Experimental Tests of Lorentz- and CPT-Symmetry

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Preliminaries

• CPT- and Lorentz invariance are fundamental ingredients in the standard model of particle physics
• open questions remain to be solved (Matter/Antimatter asymmetry, quantum gravity)
• Theory at least allows for and sometimes suggests Lorentz and/or CPT to be violated

CPT and Lorentz violation could be a distinguishable experimental signature for new physics.
The Standard Model Extension

• The Matter Sector: Modified Dirac Equation:

\[
\begin{align*}
&\left( i\gamma^\mu D_\mu - M - a_\mu \gamma^\mu - b_\mu \gamma_5 \gamma^\mu \right. \\
&\left. - \frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu} + ic_{\mu\nu} \gamma^\mu D^\nu + id_{\mu\nu} \gamma_5 \gamma^\mu D^\nu \right) \psi = 0
\end{align*}
\]

44 parameters per particle: \( a_\mu, b_\mu \) 
CPT and Lorentz violating
\( H_{\mu\nu}, c_{\mu\nu}, d_{\mu\nu} \) 
Lorentz violating only

• The photon sector: modified Maxwell equations:

\[
\mathcal{L} = \frac{1}{2} \left[ (1 + \tilde{\kappa}_{tr}) \vec{E}^2 - (1 - \tilde{\kappa}_{tr}) \vec{B}^2 \right] + \frac{1}{2} \vec{E} \cdot (\tilde{\kappa}_{e+} + \tilde{\kappa}_{e-}) \cdot \vec{E}
\]
\[
- \frac{1}{2} \vec{B} \cdot (\tilde{\kappa}_{e+} - \tilde{\kappa}_{e-}) \cdot \vec{B} + \vec{E} \cdot (\tilde{\kappa}_{o+} + \tilde{\kappa}_{o-}) \cdot \vec{B}
\]

19 parameters
CPT-violation shortly: Pennning Trap experiments

Lorentz violation: focus on precision experiments → restriction to QED (electron, proton, neutron, photon sector)
- matter sector: e.g. clock-comparison experiments
- photon sector: The classic experiments of Special relativity:
  - Michelson-Morley
  - Kennedy-Thorndike
  - Ives-Stilwell: Test of time dilation

Summary
CPT-Tests: Penning Traps

CPT requires certain properties of particles and their antiparticles to be the same e.g.: charge-to-mass ratio $q/m$

\[ \nu_c = \frac{qB}{2\pi m} \]

magnetic filed confines particle in a plane perpendicular to $B$. 
add axial confinement by superimposing electric dc quadrupole field: Penning Trap

\[ \nu_c^2 = \nu_c'^2 + \nu_z^2 + \nu_m^2 \]

\[ \nu_c = \frac{qB}{2\pi m} \]

measure \( \nu_c \) for particle and antiparticle at the same \( B \):

extract: \( r = \frac{\bar{q}/\bar{m}}{q/m} \)
CPT-Violation: Penning Traps

Gabrielse et al, PRL 82, 3198 (1999)

LCR circuits at frequencies $\nu_c$, $\nu_m$, and $\nu_a$ for cooling and accurate measurement.

$$r = \frac{\bar{q}/\bar{m}}{q/m} = -0.99999999991(9)$$

most precise test of CPT in baryonic system
frequent scheme: observe transition energies as the lab rotates
→ Lorentz violation would induce **sidereal variations**
The Hydrogen Maser Clock Comparison

Zeeman frequency is sensitive to Lorentz violation:

$$|\Delta \nu_z| = |\vec{b}^e_z + \vec{b}^p_z|/h$$

$$|\vec{b}^p + \vec{b}^e| \leq 2 \times 10^{-18} \text{ eV}$$

known to $5 \times 10^{-21} \text{ eV}$ from spin torsion pendulum [Heckel et al., PRL 97, 021603]
Neutron Sector

