Truncated Unity Functional renormalization group (TUfRG) for 2D lattices: getting more quantitative

1. **fRG: quantitative issues**
2. **TUfRG in momentum space: recent results**
3. **TUfRG for frquency dependence: outlook**

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Support DFG FOR 723, FOR 912, SPP1458/9 & Ho2422/x-x, RTG 1995

Functional RG for Hubbard-type models

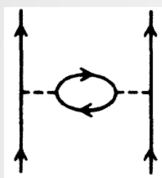
Model bandwidth Λ

bandwidth (few eVs)

$$H_{\text{int}} = U\sum_i n_{i,\uparrow}n_{i,\downarrow} + V_1 \sum_{\langle i,j\rangle,s,s'} n_{i,s}n_{j,s'} + V_2 \sum_{\langle\langle i,j\rangle\rangle,s,s'} n_{i,s}n_{j,s'} + V_3 \sum_{\langle\langle\langle i,j\rangle\rangle\rangle,s,s'} n_{i,s}n_{j,s'}$$

Interaction at scale ~ eV
not much structure, mean-field decoupling ambiguous/impossible

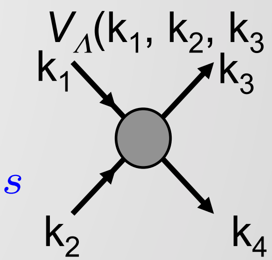
functional renormalization group (fRG): **lower Λ**



Intermediate energy scales: particle-hole pairs, particle-particle **loop corrections** generate structure in **effective low-energy interaction**

10-100 meV
$\geq T_c$

$$H_{\text{eff}} = \frac{1}{2}\sum_{\substack{\vec{p},\vec{p}',\vec{q}\\ s,s'}} V(\vec{p},\vec{p}',\vec{p}+\vec{q})\, c^{\dagger}_{\vec{p}+\vec{q},s}\, c^{\dagger}_{\vec{p}'-\vec{q},s'}\, c_{\vec{p}',s'}\, c_{\vec{p},s}$$



$\Rightarrow$ e.g. guided mean-field decoupling

0

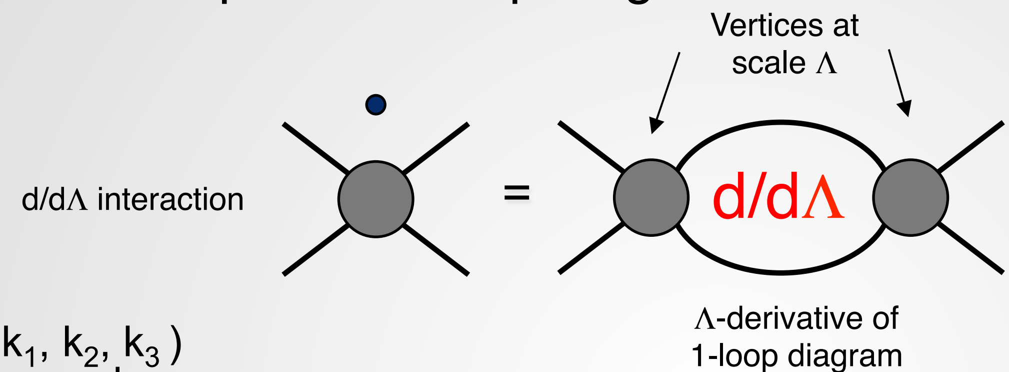
Functional Renormalization Group (fRG):

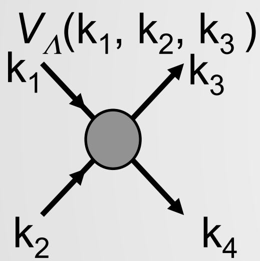
Provides low-energy effective action & momentum structure $V_\Lambda(k,k',k+q)$!

Removes ambiguities of mean-field decouplings.

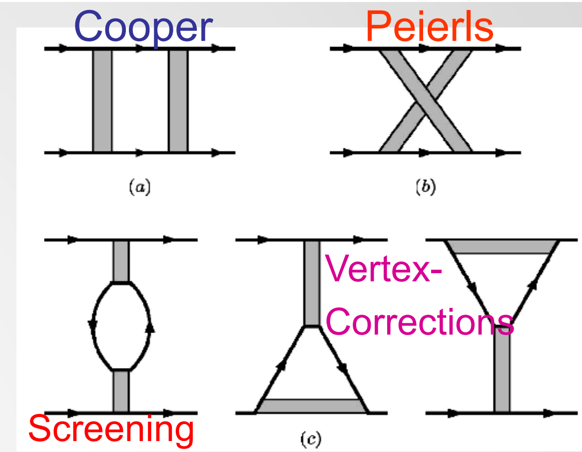
Functional RG

fRG captures all one-loop contributions: unbiased description of competing orders

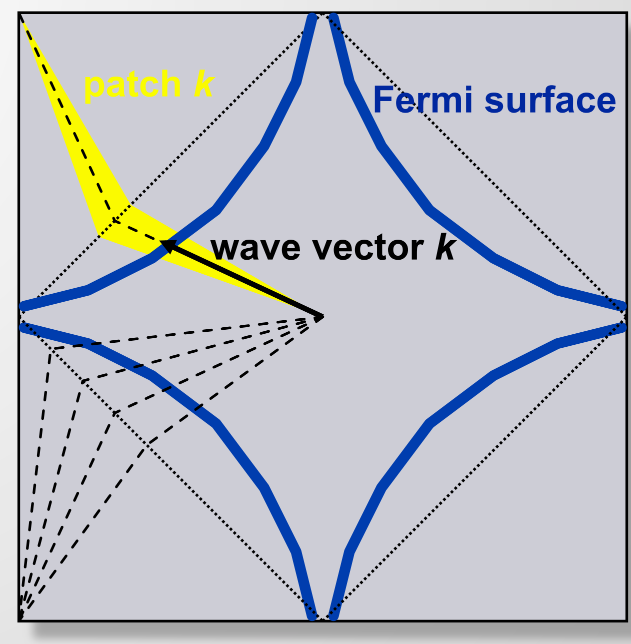




k = wavevector, band, frequency



Brillouin zone



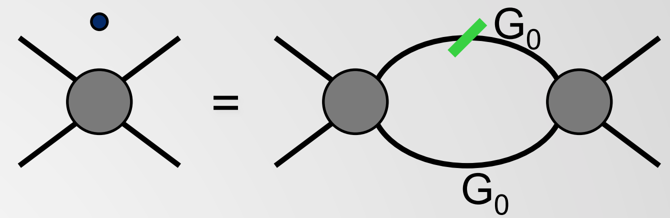
Keep track of wavevector structure: ***N*-patch**

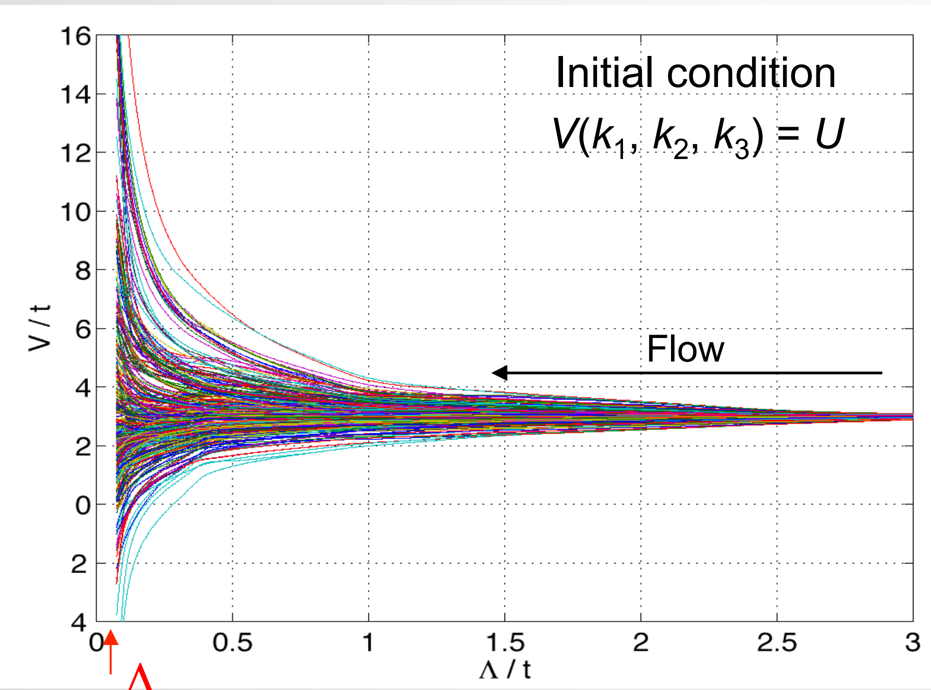
- Discretize Brillouin zone into *N* patches
- More recently: channel decomposition & form factor expansion

Often neglected: self-energy, higher-order interactions, frequency dependence

Flow to strong coupling

Standard cases without self-energy feedback:
Flow to strong coupling





Λ_c
= estimate for gaps
in electronic spectrum

Leading low-energy correlations

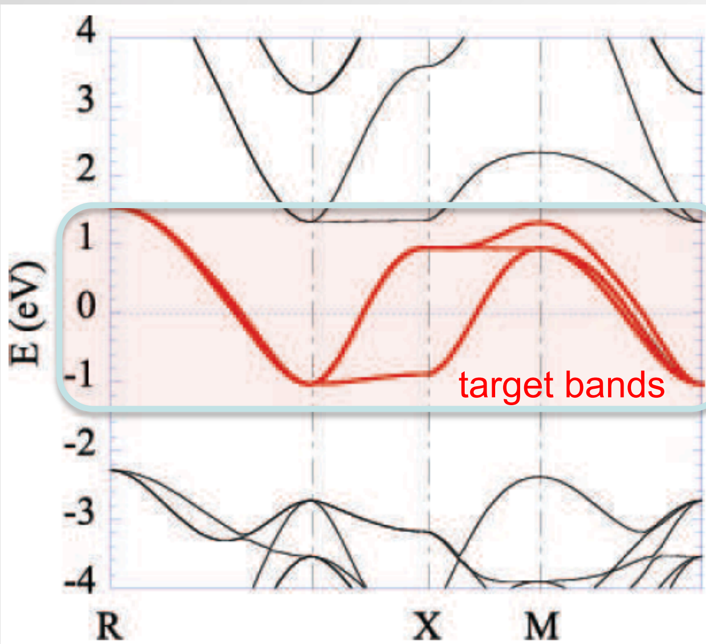
Energy scales

➔ **'Weather forecast'**

Metzner, Salmhofer et al.
RMP 2012

1. Quantitative issues: testing fRG for materials

Take model Hamiltonian with parameters given, e.g., by DFT & cRPA



Single-particle parameters, fit or Wannier matrix elements

$$H = H_K + H_U \, ,$$

$$H_K = \sum_{Rn,R'n'} c^\dagger_{Rn} \boxed{t_{Rn,R'n'}} c_{R'n'} \, ,$$

$$H_U = \frac{1}{2} \sum_{R,nn',mm'} c^\dagger_{Rn} c_{Rn'} \boxed{U_{nn'R,mm'R'}} c^\dagger_{R'm} c_{R'm}$$

Interaction parameters, e.g., Wannier matrix elements, cRPA

- **Can fRG become quantitative low-energy frontend of ab-initio theory?**
- Besides groundstate: Energy scales for phase transitions & relevant excitations? Trends within material families?

