{\mu_2}{q_2}\theta(\mathbf{q}_2).. \int \frac{d^3q_n}{(2\pi)^3}\frac{\mu_n}{q_n}\theta(\mathbf{q}_n)e^{i(\mathbf{k}-\sum_i^n \mathbf{q}_i)\mathbf{r}}\}$$

Recap III

More general bias

$$\delta_g = b_1\delta + \frac{b_2^2}{2}\delta^2 + b_G \mathcal{G}_2 + b_\Gamma \Gamma_3 + \ldots$$

More general kernels!

$$Z_2(\mathbf{q}_a, \mathbf{q}_b) = b_1\{F_2(\mathbf{q}_a, \mathbf{q}_b) + \beta\mu^2 G_2(\mathbf{q}_a, \mathbf{q}_b) + \frac{\beta b \mu k}{2}\left[\frac{\mu_{az}}{q_a}(1+\beta\mu_{bz}^2) + \frac{\mu_{bz}}{q_b}(1+\beta\mu_{az}^2)\right]\} + \frac{b_2}{2} + b_G S_1(\mathbf{q}_a, \mathbf{q}_b)$$

$$\begin{aligned} Z_3(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) = b_1\{&F_3(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) + \beta\mu^2 G_3(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) + \beta\mu k b[F_2(\mathbf{q}_1, \mathbf{q}_2) + \beta\mu_{12z}^2 G_2(\mathbf{q}_1, \mathbf{q}_2)]\frac{\mu_{3z}}{q_3} \\ &+ \beta\mu k b(1+\beta\mu_{1z}^2)\frac{\mu_{23z}}{q_{23}} G_2(\mathbf{q}_2, \mathbf{q}_3) + \frac{(\beta\mu k)^2}{2} b_1^2 (1+\beta\mu_{1z}^2)\frac{\mu_{2z}}{q_2}\frac{\mu_{3z}}{q_3}\} \\ &+ 2b_G S_1(\mathbf{q}_1, \mathbf{q}_2 + \mathbf{q}_3) F_2(\mathbf{q}_2, \mathbf{q}_3) + b_G b_1 \beta\mu k \frac{\mu_{1z}}{q_1} S_1(\mathbf{q}_2, \mathbf{q}_3) \\ &+ 2b_\Gamma S_1(\mathbf{q}_1, \mathbf{q}_1 + \mathbf{q}_3)(F_2(\mathbf{q}_2, \mathbf{q}_3) - G_2(\mathbf{q}_2, \mathbf{q}_3)) \end{aligned}$$

Quiz time

- What’s next?

What’s next?

Now we use all this technology to derive the NL correction to the power spectrum

So far:

$$\delta_g^{(1)}(\mathbf{k}) = \delta^{(1)}(\mathbf{k}) Z_1(\mathbf{k})$$

$$\delta_g^{(2)}(\mathbf{k}) = \int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\delta^{(1)}(\mathbf{q}_1)\delta^{(1)}(\mathbf{q}_2) Z_2(\mathbf{q}_1,\mathbf{q}_2)\delta_D(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)$$

$$\delta_g^{(3)}(\mathbf{k}) = \int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\frac{d^3q_3}{(2\pi)^3}\delta^{(1)}(\mathbf{q}_1)\delta^{(1)}(\mathbf{q}_2)\delta^{(1)}(\mathbf{q}_3) Z_3(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3)\delta_D(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_3)$$

$$Z_1(\mathbf{k}) = b_1 + f\mu^2$$

$$Z_2(\mathbf{q}_a,\mathbf{q}_b) = b_1\{F_2(\mathbf{q}_a,\mathbf{q}_b) + \beta\mu^2 G_2(\mathbf{q}_a,\mathbf{q}_b) + \frac{\beta b\mu k}{2}\left[\frac{\mu_{az}}{q_a}(1+\beta\mu_{bz}^2) + \frac{\mu_{bz}}{q_b}(1+\beta\mu_{az}^2)\right]\} + \frac{b_2}{2} + b_G S_1(\mathbf{q}_a,\mathbf{q}_b)$$

$$\begin{aligned} Z_3(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3) = b_1\{&F_3(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3) + \beta\mu^2 G_3(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3) + \beta\mu k b[F_2(\mathbf{q}_1,\mathbf{q}_2) + \beta\mu_{12z}^2 G_2(\mathbf{q}_1,\mathbf{q}_2)]\frac{\mu_{3z}}{q_3} \\ &+ \beta\mu k b(1+\beta\mu_{1z}^2)\frac{\mu_{23z}}{q_{23}}G_2(\mathbf{q}_2,\mathbf{q}_3) + \frac{(\beta\mu k)^2}{2}b_1^2(1+\beta\mu_{1z}^2)\frac{\mu_{2z}}{q_2}\frac{\mu_{3z}}{q_3}\} \\ &+ 2b_G S_1(\mathbf{q}_1,\mathbf{q}_2+\mathbf{q}_3)F_2(\mathbf{q}_2,\mathbf{q}_3) + b_G b_1\beta\mu k\frac{\mu_{1z}}{q_1}S_1(\mathbf{q}_2,\mathbf{q}_3) \\ &+ 2b_\Gamma S_1(\mathbf{q}_1,\mathbf{q}_1+\mathbf{q}_3)(F_2(\mathbf{q}_2,\mathbf{q}_3) - G_2(\mathbf{q}_2,\mathbf{q}_3)) \end{aligned}$$

Wick theorem: covariance is all that matters

Higher order moments of Gaussian fields

$$\langle \delta_1 ... \delta_{2p+1} \rangle = 0$$

$$\langle \delta_1 ... \delta_{2p} \rangle = \sum_{\text{all pairs}} \prod_{p \text{ pairs}} \langle \delta_i \delta_j \rangle$$