Dual Species Maser Clock Comparison

• combined $^3$He and $^{129}$Xe magnetometers
• no net electron spin
• nuclear spin from Neutron in both cases
• Maser frequency $\omega_{He} = \gamma_{He} B_0 + \omega_{LV}$
• Difficult to keep $B_0$ stable
• fix $B_0$ by locking it to Xe Maser frequency $\omega_{Xe} = \gamma_{Xe} B_0 + \omega_{LV}$

$$\eta = \frac{\gamma_{He}}{\gamma_{Xe}} = 2.75$$

$$\omega_{He} = \eta \omega_{Xe} + (1 - \eta) \omega_{LV}$$

look for sidereal variations due to 2nd term

$$\tilde{b}^n = 6.4 \pm 5.4 \times 10^{-23} \text{ eV}$$

D. Bear et al. PRL 85, 5038 (2000)
Photon Sector: Experiments with Light

This is the domain of the classic experiments of Special Relativity:

• Michelson-Moreley: Isotropy of light speed
• Kennedy-Thorndike: Velocity Independence of light speed
• Ives-Stilwell: Time Dilation

**kinematic test theory:** modify Lorentz transformation
   three parameters, intuitive but inconsistent

**dynamic test theory:** modify Maxwell equations
   19 parameters, less intuitive but consistent
A Kinematic Test Theory of Special Relativity
Robertson (1949), Mansouri & Sexl (1977)

Assumptions:

- preferred ("ether") system $\Sigma(T, \vec{X})$
- speed of light $c_0$ isotropic in $\Sigma$
- lab. system $S(t, \vec{x})$ moving at velocity $\vec{w}c_0$ w.r.t. $\Sigma$ (along $X$)
- assume linear transformation

Generalized Lorentz Transformation:

$$T = \frac{1}{a} \left( t + \frac{wx}{c_0} \right)$$
$$X = \frac{x}{b} + \frac{wc_0}{a} \left( t + \frac{wx}{c_0} \right)$$
$$Y = \frac{y}{d}, \quad Z = \frac{z}{d}$$

SR: $a = 1/b = \sqrt{1 - w^2}, d = 1$

Low-velocity limit:

$$a(w^2) = (1 - w^2)^{1/2} \left( 1 + \delta \tilde{\alpha} \cdot w^2 + \ldots \right)$$

Time Dilation

$$b(w^2) = (1 - w^2)^{-1/2} \left( 1 + \delta \tilde{\beta} \cdot w^2 + \ldots \right)$$

Longitudinal Length Contraction

$$d(w^2) = (1 + \bar{\delta} \cdot w^2 + \ldots)$$

Transverse Length Contraction

$$\gamma_{MS} \approx (1 - w)^{-1/2} - \delta \tilde{\alpha}$$
Three Parameters – Three Experiments

Speed of light $c(\theta, w^2)$ in the moving frame $S$ within the Mansouri-Sexl theory ($\theta = \hat{\theta}(c, w)$):

$$\frac{c(\theta, w)}{c_0} = 1 + (\delta \hat{\beta} - \hat{\delta}) w^2 \sin^2(\theta) + (\delta \hat{\alpha} - \delta \hat{\beta}) w^2$$

- Michelson-Morley experiments ! Isotropy of $c$:

- Kennedy-Thorndike experiments ! Velocity-independence of $c$:

- Ives-Stilwell experiment ! time dilation:
Modern Michelson-Morley Experiment

Brillet, Hall 1979, PRL 42, 549 (1979)

\[
\frac{c(\theta,w)}{c_0} = 1 + (\delta\beta - \delta)w^2 \sin^2(\theta) + (\delta\alpha - \delta\beta)w^2
\]

- Laser 1 locked to rotating cavity
- Laser 2 (reference) locked to Iodine
- high finesse cavity enhance effective path length
- Measure \( \Delta \nu = \nu_1 - \nu_2 \) as cavity rotates

\[
\nu_1 = \frac{n c(\theta)}{2L} = (1 + (\delta\beta - \delta)w^2 \sin^2(\theta) + (\delta\alpha - \delta\beta)w^2) \frac{n c_0}{2L}
\]

\( w \approx \text{const}; \sin^2\theta \) dependence of \( \nu_1 \) would indicate anisotropic light speed

\[|\delta\beta - \delta| < 5 \times 10^{-9}\]
Modern Kennedy-Thorndike Experiment
[Hils, Hall 1990, PRL 64, 1697 (1990)]