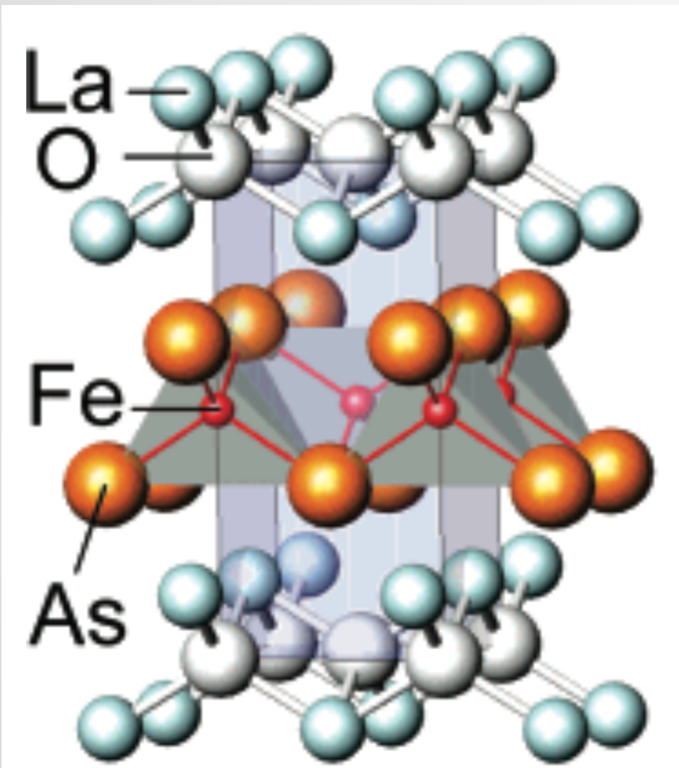
Trends in 1111 iron arsenide superconductors

J|A|C|S
COMMUNICATIONS

Published on Web 02/23/2008

Iron-Based Layered Superconductor La[$O_{1-x}F_x$]FeAs ($x = 0.05-0.12$) with $T_c = 26$ K

Yoichi Kamihara,*,† Takumi Watanabe,‡ Masahiro Hirano,†,§ and Hideo Hosono†,‡,§



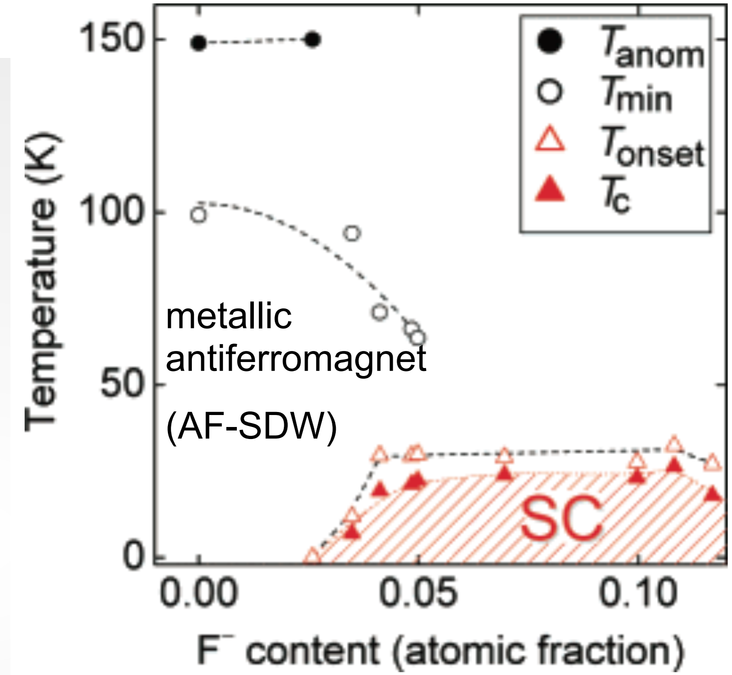


Table I. Maximum T_c in each RFeAs($O_{1-x}F_x$). The F concentration x, which gives the maximum T_c is shown. T_c^{Max} is determined at the onset temperature of superconducting transition in resistivity measurements.

	R							
	La	Ce	Pr	Nd	Sm	Gd	Tb	Dy
T_c^{Max} (K)	28	41	52	52	55	36	46	45
x	0.11	0.16	0.11	0.11	0.1	0.17	0.1	0.1

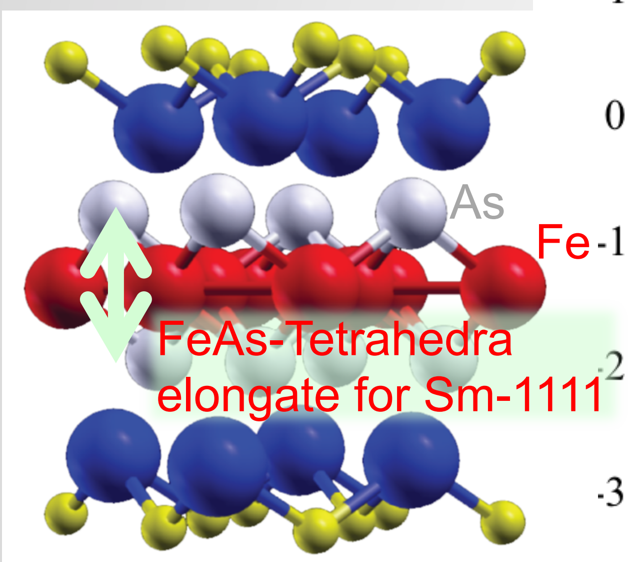
Ishida, K., Nakaii, Y. & Hosono, H. To what extent iron-pnictide new superconductors have been clarified: A progress report. *J. Phys. Soc. Jpn* **78,** 062001 (2009).

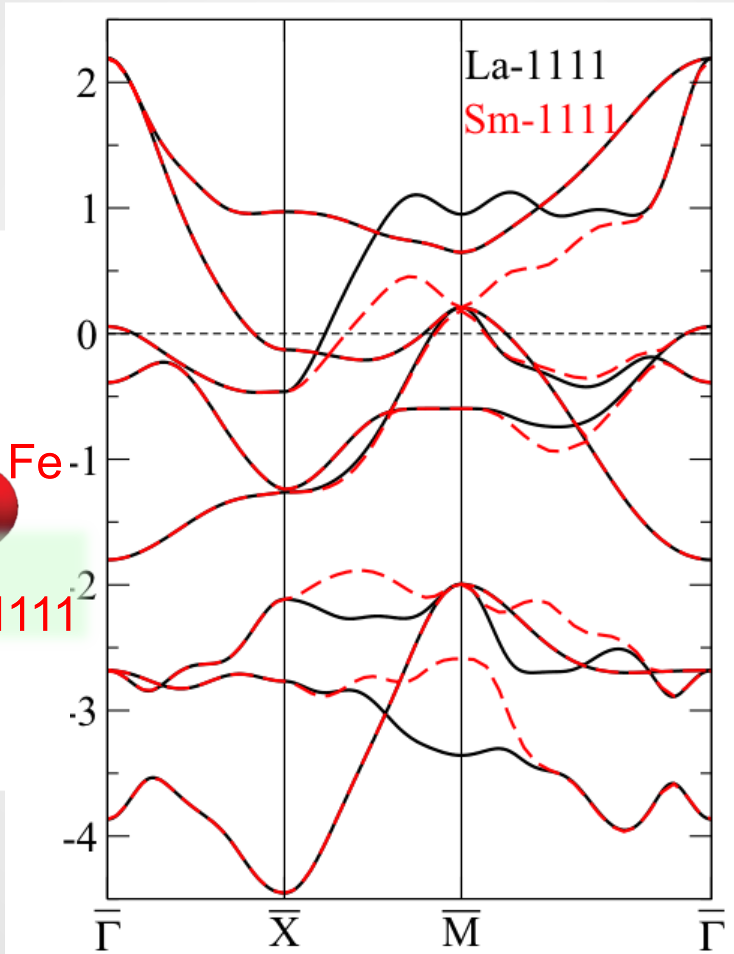
Z.-A. Ren, W. Lu, J. Yang, W. Yi, X.-L. Shen, Z.-C. Li, G.-C. Che, X.-L. Dong, L.-L. Sun, F. Zhou, and Z.-X. Zhao: Chin. Phys. Lett. **25** (2008) 2215.

La-1111 versus Sm-1111

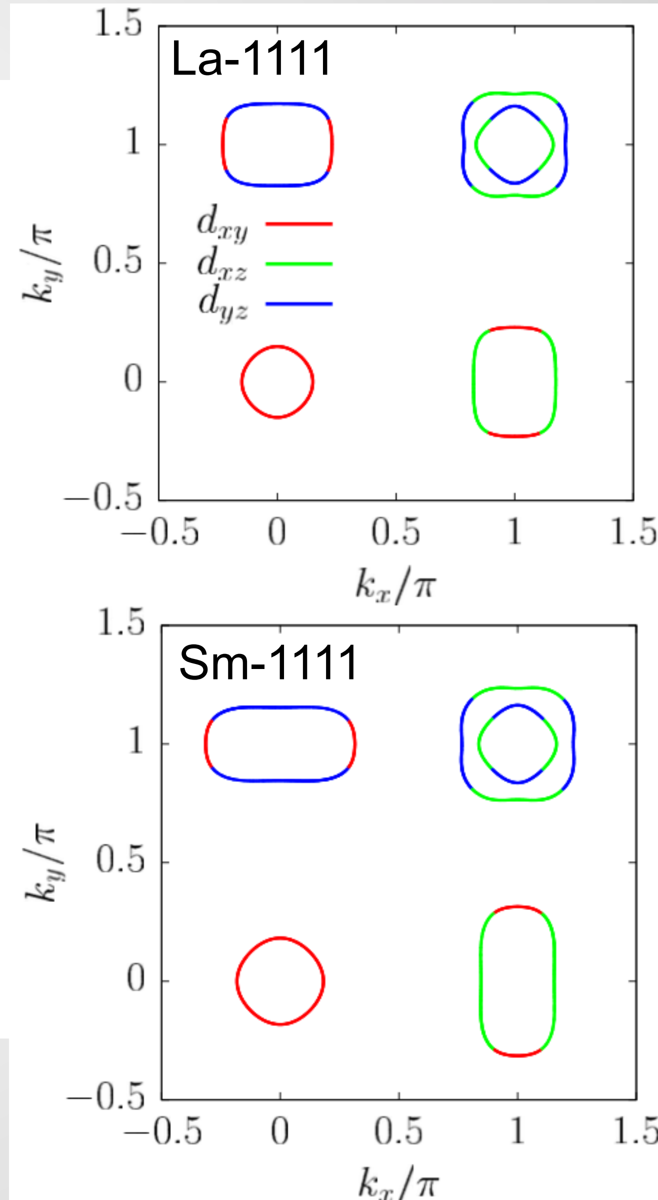
Why is T_c in La-1111 much lower than in Sm-1111?

RE-OFeAs
'*RE*-1111'
RE=La,Sm, …



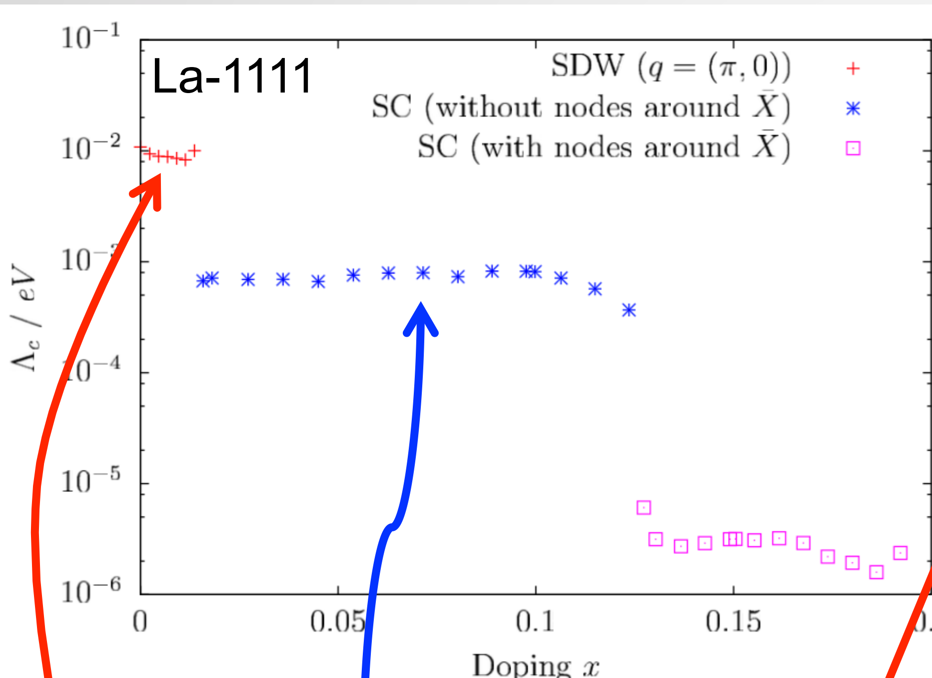


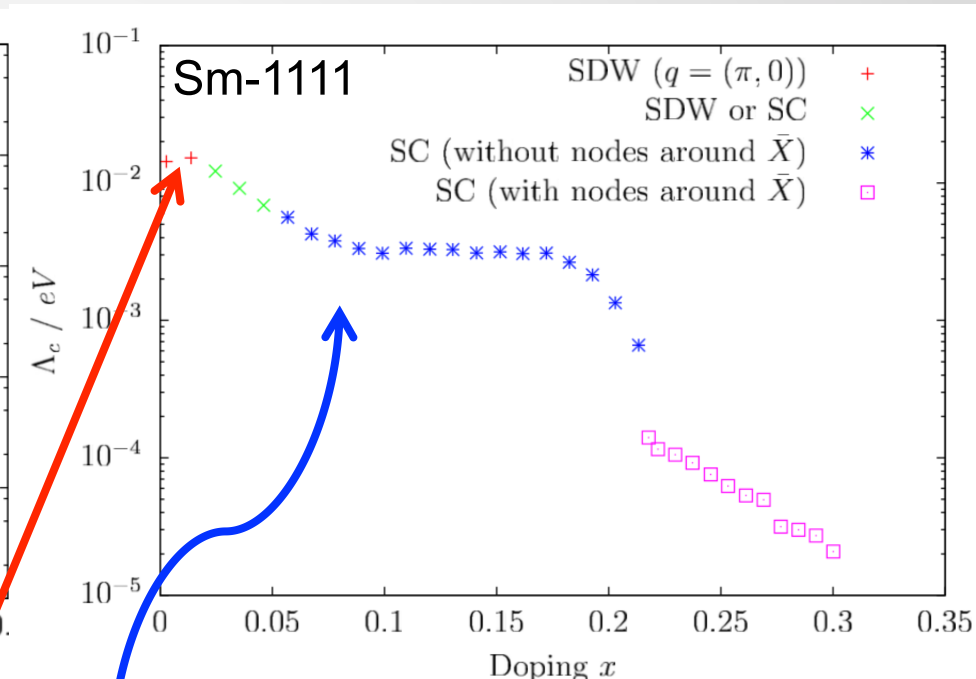
Andersen & Boeri 2011



Trends in 1111 iron arsenides

fRG for 8-band model reproduces sizable T_c-difference for pairing, while keeping AF-SDW scale unchanged