For instance:

$$\langle \delta_1 \delta_2 \delta_3 \delta_4 \rangle = \langle \delta_1 \delta_2 \rangle \langle \delta_3 \delta_4 \rangle + \langle \delta_1 \delta_3 \rangle \langle \delta_2 \delta_4 \rangle + \langle \delta_1 \delta_4 \rangle \langle \delta_2 \delta_3 \rangle$$

NL power spectrum

$$\delta_g^{(1)}(\mathbf{k}) = \delta^{(1)}(\mathbf{k}) Z_1(\mathbf{k})$$

$$\delta_g^{(2)}(\mathbf{k}) = \int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\delta^{(1)}(\mathbf{q}_1)\delta^{(1)}(\mathbf{q}_2) Z_2(\mathbf{q}_1,\mathbf{q}_2)\delta_D(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)$$

$$\delta_g^{(3)}(\mathbf{k}) = \int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\frac{d^3q_3}{(2\pi)^3}\delta^{(1)}(\mathbf{q}_1)\delta^{(1)}(\mathbf{q}_2)\delta^{(1)}(\mathbf{q}_3) Z_3(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3)\delta_D(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_3)$$

$$P_{gg}(k) = \langle \delta_g(\mathbf{k})\delta_g^*(\mathbf{k})\rangle = \langle (\delta^{(1)}+\delta^{(2)}+\delta^{(3)})_g(\delta^{(1)*}+\delta^{(2)*}+\delta^{(3)*})_g\rangle$$

$$= \langle \delta_g^{(1)}\delta_g^{(1)*}\rangle + \langle \delta_g^{(2)}\delta_g^{(2)*}\rangle + \langle \delta_g^{(1)}\delta_g^{(3)*}\rangle + \langle \delta_g^{(3)}\delta_g^{(1)*}\rangle = (b+f\mu^2)^2 P_L \qquad (6.1.9)$$

$$+ \int \langle \delta_{k_1}^{*(1)}\delta_{k_2}^{*(1)}\delta_{k_3}^{(1)}\delta_{k_4}^{(1)}\rangle Z_2(\mathbf{k}_1,\mathbf{k}_2)Z_2(\mathbf{k}_3,\mathbf{k}_4)\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\delta_D(\mathbf{k}_3+\mathbf{k}_4-\mathbf{k})d^3k_1d^3k_2d^3k_3d^3k_4$$

$$+ 2(b+f\mu^2)\int \langle \delta_k^{*(1)}\delta_{k_1}^{(1)}\delta_{k_2}^{(1)}\delta_{k_3}^{(1)}\rangle Z_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3-\mathbf{k})d^3k_1d^3k_2d^3k_3 \qquad (6.1.10)$$

Quiz:

Why $\langle \delta^{(1)}\delta^{(2)*}\rangle$ disappears?

NL power spectrum

$$P_{gg}(k) = \langle \delta_g(\mathbf{k})\delta_g(\mathbf{k})\rangle = \langle (\delta^{(1)} + \delta^{(2)} + \delta^{(3)})_g(\delta^{(1)*} + \delta^{(2)*} + \delta^{(3)*})_g\rangle$$

$$= \langle \delta_g^{(1)}\delta_g^{(1)*}\rangle + \langle \delta_g^{(2)}\delta_g^{(2)*}\rangle + \langle \delta_g^{(1)}\delta_g^{(3)*}\rangle + \langle \delta_g^{(3)}\delta_g^{(1)*}\rangle = (b + f\mu^2)^2 P_L \qquad (6.1.9)$$

$$+ \int \langle \delta_{k_1}^{*(1)}\delta_{k_2}^{*(1)}\delta_{k_3}^{(1)}\delta_{k_4}^{(1)}\rangle Z_2(\mathbf{k}_1, \mathbf{k}_2) Z_2(\mathbf{k}_3, \mathbf{k}_4)\delta_D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k})\delta_D(\mathbf{k}_3 + \mathbf{k}_4 - \mathbf{k}) d^3k_1 d^3k_2 d^3k_3 d^3k_4$$

$$+ 2(b + f\mu^2) \int \langle \delta_k^{*(1)}\delta_{k_1}^{(1)}\delta_{k_2}^{(1)}\delta_{k_3}^{(1)}\rangle Z_3(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)\delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 - \mathbf{k}) d^3k_1 d^3k_2 d^3k_3 \qquad (6.1.10)$$

Wick:

$$\langle \delta_{k_1}^* \delta_{k_2}^* \delta_{k_3} \delta_{k_4}\rangle = \langle \delta_{k_1}^* \delta_{k_2}^*\rangle\langle \delta_{k_3}\delta_{k_4}\rangle + \langle \delta_{k_1}^* \delta_{k_3}\rangle\langle \delta_{k_2}^* \delta_{k_4}\rangle + \langle \delta_{k_1}^* \delta_{k_4}\rangle\langle \delta_{k_2}^* \delta_{k_3}\rangle$$

$$\langle \delta_{k_1}\delta_{k_2}^*\rangle = \langle \delta_{k_1}\delta_{-k_2}\rangle = P_L(k_1)\delta_D(\mathbf{k}_1 - \mathbf{k}_2)$$

