Cosmological large-scale structure

Lecture 7

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Workshop/course 10.02

Philosophenweg 16, SR

you are very welcom to join!

Testing Fundamental Physics with Cosmology

One-day block course 10.02.25

9:30-10:20 Ziyang Zheng "*Measuring the anisotropic stress*"
10:20-11:10 Tatiane de Paula Moraes "*Cosmology with dark sirens and galaxy Catalogs*"
11:10-11:25 coffee break
11:25-12:15 Luca Amendola "*A Frequentist-Bayesian approach*"
12:15-12:45 Hanqiong Jia "*Non-linear lensing*"
12:45-14:00 lunch
14:00-14:50 Malte Schneider "*The Raychaudhuri equation and its implications*"
14:50-15:40 Matteo Maturi "*Photometric surveys and cosmology, an example: Euclid*"
15:40-16:00 coffee break
16:00-16:50 Bilal Tüdes "*Non-linear coupled dark energy*"
16:50-17:40 Felipe A. Da Silva Barbosa (online, TBA)

The structure of the large scale structure

large scales → **small scales**

super-horizon scales		linear scales		mildly non-linear scales		strongly non-linear scales
relativistic corrections	$k \approx aH \approx 0.002$ h/Mpc $\lambda \approx 3000\ Mpc/h$	*linear pert. theory*	$k \approx 0.1\ h/Mpc$ $\lambda \approx 60\ Mpc/h$	*non-linear pert. theory*	$k \approx 0.3\ h/Mpc$ $\lambda \approx 20\ Mpc/h$	*N-body simulations*

Recap I

1-loop spectrum

$$P_{gg}(\mathbf{k}, z) = (b + f\mu^2)^2 P_L(\mathbf{k}, z) + 2P_{22} + 6(b + f\mu^2)P_{31}(\mathbf{k}, z)$$

stress tensor

$$\frac{1}{\rho_b}\partial_j \sigma^{ij} \to ik^i(c_s^2\delta - c_v^2\theta) = ik^i(c_s^2 + f c_v^2)\delta$$

NL spectrum with UV correction

$$P_{gg}(k, \mu, z) = Z_1^2 P_L + 2P_{22} + 6Z_1 P_{31} - 2P(k)k^2(c_0 + c_2\beta\mu^2 + c_4\beta^2\mu^4)$$

Recap II

bispectrum

$$(2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_g(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \equiv \langle\delta_g(\mathbf{k}_1)\delta_g(\mathbf{k}_2)\delta_g(\mathbf{k}_3)\rangle$$
$$= \langle(\delta^{(1)}+\delta^{(2)}+..)_{g\mathbf{k}_1}(\delta^{(1)}+\delta^{(2)}+...)_{g\mathbf{k}_2}(\delta^{(1)}+\delta^{(2)}+...)_{g\mathbf{k}_3}\rangle$$

Tree-level bispectrum

$$B_g(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = 2Z_1(\mathbf{k}_1)Z_1(\mathbf{k}_2)\times \quad (6.4.12)$$
$$\int\frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}P_L(\mathbf{k}_1)\delta_D(\mathbf{k}_1+\mathbf{q}_1)P_L(\mathbf{k}_2)\delta_D(\mathbf{k}_2+\mathbf{q}_2)Z_2(\mathbf{q}_1,\mathbf{q}_2)\delta_D(-\mathbf{k}_1-\mathbf{k}_2-\mathbf{q}_1-\mathbf{q}_2)+\text{cyclic} \quad (6.4.13)$$
$$= 2Z_1(\mathbf{k}_1)Z_2(\mathbf{k}_2)Z_2(\mathbf{k}_1,\mathbf{k}_2)P_L(\mathbf{k}_1)P_L(\mathbf{k}_2)+\text{cyclic} \quad (6.4.14)$$

Roadmap for today

- Symmetry conditions
- Generalized Kernels
- Comparing to observations

Simmetry conditions

General building blocks

$$\alpha = \frac{(\mathbf{k}_1 + \mathbf{k}_2)}{2}(\frac{\mathbf{k}_1}{k_1^2} + \frac{\mathbf{k}_2}{k_2^2}) = 1 + \frac{1}{2}\mathbf{k}_1\mathbf{k}_2(\frac{1}{k_1^2} + \frac{1}{k_2^2})$$

$$\beta = \frac{(\mathbf{k}_1 + \mathbf{k}_2)^2(\mathbf{k}_1\mathbf{k}_2)}{2k_1^2k_2^2}$$

Simmetry conditions

(Kehagias, Riotto 2013 1302.0130)

1. Equivalence principle
2. Invariance under a general time-dependent translation, dubbed Extended Galilean Invariance (EGI)

$$\tau \to \tilde{\tau} = \tau; \quad \mathbf{x} \to \tilde{\mathbf{x}} + d(\tau) \tag{6.5.3}$$

   The standard fluidodynamics equations satisfy this invariance, up to a redefinition of $\mathbf{v}$ and Φ.

3. Mass and momentum invariance

$$\int d^3x\delta(x,\tau) = 0 \tag{6.5.4}$$

$$\int d^3xx^i\delta(x,\tau) = 0 \tag{6.5.5}$$

4. Rotational invariance

boostrap method:

G. D'Amico, M. Marinucci, M. Pietroni, and F. Vernizzi. The large scale structure bootstrap: perturbation theory and bias expansion from symmetries. *Journal of Cosmology and Astroparticle Physics*, 2021(10):069, October 2021.

Mass and momentum

3. Mass and momentum invariance

$$\int d^3x\delta(x,\tau) = 0$$

$$\int d^3x x^i \delta(x,\tau) = 0$$

$$\int \delta(x) d^3x = \frac{\frac{1}{V}\int \rho(x) d^3x - \frac{1}{V}\int \rho_0\, d^3x}{\rho_0/V} = \frac{\rho_0 - \rho_0}{\rho_0/V} = 0$$

$$\int \mathbf{x}\,\delta(x) d^3x = \frac{\frac{1}{M}\int \mathbf{x}\,\rho(x) d^3x - \frac{1}{M}\int \mathbf{x}\,\rho_0\, d^3x}{\frac{1}{M}\rho_0} = \frac{\mathbf{R}}{1/V} \quad \to 0$$

(choosing the center of mass as the origin)

Mass invariance

3. Mass and momentum invariance

$$\int d^3x\,\delta(x,\tau) = 0$$

$$\int d^3x\,x^i\delta(x,\tau) = 0$$

Mass conservation in Fourier space

$$\int \delta^{(2)}(x)\,d^3x = \frac{1}{(2\pi)^3}\int \delta_k^{(2)}\,e^{i\,\mathbf{k}\,\mathbf{x}}\,d^3k\,d^3x = \int \delta_k^{(2)}\,d^3k\left[\frac{1}{(2\pi)^3}\int e^{i\,\mathbf{k}\,\mathbf{x}}\,d^3x\right]$$

$$\square = \int \delta_k^{(2)}\,d^3k\,\delta_D(\mathbf{k}) = \delta_{\mathbf{k}=0}^{(2)}$$