Superconducting scale differs ~ factor 3

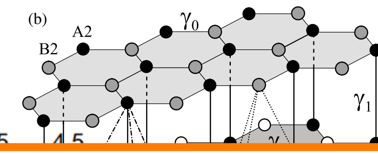
AF-SDW scale comparable

Experimental trend reproduced

Overall energy scale ok, or a little too small …

Lichtenstein, Maier, Platt, Thomale, CH, Boeri, Andersen PRB 2014

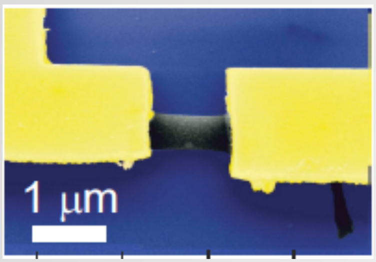
Gaps in bi- & trilayer graphene



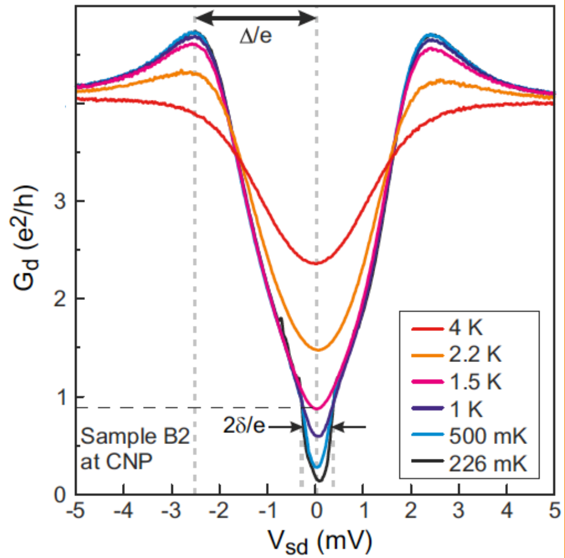
Spontaneously gapped ground state in suspended bilayer graphene

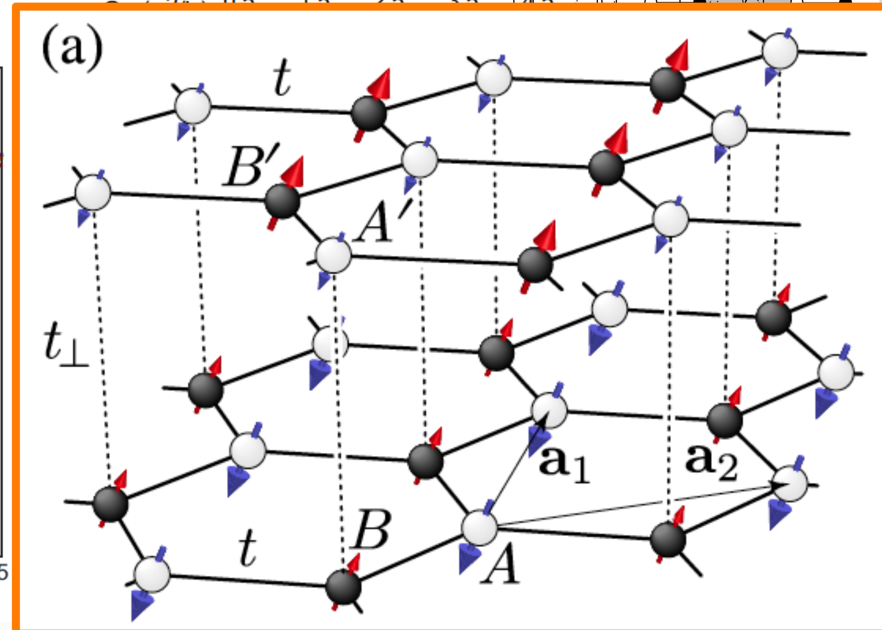
F. Freitag,[1] J. Trbovic,[1] M. Weiss,[1] and C. Schönenberger[1,*]

Phys. Rev. Lett. 108, 076602 (2012)



Clean current-annealed suspendend BLG





Evidence for a spontaneous gapped state in ultraclean bilayer graphene

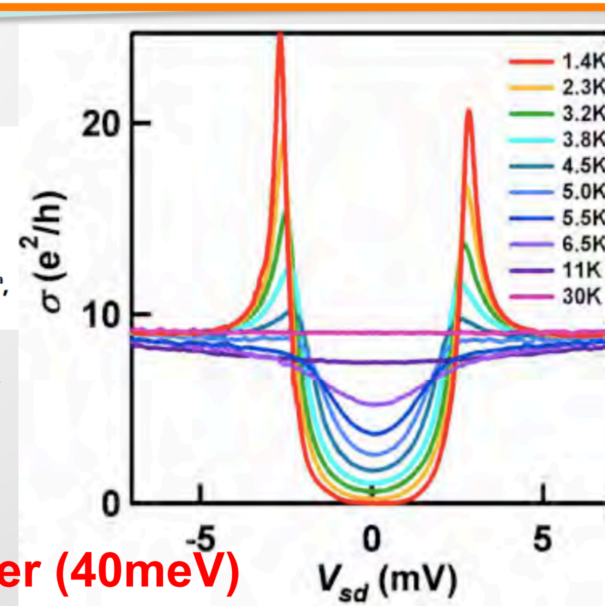
Wenzhong Bao[a,b,1], Jairo Velasco, Jr.[a,1], Fan Zhang[c,d,1], Lei Jing[a], Brian Standley[e], Dmitry Smirnov[f], Marc Bockrath[a], Allan H. MacDonald[c,2], and Chun Ning Lau[a,2]

Proc. Nat. Acad. Sci., 109, 10802 (2012)
Trilayer: Nature Physics, 7, 948 (2011),
Lee, C.N. Lau et al. 2014

Also: Nijmegen (Maan) group

Gap scale ≈ 2-3meV
T_c ≈ 5K in bilayer,

Even larger in trilayer (40meV)

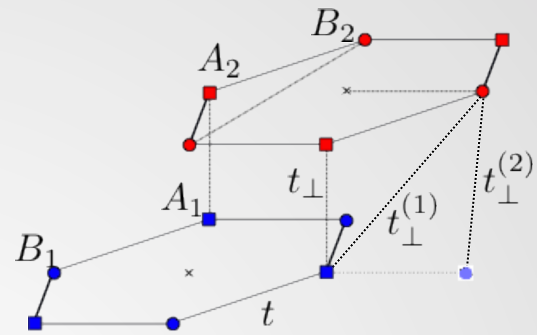


Model for layered graphene

E.g. AB (bernal) stacked bilayer:

$$H_l^{\parallel} = -t \sum_{\sigma,\vec{R},\vec{\delta}_i} \left(b_{l,\sigma}^{\dagger}(\vec{R} + (-)^{l-1}\vec{\delta}_i) a_{l,\sigma}(\vec{R}) + \text{h.c.} \right)$$

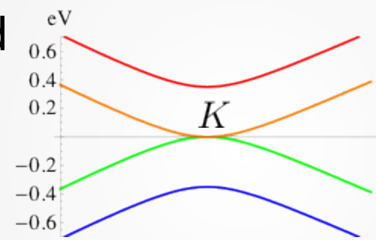
$$H^{\perp} = -t_{\perp} \sum_{\sigma,\vec{R}} \left(a_{1,\sigma}^{\dagger}(\vec{R}) a_{2,\sigma}(\vec{R}) + \text{h.c.} \right)$$

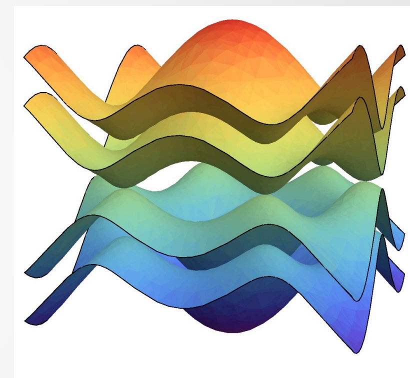


$t \approx 3.0\text{eV}$

$t_\perp \approx 0.4\text{eV}$

Four bands, 2 quadratic band crossing points @ *K*,*K'*





Take ab-initio-derived interaction parameters ('constrained RPA'), interpolate between mono-layer and graphite values

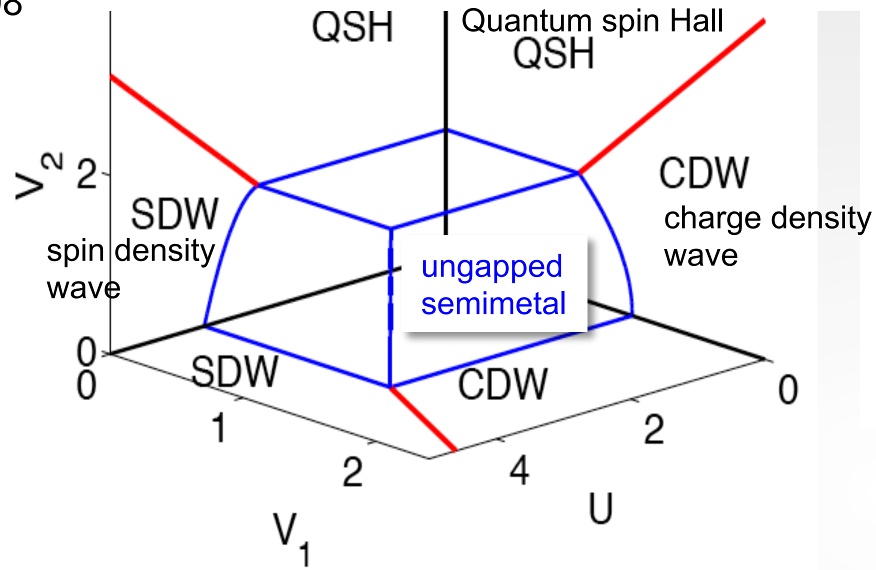
	Graphene		Graphite	
	Bare	cRPA	Bare	cRPA
$U_{00}^{A \text{ or } B}$ (eV)	17.0	9.3	17.5, 17.7	8.0, 8.1
U_{01} (eV)	8.5	5.5	8.6	3.9
$U_{02}^{A \text{ or } B}$ (eV)	5.4	4.1	5.4, 5.4	2.4, 2.4
U_{03} (eV)	4.7	3.6	4.7	1.9

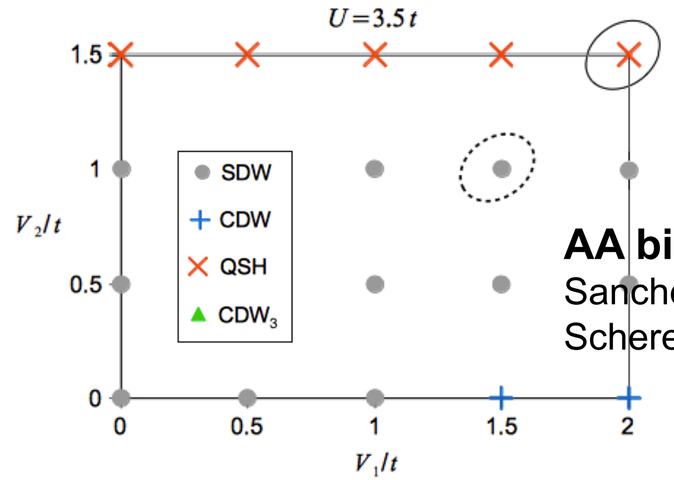
Wehling et al. PRL 2011

$$\begin{aligned} H_{\text{int}} &= U \sum_i n_{i,\uparrow} n_{i,\downarrow} + V_1 \sum_{\langle i,j\rangle,s,s'} n_{i,s} n_{j,s'} \\ &+ V_2 \sum_{\langle\langle i,j\rangle\rangle,s,s'} n_{i,s} n_{j,s'} + V_3 \sum_{\langle\langle\langle i,j\rangle\rangle\rangle,s,s'} n_{i,s} n_{j,s'} \end{aligned}$$