NL power spectrum

$$P_{gg}(k) = \langle \delta_g(\mathbf{k})\delta_g(\mathbf{k})\rangle = \langle (\delta^{(1)} + \delta^{(2)} + \delta^{(3)})_g (\delta^{(1)*} + \delta^{(2)*} + \delta^{(3)*})_g \rangle$$

$$= \langle \delta_g^{(1)}\delta_g^{(1)*}\rangle + \langle \delta_g^{(2)}\delta_g^{(2)*}\rangle + \langle \delta_g^{(1)}\delta_g^{(3)*}\rangle + \langle \delta_g^{(3)}\delta_g^{(1)*}\rangle = (b + f\mu^2)^2 P_L \qquad (6.1.9)$$

$$+ \int \langle \delta_{k_1}^{*(1)}\delta_{k_2}^{*(1)}\delta_{k_3}^{(1)}\delta_{k_4}^{(1)}\rangle Z_2(\mathbf{k}_1,\mathbf{k}_2)Z_2(\mathbf{k}_3,\mathbf{k}_4)\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\delta_D(\mathbf{k}_3+\mathbf{k}_4-\mathbf{k})d^3k_1 d^3k_2 d^3k_3 d^3k_4$$

$$+ 2(b+f\mu^2)\int \langle \delta_k^{*(1)}\delta_{k_1}^{(1)}\delta_{k_2}^{(1)}\delta_{k_3}^{(1)}\rangle Z_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3-\mathbf{k})d^3k_1 d^3k_2 d^3k_3 \qquad (6.1.10)$$

Consider this integral first

$$\langle \delta_{k_1}^*\delta_{k_2}^*\delta_{k_3}\delta_{k_4}\rangle = \langle \delta_{k_1}^*\delta_{k_2}^*\rangle\langle \delta_{k_3}\delta_{k_4}\rangle + \langle \delta_{k_1}^*\delta_{k_3}\rangle\langle \delta_{k_2}^*\delta_{k_4}\rangle + \langle \delta_{k_1}^*\delta_{k_4}\rangle\langle \delta_{k_2}^*\delta_{k_3}\rangle$$

The first term vanishes:

using $\langle \delta_{k_1}^*\delta_{k_2}^*\rangle = P_L(k_1)\delta_D(\mathbf{k}_1+\mathbf{k}_2)$

$$\int \langle \delta_{k_1}^*\delta_{k_2}^*\rangle\langle \delta_{k_3}\delta_{k_4}\rangle Z_2(\mathbf{k}_1,\mathbf{k}_2)Z_2(\mathbf{k}_3,\mathbf{k}_4)\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\delta_D(\mathbf{k}_3+\mathbf{k}_4-\mathbf{k})d^3k_1 d^3k_2 d^3k_3 d^3k_4 =$$

$$\int P_L(k_1)\delta_D(\mathbf{k}_1+\mathbf{k}_2)P_L(k_3)\delta_D(\mathbf{k}_3+\mathbf{k}_4)Z_2(\mathbf{k}_1,\mathbf{k}_2)Z_2(\mathbf{k}_3,\mathbf{k}_4)\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\delta_D(\mathbf{k}_3+\mathbf{k}_4-\mathbf{k})d^3k_1 d^3k_2 d^3k_3 d^3k_4 =$$

$$\int P_L(k_1)P_L(k_3)Z_2(\mathbf{k}_1,-\mathbf{k}_1)Z_2(\mathbf{k}_3,-\mathbf{k}_3)\delta_D(-\mathbf{k})\delta_D(-\mathbf{k})d^3k_1 d^3k_3 = 0$$

NL power spectrum

$$P_{gg}(k) = \langle \delta_g(\mathbf{k})\delta_g(\mathbf{k})\rangle = \langle (\delta^{(1)} + \delta^{(2)} + \delta^{(3)})_g(\delta^{(1)*} + \delta^{(2)*} + \delta^{(3)*})_g\rangle$$

$$= \langle \delta_g^{(1)}\delta_g^{(1)*}\rangle + \langle \delta_g^{(2)}\delta_g^{(2)*}\rangle + \langle \delta_g^{(1)}\delta_g^{(3)*}\rangle + \langle \delta_g^{(3)}\delta_g^{(1)*}\rangle = (b + f\mu^2)^2 P_L \quad (6.1.9)$$

$$+ \int \langle \delta_{k_1}^{*(1)}\delta_{k_2}^{*(1)}\delta_{k_3}^{(1)}\delta_{k_4}^{(1)}\rangle Z_2(\mathbf{k}_1,\mathbf{k}_2)Z_2(\mathbf{k}_3,\mathbf{k}_4)\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\delta_D(\mathbf{k}_3+\mathbf{k}_4-\mathbf{k})d^3k_1d^3k_2d^3k_3d^3k_4$$

$$+ 2(b + f\mu^2)\int \langle \delta_k^{*(1)}\delta_{k_1}^{(1)}\delta_{k_2}^{(1)}\delta_{k_3}^{(1)}\rangle Z_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3-\mathbf{k})d^3k_1d^3k_2d^3k_3 \quad (6.1.10)$$

Consider this integral first

The other two terms of the first integral are identical and give

$$\int P(k_1)P(k_4)Z_2(\mathbf{k}_1,\mathbf{k}_4)Z_2(\mathbf{k}_1,\mathbf{k}_4)\delta_D(\mathbf{k}_1+\mathbf{k}_4-\mathbf{k})\delta_D(\mathbf{k}_1+\mathbf{k}_4-\mathbf{k})d^3k_1d^3k_4 =$$

$$\int P_L(k_1)P_L(|\mathbf{k}-\mathbf{k}_1|)Z_2^2(\mathbf{k}_1,\mathbf{k}-\mathbf{k}_1)\frac{d^3k_1}{(2\pi)^3} \equiv P_{22}$$