Since we know that…

$$\delta_k^{(2)} = G^2\int \delta_1\,\delta_2\,F_2(\mathbf{k}_1,\mathbf{k}_2)\,\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\,d^3k_1\,d^3k_2$$

…we obtain

$$\delta_{\mathbf{k}=0}^{(2)} = G^2\int \delta_1\,\delta_2\,F_2(\mathbf{k}_1,\mathbf{k}_2)\,\delta_D(\mathbf{k}_1+\mathbf{k}_2)\,d^3k_1\,d^3k_2$$

$$\square = G^2\int \delta_1\,\delta_2\,F_2(\mathbf{k}_1,-\mathbf{k}_1)\,d^3k_1 = 0$$

$$\square \rightarrow F_2(\mathbf{k}_1,-\mathbf{k}_1) = 0$$

Momentum invariance

3. Mass and momentum invariance

$$\int d^3x\,\delta(x,\tau) = 0$$

$$\int d^3x\,x^i\,\delta(x,\tau) = 0$$

Momentum invariance in Fourier space

$$\int \mathbf{x}\,\delta^{(2)}(x)d^3x = \frac{1}{(2\pi)^3}\int \delta_k^{(2)}\,\mathbf{x}\,e^{i\mathbf{k}\,\mathbf{x}}\,d^3k\,d^3x = \int \delta_k^{(2)}\,d^3k\left[\frac{-i}{(2\pi)^3}\frac{\partial}{\partial \mathbf{k}}\int e^{i\mathbf{k}\,\mathbf{x}}\,d^3x\right]$$

$$= -i\int \delta_k^{(2)}\,d^3k\frac{\partial}{\partial \mathbf{k}}\delta_D(\mathbf{k}) = i\int\left(\frac{\partial}{\partial \mathbf{k}}\delta_k^{(2)}\right)d^3k\,\delta_D(\mathbf{k}) = 0$$

...we obtain

$$\lim_{\mathbf{k}\to 0}\frac{\partial}{\partial \mathbf{k}}\delta_k^{(2)} = 0$$

$$G^2\lim_{\mathbf{k}\to 0}\frac{\partial}{\partial \mathbf{k}}\int \delta_1\,\delta_2\,F_2(\mathbf{k}_1,\mathbf{k}-\mathbf{k}_1)d^3k_1 =$$

$$G^2\lim_{\mathbf{k}\to 0}\int \delta_1\,\delta_2\frac{\partial}{\partial \mathbf{k}}F_2(\mathbf{k}_1,\mathbf{k}-\mathbf{k}_1)d^3k_1 = 0$$

$$\to \lim_{\mathbf{k}\to 0}\frac{\partial}{\partial \mathbf{k}}F_2(\mathbf{k}_1,\mathbf{k}-\mathbf{k}_1) = 0$$

Momentum invariance

$$\rightarrow \lim_{\mathbf{k}\rightarrow 0} \frac{\partial}{\partial \mathbf{k}} F_2(\mathbf{k}_1, \mathbf{k} - \mathbf{k}_1) = 0$$

equivalent to

$$\lim_{\sum \mathbf{k}_i \rightarrow 0} \frac{\partial}{\partial \mathbf{k}_1} F_2(\mathbf{k}_1, \mathbf{k}_2) = 0$$

$$\lim_{\sum \mathbf{k}_i \rightarrow 0} \frac{\partial}{\partial \mathbf{k}_2} F_2(\mathbf{k}_1, \mathbf{k}_2) = 0$$

Simmetry conditions

3. Mass and momentum invariance

$$\int d^3x \delta(x, \tau) = 0$$

$$\int d^3x x^i \delta(x, \tau) = 0$$

Generalizing, they impose these conditions:

$$Q_n = \sum k_i$$

$$\lim_{\mathbf{Q}_{n,0} \to 0} F_n(\mathbf{q}_1, \cdots \mathbf{q}_n; \eta) = 0\,,$$

$$\lim_{\mathbf{Q}_{n,0} \to 0} \frac{\partial}{\partial q_1^i} F_n(\mathbf{q}_1, \cdots \mathbf{q}_n; \eta) = 0\,,$$

for all the dark matter kernels
but not in general for the tracer kernels !

Extended Galileian Invariance

1. Equivalence principle
2. Invariance under a general time-dependent translation, dubbed Extended Galilean Invariance (EGI)

$$\tau \to \tilde{\tau} = \tau; \quad \mathbf{x} \to \tilde{\mathbf{x}} + d(\tau) \qquad (6.5.3$$

The standard fluidodynamics equations satisfy this invariance, up to a redefinition of $\mathbf{v}$ and Φ.

The EGI means that the equations for $\delta^{(n)}$ as a function of the $\delta^{(1)}$ must obey the property. This produces consistency relations between kernels of different order.

G. D'Amico, M. Marinucci, M. Pietroni, and F. Vernizzi. The large scale structure bootstrap: perturbation theory and bias expansion from symmetries. *Journal of Cosmology and Astroparticle Physics*, 2021(10):069, October 2021.

$$\lim_{\mathbf{q}_1,\cdots,\mathbf{q}_m \to 0} K_n(\mathbf{q}_1,\cdots,\mathbf{q}_m,\mathbf{q}_{m+1}\cdots\mathbf{q}_n) = \frac{\mathbf{q}_1\cdot\mathbf{Q}_{n,m}}{q_1^2}\cdots\frac{\mathbf{q}_m\cdot\mathbf{Q}_{n,m}}{q_m^2} K_{n-m}(\mathbf{q}_{m+1}\cdots\mathbf{q}_n) + \mathcal{O}((1/q)^{m-1}) \, .$$

$$\mathbf{Q}_{n,m} \equiv \sum_{i=m+1}^{n} \mathbf{q}_i \, .$$

e.g. for n=2

$$\lim_{\mathbf{q}_2 \to 0} K_2(\mathbf{q}_1,\mathbf{q}_2) = \frac{\mathbf{q}_1\cdot\mathbf{q}_2}{q_2^2} K_1(\mathbf{q}_1)$$

Rotational invariance

$$\mathbf{k}_1 \cdot \mathbf{k}_1 = k_1^2, \mathbf{k}_2 \cdot \mathbf{k}_2 = k_2^2, \mathbf{k}_1 \cdot \mathbf{k}_2$$

all possible combinations that
are dimensionless, rotationally invariant,
at most quadratic in each momentum,
and have poles for $k_i \to 0$ of order 1 at most

$$1, \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_1^2}, \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_2^2}, \frac{(\mathbf{k}_1 \cdot \mathbf{k}_2)^2}{k_1^2 k_2^2}$$

we include only powers up to k^2 in each momentum, because the conservation equations are quadratic in k

Basis functions

At second order, only
four basis function can be realized

$$1, \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_1^2}, \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_2^2}, \frac{(\mathbf{k}_1 \cdot \mathbf{k}_2)^2}{k_1^2 k_2^2}$$

They are functions of order zero

$$f(\lambda \mathbf{k}_1, \lambda \mathbf{k}_2) = f(\mathbf{k}_1, \mathbf{k}_2).$$