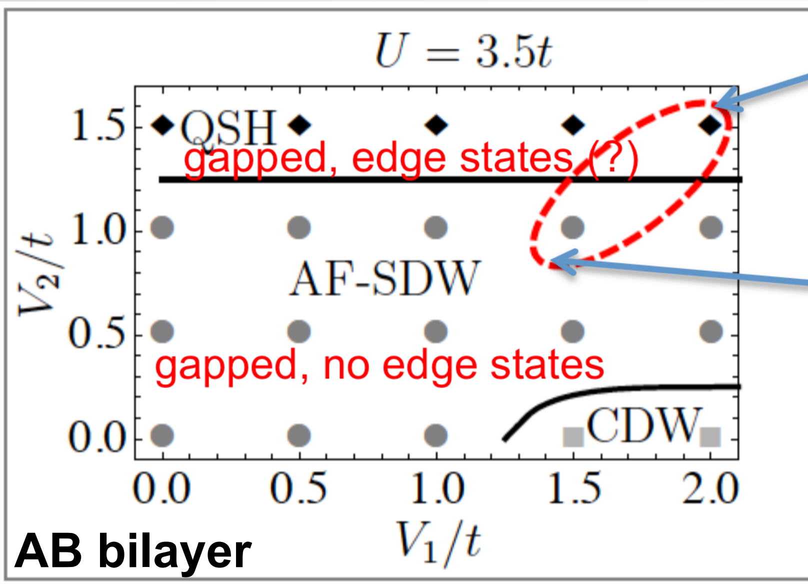
N-Layer graphene @ charge neutrality

Single layer: Raghu, Scherer[0], CH et al., PRL 2008



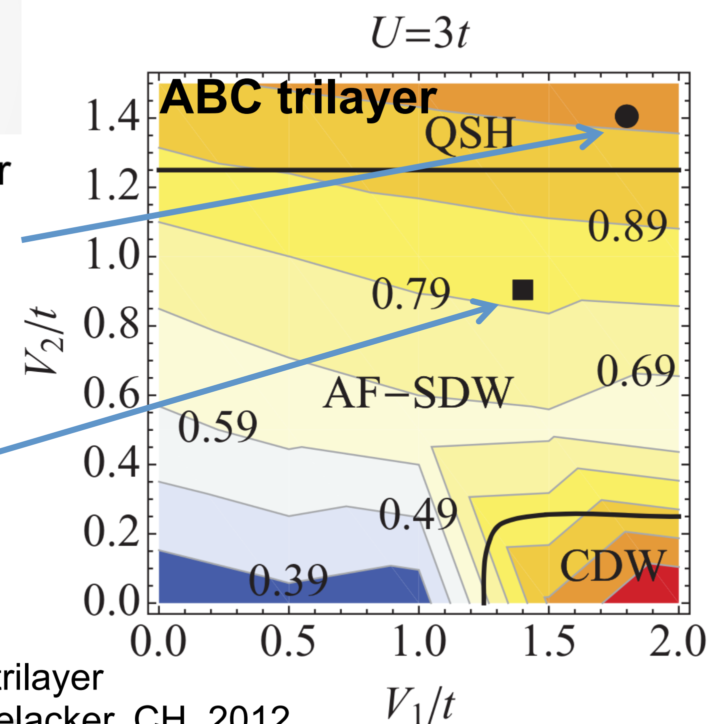


AA bilayer, Sanchez de la Pena, Scherer, CH, 2014



Single-layer cRPA parameters

Graphite cRPA parameters



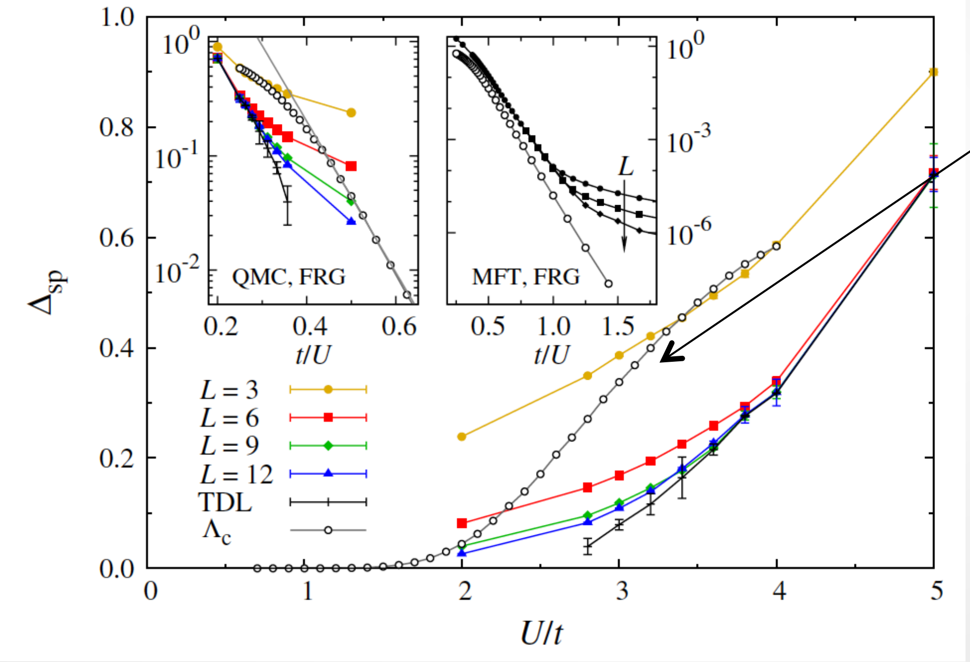
AB bilayer, ABC trilayer
Scherer[(N-1)], Uebelacker, CH, 2012

The ‘scale challenge’

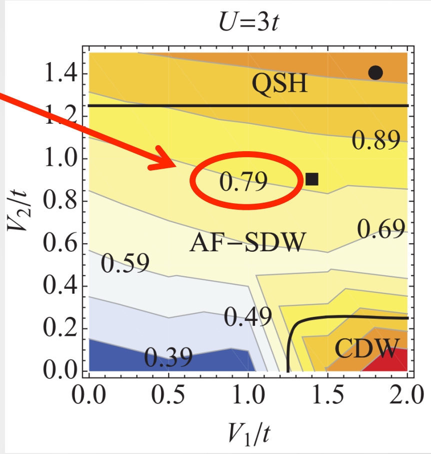
fRG scales for gaps in layered graphene seems **far too large** compared to experiment, even with ‘realistic’ model parameters

Sources of error:

- N-patch fRG (in-)sufficient approximation?



fRG

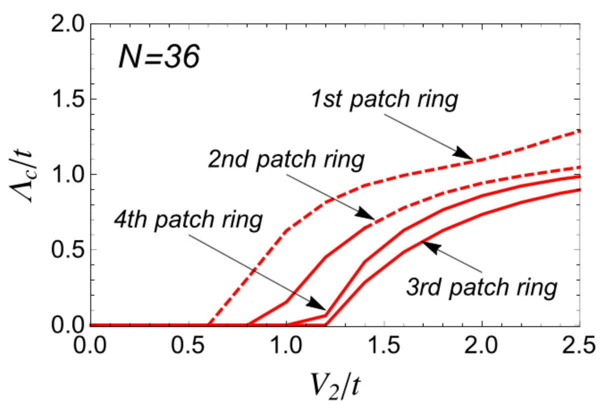


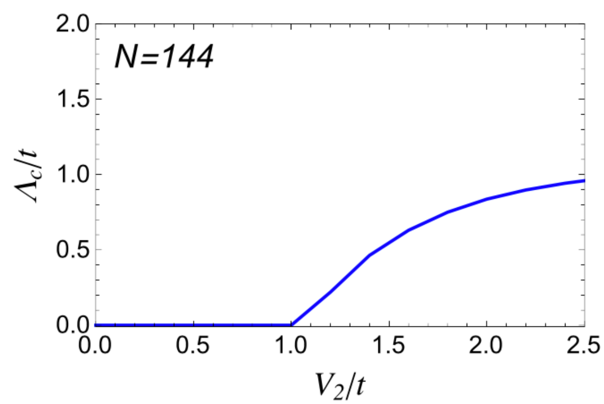
Th. Lang et al. PRL 2012, compares QMC gaps with fRG scale, pure onsite Hubbard U

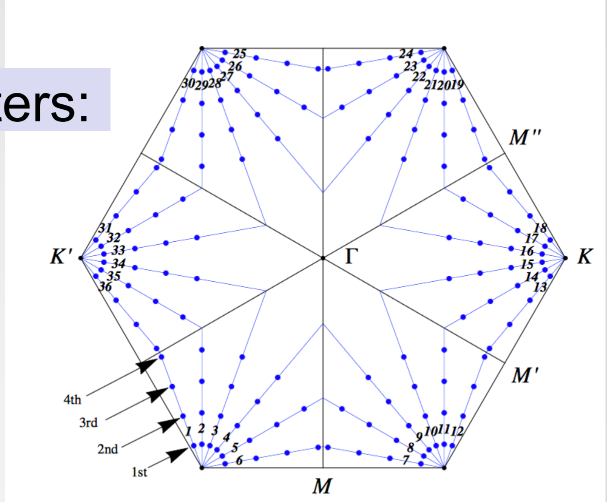
- Model incorrect? Other interactions? Long-range Coulomb!
- Model parameters incorrect? cfRG instead of cRPA?

Resolve patching ambiguities

Choice of representative wavevectors for patches matters:



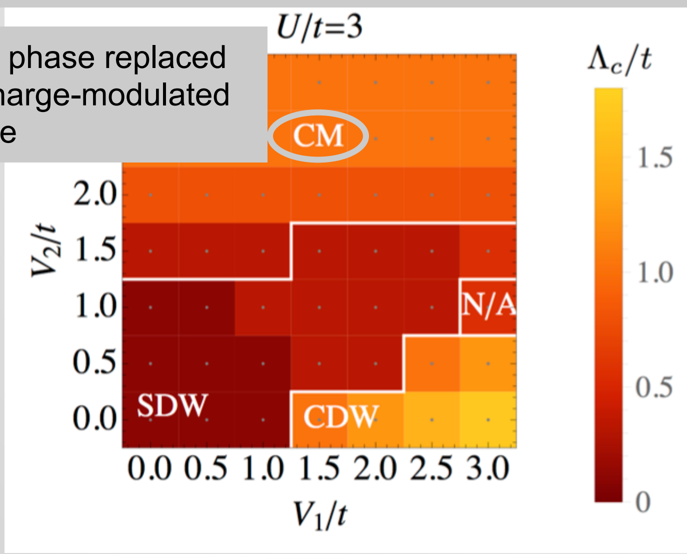




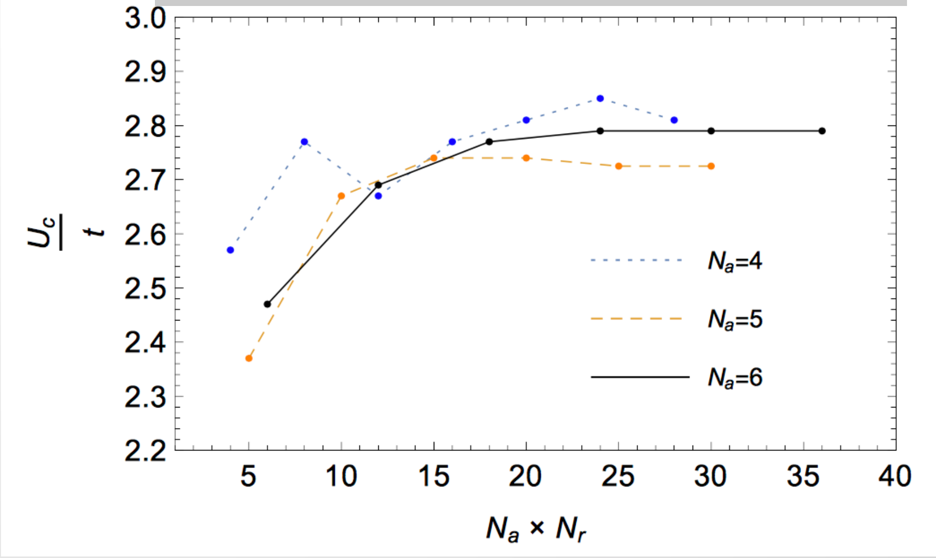
Daniel D. Scherer, Michael M. Scherer, C. Honerkamp, Phys. Rev. B **92** (2015)

Yanick Volpez, Daniel D. Scherer, and Michael M. Scherer
Phys. Rev. B **94**, 165107 (2016)

QSH phase replaced by charge-modulated phase



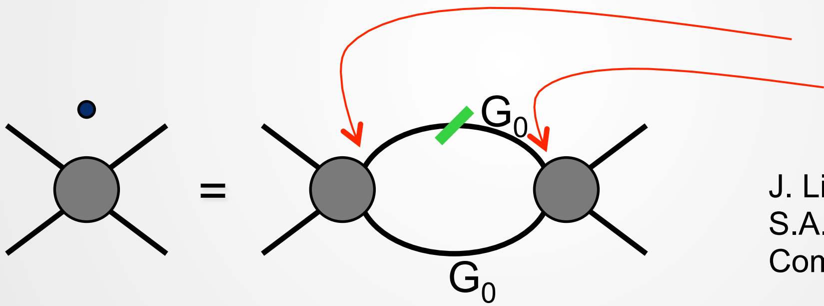
Convergence requires several patch rings



2. Truncated unity fRG in momentum space

- Builds on **channel decomposition** à la Salmhofer et al. (Husemann, Salmhofer, PRB 2009)
- Incorporates numerical advantages of **singular-mode (SM-)fRG**, Q.H. Wang et al. PRB 21012
- Idea: insert resolutions of unity in momentum space factor basis into one-loop RG eqns

$$\delta_{kk'} = \frac{1}{N} \sum_x e^{i(k-k')x}$$



J. Lichtenstein, D. Sanchez dlP, D. Rohe, CH, S.A. Maier
Computer Physics Communications 2017

- **Truncation of basis provides physically transparent approximation & high momentum resolution**
- **Parallelizes nicely** on high-performance architectures (= headroom for attacking frequency-dependence, selfenergies, $\cdots$)

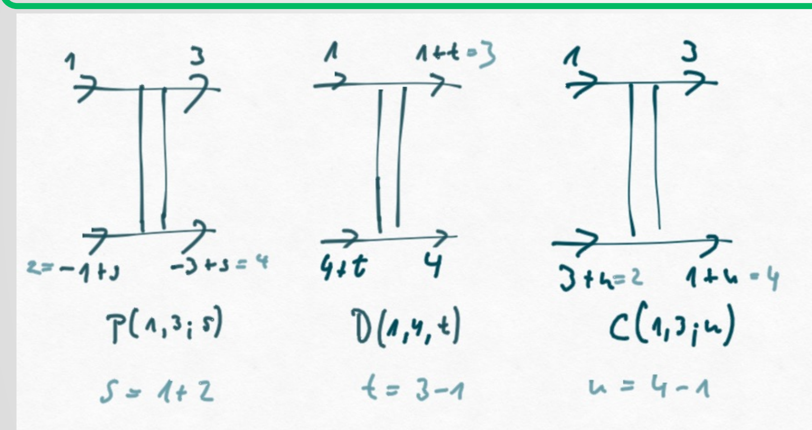
Channel decomposition

Husemann, Salmhofer, Giering, Eberlein & Metzner , Maier &CH ... Karrasch et al.