- similar setup as for Michelson-Morley
- this time without rotation
- assume $|\delta \beta - \delta| = 0$ from MM:

\[
\frac{c(\theta, w)}{c_0} = 1 + (\delta \alpha - \delta \beta) w^2
\]

- $w^2$ changes as earth rotates
  → look for sidereal variations of

\[
\nu_1 = \frac{nc(\theta)}{2L} = (1 + (\delta \alpha - \delta \beta) w^2) \frac{nc_0}{2L}
\]

\[
|\delta \alpha - \delta \beta| < 6 \times 10^{-5}
\]
### Status of MM and KT

| Michelson-Morley                          | limit on $|\delta \beta - \delta |$ |
|------------------------------------------|-------------------------------------|
| Brillet, Hall 1979, PRL 42, 549 (1979)   | 5x10^{-9}                           |
| Müller et al. PRL 91, 020401 (2003)      | 1.5x10^{-9}                         |
| Antonini et al., PRA 71, 050101 (2005)   | 3x10^{-10}                          |
| Stanwix et al., PRD 74, 081101 (2006)    | 8x10^{-11}                          |

| Kennedy-Thorndike                         | limit on $|\delta \beta - \delta \alpha |$ |
|------------------------------------------|-------------------------------------|
| Hils, Hall 1990, PRL 64, 1697 (1990)     | 6x10^{-5}                           |
| Braxmaier et al., PRL 88, 010401 (2002)  | 2x10^{-5}                           |
| Wolf et al., PRL 90, 060402 (2003)       | 7x10^{-7}                           |
Michelson-Morley and Kennedy-Thorndike experiments are sensitive to the tensor components:

\[
\begin{align*}
L &= \frac{1}{2}[(1 + \tilde{\kappa}_{tr}) \tilde{E}^2 - (1 - \tilde{\kappa}_{tr}) \tilde{B}^2] + \frac{1}{2} \tilde{E} \cdot (\tilde{\kappa}_{e+} + \tilde{\kappa}_{e-}) \cdot \tilde{E} \\
&\quad - \frac{1}{2} \tilde{B} \cdot (\tilde{\kappa}_{e+} - \tilde{\kappa}_{e-}) \cdot \tilde{B} + \tilde{E} \cdot (\tilde{\kappa}_{o+} + \tilde{\kappa}_{o-}) \cdot \tilde{B}
\end{align*}
\]

\[
\begin{align*}
(\tilde{\kappa}_{e-})_{xy,xz,yz} &< 10^{-16} \\
(\tilde{\kappa}_{e-})_{zz} &< 10^{-14} \\
(\tilde{\kappa}_{o+})_{xy,xz,yz} &< 10^{-12}
\end{align*}
\]

\[
\tilde{\kappa}_{e+}, \tilde{\kappa}_{o-} < 10^{-32}
\]

from astrophysical constraints on the birefringence of space

However these experiments are insensitive to \( \tilde{\kappa}_{tr} \) → Ives-Stilwell
Generalized time dilation for a clock moving at a velocity \( \bar{w}' \) w.r.t. the ether

\[
\gamma = a^{-1} = \gamma_{SR}(1 + \delta \hat{\alpha} \cdot \bar{w}'^2 + \ldots)
\]

\( \bar{w}' \approx \bar{w} + \bar{\beta} \)

\( \bar{w} = 350 \text{ km/s} \) if \( \Sigma = \text{CMB rest-frame} \)

\[
\gamma_{MS} = \gamma_{SR}[1 + \delta \hat{\alpha}(\beta^2 + 2\bar{\beta}\bar{w}) + \ldots]
\]

we neglect sidereal term
Measuring Time Dilation via the Optical Doppler Effect

The relativistic Doppler Effect:

- Einstein (1907): \( \theta = 90^\circ \) (transverse Doppler effect)

\[
\nu = \nu_0 \frac{1}{\gamma (1 - \beta \cos \theta)}
\]

\[
\gamma = \frac{1}{\sqrt{1 - \beta^2}}
\]

