KPZ dynamics with correlated noise: Emergent symmetries and non-universal observables

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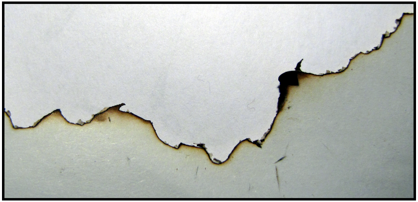
PRE **95**, 032117 (2017), arXiv:1611.02295[cond-mat.stat-mech]

Interface dynamics



The **Kardar–Parisi–Zhang (KPZ)** equation is a model for the dynamics of interfaces with

- Non-Equilibrium scale invariance
- a mathematically exact solution

Motivation

The main objective is to understand the effect of a correlated noise on the dynamics of the Kardar–Parisi–Zhang (KPZ) steady-state.

Spatial correlations can be used to model

- existing microscopic correlations.
- a large scales driving mechanism.

Plan

Introduction

KPZ dynamics

Renormalising KPZ equation

Universal and Non-Universal Observables

Conclusions

Based on...

This exploits the formalism, approximation scheme and numerical code developed in

- L. Canet, arXiv:cond-mat/0509541v4 [cond-mat.stat-mech]
- L. Canet, H. Chaté, B. Delamotte, N. Wschebor, Phys.Rev.Lett.104:150601,2010, arXiv:0905.1025v2 [cond-mat.stat-mech]
- L. Canet, H. Chaté, B. Delamotte, N. Wschebor, Phys. Rev. E 84, 061128 (2011); Phys. Rev. E 86, E019904 (2012), arXiv:1107.2289v3 [cond-mat.stat-mech]
- T. Kloss, L. Canet, N. Wschebor, Phys. Rev. E 86, 051124 (2012), arXiv:1209.4650v2 [cond-mat.stat-mech]
- T. Kloss, L. Canet, B. Delamotte, N. Wschebor, Phys. Rev. E 89, 022108 (2014), arXiv:1312.6028v2 [cond-mat.stat-mech]
- T. Kloss, L. Canet, N. Wschebor, Phys. Rev. E 90, 062133 (2014), arXiv:1409.8314v2 [cond-mat.stat-mech]

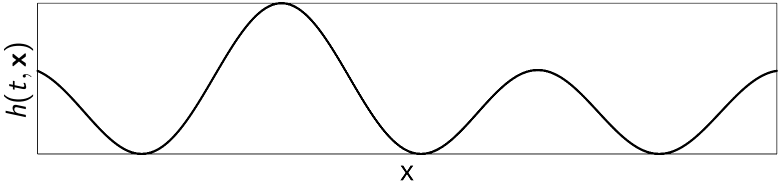
Kardar–Parisi–Zhang (KPZ) equation

A model for interface growth,

$$\partial_t h = \frac{\lambda}{2}\left[\boldsymbol{\nabla} h\right]^2 + \nu \nabla^2 h + \eta$$

with diffusion, perpendicular expansion and stochastic noise,

$$\langle \eta \rangle = 0 \quad \langle \eta(\mathbf{x}, t)\eta(\mathbf{x}', t')\rangle = D\,\delta(t - t')\,\delta(\mathbf{x} - \mathbf{x}')$$



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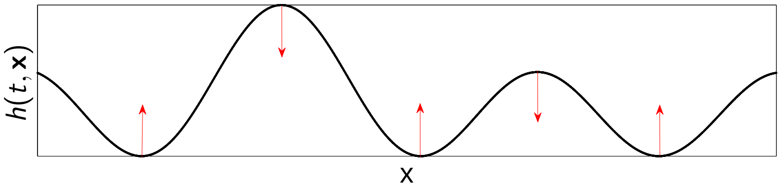

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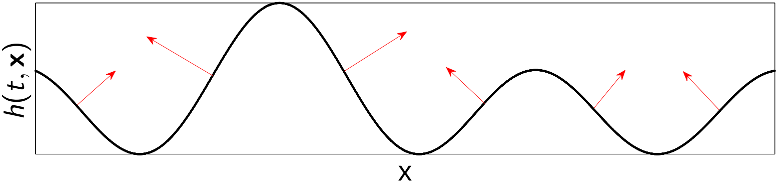