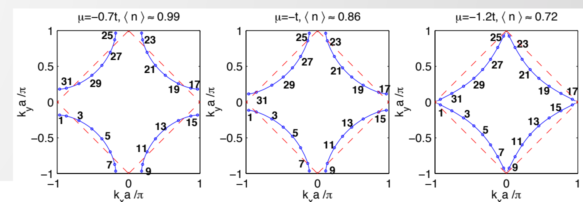
Instead of one function of three variables, use three functions *P,D, C* of
one 'strong/bosonic' variable

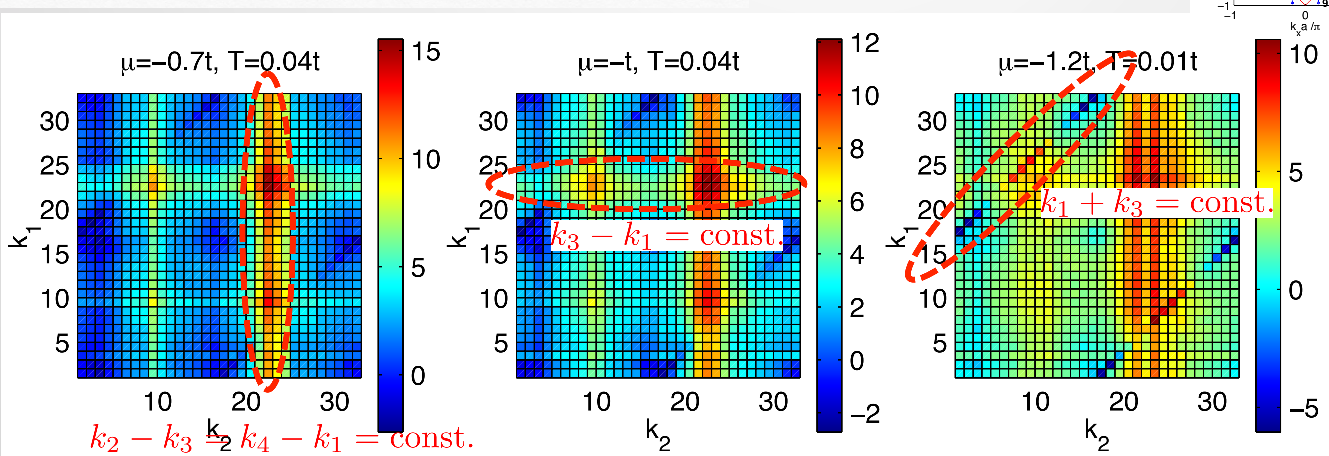
$$s = k_1 + k_2 \,, \quad t = k_3 - k_1 \,, u = k_4 - k_1$$

$$\boxed{V_\Lambda(k_1,k_2,k_3) = V_0(k_1,k_2,k_3) + P_\Lambda(k_1,k_3;s) + D_\Lambda(k_1,k_4;t) + C_\Lambda(k_1,k_3;u)}$$



'weak/fermionic' variables, smooth dependence





Data for $V_\Lambda(k_1,k_2,k_3)$ from 2D Hubbard model, CH (2000)

Channel decomposition

Husemann, Salmhofer, Giering, Eberlein & Metzner , Maier & CH ... Karrasch et al.

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$$s = k_1 + k_2 \,, \quad t = k_3 - k_1 \,, u = k_4 - k_1$$

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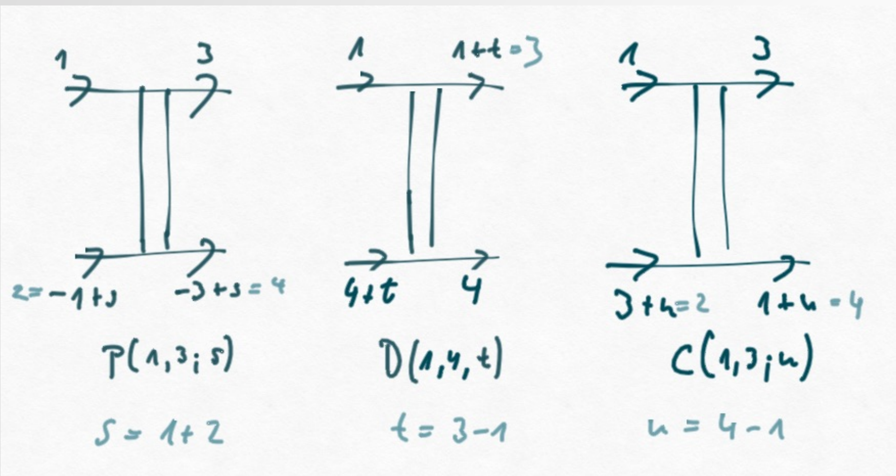
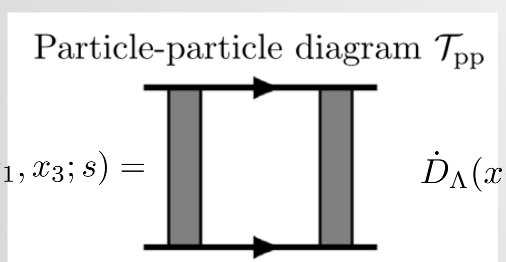


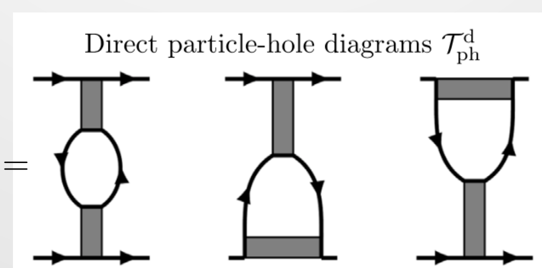
'weak/fermionic variables', captured by smooth form factors $f_x(k)$, form factor expansion:

$$P_\Lambda(k_1,k_3;s) = \sum_{x_1,x_3} f_{x_1}(k_1) f^*_{x_3}(k_3) P_\Lambda(x_1,x_3;s)$$

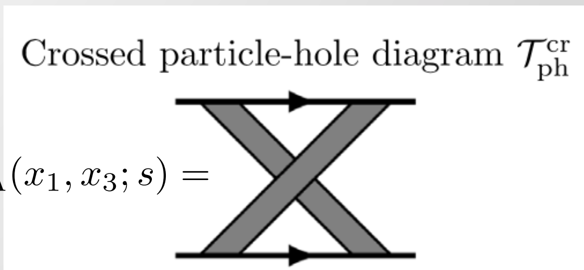
$\dot{P}_\Lambda(x_1,x_3;s) =$



$\dot{D}_\Lambda(x_1,x_3;s) =$



$\dot{C}_\Lambda(x_1,x_3;s) =$



$$\dot{P}_\Lambda(k_1,k_3;s) = \frac{T}{N_L} \sum_k V_\Lambda(k_1,k;s) L_{\mathrm{PP}}(k;s) V_\Lambda(k,k_3;s)$$

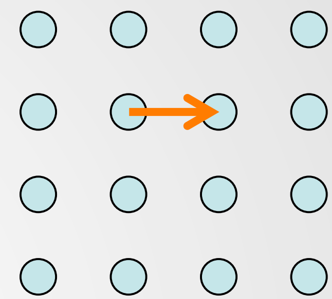
Form factor basis: bonds on real space lattice

$$P_\Lambda(k_1,k_3;s) = \sum_{x_1,x_3} f_{x_1}(k_1) f^*_{x_3}(k_3) P_\Lambda(x_1,x_3;s)$$

$$x_i = \vec{b}_i$$

Form factors/basis functions $f_n(\boldsymbol{k})$ most easily organized on real space Bravais lattice spanned by bond vectors

$$\vec{b} = b_1\vec{e}_1 + b_2\vec{e}_2$$



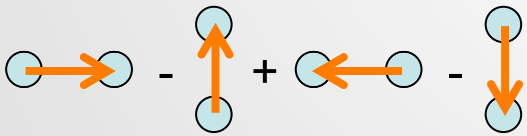
real lattice

$f_{\vec{b}}(\vec{r}) = \delta_{\vec{r},\vec{b}}$ bond functions

Symmetrize wrt IRREPs of point group G

$$f_l(\vec{r}) = \sum_{R\in\mathcal{G}} a_l(R)\delta_{\vec{r},R\vec{b}}$$

e.g. ‘*d*-wave‘



reciprocal lattice

$f_{\vec{b}}(\vec{k}) = e^{i\vec{k}\cdot\vec{b}}$ bond exponentials

$$f_l(\vec{k}) = \sum_{R\in\mathcal{G}} a_l(R) f_{R\vec{b}}(\vec{k})$$

$$f_{d_{x^2-y^2}}(\vec{k}) \propto \cos k_x - \cos k_y$$

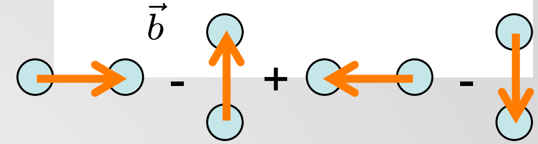
For most cases:
Short bonds ***b*** most important
<=> form factors $f_n(\boldsymbol{k})$ smooth

C. Platt, W. Hanke, R. Thomale, Adv. Phys. 2013

Fermion bilinear interaction

$$f_l(\vec{r}) = \sum_{\vec{b}} a_l(\vec{b})\, \delta_{\vec{r},\vec{b}}$$

e.g.
'*d*-wave'



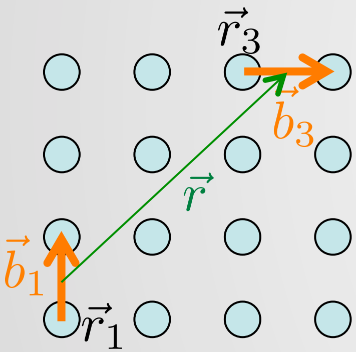
In real space, *P*-interaction becomes pair-pair scattering:

$$H_V = \frac{1}{2} \sum_{\substack{l_1,l_3 \\ s,s'}} \left[\sum_{\vec{b}_3} a^*_{l_3}(\vec{b}_3) c^\dagger_{\vec{r}_3,s} c^\dagger_{\vec{r}_3+\vec{b}_3,s'} \right] V^P_{l_1,l_3} \left(\underbrace{\vec{r}_1 - \vec{r}_3 + \frac{\vec{b}_1 - \vec{b}_3}{2}}_{\vec{r}} \right) \left[\sum_{\vec{b}_1} a_{l_1}(\vec{b}_1) c_{\vec{r}_1+\vec{b},s'} c_{\vec{r}_1,s} \right]$$

outgoing pair,
short ranged

pair distance, can get long
~ exchange boson

incoming pair,
short ranged



Channel decomposition is way of rewriting full interaction as sum of interactions between all possible/necessary fermion bilinears!