- **Advantage:** independent of first-order Doppler effect
- **Disadvantage:** sensitive to misalignment (cos linear @ \( \pi /2 \))
  - particle velocity has to be measured separately
measure with \textbf{and} against the particle motion $\theta = 0^\circ$ and $\theta = 180^\circ$

\[ \nu_p = \frac{\nu_0}{\gamma(1 - \beta)} \]
\[ \nu_a = \frac{\nu_0}{\gamma(1 + \beta)} \]

\[ \beta = \frac{\nu_p - \nu_a}{\nu_p + \nu_a} \]
\[ \gamma_{\text{exp}} = \frac{(\nu_p + \nu_a)}{2} \]

- **Special Relativity:**
  \[ \gamma \longrightarrow \gamma_{\text{SR}} = (1 - \beta^2)^{-1/2} \]
  \[ \Rightarrow \nu_p \nu_a = \nu_0^2 \]

- **Mansouri-Sexl Test Theory:**
  \[ \gamma \longrightarrow \gamma_{\text{MS}} = \gamma_{\text{SR}} (1 + \delta \alpha \cdot (\beta^2 + 2 \bar{\beta} \bar{w}) + \ldots) \]
  \[ \Rightarrow \nu_p \nu_a = \nu_0^2 (1 + 2 \delta \alpha (\beta^2 + 2 \bar{\beta} \bar{w})) \]
Non-Storage-Ring Ives-Stilwell Experiments

Typically two types of measurements:

- slow particles and high accuracy: use $\vec{\beta} \cdot \vec{w}$ term looking for sidereal variations
- spectroscopy with fast particles is sensitive to $\beta^2$ term

<table>
<thead>
<tr>
<th>experiment</th>
<th>limit on $\delta \alpha$</th>
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<tbody>
<tr>
<td>Ives and Stilwell (1938)</td>
<td>1.0 \times 10^{-2}</td>
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<tr>
<td>relativistic H\textsuperscript+ beam (1986)</td>
<td>1.9 \times 10^{-4}</td>
</tr>
<tr>
<td>Gravity Probe A (1980)</td>
<td>2.1 \times 10^{-6}</td>
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<td>Two-photon transition in neon (1988)</td>
<td>1.4 \times 10^{-6}</td>
</tr>
<tr>
<td>Two-photon transition in neon (1993)</td>
<td>2.3 \times 10^{-6}</td>
</tr>
</tbody>
</table>
The original Ives-Stilwell experiment

Moving source: Hydrogen (H) produced in canal ray tube ($\beta = 0.0005$, $(\gamma -1)<1\times10^{-5}$)
Observer: High resolution grating spectrometer

Observing the $H_\beta$ line of the Balmer series ($n=4 \rightarrow n=2$)

$v = c$ in all frames \( \lambda (\theta) = \lambda_0 \gamma (1 - \beta \cos \theta) \)

\[ \lambda(0^0) = \lambda_0 (1 - \beta + 1/2 \beta^2 + ...) \]
\[ \lambda(180^0) = \lambda_0 (1 + \beta + 1/2 \beta^2 + ...) \]
Modern Ives-Stilwell experiment

Improvements by:

- Faster particles \( \uparrow \) accelerator, ion storage ring
- Higher accuracy \( \uparrow \) laser spectroscopy

2 sources: Frequency-stabilized lasers \( (\Delta \nu / \nu < 10^{-9}) \)

1 moving observer: Fast ion with narrow resonance \( (\Delta \nu / \nu < 10^{-8}) \)

\[
\nu_a = \gamma (1-\beta) \nu_0
\]

Frequency measurement now limited by Doppler broadening
Doppler-free Saturation Spectroscopy

\[ \nu_p = \gamma (1+\beta_0) \nu_0 \]

fixed frequency laser  
(marking ions of velocity \( \beta_0 \))

\[ \nu_a = \gamma (1-\beta) \nu_0 \]

tunable laser

Lamb-dip: \( \nu_p \nu_a = \nu_0^2 \)

Lamb dip: \( \nu_p \nu_a = \nu_0^2 \)
Metastable $^7\text{Li}^+$: A Suitable Candidate

Issues:
- only 10-20% metastables in beam
- decay of metastable fraction: 8-16s in storage ring
- natural linewidth: 4 MHz

Doppler Shift of up to 30 000 GHz
The MPIK Accelerator Facility

$^7\text{Li}^+$

($\sim10\%$ in $^3S_1$)

