Homework 2

due November 9$^{\text{nd}}$, 2009, at the beginning of class.

1. Vectors and matrices. Let A and B be the following matrices:

$$A = \begin{pmatrix} 1+i & 1 & -1+i \\ 1 & 1-i & 1+i \\ -1+i & 1+i & -1 \end{pmatrix}, \qquad B = \begin{pmatrix} 2+i & 4i & -2-i \\ 2 & 3+2i & 3+2i \\ -3i & 1 & 5 \end{pmatrix}.$$

a. Calculate A^*B and B^*A (where $(A^*)_{ij} = \overline{A_{ji}}$ and $^-$ denotes the complex conjugation) .

b. Solve the system of equations $Ax = v$ for $v = \begin{pmatrix} -2-2i \\ -1+2i \\ 1-3i \end{pmatrix}$.

2. Wave equation.

a. Verify that the functions

$$\sin(\mathbf{k}\cdot\mathbf{x} - \omega t), \quad \cos(\mathbf{k}\cdot\mathbf{x} - \omega t), \quad e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}$$

are solutions of the wave equation $\left(\frac{1}{v^2}\frac{\partial}{\partial t^2} - \Delta\right) f(t, \mathbf{x}) = 0$. Is there an additional requirement for them to be a solution ?

b. Show that if f and g solve the wave equation, and $\alpha, \beta \in \mathbb{C}$, then $\alpha f + \beta g$ also solves the wave equation.

c. Show that if f and g solve the Schrödinger equation with Hamiltonian $H = -\frac{\hbar^2}{2m}\Delta + V(x)$, and $\alpha, \beta \in \mathbb{C}$, then $\alpha f + \beta g$ also solves the Schrödinger equation.

3. Orders of magnitude. Calculate the de Broglie wavelength of an electron with energy 1000 eV and that of a flower pot (mass 500 g) that falls freely from 20 m above ground, at the moment of impact.