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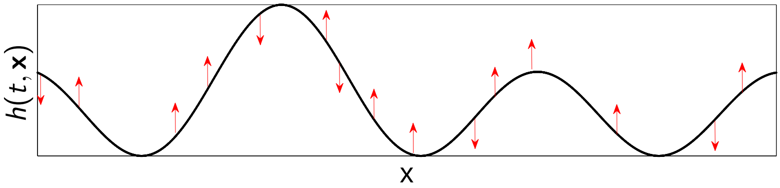


Set-up

$$\partial_t h = \frac{\lambda}{2}\,[\boldsymbol{\nabla} h]^2 + \nu \Delta h + \eta\,,$$

$$\langle \eta \rangle = 0\,, \qquad \langle \eta(t,\mathbf{x})\eta(t',\mathbf{x}')\rangle = 2D\delta(t-t')\,R_\xi(\mathbf{x}-\mathbf{x}')\,.$$

The interface is propagating in a **correlated environment**.

$$R_\xi(\mathbf{r}) = \frac{1}{\left(\sqrt{2\pi}\xi\right)^d}\mathrm{e}^{-\frac{r^2}{2\xi^2}}\,, \qquad R_\xi(\mathbf{p}) = \mathrm{e}^{-\frac{\xi^2 p^2}{2}}\,.$$

Set-up

$$\partial_t h = \frac{1}{2}\,[\nabla h]^2 + \quad \Delta h + \eta\,,$$

$$\langle \eta \rangle = 0\,, \qquad \langle \eta(t,\mathbf{x})\eta(t',\mathbf{x}')\rangle = 2 \quad \delta(t-t')\,R_\xi(\mathbf{x}-\mathbf{x}')\,.$$

The interface is propagating in a **correlated environment**

$$R_\xi(\mathbf{r}) = \quad \frac{1}{\sqrt{2\pi}\;\xi}\,\mathrm{e}^{-\frac{r^2}{2\xi^2}}\,, \qquad\qquad R_\xi(\mathbf{p}) = \mathrm{e}^{-\frac{\xi^2 p^2}{2}}\,.$$

Choose $d = 1$ and pick simpler units: $\qquad \xi \to \xi\,\frac{D\lambda^2}{\nu^3}\,.$

KPZ field theory

Stationary-state observables are generated by

$$Z[J] = \left\langle e^{\int_{t,\mathbf{x}} J(t,\mathbf{x}) h(t,\mathbf{x})} \right\rangle$$

KPZ field theory

Stationary-state observables are generated by

$$Z\left[J\right] = \langle \mathrm{e}^{\int_{t,\mathbf{x}} J(t,\mathbf{x})h(t,\mathbf{x})} \rangle \sim \int DhD\tilde{h}\, \mathrm{e}^{-S[h,\tilde{h}]+\int_{t,\mathbf{x}} J(t,\mathbf{x})h(t,\mathbf{x})}\,,$$

with

$$S = \int_{t,\mathbf{x}} \tilde{h} \left\{ \partial_t h - \frac{1}{2}\left[\boldsymbol{\nabla} h\right]^2 - \nabla^2 h \right\} - \int_{t,\mathbf{x},\mathbf{y}} \tilde{h}(t,\mathbf{x})\tilde{h}(t,\mathbf{y})\, R_\xi(\mathbf{x}-\mathbf{y})\,.$$

Symmetries

$$S = \int_{t,\mathbf{x}} \tilde{h} \left\{ \partial_t h - \frac{1}{2} [\boldsymbol{\nabla} h]^2 - \nabla^2 h \right\} - \int_{t,\mathbf{x},\mathbf{y}} \tilde{h}(t,\mathbf{x}) \tilde{h}(t,\mathbf{y}) \, R_\xi(\mathbf{x}-\mathbf{y}) \,,$$

is symmetric under

$$\left. \begin{array}{l} h'(t,\mathbf{x}) = h(t+\tau, \mathbf{x}+\mathbf{r}) \\ \tilde{h}'(t,\mathbf{x}) = \tilde{h}(t+\tau, \mathbf{x}+\mathbf{r}) \end{array} \right\} \quad \text{space-time translation}$$

$$\left. \begin{array}{l} h'(t,\mathbf{x}) = h(t, R\mathbf{x}) \\ \tilde{h}'(t,\mathbf{x}) = \tilde{h}(t, R\mathbf{x}) \end{array} \right\} \quad \text{spatial rotation}$$

$$\left. \begin{array}{l} h'(t,\mathbf{x}) = h(t,\mathbf{x}) + c \\ \tilde{h}'(t,\mathbf{x}) = \tilde{h}(t,\mathbf{x}) \end{array} \right\} \quad \text{height shift}$$