Equivalent basis

$$1, \quad \gamma = 1 - \frac{(\mathbf{k}_1 \cdot \mathbf{k}_2)^2}{k_1^2 k_2^2}, \quad \beta = \frac{(\mathbf{k}_1 + \mathbf{k}_2)^2 \mathbf{k}_1 \cdot \mathbf{k}_2}{2 k_1^2 k_2^2}, \quad \alpha_a = \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_1^2} - \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_2^2}$$

For instance:

$$\alpha = \gamma + \beta + \frac{\alpha_a}{2}$$

General kernels

$$1, \quad \gamma = 1 - \frac{(\mathrm{k}_1 \cdot \mathrm{k}_2)^2}{k_1^2 k_2^2}, \quad \beta = \frac{(\mathrm{k}_1 + \mathrm{k}_2)^2 \mathrm{k}_1 \cdot \mathrm{k}_2}{2 k_1^2 k_2^2}, \quad \alpha_a = \frac{\mathrm{k}_1 \cdot \mathrm{k}_2}{k_1^2} - \frac{\mathrm{k}_1 \cdot \mathrm{k}_2}{k_2^2}$$

Most general 2nd order symmetric kernel

$$F_2(\mathrm{k}_1, \mathrm{k}_2) = a_0^{(2)} + a_1^{(2)} \gamma(\mathrm{k}_1, \mathrm{k}_2) + a_2^{(2)} \beta(\mathrm{k}_1, \mathrm{k}_2)$$

Quiz:
Why is α_a absent?

General kernels

$$F_2(\mathbf{k}_1,\mathbf{k}_2) = a_0^{(2)} + a_1^{(2)}\gamma(\mathbf{k}_1,\mathbf{k}_2) + a_2^{(2)}\beta(\mathbf{k}_1,\mathbf{k}_2)$$

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3. Mass and momentum invariance

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$$\int d^3x x^i \delta(x,\tau) = 0 \qquad (6.5.5)$$

4. Rotational invariance

$$a_0^{(2)} = 0 \qquad a_2^{(2)} = 2$$

so $a_1^{(2)}$ is the only free parameter for F_2

Apply the mass conservation

$$F_2(\mathbf{k}_1,\mathbf{k}_2) = a_0^{(2)} + a_1^{(2)}\gamma(\mathbf{k}_1,\mathbf{k}_2) + a_2^{(2)}\beta(\mathbf{k}_1,\mathbf{k}_2)$$

$$\theta^{(2)} = -G^2 f \int \delta_1\delta_2 G_2(\mathbf{k}_1,\mathbf{k}_2)\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}$$
$$\delta^{(2)} = G^2 \int \delta_1\delta_2 F_2(\mathbf{k}_1,\mathbf{k}_2)\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}$$

mass conservation…

$$\int \delta(x) d^3x = 0$$

…in Fourier space

$$\int \delta_k e^{ikx}\frac{d^3k}{(2\pi)^3}d^3x = \int \delta_k \frac{d^3k}{(2\pi)^3}e^{ikx}d^3x = \int \delta_k \delta_D(k) d^3k = \delta_{k=0}$$

$$\delta_{k=0}^{(2)} = G^2\int \delta_{k_1}\delta_{k_2}F(k_1,k_2)\delta_D(k_1+k_2)\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}$$
$$= G^2\int \delta_{k_1}\delta_{k_2}F(k_1,-k_1)\frac{d^3k_1}{(2\pi)^3}$$
$$= G^2\int \delta_{k_1}\delta_{k_2}[a_0^{(2)} + a_1^{(2)}\gamma(k_1,-k_1) + a_2^{(2)}\beta(k_1,-k_1)]\frac{d^3k_1}{(2\pi)^3}$$
$$= G^2\int \delta_{k_1}\delta_{k_2}[a_0^{(2)}]\frac{d^3k_1}{(2\pi)^3} = 0$$

Apply the mass conservation

$$F_2(\mathbf{k}_1, \mathbf{k}_2) = a_0^{(2)} + a_1^{(2)}\gamma(\mathbf{k}_1, \mathbf{k}_2) + a_2^{(2)}\beta(\mathbf{k}_1, \mathbf{k}_2)$$

Analogously

$$G_2(\mathbf{k}_1, \mathbf{k}_2) = d_1^{(2)}\gamma(\mathbf{k}_1, \mathbf{k}_2) + 2\beta(\mathbf{k}_1, \mathbf{k}_2)$$

In EdS:

$$a_{1\mathrm{EdS}}^{(2)} = \frac{10}{7}.$$

$$d_{1\mathrm{EdS}}^{(2)} = \frac{6}{7}.$$

Third order

$$F_3(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3) = 2\beta(\mathbf{q}_1,\mathbf{q}_2)\beta(\mathbf{q}_{12},\mathbf{q}_3) + a_5^{(3)}\gamma(\mathbf{q}_1,\mathbf{q}_2)\gamma(\mathbf{q}_{12},\mathbf{q}_3) - 2\left(a_{10}^{(3)} - h\right)\gamma(\mathbf{q}_1,\mathbf{q}_2)\beta(\mathbf{q}_{12},\mathbf{q}_3) \tag{6.5.12}$$

$$+ 2\left(a_1^{(2)} + 2a_{10}^{(3)} - h\right)\beta(\mathbf{q}_1,\mathbf{q}_2)\gamma(\mathbf{q}_{12},\mathbf{q}_3) + a_{10}^{(3)}\gamma(\mathbf{q}_1,\mathbf{q}_2)\alpha_a(\mathbf{q}_{12},\mathbf{q}_3) + \text{ cyclic} \tag{6.5.13}$$

$$G_3(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3) = -2\beta(\mathbf{q}_1,\mathbf{q}_2)\beta(\mathbf{q}_{12},\mathbf{q}_3) - d_5^{(3)}\gamma(\mathbf{q}_1,\mathbf{q}_2)\gamma(\mathbf{q}_{12},\mathbf{q}_3) + 2\left(d_{10}^{(3)} - h\right)\gamma(\mathbf{q}_1,\mathbf{q}_2)\beta(\mathbf{q}_{12},\mathbf{q}_3) \tag{6.5.14}$$

$$- 2\left(d_1^{(2)} + 2d_{10}^{(3)} - h\right)\beta(\mathbf{q}_1,\mathbf{q}_2)\gamma(\mathbf{q}_{12},\mathbf{q}_3) - d_{10}^{(3)}\gamma(\mathbf{q}_1,\mathbf{q}_2)\alpha_a(\mathbf{q}_{12},\mathbf{q}_3) + \text{ cyclic.} \tag{6.5.15}$$

$$h(\tau) = \int^{\tau} d\tau' f(\tau') \left(\frac{G(\tau')}{G(\tau)}\right)^2 d_1^{(2)}(\tau')$$

(G = growth function)

Free functions

Six time-dependent free functions (plus 4 bias parameters)

$$a_1^{(2)}, d_1^{(2)}, a_5^{(3)}, a_{10}^{(3)}, d_5^{(3)}, d_{10}^{(3)}.$$

EdS values

$$a_1^{(2)} = \frac{10}{7}, \quad d_1^{(2)} = \frac{6}{7}, \quad a_5^{(3)} = \frac{8}{9}, \quad d_5^{(3)} = \frac{8}{21}, \quad a_{10}^{(3)} = -\frac{1}{9}, \quad d_{10}^{(3)} = -\frac{1}{21}$$

how do we calculate them in general?