NL power spectrum

$$P_{gg}(k) = \langle \delta_g(\mathbf{k})\delta_g(\mathbf{k})\rangle = \langle (\delta^{(1)} + \delta^{(2)} + \delta^{(3)})_g (\delta^{(1)*} + \delta^{(2)*} + \delta^{(3)*})_g \rangle$$

$$= \langle \delta_g^{(1)}\delta_g^{(1)*}\rangle + \langle \delta_g^{(2)}\delta_g^{(2)*}\rangle + \langle \delta_g^{(1)}\delta_g^{(3)*}\rangle + \langle \delta_g^{(3)}\delta_g^{(1)*}\rangle = (b + f\mu^2)^2 P_L \quad (6.1.9)$$

$$+ \int \langle \delta_{k_1}^{*(1)}\delta_{k_2}^{*(1)}\delta_{k_3}^{(1)}\delta_{k_4}^{(1)}\rangle Z_2(\mathbf{k}_1,\mathbf{k}_2) Z_2(\mathbf{k}_3,\mathbf{k}_4)\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\delta_D(\mathbf{k}_3+\mathbf{k}_4-\mathbf{k}) d^3k_1 d^3k_2 d^3k_3 d^3k_4$$

$$+ 2(b + f\mu^2) \int \langle \delta_k^{*(1)}\delta_{k_1}^{(1)}\delta_{k_2}^{(1)}\delta_{k_3}^{(1)}\rangle Z_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3-\mathbf{k}) d^3k_1 d^3k_2 d^3k_3 \quad (6.1.10)$$

The other two terms of the first integral give

$$\int P(k_1)P(k_4) Z_2(\mathbf{k}_1,\mathbf{k}_4) Z_2(\mathbf{k}_1,\mathbf{k}_4)\delta_D(\mathbf{k}_1+\mathbf{k}_4-\mathbf{k})\delta_D(\mathbf{k}_1+\mathbf{k}_4-\mathbf{k}) d^3k_1 d^3k_4 =$$

$$\int P_L(k_1) P_L(|\mathbf{k}-\mathbf{k}_1|) Z_2^2(\mathbf{k}_1,\mathbf{k}-\mathbf{k}_1)\frac{d^3k_1}{(2\pi)^3} \equiv P_{22}$$

Now the second integral:
3 identical terms

$$\int \langle \delta_k^*\delta_{k_1}\rangle\langle \delta_{k_2}\delta_{-k_3}^*\rangle \delta_D(\mathbf{k}-\mathbf{k}_1)\delta_D(\mathbf{k}_2+\mathbf{k}_3) Z_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3-\mathbf{k}) d^3k_1 d^3k_2 d^3k_3 =$$

$$\int P_L(k_1) P_L(k_2)\delta_D(\mathbf{k}_2+\mathbf{k}_3) Z_3(\mathbf{k},\mathbf{k}_2,\mathbf{k}_3)\delta_D(\mathbf{k}_2+\mathbf{k}_3) d^3k_2 d^3k_3 =$$

$$P_L(k)\int P_L(k_1) Z_3(\mathbf{k},\mathbf{k}_1,-\mathbf{k}_1)\frac{d^3k_1}{(2\pi)^3} \equiv P_{31}$$

$$P_{gg}(\mathbf{k},z) = (b+f\mu^2)^2 P_L(\mathbf{k},z) + 2P_{22} + 6(b+f\mu^2)P_{31}(\mathbf{k},z)$$

NL power spectrum

$$P_{gg}(k) = \langle \delta_g(\mathbf{k})\delta_g(\mathbf{k})\rangle = \langle (\delta^{(1)} + \delta^{(2)} + \delta^{(3)})_g (\delta^{(1)*} + \delta^{(2)*} + \delta^{(3)*})_g \rangle$$

$$= \langle \delta_g^{(1)}\delta_g^{(1)*}\rangle + \langle \delta_g^{(2)}\delta_g^{(2)*}\rangle + \langle \delta_g^{(1)}\delta_g^{(3)*}\rangle + \langle \delta_g^{(3)}\delta_g^{(1)*}\rangle = (b + f\mu^2)^2 P_L \quad (6.1.9)$$

$$+ \int \langle \delta_{k_1}^{*(1)}\delta_{k_2}^{*(1)}\delta_{k_3}^{(1)}\delta_{k_4}^{(1)}\rangle Z_2(\mathbf{k}_1,\mathbf{k}_2) Z_2(\mathbf{k}_3,\mathbf{k}_4)\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\delta_D(\mathbf{k}_3+\mathbf{k}_4-\mathbf{k}) d^3k_1 d^3k_2 d^3k_3 d^3k_4$$

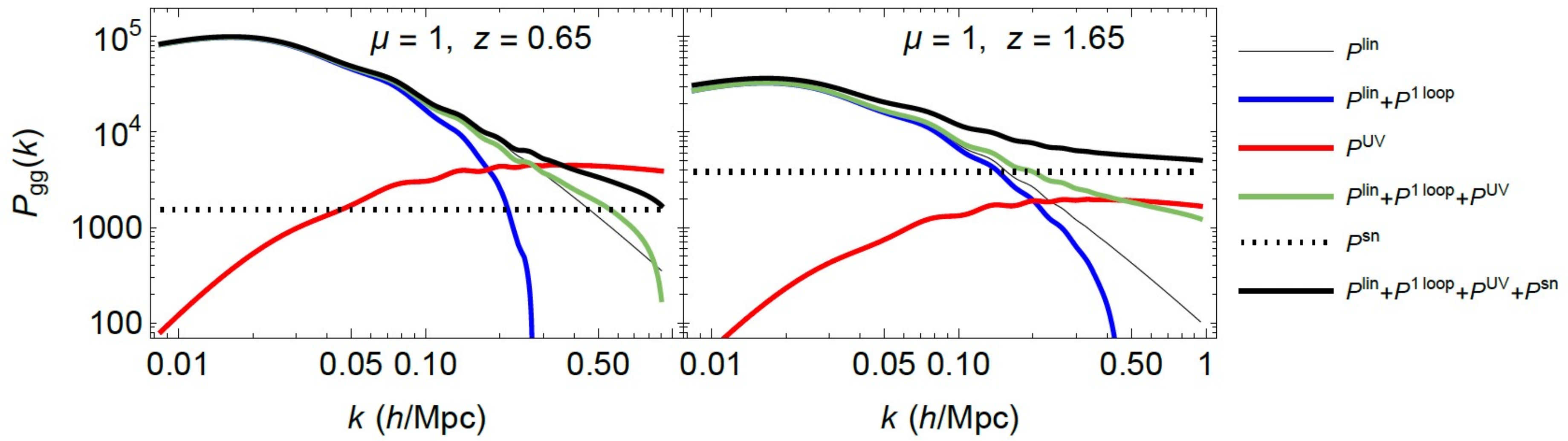
$$+ 2(b+f\mu^2)\int \langle \delta_k^{*(1)}\delta_{k_1}^{(1)}\delta_{k_2}^{(1)}\delta_{k_3}^{(1)}\rangle Z_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3-\mathbf{k}) d^3k_1 d^3k_2 d^3k_3 \quad (6.1.10)$$