Symmetries

$$S = \int_{t,\mathbf{x}} \tilde{h} \left\{ \partial_t h - \quad \frac{1}{2} [\boldsymbol{\nabla} h]^2 - \quad \nabla^2 h \right\} - \quad \int_{t,\mathbf{x},\mathbf{y}} \tilde{h}(t,\mathbf{x}) \tilde{h}(t,\mathbf{y}) \, R_\xi(\mathbf{x}-\mathbf{y}) \, ,$$

is symmetric under

$$\left.\begin{aligned} h'(t,\mathbf{x}) &= h(t,\mathbf{x}+ \quad \mathbf{v}t) + \mathbf{v}\cdot\left(\mathbf{x}+\tfrac{t}{2}\mathbf{v}\right) \\ \tilde{h}'(t,\mathbf{x}) &= \tilde{h}(t,\mathbf{x}+ \quad \mathbf{v}t) \end{aligned}\right\} \quad \text{Galilee} \rightarrow \mathbf{u} = \quad \boldsymbol{\nabla} h$$

$$\left.\begin{aligned} h'(t,\mathbf{x}) &= -h(-t,\mathbf{x}) \\ \tilde{h}'(t,\mathbf{x}) &= \tilde{h}(-t,\mathbf{x}) + \quad \nabla^2 h(-t,\mathbf{x}) \end{aligned}\right\} \quad \begin{array}{l} \text{Time Reversal only} \\ \text{for } d=1 \text{ and } \xi = 0 \end{array}$$

Approximation scheme

The action for the stationary state fluctuations is

$$S \; = \int_{t,\mathbf{x}} \left\{ \tilde{h} D_t h - \frac{1}{2} \left[\nabla^2 h \quad \tilde{h} + \tilde{h} \quad \nabla^2 h \right] \right\} - \int_{t,\mathbf{x}, \quad \mathbf{y}} \tilde{h}(t,\mathbf{x}) \tilde{h}(t \;, \mathbf{y}) \, R_\xi(\mathbf{x} - \mathbf{y}) \,,$$

with

$$D_t h = \partial_t h - \frac{1}{2} \left(\boldsymbol{\nabla} h \right)^2 \; .$$

Approximation scheme

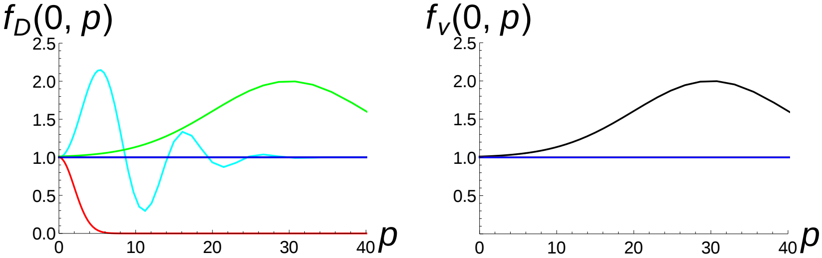
The ansatz for the flowing effective action is

$$\Gamma_k = \int_{t,\mathbf{x}} \left\{ \tilde{h} D_t h - \frac{1}{2} \left[\nabla^2 h f_k^{\nu} \tilde{h} + \tilde{h} f_k^{\nu} \nabla^2 h \right] \right\} - \int_{t,\mathbf{x},t',\mathbf{y}} \tilde{h}(t,\mathbf{x}) \tilde{h}(t',\mathbf{y}) f_k^D \,,$$

with effective noise and dissipation

$$D_t h = \partial_t h - \frac{1}{2} (\boldsymbol{\nabla} h)^2 \,, \quad f_k^X = f_k^X(-\tilde{D}_t^2, -\nabla^2) \,, \quad \tilde{D}_t = \partial_t - \boldsymbol{\nabla} h \cdot \boldsymbol{\nabla} \,.$$

RG flow



Different models correspond to different RG flow initial conditions.