Remember what we did to go beyond EdS?

$$\delta'' + F\delta' - S\delta = G^2(2f^2 + f' + Ff)\int \delta_1\delta_2\alpha(\mathbf{k}_1,\mathbf{k}_2)\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}$$
$$+ G^2f^2\int \delta_1\delta_2\beta(\mathbf{k}_1,\mathbf{k}_2)\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}$$

$$\delta^{(2)} = G_{2A}(a)A(\mathbf{k}) + G_{2B}(a)B(\mathbf{k})$$

$$A(k) = \frac{5}{7}G^2\int \delta_1\delta_2\alpha(\mathbf{k}_1,\mathbf{k}_2)(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}$$
$$B(k) = \frac{2}{7}G^2\int \delta_1\delta_2\beta(\mathbf{k}_1,\mathbf{k}_2)(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}$$

we got two equations:

$$G''_{2A} + FG'_{2A} - SG_{2A} = \frac{7}{5}G^2(2f^2 + f' + Ff) = \frac{7}{5}G^2(f^2 + S)$$
$$G''_{2B} + FG'_{2B} - SG_{2B} = \frac{7}{2}G^2f^2$$

…to be solved numerically!

…we now do the same!

Insert the expressions for. $\delta^{(2)}, \theta^{(2)}, \delta^{(3)}, \theta^{(3)}$ inside the continuity and Euler eqs, and obtain differential equations for the free functions by equating terms with the same basis functions

$$\partial_\tau a_1^{(2)} = f(2 - 2a_1^{(2)} + d_1^{(2)})$$
$$\partial_\tau d_1^{(2)} = -f d_1^{(2)} + \frac{S}{f}\left(a_1^{(2)} - d_1^{(2)}\right)$$
$$\partial_\tau a_5^{(3)} = f\left(a_1^{(2)} + d_1^{(2)} - 3a_5^{(3)} + d_5^{(3)}\right)$$
$$\partial_\tau d_5^{(3)} = -2f d_5^{(3)} + \frac{S}{f}\left(a_5^{(3)} - d_5^{(3)}\right)$$
$$\partial_\tau a_{10}^{(3)} = -f\frac{1}{2}\left(a_1^{(2)} - d_1^{(2)} + 6a_{10}^{(3)} - 2d_{10}^{(3)}\right)$$
$$\partial_\tau d_{10}^{(3)} = -2f d_{10}^{(3)} + \frac{S}{f}\left(a_{10}^{(3)} - d_{10}^{(3)}\right)$$

Solving

$$\partial_\tau a_1^{(2)} = f(2 - 2a_1^{(2)} + d_1^{(2)})$$
$$\partial_\tau d_1^{(2)} = -f d_1^{(2)} + \frac{S}{f}\left(a_1^{(2)} - d_1^{(2)}\right)$$
$$\partial_\tau a_5^{(3)} = f\left(a_1^{(2)} + d_1^{(2)} - 3a_5^{(3)} + d_5^{(3)}\right)$$
$$\partial_\tau d_5^{(3)} = -2f d_5^{(3)} + \frac{S}{f}\left(a_5^{(3)} - d_5^{(3)}\right)$$
$$\partial_\tau a_{10}^{(3)} = -f\frac{1}{2}\left(a_1^{(2)} - d_1^{(2)} + 6a_{10}^{(3)} - 2d_{10}^{(3)}\right)$$
$$\partial_\tau d_{10}^{(3)} = -2f d_{10}^{(3)} + \frac{S}{f}\left(a_{10}^{(3)} - d_{10}^{(3)}\right)$$

plus…

Equation for *f*

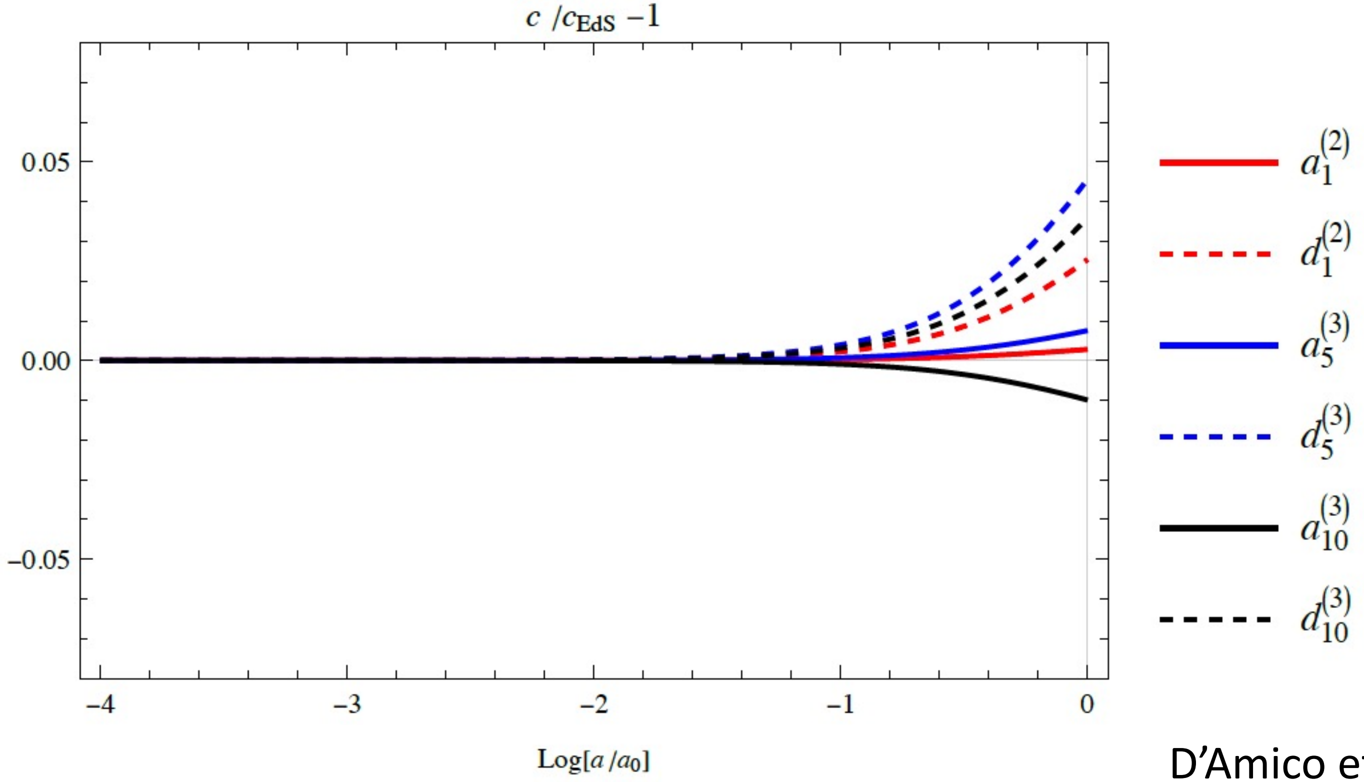
$$f' + f^2 + Ff - S = 0$$

Equation for *S*

$$S = 3\Omega_m/2.$$

Once we specify the cosmological functions *S,f* we can obtain numerically the six free functions