particle-particle-pairs

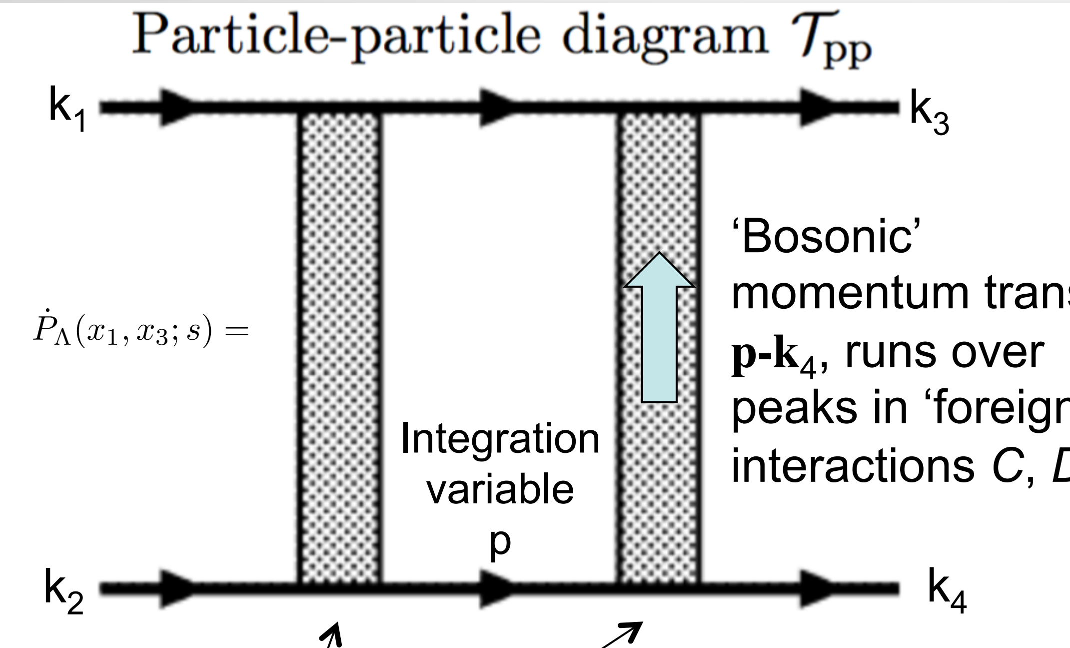
particle-hole-pairs, no spin flip

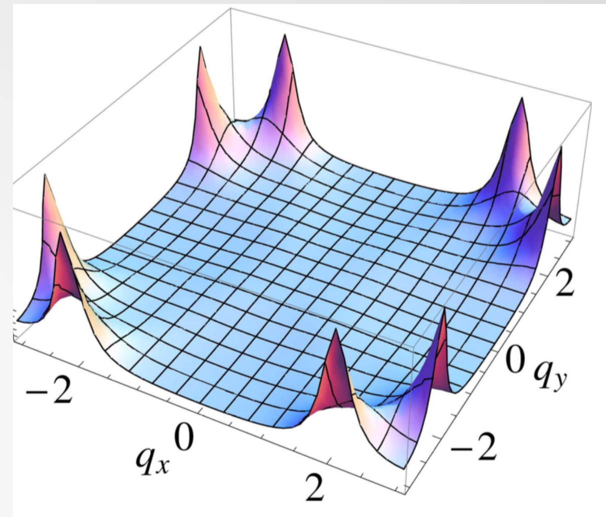
particle-hole-pairs, spin flip

$$V_\Lambda(k_1,k_2,k_3) = V_0(k_1,k_2,k_3) + P_\Lambda(k_1,k_3;s) + D_\Lambda(k_1,k_4;t) + C_\Lambda(k_1,k_3;u)$$

Intuitive representation with meaningful truncations

Numerically still hard





Typical C(k_1,k_3;u) vs. strong momentum *u*

➔ need to integrate over sharp peaks

$$V_\Lambda(k_1, k_2, k_3) = V_0(k_1, k_2, k_3) + P_\Lambda(k_1, k_3; s) + D_\Lambda(k_1, k_4; t) + C_\Lambda(k_1, k_3; u)$$

(other channels similar)

Truncated unity fRG: matrix flow equations

S. Maier

Consider flow of pairing interaction

$$\dot{P}_\Lambda(k_1,k_3;s) = \frac{T}{N_L}\sum_k V_\Lambda(k_1,k;s)L_{\mathrm{PP}}(k;s)V_\Lambda(k,k_3;s)$$

Truncate sum, only take relevant form factors, bonds |x|<*R*

$$\delta_{kk'} = \frac{1}{N}\sum_x e^{i(k-k')x}$$

Insert (truncated) unities:

$$V_\Lambda(k_1,k;s) = \sum_{k'}\delta_{k,k'}V_\Lambda(k_1,k';s) = \frac{1}{N}\sum_{x'} e^{ikx'}\sum_{k'} e^{-ik'x'}V_\Lambda(k_1,k';s)$$

$$\dot{P}_\Lambda(k_1,k_3;s) = \sum_{x',x''}\left[\frac{1}{N}\sum_{k'}V_\Lambda(k_1,k';s)e^{-ik'x'}\right]\cdot\left[\frac{T}{N_L}\sum_k e^{ik(x'-x'')}L_{\mathrm{PP}}(k;s)\right]\cdot\left[\frac{1}{N}\sum_{k''}e^{ik''x''}V_\Lambda(k'',k_3;s)\right]$$

Project both sides on form factor basis: $\dot{P}_\Lambda(x_1,x_3;s) = \frac{1}{N}\sum_{k_1,k_3} e^{ik_1x_1}\dot{P}_\Lambda(k_1,k_3;s)e^{-ik_3x_3}$

$$\Longrightarrow \boxed{\dot{P}_\Lambda(x_1,x_3;s) = \frac{1}{N}\sum_{x',x''}P[V]_\Lambda(x_1,x';s)L_{\mathrm{PP}}(x',x'';s)P[V]_\Lambda(x'',x_3;s)}$$

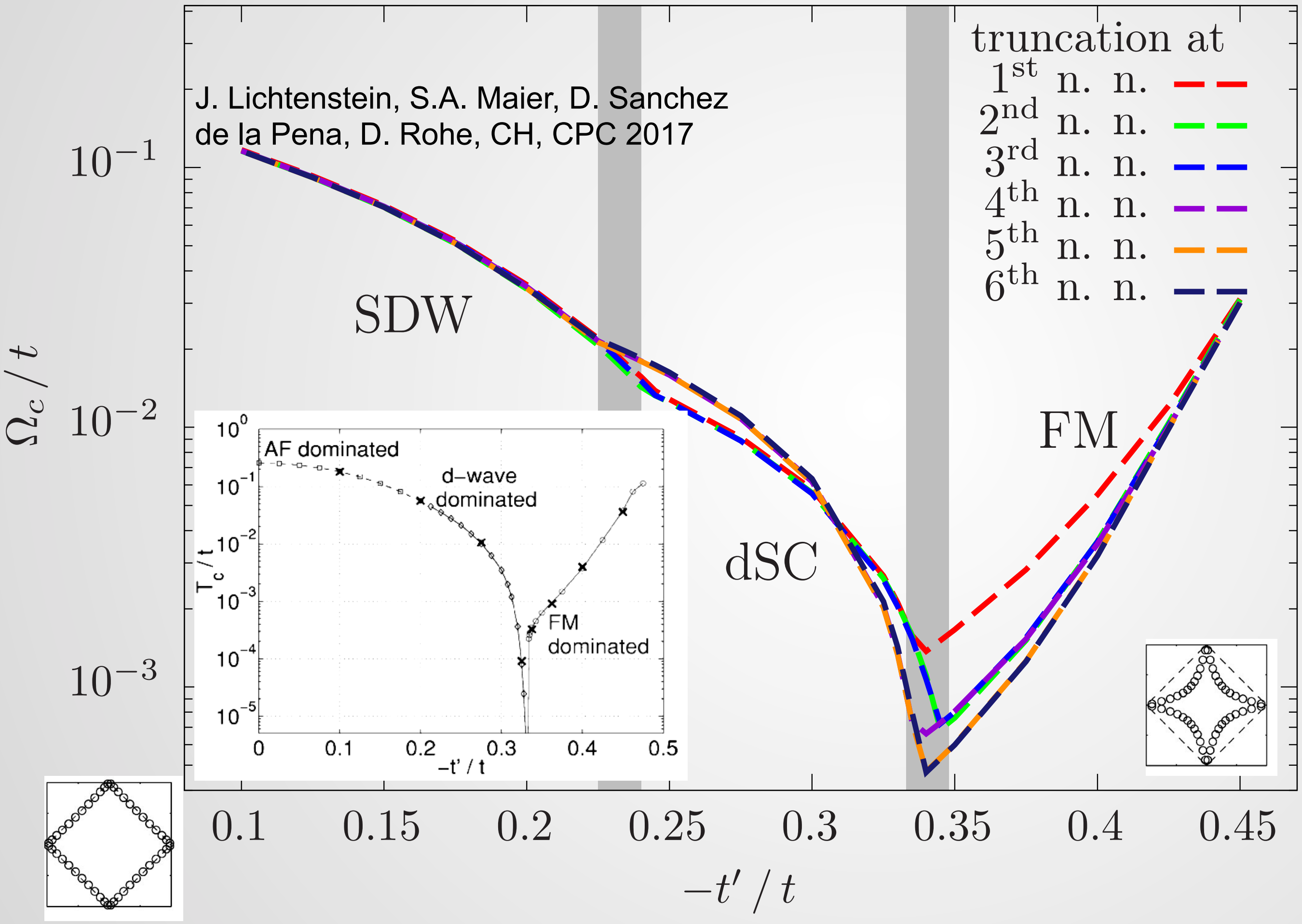
Flow eqns become matrix products of projected couplings and loops (no slow integrals)

$$L_{\mathrm{PP}}(x',x'';s) = \frac{T}{N_L}\sum_k e^{ik(x'-x'')}L_{\mathrm{PP}}(k;s)$$

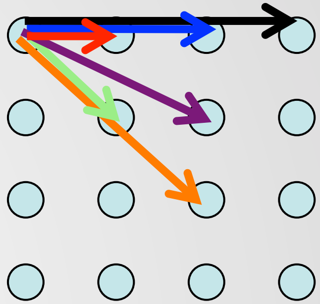
How does well it work?

J. Lichtenstein, D. Sanchez dlP, D. Rohe, CH, S.A. Maier
Computer Physics Communications 2017

Phases and convergence in *t-t'* Hubbard model



Bonds kept for form factors



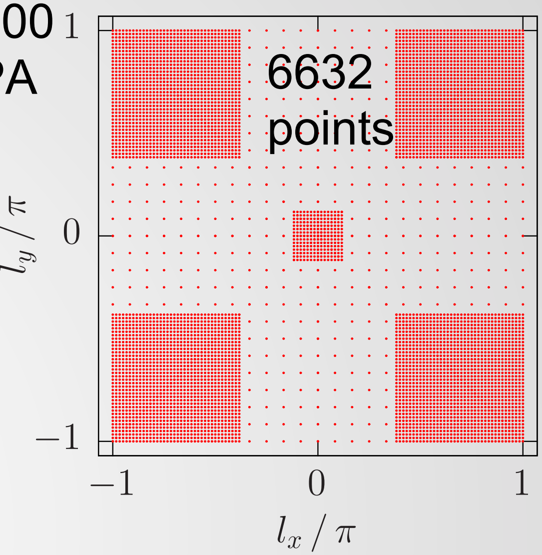
Bosonic momentum discretized with 1000 (D) or 6600 (C,P) points

Look at momentum-resolved response function (beyond RPA)

Include long-range Coulomb interactions

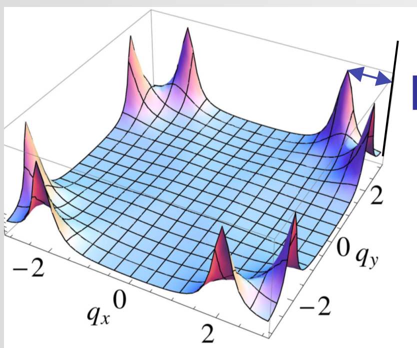
Response functions beyond RPA

High momentum resolution for ‘bosonic‘ variables (~6000 points) permits study of response functions beyond RPA

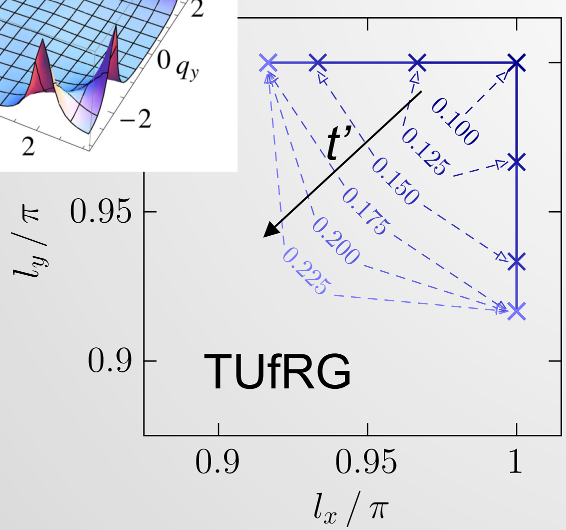


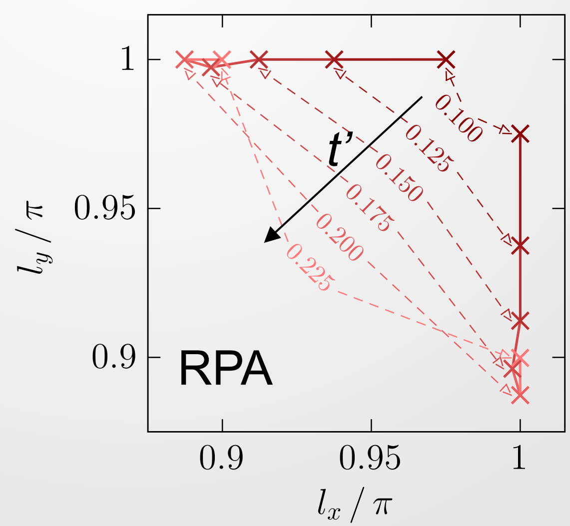
Peaks of C-channel (=spin channel) at van Hove filling for different *t’*:

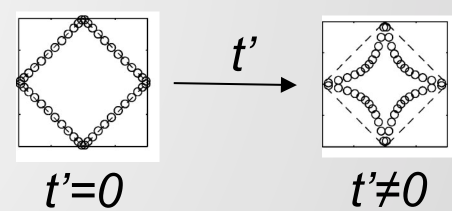
TUfRG has weaker **peak splitting** than random phase approximation (RPA)



peak splitting away from (π,π)

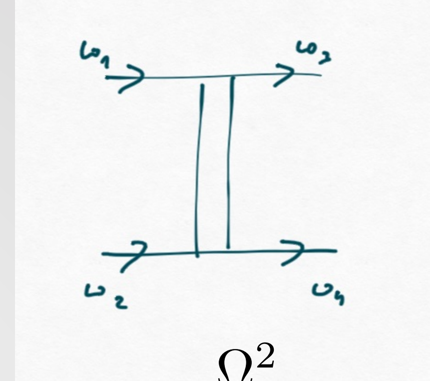






J. Lichtenstein, S.A. Maier, D. Sanchez de la Pena, D. Rohe, CH, CPC 2017

3. TUfRG for frequencies: Why?



- Width Ω of interactions on frequency axis matters for critical scales, see e.g. BCS

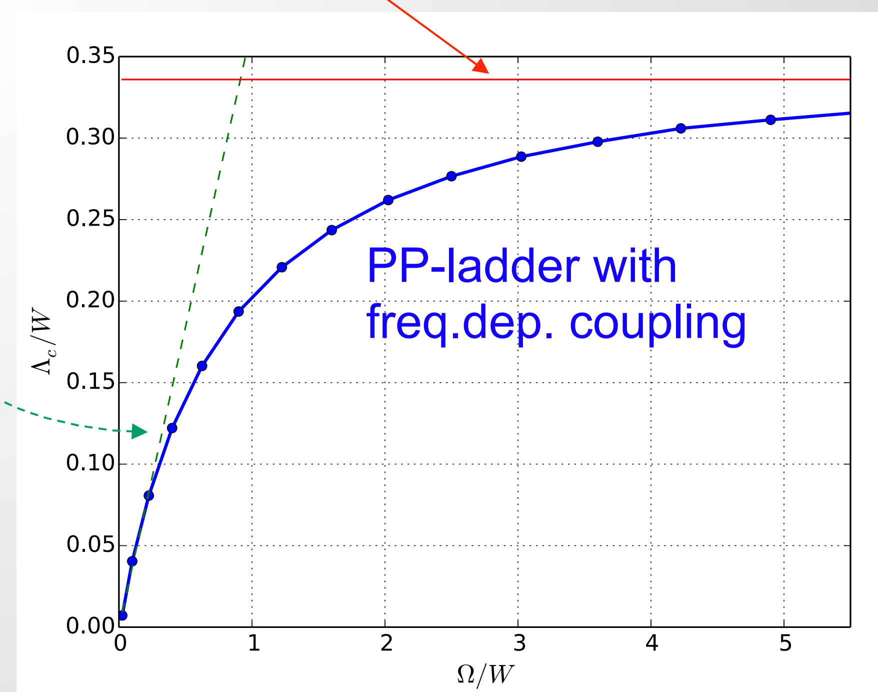
$$V_{\mathrm{ph-med.}}(\omega_1, \omega_3) = V_0 \, \frac{\Omega^2}{(\omega_1 - \omega_3)^2 + \Omega^2}$$

- w/o frequency dependence, as previous fRG (Ω = ∞)

$$T_c^{\infty} = W \, e^{-\frac{1}{\rho_0 |V_0|}}$$

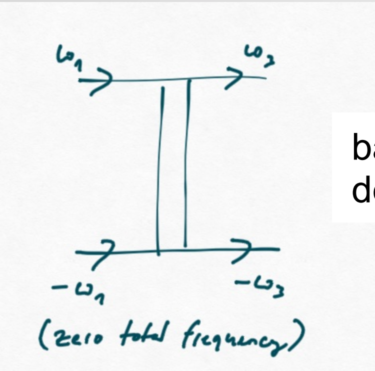
- with frequency dependence

$$T_c^{\Omega} = \Omega \, e^{-\frac{1}{\rho_0 |V_0|}}$$

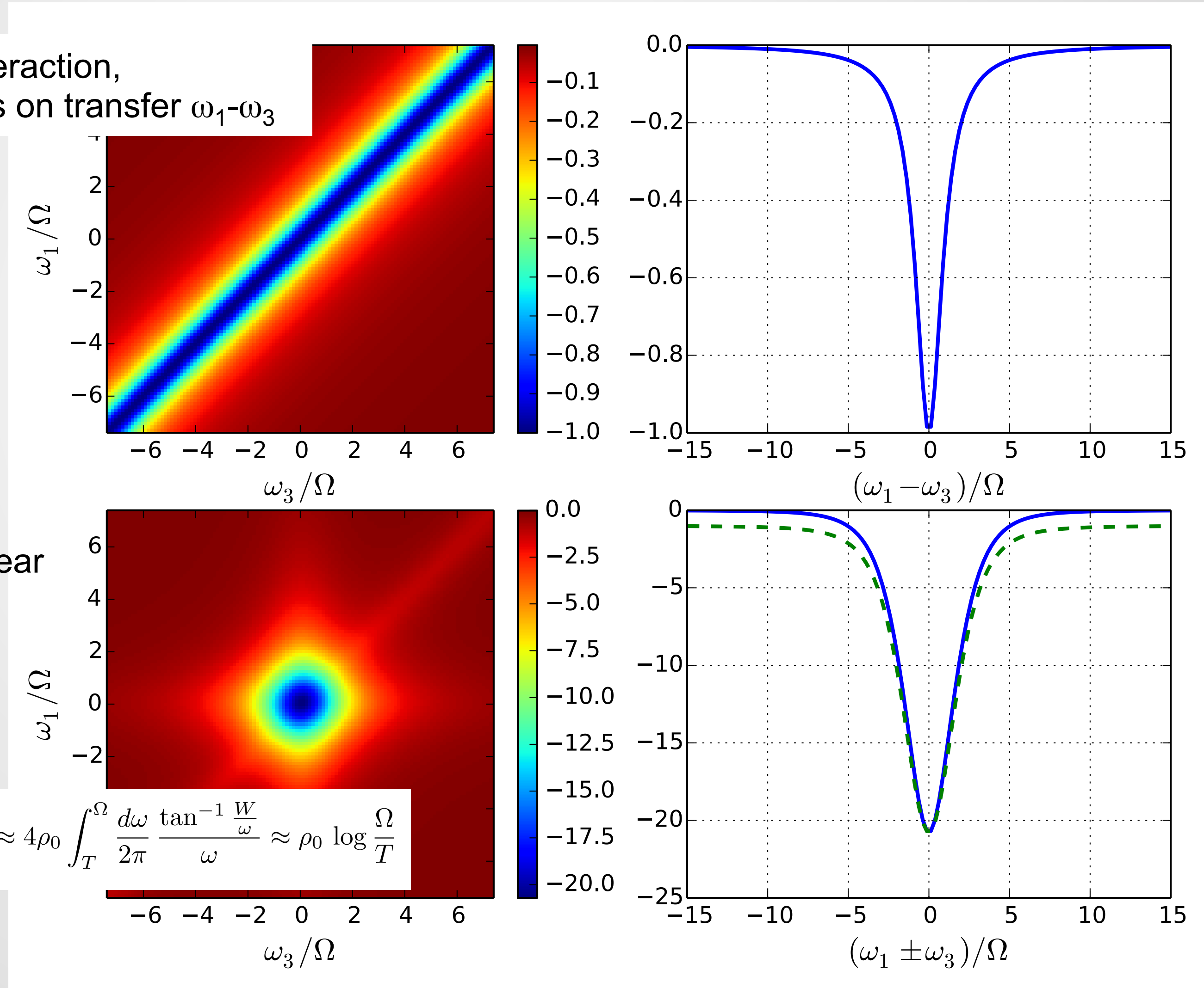


Frequency structure of phonon-mediated interaction

$$V_{\mathrm{ph-med.}}(\omega_1, \omega_3) = V_0 \, \frac{\Omega^2}{(\omega_1 - \omega_3)^2 + \Omega^2}$$



bare interaction,
depends on transfer ω_1-ω_3



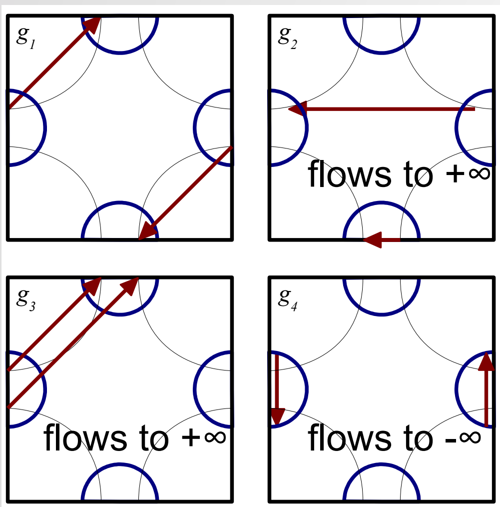
Effective interaction near
Cooper instability,
large for $\omega_1, \omega_3 < \Omega$

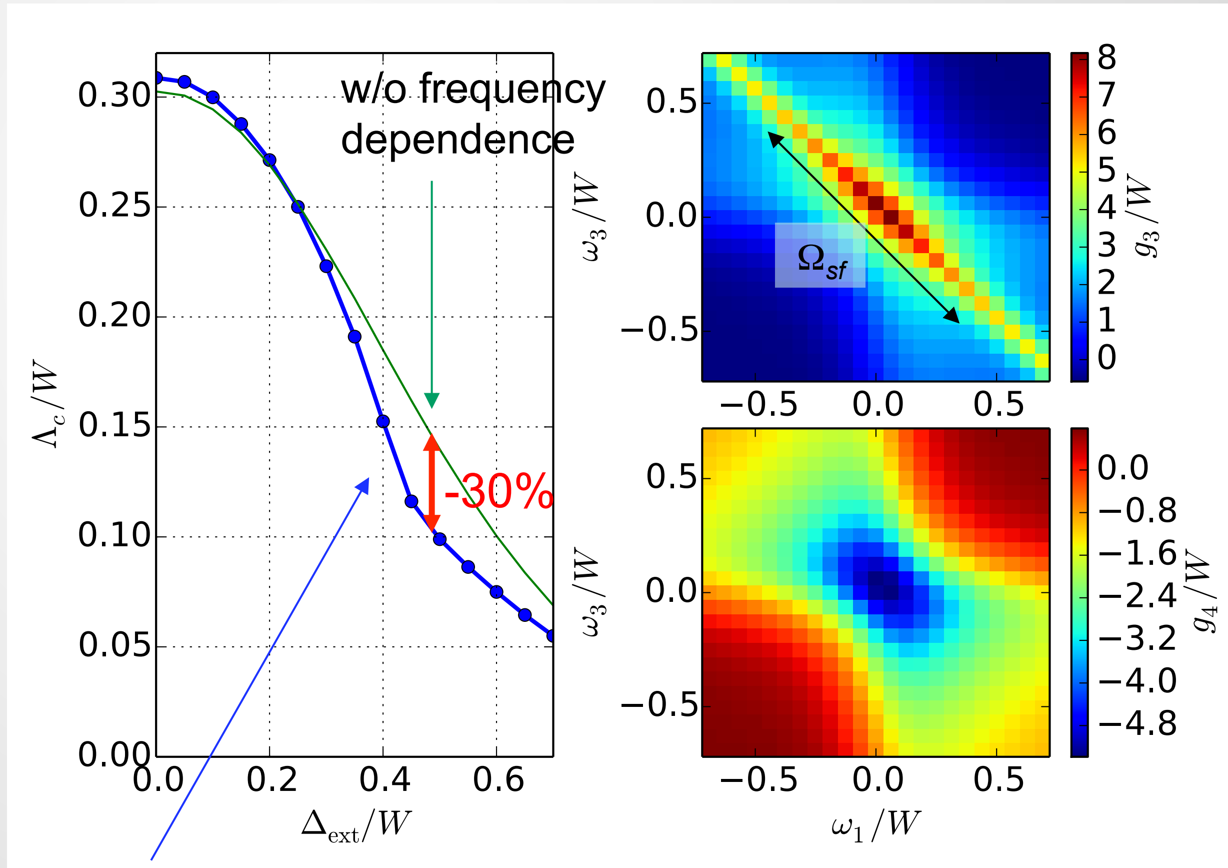
$$L_{\mathrm{PP}}(T) = T \sum_{|\omega_n| < \Omega} \rho_0 \int_{-W}^{W} d\epsilon \, \frac{1}{\omega_n^2 + \epsilon^2} \approx 4\rho_0 \int_T^{\Omega} \frac{d\omega}{2\pi} \, \frac{\tan^{-1} \frac{W}{\omega}}{\omega} \approx \rho_0 \log \frac{\Omega}{T}$$

T. Reckling

Critical scales with frequency dependence, for spin-fluctuation pairing, the hard way