$f^D_\Lambda(\omega, \mathbf{p})$	$f^\nu_\Lambda(\omega, \mathbf{p})$
1	1
$e^{-(p/2)^2/2}$	1
$1 + e^{-[(p-30)/10]^2/2}$	1
1	$1 + e^{-[(p-30)/10]^2/2}$
$1 + \frac{4}{3} e^{-(p/10)^2/2} \sin\left[\pi(\sqrt{(p/5)^2+1} - 1)\right]$	1

Two-Point correlation function

The stationary-state two-point correlation function is

$$G_\xi(\tau, \mathbf{r}) = \langle h(t+\tau, \mathbf{x}+\mathbf{r}) h(t, \mathbf{x}) \rangle - \langle h(t+\tau, \mathbf{x}+\mathbf{r}) \rangle \langle h(t, \mathbf{x}) \rangle \,.$$

The FRG provides directly its Fourier transform

$$G_\xi(\omega, \mathbf{p}) = \int_{\tau,\mathbf{r}} e^{i(\omega t - \mathbf{p}\cdot\mathbf{r})} G_\xi(\tau, \mathbf{r}) = \lim_{k\to 0} \frac{2 f_k^D(\omega, \mathbf{p})}{\omega^2 + \left[f_k^\nu(\omega, \mathbf{p}) p^2 \right]^2} \,.$$

Infrared (IR) data collapse

Large scale physics is described by the usual KPZ fixed point. Then

$$G_\xi(\omega, \mathbf{p}) = p^{-7/2}\, G_\xi\left(\frac{\omega}{p^{3/2}}\right) \quad \text{for } p \ll 1/\xi \text{ and } \omega \ll (1/\xi)^{3/2}\,.$$

$G(x)$ is universal up to normalisation factors

$$G_\xi(x) = \alpha_\xi\, G_0(\beta_\xi x)\,.$$

Kinetic energy spectrum

The interpretation of $\mathbf{u} = \boldsymbol{\nabla} h$ as a velocity field provides an interpretation for,

$$E = \langle \boldsymbol{\nabla} h^2 \rangle \,,$$

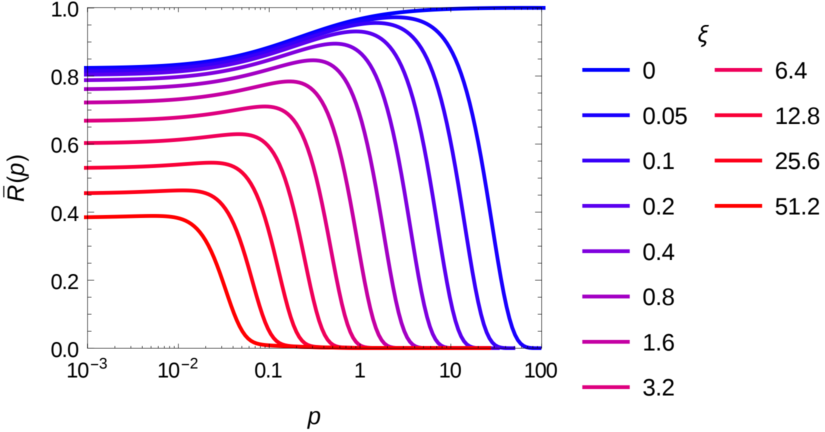
as the **kinetic energy density** which is decomposed as

$$E = \int_p \bar{R}(p) \,.$$

$\bar{R}(p)$ is the **kinetic energy spectrum**,

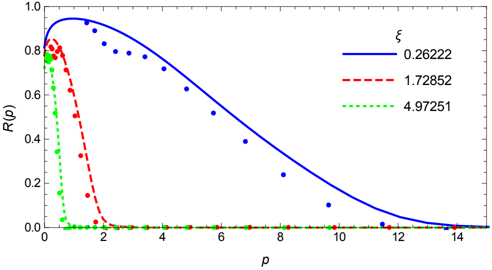
$$\bar{R}(\mathbf{p}) = p^2 \int_\omega G_\xi(\omega, \mathbf{p}) \,.$$