Solving



D'Amico et al. 2019

The deviation from EdS is typically quite small

For instance: modified gravity

coupled dark energy

$$\nabla_\mu T^\mu_{v(\phi)} = \beta T_m \nabla_\nu \phi, \qquad \nabla_\mu T^\mu_{\nu(m)} = -\beta T_m \nabla_\nu \phi$$

DE field

DM fluid

coupling constant

conservation equations

$$\rho'_m + 3(P_m + \rho_m) = -\beta \rho_m \phi'$$
$$\rho'_\phi + 3(P_\phi + \rho_\phi) = \beta \rho_m \phi'$$
$$\rho'_r + 4\rho_r = 0$$

Coupled Dark Energy kernels

modified growth equation

$$f' + f^2 + \frac{1}{2}(1 - 3w_{\rm eff} - 2\beta\phi')f - \frac{3}{2}[(1+2\beta^2)\Omega_c + \Omega_b] = 0$$

$$x = \frac{\phi'}{\sqrt{6}}, \quad y = \frac{1}{H}\sqrt{\frac{V}{3}}, \quad z = \sqrt{\Omega_r}, \quad v = \sqrt{\Omega_b}$$

$$F = \frac{1}{2} - \frac{3}{2}(x^2 - y^2) - \sqrt{6}\beta x$$

$$S = \frac{3}{2}[(1+2\beta^2)(1 - x^2 - y^2 - v^2) + v^2]$$

solving the NL equations we get (asymptotically in the past)

$$a_1^{(2)} = \frac{4\beta^2 + 10}{6\beta^2 + 7}, \quad d_1^{(2)} = \frac{6 - 4\beta^2}{6\beta^2 + 7}, \quad a_5^{(3)} = \frac{8}{10\beta^2 + 9},$$

$$d_5^{(3)} = \frac{24 - 16\beta^2}{60\beta^4 + 124\beta^2 + 63}, \quad a_{10}^{(3)} = -\frac{2\beta^2 + 1}{10\beta^2 + 9}, \quad d_{10}^{(3)} = \frac{4\beta^4 - 4\beta^2 - 3}{60\beta^4 + 124\beta^2 + 63}$$

B. Tüdes and L.A. 2411.06014

they reduce to EdS for $\beta = 0$

Quiz time

- Why are we implementing these symmetry conditions?
- Why the generalized kernels are still not completely general?

Comparing to Observations

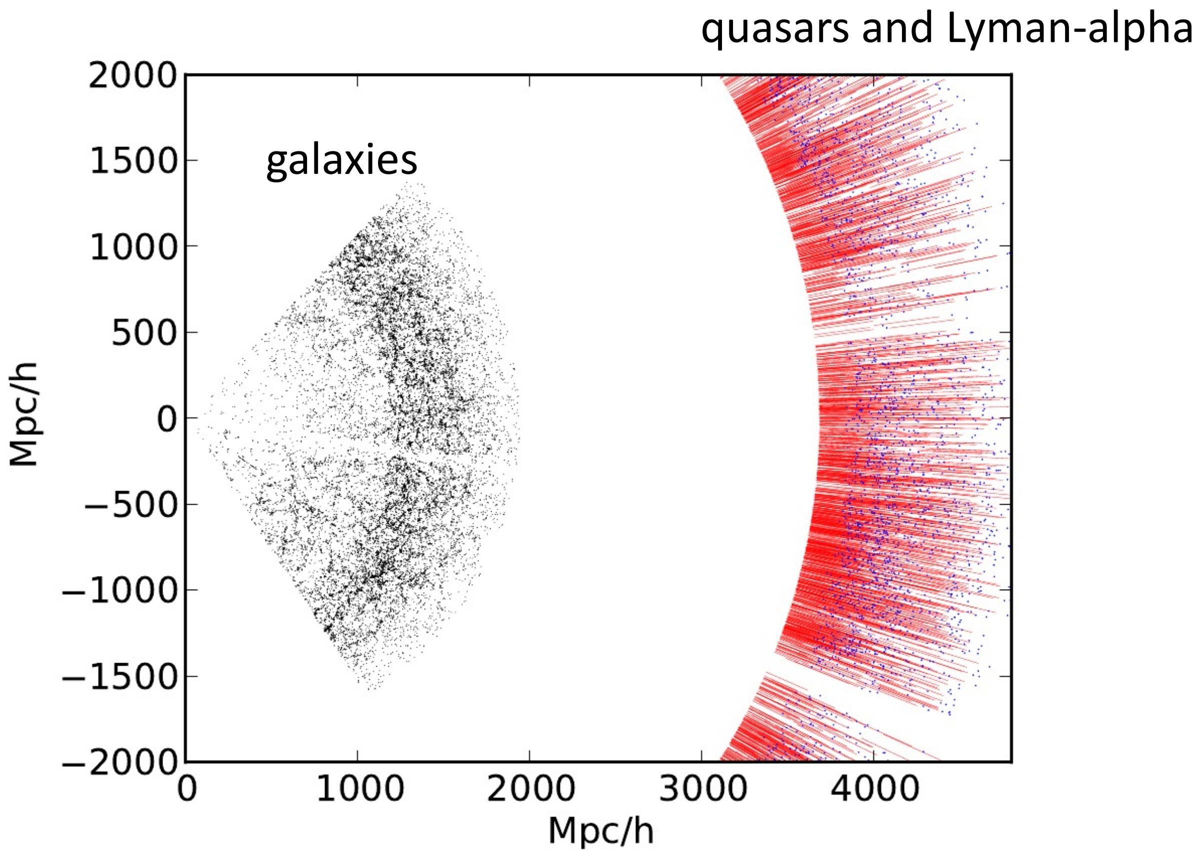
BOSS

The Baryon Oscillation Spectroscopic Survey (BOSS) is a key component of the Sloan Digital Sky Survey III (SDSS-III), which ran from 2009 to 2014.

The primary goal of BOSS was to create a highly detailed three-dimensional map of the large-scale structure of the universe, focusing on the detection of baryon acoustic oscillations (BAOs).

The BOSS survey used the 2.5-meter Sloan Foundation Telescope at Apache Point Observatory in New Mexico, USA, as well as the 4.1-meter Southern Astrophysical Research (SOAR) telescope in Chile, for spectroscopic observations. The survey targeted a large sample of galaxies and quasars, specifically focusing on a volume of space that spanned about 10 billion light-years. In total, BOSS measured the redshifts of over 1.5 million galaxies, producing a high-precision catalog that covers approximately one-third of the entire sky.