Finally we obtain the one-loop spectrum:

$$P_{gg}(\mathbf{k},z) = (b+f\mu^2)^2 P_L(\mathbf{k},z) + 2P_{22} + 6(b+f\mu^2)P_{31}(\mathbf{k},z)$$

NL power spectrum

$$P_{gg}(\mathbf{k}, z) = (b + f\mu^2)^2 P_L(\mathbf{k}, z) + 2P_{22} + 6(b + f\mu^2)P_{31}(\mathbf{k}, z)$$



UV correction: beyond single-stream

$$P_{gg}(\mathbf{k},z) = (b + f\mu^2)^2 P_L(\mathbf{k},z) + 2P_{22} + 6(b + f\mu^2)P_{31}(\mathbf{k},z)$$

Stress tensor

$$\sigma_{ij} = p\delta_{ij} - \eta(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v^k) - \zeta\delta_{ij}\partial_k v^k$$

Perturbed pressure

$$p = p_b + c_s^2 \rho_b \delta.$$

New free shear parameters

$$\eta = \frac{3}{4}\rho_b \frac{c_{sv}^2}{\mathcal{H}}$$

$$\zeta = \rho_b \frac{c_{bv}^2}{\mathcal{H}}$$

…rewritten

$$\sigma^{ij} = p_b\delta^{ij} + \rho_b[c_s^2\delta\delta^{ij} - \frac{3}{4}\frac{c_{sv}^2}{\mathcal{H}}(\partial^i v^j + \partial^j v^i - \frac{2}{3}\delta^{ij}\partial_k v^k) - \frac{c_{bv}^2}{\mathcal{H}}\delta^{ij}\partial_k v^k] + \ldots$$

UV correction: beyond single-stream

$$\sigma^{ij} = p_b\delta^{ij} + \rho_b[c_s^2\delta\delta^{ij} - \frac{3}{4}\frac{c_{sv}^2}{\mathcal{H}}(\partial^i v^j + \partial^j v^i - \frac{2}{3}\delta^{ij}\partial_k v^k) - \frac{c_{bv}^2}{\mathcal{H}}\delta^{ij}\partial_k v^k] + \ldots$$

Kronecker

$$\theta = ik_i v^i/\mathcal{H}$$

Fourier space

$$\frac{1}{\rho_b}\partial_j\sigma^{ij}(\mathbf{k}) = i[c_s^2 k^i\delta - \frac{3}{4}c_{sv}^2(k^i\theta + k^i\theta - \frac{2}{3}k^i\theta) - c_{bv}^2 k^i\theta] = ik^i[c_s^2\delta(\mathbf{k}) - c_v^2\theta(\mathbf{k})]$$

redefinition $c_v^2 = c_{sv}^2 + c_{bv}^2$

$$\theta = -f\delta$$

therefore:

$$\frac{1}{\rho_b}\partial_j\sigma^{ij} \to ik^i(c_s^2\delta - c_v^2\theta) = ik^i(c_s^2 + f c_v^2)\delta$$

UV correction: beyond single-stream

Euler equation with stress tensor

$$\frac{\partial}{\partial t} v_i + v_j \nabla^j v_i = -\nabla_i \Phi - \frac{1}{\rho} \nabla^j \sigma_{ij}$$

$$\frac{1}{\rho_b} \partial_j \sigma^{ij} \rightarrow ik^i (c_s^2 \delta - c_v^2 \theta) = ik^i (c_s^2 + f c_v^2) \delta$$

New Euler equation:

$$\theta'^{(2)} + F\theta^{(2)} + S\delta^{(2)} - (c_s^2 + c_v^2 f) k^2 \delta^{(1)} = -G^2 f^2 \int \delta_1^{(1)} \delta_2^{(1)} \beta(\mathbf{k}_1, \mathbf{k}_2) \delta_D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) d^3k_1 d^3k_2$$

Linear in δ !
UV: Important at high k!