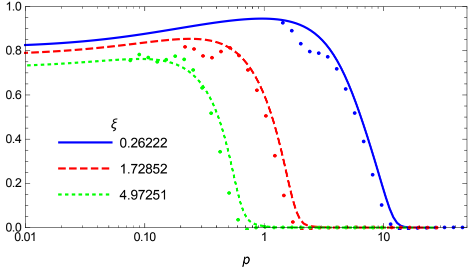
Kinetic energy spectrum



Kinetic energy spectrum

The FRG results can be compared to direct numerical simulations

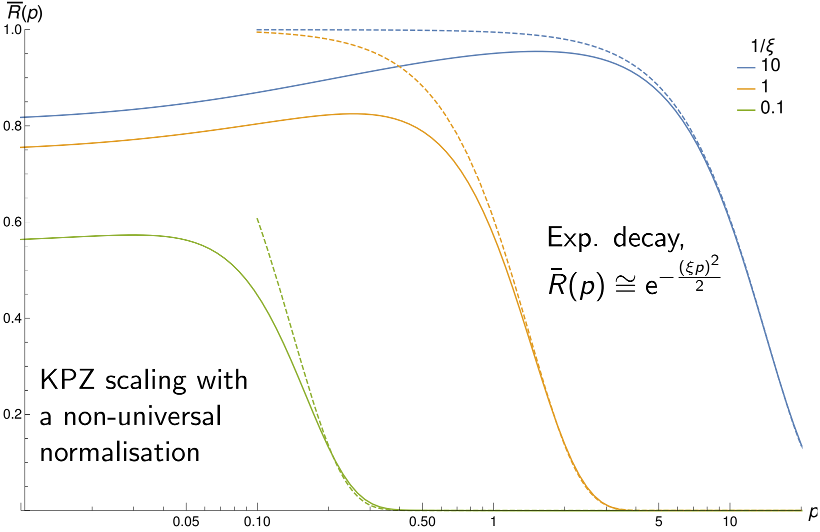




The numerics contain a correlation-time: Galilee symmetry is emergent.

E. Agoritsas, V. Lecomte, and T. Giamarchi, Phys. Rev. E **87**, 062405 (2013), arXiv:1305.2364[cond-mat.dis-nn]

Kinetic energy spectrum



Interface roughness amplitude

The interface roughness,

$$\bar{C}(\mathbf{r}) = \langle [h(t,\mathbf{x}+\mathbf{r}) - h(t,\mathbf{x}) - \langle h(t,\mathbf{x}+\mathbf{r})\rangle + \langle h(t,\mathbf{x})\rangle]^2 \rangle$$

measures the spatial fluctuations of h. It behaves as

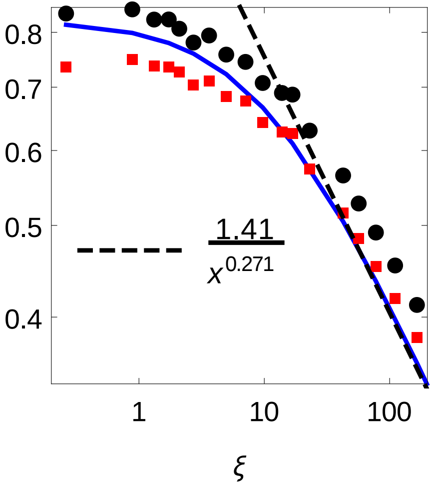
$$\bar{C}(\mathbf{r}) = \tilde{D}(\xi)\,|r|_\xi \ ,$$

with $\tilde{D}(\xi) = \bar{R}(p \to 0)$.

$\tilde{D}(\xi)$ is

- a large-scale signature of the microscopic correlations.
- experimentally observable.

Interface roughness amplitude



E. Agoritsas, V. Lecomte, and T. Giamarchi, Phys. Rev. E **87**, 062405 (2013), arXiv:1305.2364[cond-mat.dis-nn]

Conclusions

- The TR symmetry is **emergent** at large spatial scales.
- Up to normalisation factors, the IR physics can be extracted from the known $\xi = 0$ case.
- $\tilde{D}(\xi)$ contains an experimentally accessible large-scale signature of the finite correlation length.
- An interesting extensions is to consider $\xi \gg 1$ and study **Burgers turbulence**.