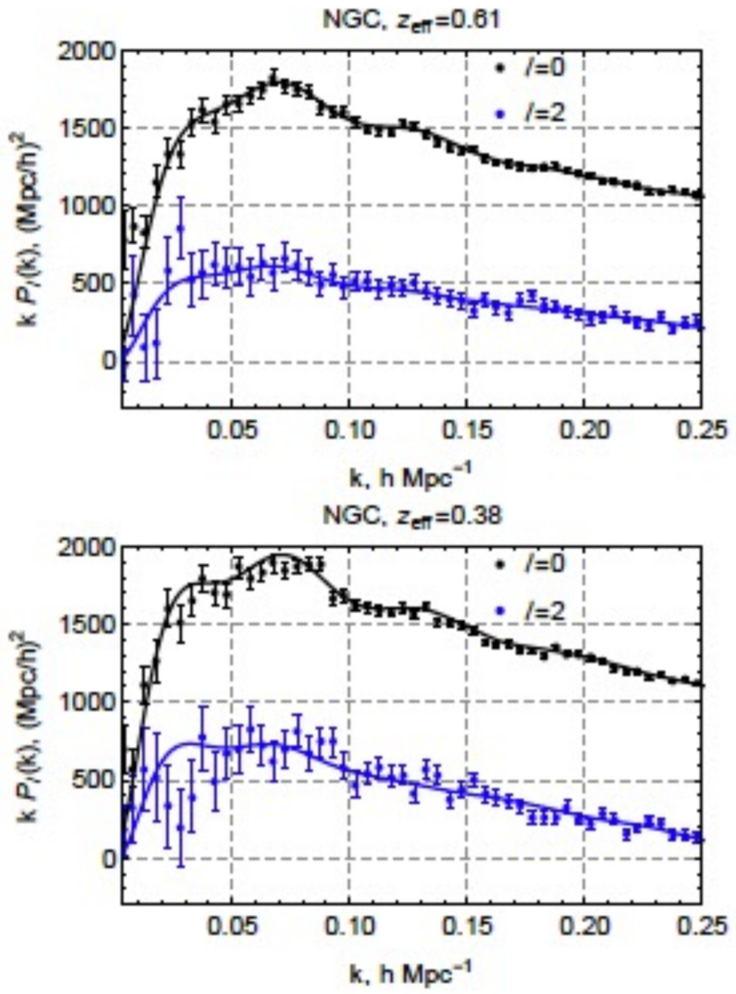
BOSS map



Slosar et al. 2011

Comparing to Observations

monopole + quadrupole



BOSS multipoles

$$P_{g,\ell}(k) \equiv \frac{2\ell+1}{2}\int_{-1}^{1} d\mu\; P_g(k,\mu)\mathcal{P}_\ell(\mu)\ ,$$

Legendre polynomials
$P_0=1$
$P_2=(3\mu^2-1)/2$

ℓ=0 monopole
ℓ=1 dipole
ℓ=2 quadrupole
ℓ=3 octupole
ℓ=4 hexadecapole

Ivanov et al. 2019

Comparing to Observations

Euclid

The Euclid mission is a major space-based observatory dedicated to understanding the nature of dark energy and dark matter, and to mapping the large-scale structure of the universe. Launched by the European Space Agency (ESA) in July 2023, Euclid is designed to probe the geometry of the universe and the role of dark energy in its expansion, aiming to refine our understanding of the cosmos over the next decade.

Spacecraft and Instruments
Euclid is equipped with two main instruments:
1. **Visible Imaging Channel (VIS):** A large visible-light camera that will capture high-resolution images of galaxies across a wide range of wavelengths (from 550 to 900 nm). This allows it to trace the geometry of the universe through galaxy clustering and the patterns of weak gravitational lensing.
2. **Near-Infrared Imaging Channel (NISP):** An infrared camera that will observe galaxies at longer wavelengths (from 1 to 2 microns). This provides crucial information on galaxy evolution, especially for distant, faint galaxies that are redshifted to longer wavelengths due to the expansion of the universe.

Observing Strategy and Survey Area
Euclid's observations will focus on a large section of the sky spanning approximately 15,000 square degrees, up to redshift 3. This vast survey area will provide a statistically significant sample of galaxies to investigate cosmic structures at various scales.

Comparing to Observations

data theory

likelihood

$$L = \frac{1}{(2\pi)^{N/2}|C|} \exp -\frac{1}{2}(P_i - \hat{P}_i)C_{ij}^{-1}(P_j - \hat{P}_j)$$

***i,j* sum over *k*-modes**

Variance of the spectrum

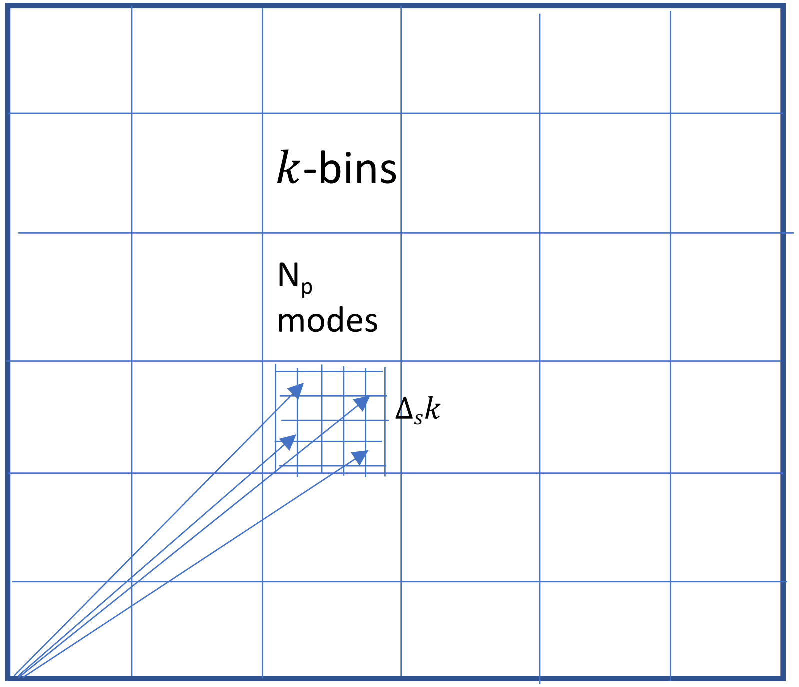
$$\langle \delta_k \delta_k^* \delta_k \delta_k^* \rangle = \langle \delta_k \delta_k^* \rangle \langle \delta_k \delta_k^* \rangle + \langle \delta_k \delta_k^* \rangle \langle \delta_k^* \delta_k \rangle + \langle \delta_k \delta_k \rangle \langle \delta_k^* \delta_k^* \rangle = 2P(\mathbf{k})P(\mathbf{k})$$

$$(2\pi)^3 P(k)\delta_D(\mathbf{k} - \mathbf{k}') = \langle \delta_k \delta_k^* \rangle = \langle \delta_k \delta_{-k} \rangle \quad \Rightarrow \quad \langle \delta_k \delta_k \rangle = 0.$$

Comparing to Observations

$$L = \frac{1}{(2\pi)^{N/2}|C|} \exp -\frac{1}{2}(P_i - \hat{P}_i)C_{ij}^{-1}(P_j - \hat{P}_j)$$