- For spin-fluctuation-mediated pairing: $\Omega = \Omega_{sf}$ (~ mass of spin fluctuations)
- Simple two-patch model (= toy model for spin-fluctuation pairing), interactions depending on three Matsubara frequencies





g_2+g_3 leading: AF instability

g_3-g_4 leading: d-wave pairing instability

Tuning parameter $\Delta_{ext} > 0$ suppresses AF channel

Freq.dep. couplings $g_i(\omega_1, \omega_2, \omega_3)$

for 2D Hubbard: Giering, Salmhofer 2012; Uebelacker, CH 2012

Frequency basis

Label frequency basis function by imaginary time t ∈ [0,β]

$$f_\tau(i\omega_n) = \frac{1}{\sqrt{\beta}}\, e^{i\omega_n \tau} \qquad \tau_l = l\Delta\tau \quad \text{with} \quad l \in 0,\dots N_\tau - 1 \quad \text{and} \quad \Delta\tau = \frac{\beta}{N_\tau}$$

$$\Longrightarrow \quad \text{sampling rate } f_s = 1/\Delta\tau = N_\tau/\beta \qquad \omega_{\max} = 2\pi \cdot \frac{f_s}{2} = N_\tau \pi T$$

Matrix elements phonon propagator

$$D_{\tau,\tau'}(\tau_m) = \frac{1}{\sqrt{N_\tau}} \sum_\nu \delta_{l,0}\delta_{l',0} N_\tau D_0 \,\frac{\Omega^2}{\nu^2+\Omega^2}\, e^{-i\nu\tau_m}$$
$$= \beta\delta_{\tau,0}\delta_{\tau',0}\sqrt{N_\tau}\,\frac{D_0\Omega}{2}\, n_B(\Omega)\left\{e^{\Omega\tau_m} + e^{-\Omega\tau_m} e^{\beta\Omega}\right\}$$

Phonon propagator projected on P channel

$$P_{\tau,\tau'}(s)\left[V^{(D_0,0)}\right] = \delta_{\tau,\tau'}\,\beta\,\frac{D_0\Omega}{2}\, n_B(\Omega)\left\{e^{\Omega\tau_l} + e^{-\Omega(\tau_l-\beta)}\right\}$$

P channel flow equation

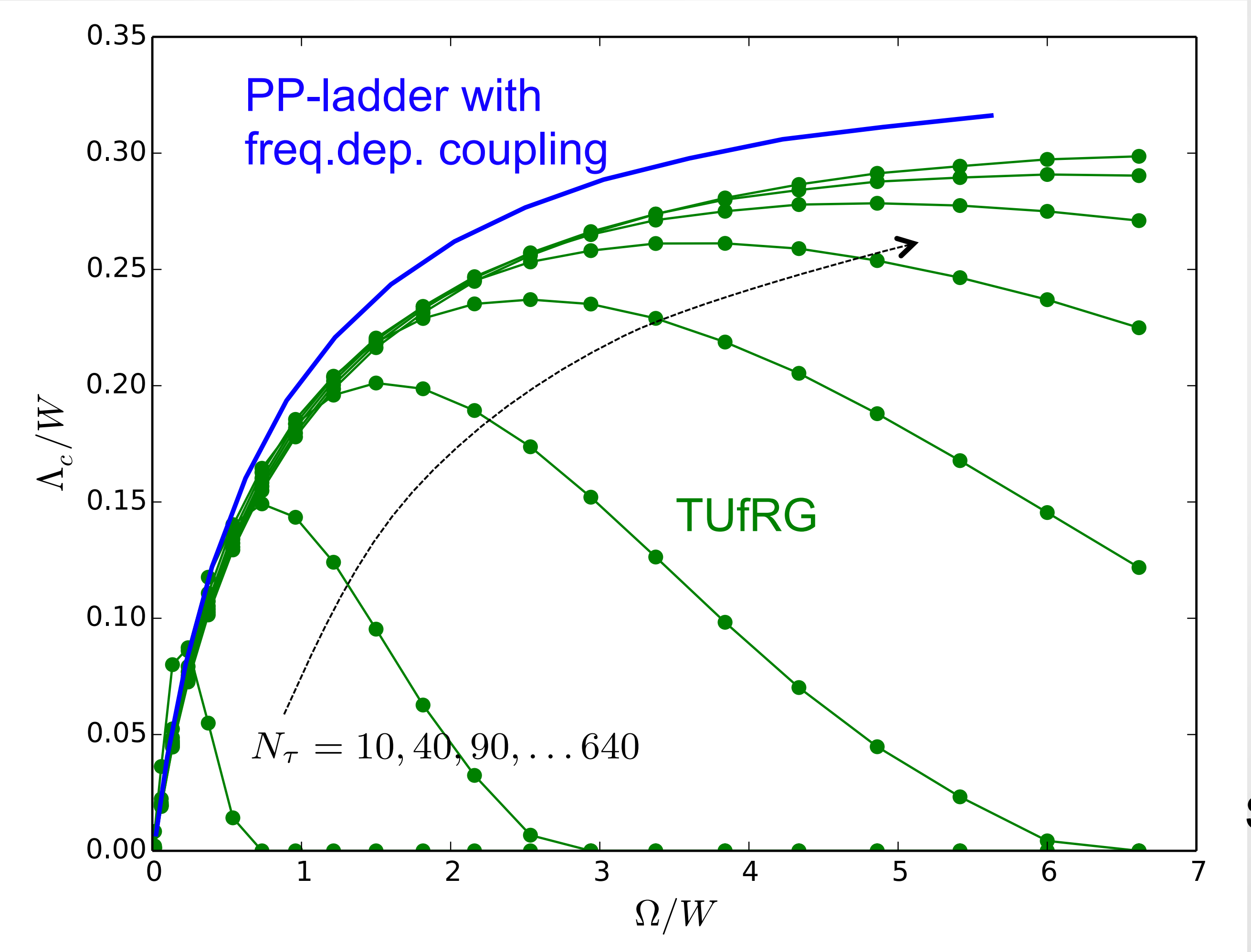
$$\frac{d}{d\Lambda} P_\Lambda(\tau,\tau';s) = N_\tau^{-1} \sum_{\tau'',\tau'''} P_\Lambda(\tau,\tau'';s)\dot{L}^\Lambda_{\mathrm{PP}}(\tau'',\tau''';s)P_\Lambda(\tau''',\tau';s)$$

Projected bubble

$$L^{\epsilon,\epsilon'}_{\mathrm{PP}}(\tau,\tau';s) = T\sum_{\omega_n} \frac{1}{i\omega-\epsilon}\,\frac{1}{-i\omega+is-\epsilon'}\, e^{i\omega_n(\tau_l-\tau')}$$
$$= \frac{1}{-is+\epsilon+\epsilon'}\left\{[1-n_F(\epsilon')]\, e^{is|\tau-\tau'|}e^{-\epsilon'|\tau-\tau'|} - n_F(\epsilon)e^{\epsilon|\tau-\tau'|}\right\}$$

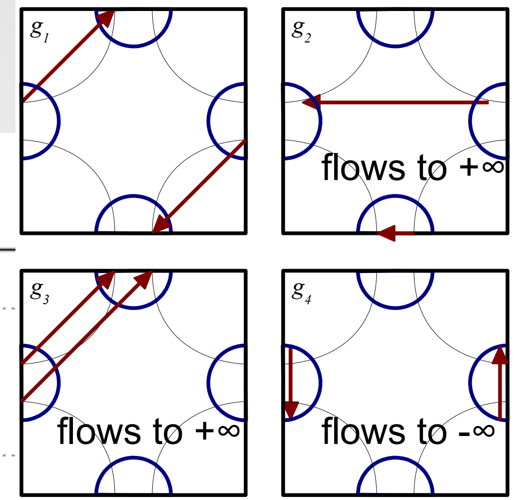
Critical scales in BCS model

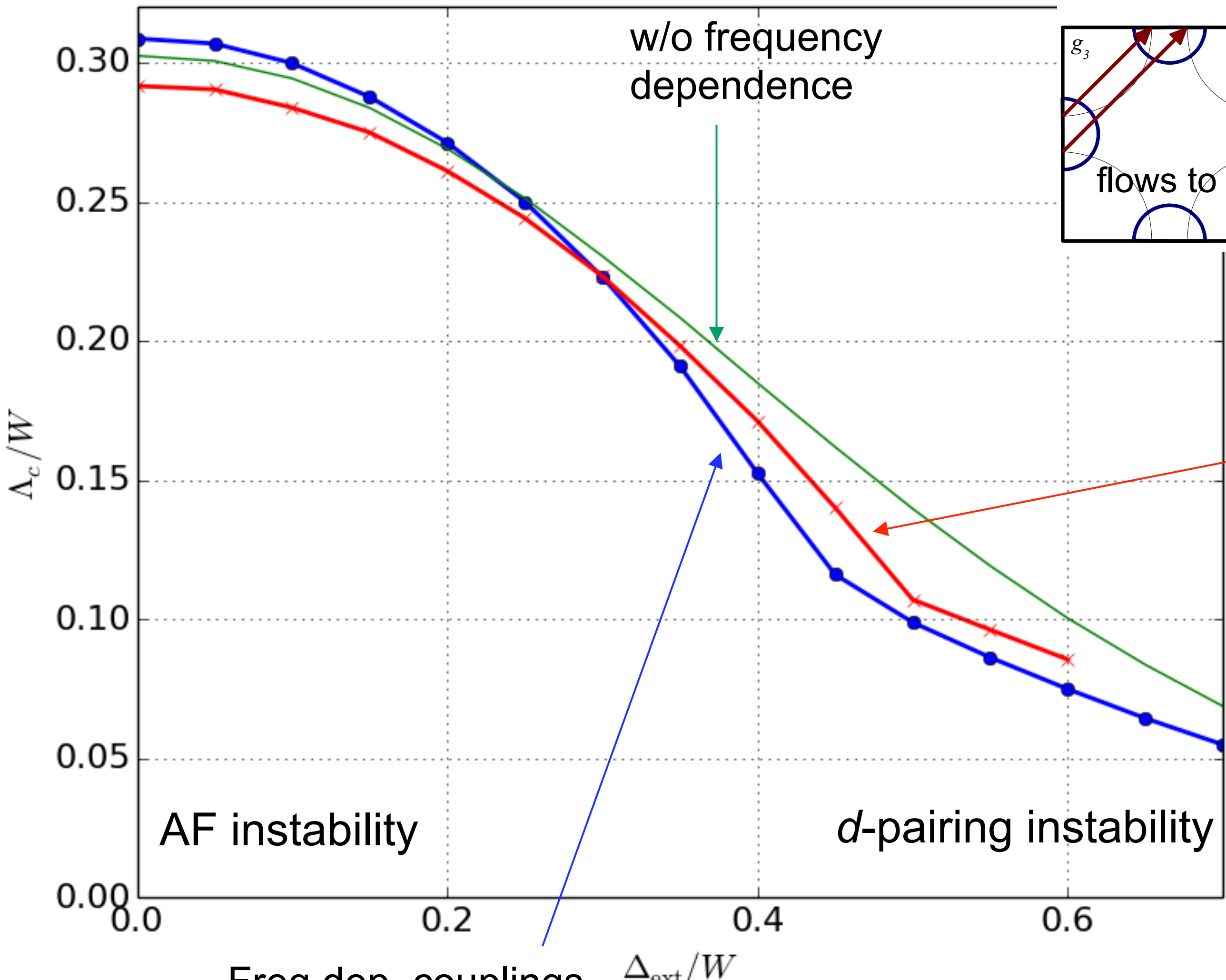
TUfRG with frequency basis: test for phonon-mediated pairing



Seems to work ok!

Test in 2-patch model





(Fast) frequency-TUfRG with N_τ=12

Seems to work ok for first try!

Freq.dep. couplings $g_i(\omega_1, \omega_2, \omega_3)$

Tuning parameter $\Delta_{ext} > 0$ suppresses AF channel

Conclusions

- Functional RG is a versatile tool to explore low-energy physics of interacting fermions in low dimension, for material studies qualitatively useful (see R. Thomale)
- Quantitative precision is currently improved
- Wavevector-TUfRG allows to reach high resolution and convergence wrt to form factor basis
- Frequency-TUfRG should work as well ⋯

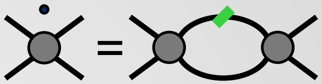


Table I. Maximum T_c in each RFeAs($O_{1-x}F_x$). The F concentration x, which gives the maximum T_c is shown. T_c^{Max} is determined at the onset temperature of superconducting transition in resistivity measurements.

	R							
	La	Ce	Pr	Nd	Sm	Gd	Tb	Dy
T_c^{Max} (K)	28	41	52	52	55	36	46	45
x	0.11	0.16	0.11	0.11	0.1	0.17	0.1	0.1

Ishida, K., Nakaii, Y. & Hosono, H. To what extent iron-pnictide new superconductors have been clarified: A progress report. *J. Phys. Soc. Jpn* **78**, 062001 (2009).

Z.-A. Ren, W. Lu, J. Yang, W. Yi, X.-L. Shen, Z.-C. Li, G.-C. Che, X.-L. Dong, L.-L. Sun, F. Zhou, and Z.-X. Zhao: Chin. Phys. Lett. 25 (2008) 2215.