UV correction: beyond single-stream

$$\theta'^{(2)} + F\theta^{(2)} + S\delta^{(2)} - (c_s^2 + c_v^2 f)k^2\delta^{(1)} = -G^2 f^2 \int \delta_1^{(1)}\delta_2^{(1)}\beta(\mathbf{k}_1, \mathbf{k}_2)\delta_D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k})d^3k_1 d^3k_2$$

In the power spectrum,
The new terms give

$$P_{\mathrm{ctr}} = -2\,c_0 P(k)k^2$$

$$c_0 \equiv -\tfrac{1}{2}(c_s^2 + c_v^2 f).$$

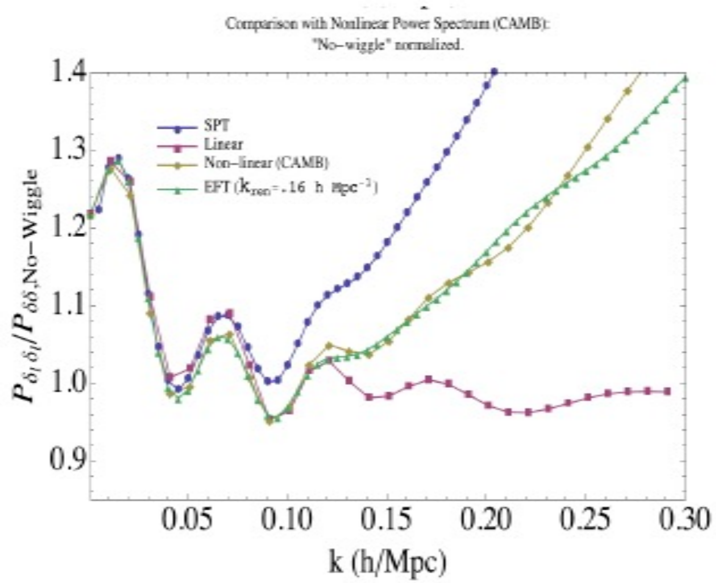
"counterterms"

More general counterterms with FoG corrections

$$P_{\mathrm{ctr}} = -2P(k)k^2(c_0 + c_2\beta\mu^2 + c_4\beta^2\mu^4)$$

One-loop spectrum with UV corrections

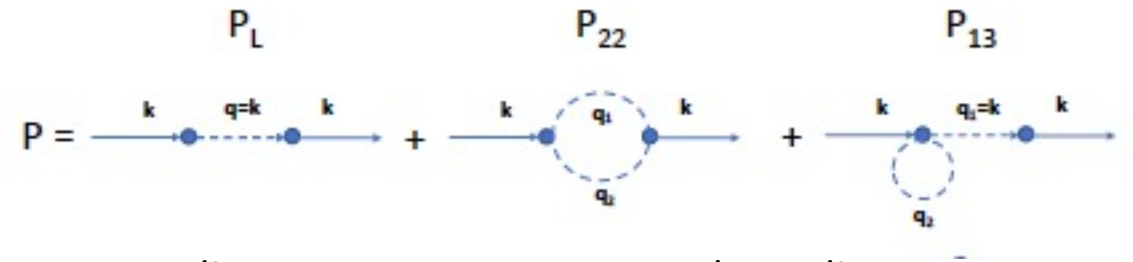
$$P_{gg}(k,\mu,z) = Z_1^2 P_L + 2P_{22} + 6Z_1 P_{31} - 2P(k)k^2(c_0 + c_2\beta\mu^2 + c_4\beta^2\mu^4)$$



Counterterms are very important at small scales!

Diagrams for the spectrum

A spectrum is a two-point correlator.
Rule: Draw two points and connect them in all possible ways
so that you form zero or one loop. This gives the one-loop NL spectrum:



Tree diagram

1-loop diagrams

$$\int P_L(k_1)P_L(|\mathbf{k}-\mathbf{k}_1|)Z_2^2(\mathbf{k}_1,\mathbf{k}-\mathbf{k}_1)\frac{d^3k_1}{(2\pi)^3} \equiv P_{22}$$

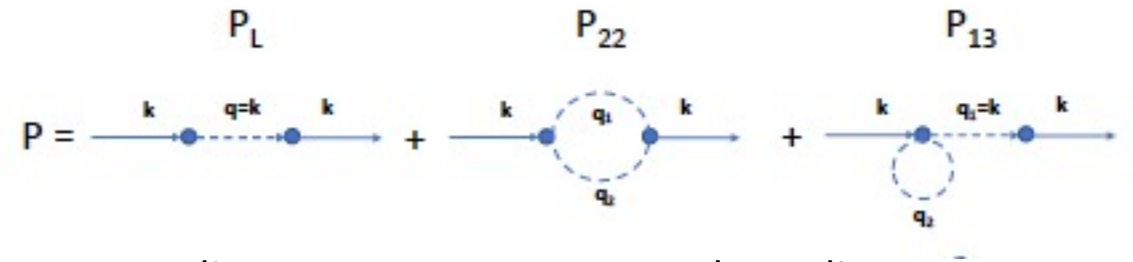
$$P_L(k)\int P_L(k_1)Z_3(\mathbf{k},\mathbf{k}_1,-\mathbf{k}_1)\frac{d^3k_1}{(2\pi)^3} \equiv P_{31}$$

External line

Internal lines

Diagrams for the spectrum

A spectrum is a two-point correlator.
Rule: Draw two points (vertices) and connect them in all possible ways so that you form zero or one loop. This gives the one-loop NL spectrum:



Tree diagram 1-loop diagrams

More rules:
Ensure momentum conservation at each vertex
Any deformation that does not cut lines does not count as additional diagram
Each diagram is multiplied by symmetry multiplicity (eg, loop on the first or second point) and a Wick factor
Every vertex with *p* internal lines is associated to $\delta^{(p)}$ and a kernel K_p (rem: K_1=1)
For every *n*-point correlator at *m*-loops, draw *n* points and trace *m* loops

Example: P_{22}

P_{22}

$\delta^{(2)}(k)$ $\delta^{(2)}(k)$

$$\int P(\mathbf{q}_1)Z_2(\mathbf{q}_1,\mathbf{q}_2)P(\mathbf{q}_2)Z_2(\mathbf{q}_1,\mathbf{q}_2)\delta_D(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)\frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}=P_{22}$$

(times a Wick factor and a graph multiplicity factor!)