Volume of a **k**-bin $\quad k^2 \Delta k \Delta \mu \Delta \phi = 2\pi k^2 \Delta k \Delta \mu,$



spacing of **k**-modes for every dimension

$$\Delta_s k = \frac{2\pi}{V^{1/3}}$$

$$\sum_{ij} \rightarrow \sum_{\text{bins}} N_P(\mathbf{k})$$

N_p modes per bin

How many modes in a k-bin?

$$L = \frac{1}{(2\pi)^{N/2}|C|} \exp -\frac{1}{2}(P_i - \hat{P}_i)C_{ij}^{-1}(P_j - \hat{P}_j)$$

Volume of a **k**-bin

$$k^2 \Delta k \Delta\mu \Delta\phi = 2\pi k^2 \Delta k \Delta\mu,$$

Spacing of 3D **k**-cells

$$(\Delta_s k)^3 = \frac{(2\pi)^3}{V}$$

total number of modes per bin

$$N_P(k,\mu) = \frac{2\pi k^2 \Delta k \Delta\mu}{(2\pi)^3/V} = \frac{V}{(2\pi)^2} k^2 \Delta k \Delta\mu$$

How many modes in a k-bin?

$$L = \frac{1}{(2\pi)^{N/2}|C|} \exp -\frac{1}{2}(P_i - \hat{P}_i)C_{ij}^{-1}(P_j - \hat{P}_j)$$

$$N_P(k,\mu) = \frac{2\pi k^2 \Delta k \Delta\mu}{(2\pi)^3/V} = \frac{V}{(2\pi)^2} k^2 \Delta k \Delta \mu$$

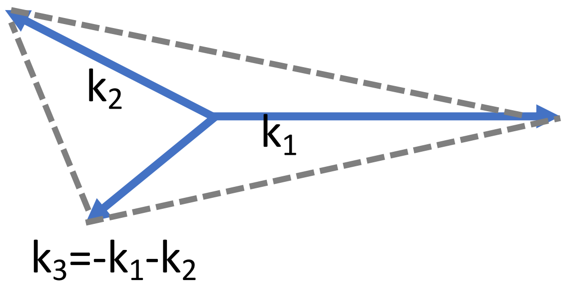
$$\sum_{ij} \to \sum_{\text{bins}} N_P(\mathbf{k})$$

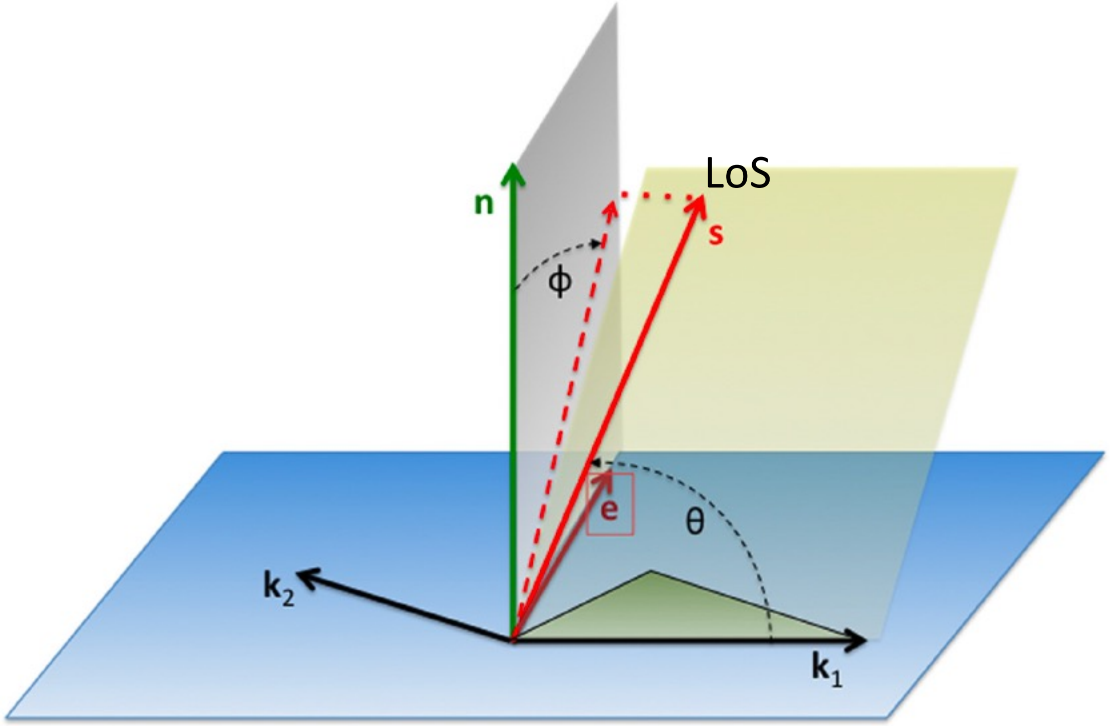
Therefore
the covariance matrix
should be redefined as

$$\bar{C}_{ab}^{(P)} \equiv \frac{C_{ab}^{(P)}}{N_P} = \frac{2}{N_P} P_{gg}(\mathbf{k}_a) P_{gg}(\mathbf{k}_b) \delta_{ab} ,$$

now the sum is over bins a,b !

Triangle geometry





Yankelevich Porciani 2018

define a triangle $k_1, k_2, k_3, \mu = \cos\theta, \ \varphi$

Similarly for the bispectrum

covariance matrix
for triangles in bins *a,b*

$$\bar{C}^{(B)}_{ab} = s_B \frac{V}{N_B} G^6 P_1(k_1) P_1(k_2) P_1(k_3) \delta_{ab} L_i \,,$$

$$s_B = 6, 2, 1 \text{ for equilateral, isosceles, and scalene triangles,}$$

$$N_B = 2 \frac{V^2}{8\pi^4} k_{a_1} k_{a_2} k_{a_3} (\Delta k)^3 \Sigma(\Omega) \Delta\Omega \,,$$

number of triangles in $\Delta\Omega = (\Delta\mu)^2$: $\Sigma(\Omega)\Delta\Omega$

bispectrum likelihood:

$$L(B) = \frac{1}{(2\pi)^{N/2} |C^{(B)}|^{1/2}} \exp -\frac{1}{2} \sum_{\text{a,b}\in\text{bins}} N_P (B_a - \hat{B}_a) [C^{(B)}_{ab}]^{-1} (B_b - \hat{B}_b)$$

Choosing the k_{max}

$$L = \frac{1}{(2\pi)^{N/2}|C|} \exp -\frac{1}{2}(P_i - \hat{P}_i)C_{ij}^{-1}(P_j - \hat{P}_j)$$

i,j sum over k-modes

We should sum only k-modes that are well represented by NL corrections: very small scales cannot be faithfully accounted for.

The k_{max} however cannot be determined by first principles, but only by comparison with N-body simulations. Typical values are:

k_{max} = 0.25 h/Mpc for the 1-loop spectrum
k_{max} = 0.1 h/Mpc for the tree-level bispectrum

(k_{min} is much less important! Why?)