Rescaled variables

When Γ_k is expressed in terms of variables that are rescaled with k,

$$\hat{f}_k^D(\hat{\omega}, \hat{\mathbf{p}}) = \frac{f_k^D(\omega, \mathbf{p})}{D_k}\,, \qquad \hat{f}_k^\nu(\hat{\omega}, \hat{\mathbf{p}}) = \frac{f_k^\nu(\omega, \mathbf{p})}{\nu_k}\,,$$

$$\hat{\mathbf{p}} = \frac{p}{k}\,, \qquad \hat{\omega} = \frac{\omega}{k^2 \nu_k}\,,$$

$$D_k = f_k^D(0, \mathbf{0})\,, \qquad \nu_k = f_k^\nu(0, \mathbf{0})\,,$$

the different microscopic theories are comparable.

IR data collapse

This can be checked numerically.

- Plot $p^{7/2}\, G_0(x\, p^{3/2}, p)$ against $x = \omega / p^{3/2}$.
-
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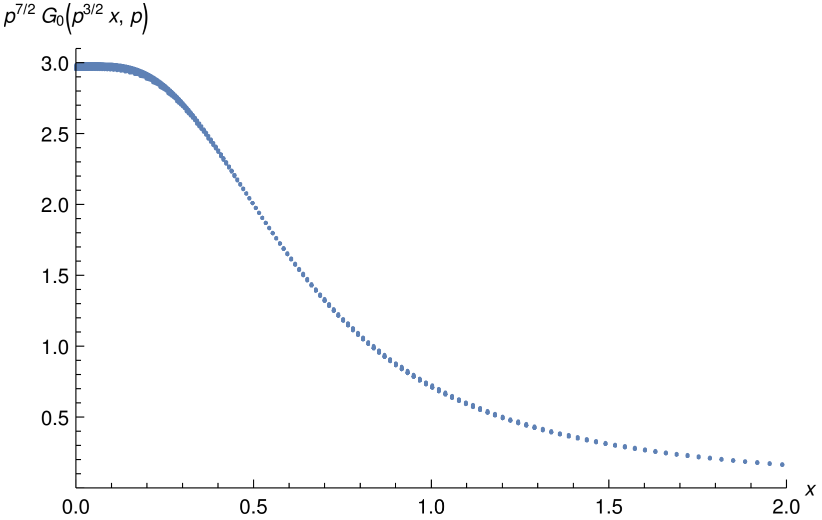
IR data collapse



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- Plot $p^{7/2}\, G_0(x\, p^{3/2}, p)$ against $x = \omega/p^{3/2}$.
- **Extract $\mathbf{G_0}(\mathbf{x})$.**
-
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IR data collapse

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- Plot $p^{7/2}\, G_0(x\, p^{3/2}, p)$ against $x = \omega / p^{3/2}$.
- **Extract $\mathbf{G_0(x)}$**.
- Pick a momentum cut-off K.
-
-
-

IR data collapse

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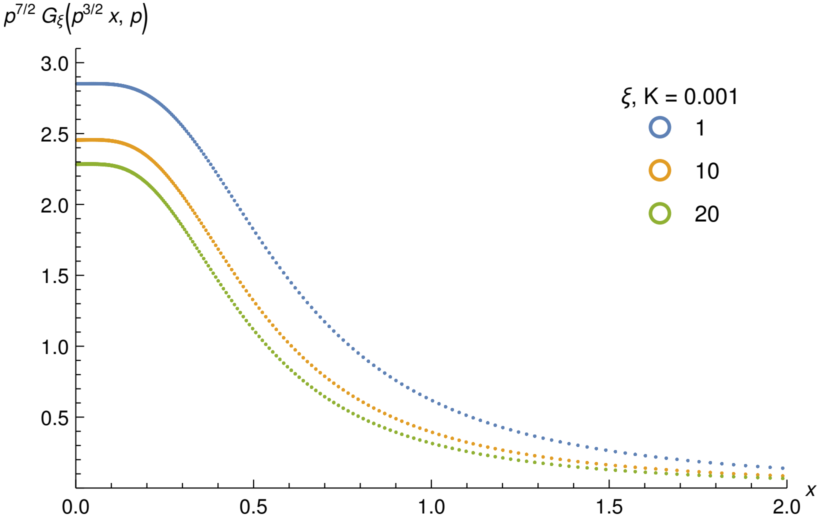
- **Extract $\mathbf{G_0}(\mathbf{x})$.**
- Pick a momentum cut-off K.
- Plot $p^{7/2}\, G_\xi(x\, p^{3/2}, p)$ **for $\mathbf{p} < \mathbf{K}$** $\lesssim 1/\xi$ and $\omega < K^{3/2}$.
-
-