Bispectrum

Definition of galaxy spectrum

$$(2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2)P(\mathbf{k}_1,\mathbf{k}_2) \equiv \langle \delta_g(\mathbf{k}_1)\delta_g(\mathbf{k}_2)\rangle$$

Definition of galaxy bispectrum

$$\begin{aligned}(2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_g(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) &\equiv \langle \delta_g(\mathbf{k}_1)\delta_g(\mathbf{k}_2)\delta_g(\mathbf{k}_3)\rangle \\ &= \langle(\delta^{(1)}+\delta^{(2)}+..)_{g\mathbf{k}_1}(\delta^{(1)}+\delta^{(2)}+...)_{g\mathbf{k}_2}(\delta^{(1)}+\delta^{(2)}+...)_{g\mathbf{k}_3}\rangle\end{aligned}$$

Bispectrum

$$(2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_g(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \equiv \langle\delta_g(\mathbf{k}_1)\delta_g(\mathbf{k}_2)\delta_g(\mathbf{k}_3)\rangle$$
$$= \langle(\delta^{(1)}+\delta^{(2)}+..)_{g\mathbf{k}_1}(\delta^{(1)}+\delta^{(2)}+...)_{g\mathbf{k}_2}(\delta^{(1)}+\delta^{(2)}+...)_{g\mathbf{k}_3}\rangle$$

First non-trivial term
(tree-level)

$$B_g(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \langle\delta_g^{(1)}(\mathbf{k}_1)\delta_g^{(1)}(\mathbf{k}_2)\delta_g^{(2)}(\mathbf{k}_3)\rangle + \mathrm{cyclic}$$

Quiz:
Why $\langle\delta^{(1)}\delta^{(1)}\delta^{(1)}\rangle$ disappears?

Bispectrum

$$B_g(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \langle \delta_g^{(1)}(\mathbf{k}_1)\delta_g^{(1)}(\mathbf{k}_2)\delta_g^{(2)}(\mathbf{k}_3)\rangle + \text{cyclic}$$

Then we need to insert:

$$\delta_g^{(1)}(\mathbf{k}) = \delta^{(1)}(\mathbf{k})Z_1(\mathbf{k})$$

$$\delta_g^{(2)}(\mathbf{k}) = \int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\delta^{(1)}(\mathbf{q}_1)\delta^{(1)}(\mathbf{q}_2)Z_2(\mathbf{q}_1, \mathbf{q}_2)\delta_D(\mathbf{k} - \mathbf{q}_1 - \mathbf{q}_2)$$

therefore

$$B_g(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = Z_1(k_1)Z_1(k_2)\langle \delta^{(1)}(\mathbf{k}_1)\delta^{(1)}(\mathbf{k}_2)\delta_g^{(2)}(\mathbf{k}_3)\rangle + \text{cyclic}$$

$$\mathbf{k}_3 = -\mathbf{k}_1 - \mathbf{k}_2,$$

Bispectrum

$$B_g(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = Z_1(k_1) Z_1(k_2) \langle \delta^{(1)}(\mathbf{k}_1) \delta^{(1)}(\mathbf{k}_2) \delta_g^{(2)}(\mathbf{k}_3) \rangle + \text{cyclic}$$

$$\mathbf{k}_3 = -\mathbf{k}_1 - \mathbf{k}_2,$$

$$\langle \delta^{(1)}(\mathbf{k}_1) \delta^{(1)}(\mathbf{k}_2) \delta_g^{(2)}(\mathbf{k}_3) \rangle = \tag{6.4.6}$$

$$\int \frac{d^3 q_1}{(2\pi)^3} \frac{d^3 q_2}{(2\pi)^3} \langle \delta^{(1)}(\mathbf{k}_1) \delta^{(1)}(\mathbf{k}_2) \delta^{(1)}(\mathbf{q}_1) \delta^{(1)}(\mathbf{q}_2) \rangle \times Z_2(\mathbf{q}_1, \mathbf{q}_2) \delta_D(-\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{q}_1 - \mathbf{q}_2) \tag{6.4.7}$$

$$= \int \frac{d^3 q_1}{(2\pi)^3} \frac{d^3 q_2}{(2\pi)^3} \{ P_L(\mathbf{k}_1) \delta_D(\mathbf{k}_1 + \mathbf{k}_2) P_L(\mathbf{q}_1) \delta_D(\mathbf{q}_1 + \mathbf{q}_2) \tag{6.4.8}$$

$$+ P_L(\mathbf{k}_1) \delta_D(\mathbf{k}_1 + \mathbf{q}_1) P_L(\mathbf{k}_2) \delta_D(\mathbf{k}_2 + \mathbf{q}_2) \tag{6.4.9}$$

$$+ P_L(\mathbf{k}_1) \delta_D(\mathbf{k}_1 + \mathbf{q}_2) P_L(\mathbf{k}_2) \delta_D(\mathbf{k}_2 + \mathbf{q}_1) \} \tag{6.4.10}$$

$$\times (Z_2(\mathbf{q}_1, \mathbf{q}_2) \delta_D(-\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{q}_1 - \mathbf{q}_2) \tag{6.4.11}$$