From likelihood to posterior:

$$L = \frac{1}{(2\pi)^{N/2}|C|} \exp -\frac{1}{2}(P_i - \hat{P}_i)C_{ij}^{-1}(P_j - \hat{P}_j)$$

$$L(B) = \frac{1}{(2\pi)^{N/2}|C^{(B)}|^{1/2}} \exp -\frac{1}{2} \sum_{\mathrm{a,b \in bins}} N_P(B_a - \hat{B}_a)[C_{ab}^{(B)}]^{-1}(B_b - \hat{B}_b)$$

The likelihood is a function
of the chosen theoretical parameters,
e.g.

$a_1^{(2)}, d_1^{(2)}, a_5^{(3)}, a_{10}^{(3)}, d_5^{(3)}, d_{10}^{(3)}$. + bias parameters + cosmological parameters

if we have external information, we can multiply the likelihood with the priors

final posterior distribution
for the theory parameters

$$P(\theta_\alpha; \mathrm{data}) = L(P; \theta_\alpha) L(B; \theta_\alpha) \pi(\theta_\alpha)$$

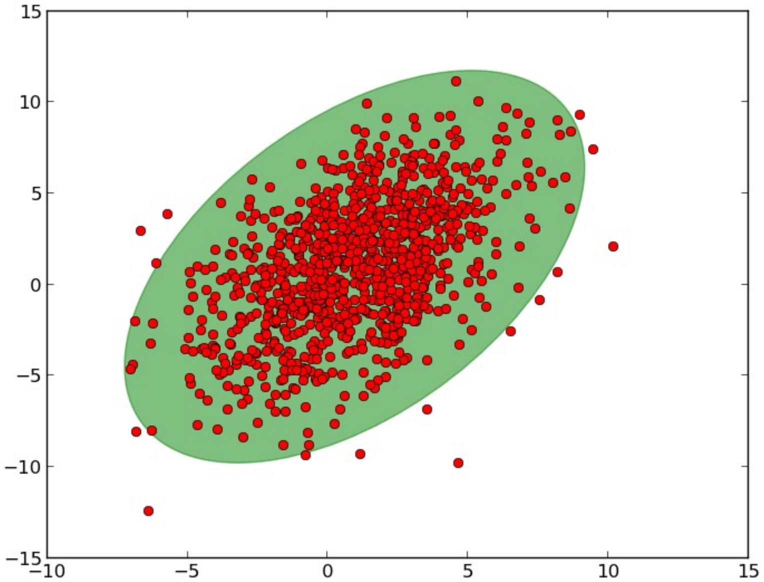
…and finally:

$$L = \frac{1}{(2\pi)^{N/2}|C|} \exp -\frac{1}{2}(P_i - \hat{P}_i)C_{ij}^{-1}(P_j - \hat{P}_j)$$

$$L(B) = \frac{1}{(2\pi)^{N/2}|C^{(B)}|^{1/2}} \exp -\frac{1}{2} \sum_{\mathrm{a,b \in bins}} N_P(B_a - \hat{B}_a)[C_{ab}^{(B)}]^{-1}(B_b - \hat{B}_b)$$

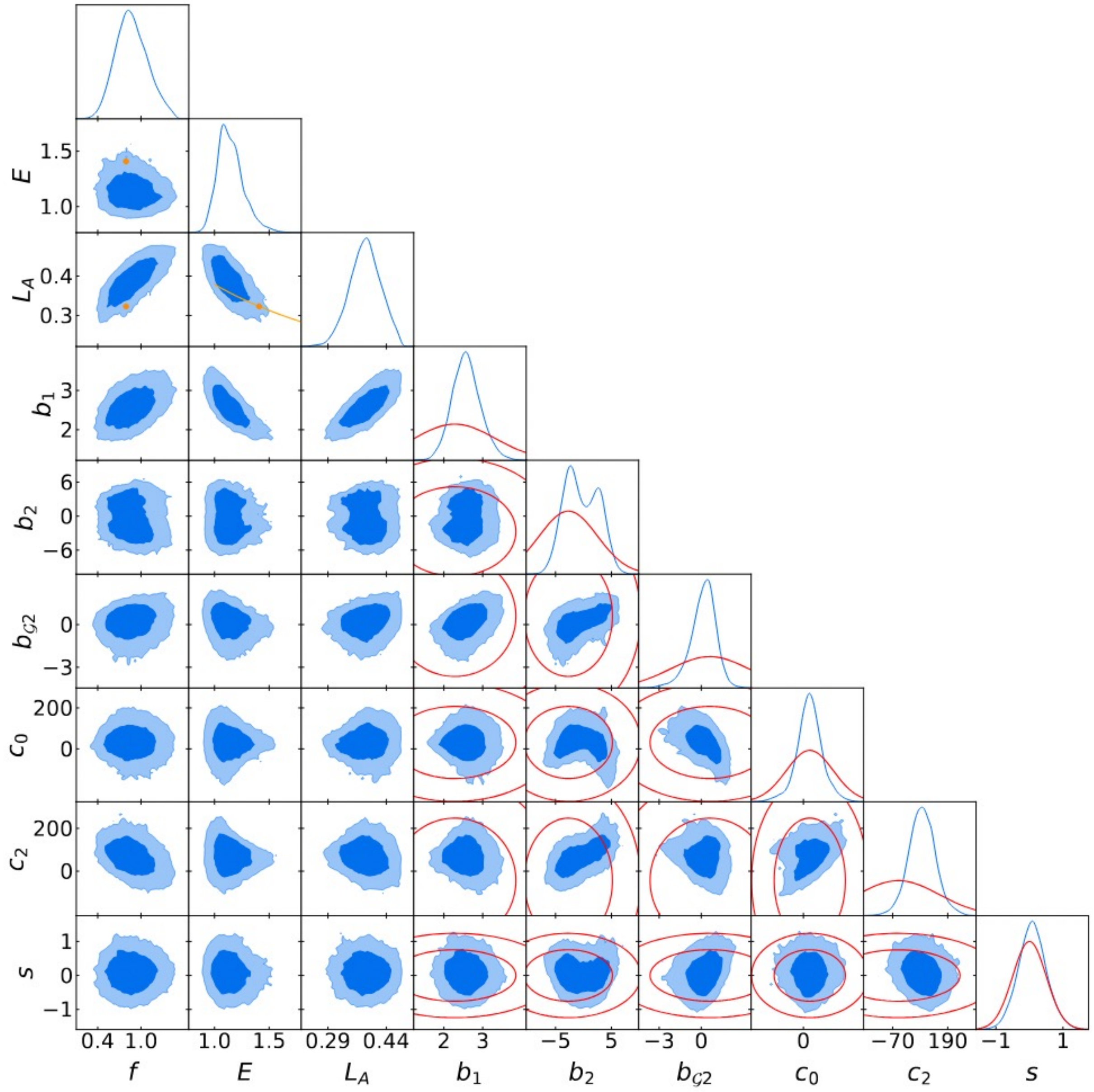
Once we have the likelihood as a function
of the chosen theoretical parameters,
the maximum gives the best fit,
the regions around the maximum the confidence
regions at the chosen level

This is done by sampling
the likelihood with a MCMC scheme



typical output

BOSS data
2406.15347v3



Next & Last:

Relativistic Corrections

by Ziyang Zheng