IR data collapse



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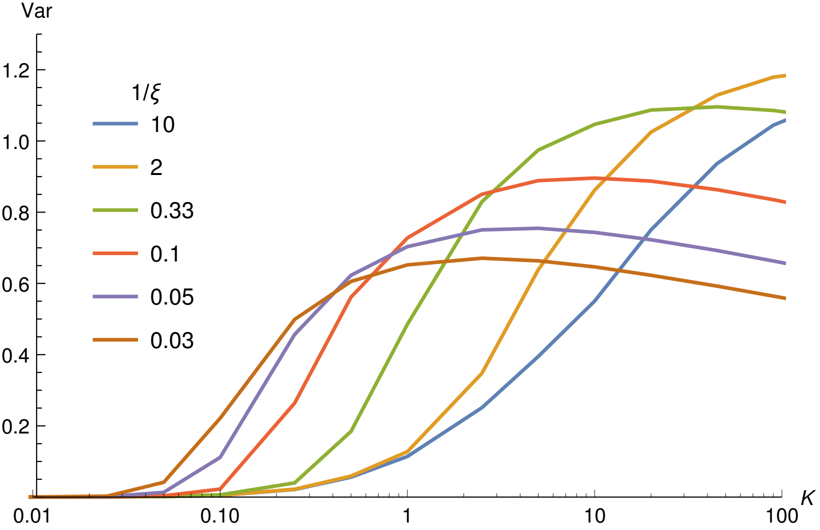
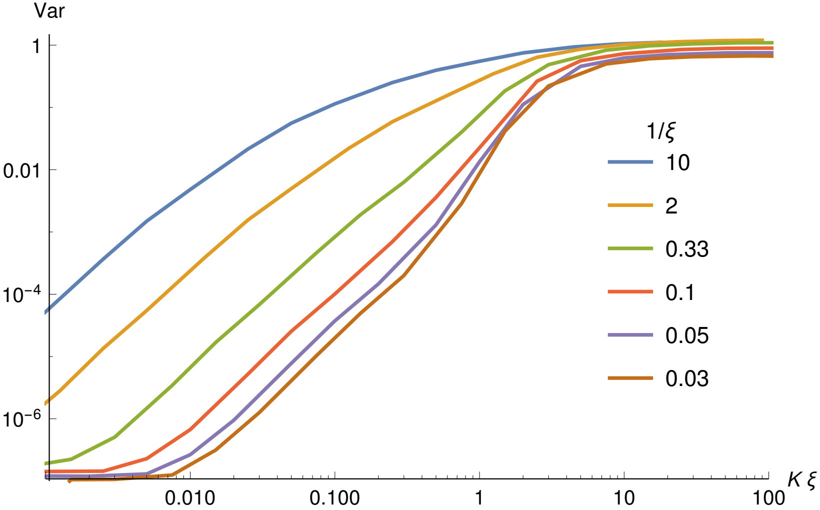
- Plot $p^{7/2}\, G_0(x\, p^{3/2}, p)$ against $x = \omega / p^{3/2}$.
- **Extract $\mathbf{G_0(x)}$.**
- Pick a momentum cut-off K.
- Plot $p^{7/2}\, G_\xi(x\, p^{3/2}, p)$ **for $\mathbf{p < K} \lesssim 1/\xi$** and $\omega < K^{3/2}$.
- **Fit this with $\alpha_\xi\, \mathbf{G_0}(\beta_\xi \mathbf{x})$.**
- The variance of the fit tells if the scaling function is $G_0(x)$.

IR data collapse



Supplementary material



Supplementary material

$$G_\xi(x) = \alpha_\xi \, G_0(\beta_\xi \, x)$$