Wick theorem

Bispectrum

$$\langle\delta^{(1)}(\mathbf{k}_1)\delta^{(1)}(\mathbf{k}_2)\delta_g^{(2)}(\mathbf{k}_3)\rangle = \tag{6.4.6}$$

$$\int\frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\langle\delta^{(1)}(\mathbf{k}_1)\delta^{(1)}(\mathbf{k}_2)\delta^{(1)}(\mathbf{q}_1)\delta^{(1)}(\mathbf{q}_2)\rangle\times Z_2(\mathbf{q}_1,\mathbf{q}_2)\delta_D(-\mathbf{k}_1-\mathbf{k}_2-\mathbf{q}_1-\mathbf{q}_2) \tag{6.4.7}$$

$$= \int\frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\{P_L(\mathbf{k}_1)\delta_D(\mathbf{k}_1+\mathbf{k}_2)P_L(\mathbf{q}_1)\delta_D(\mathbf{q}_1+\mathbf{q}_2) \tag{6.4.8}$$

$$+P_L(\mathbf{k}_1)\delta_D(\mathbf{k}_1+\mathbf{q}_1)P_L(\mathbf{k}_2)\delta_D(\mathbf{k}_2+\mathbf{q}_2) \tag{6.4.9}$$

$$+P_L(\mathbf{k}_1)\delta_D(\mathbf{k}_1+\mathbf{q}_2)P_L(\mathbf{k}_2)\delta_D(\mathbf{k}_2+\mathbf{q}_1)\} \tag{6.4.10}$$

$$\times(Z_2(\mathbf{q}_1,\mathbf{q}_2)\delta_D(-\mathbf{k}_1-\mathbf{k}_2-\mathbf{q}_1-\mathbf{q}_2) \tag{6.4.11}$$

The first term in curly brackets vanishes, since $\delta_D(\mathbf{q}_1+\mathbf{q}_2)$ implies $\mathbf{q}_2=-\mathbf{q}_1$, but $Z_2(\mathbf{q}_1,-\mathbf{q}_1)=0$ due to the property (5.3.48), that is respected also by Z_2. The second and third term are identical under exchange of $\mathbf{q}_1,\mathbf{q}_2$ and therefore

$$B_g(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = 2Z_1(\mathbf{k}_1)Z_1(\mathbf{k}_2)\times \tag{6.4.12}$$

$$\int\frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}P_L(\mathbf{k}_1)\delta_D(\mathbf{k}_1+\mathbf{q}_1)P_L(\mathbf{k}_2)\delta_D(\mathbf{k}_2+\mathbf{q}_2)Z_2(\mathbf{q}_1,\mathbf{q}_2)\delta_D(-\mathbf{k}_1-\mathbf{k}_2-\mathbf{q}_1-\mathbf{q}_2)+\text{cyclic} \tag{6.4.13}$$

$$= 2Z_1(\mathbf{k}_1)Z_2(\mathbf{k}_2)Z_2(\mathbf{k}_1,\mathbf{k}_2)P_L(\mathbf{k}_1)P_L(\mathbf{k}_2)+\text{cyclic} \tag{6.4.14}$$

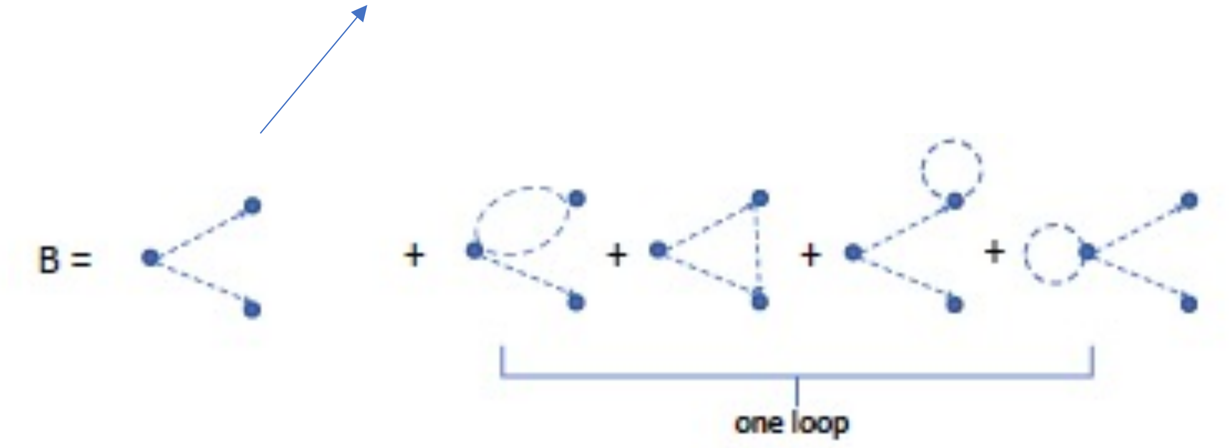
where we used the symmetry $Z_2(-\mathbf{k}_1,-\mathbf{k}_2)=Z_2(\mathbf{k}_1,\mathbf{k}_2)$. This is the tree-level bispectrum, that is proportional to the square of the linear spectra and is therefore of the same order as the spectrum 1-loop corrections. The expression for the four one-loop bispectra are given in e.g. [1], [7].

Bispectrum diagram

$$B_g(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2Z_1(\mathbf{k}_1)Z_1(\mathbf{k}_2)\times \quad (6.4.12)$$

$$\int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3} P_L(\mathbf{k}_1)\delta_D(\mathbf{k}_1+\mathbf{q}_1)P_L(\mathbf{k}_2)\delta_D(\mathbf{k}_2+\mathbf{q}_2)Z_2(\mathbf{q}_1,\mathbf{q}_2)\delta_D(-\mathbf{k}_1-\mathbf{k}_2-\mathbf{q}_1-\mathbf{q}_2) + \text{cyclic} \quad (6.4.13)$$

$$= 2Z_1(\mathbf{k}_1)Z_2(\mathbf{k}_2)Z_2(\mathbf{k}_1,\mathbf{k}_2)P_L(\mathbf{k}_1)P_L(\mathbf{k}_2) + \text{cyclic} \quad (6.4.14)$$



$\delta^2\delta^1\,\delta^2$ $\delta^3\delta^2\,\delta^1$ $\delta^2\delta^2\,\delta^2$ $\delta^2\delta^3\,\delta^1$ $\delta^4\delta^1\,\delta^1$