Tandem

+ 6.65 MV

$^7\text{Li}^-$

Stripper gas

Experimental Area

15 m

electron cooler
The Heidelberg Test Storage Ring (TSR)

- Circumference: 55 m
- Vacuum: $5 \times 10^{-11}$ mbar

$^7\text{Li}^+$-beam:
- ion number: $10^7$
- diameter: 500 $\mu$ m
- divergence: 50 $\mu$ rad
The Experimental Setup

Iodine spectroscopy:
• Frequency modulation saturation spectroscopy

Lasers are sent through AOMs:
• Iodine line can be frequency-shifted with respect to Li-signal
• Lasers can be switched on and off at a high switching frequency

(for $\beta = 0.064$ measurement)
Adjustment of the laser-ion beam angle

with new optical method: $\Delta \theta < 100\mu \text{rad}$

corresponding frequency error: 10 kHz
Measurement Scheme

Typical run containing 80 Scans:

(a+b): lasers are applied separately
! Doppler background only
(c): lasers applied simultaneously
! Doppler background + Lamb dip

Difference signal contains pure Lamb dip

\[ \delta \nu = 2.03 \pm 0.30 \text{ MHz} \]

15 MHz

Iodine signal
The Lamb dip frequency is slightly dependent on the laser power.

**Reason:**
- Laser forces locally change velocity distribution
- Modified Doppler background & ac-Stark shift

**Extrapolation to zero intensity by fitting**

\[ \delta \nu = \delta \nu_0 + m I^K \]

**Result:**
- Almost linear dependence \((\kappa = 0.93)\)
- \(\delta \nu_0 = 1.55 \pm 0.46\) MHz
Faster Laser Switching

Intensity dependence strongly decreases with increasing switching frequency.

Extrapolated frequencies from both measurements agree within the error.

! Fit uncertainty < 100 kHz
Laser Curvature Effects

Lasers are Gaussian Beams

Phase on optical axis:
\[ \phi(r = 0, z) = 2\pi\nu_L - k_L z + \arctan \frac{z}{z_R} \]

Frequency from the ions point of view:
\[ \nu = \frac{1}{2\pi} \frac{d\varphi}{dt} = \frac{1}{2\pi} \left( \frac{\partial \varphi}{\partial t} + \nu \cdot \frac{\partial \varphi}{\partial z} \right) = \nu_L + \beta \nu_L \left( -1 + \frac{c}{2\pi\nu_L \frac{z_R}{z^2 + z_R^2}} \right) \]

correction of on-axis term of the order of 50 to 500 kHz
Magnetic Fields

Vary degree of polarization:

- PM2
- PM3

θ (degrees)

2 Gauss
Lamb dip frequency $\nu_{a}^{\text{exp}}$, measured w.r.t. a well-known I$_2$-line (calibrated by Sascha Reinhardt, 2003):

$$\nu_{a}^{\text{exp}} = 512\,671\,442\,908 \pm 516\,\text{kHz}$$

dominated by intensity dependence

Lamb dip frequency $\nu_{a}^{\text{SR}} = \frac{\nu_{0}^{2}}{\nu_{a}}$ as expected from Special Relativity:

- $^7\text{Li}^+$ rest frame frequency: $\nu_{0} = 546\,466\,918\,790 \pm 400\,\text{kHz}$ [Riis et al. (1994)]

- Ar$^+$ laser, stabilized to I$_2$-line: $\nu_{p} = 582\,490\,203\,559 \pm 175\,\text{kHz}$

$$\nu_{a}^{\text{SR}} = 512\,671\,443\,249 \pm 766\,\text{kHz}$$

dominated by $\Delta \nu_{0}$

$$\delta = \nu_{a}^{\text{exp}} - \nu_{a}^{\text{SR}} = -332 \pm 924\,\text{kHz}$$

$$\delta \tilde{\alpha} < 2.2 \times 10^{-7}$$
Measurement at low ion velocity

- The experiment is limited by the knowledge of the rest frame frequency $\nu_0$
- Replace measurement of $\nu_0$ by measurement at low ion velocity

$\Delta \nu_p^{(l)}$ and $\Delta \nu_a^{(l)}$ are in the 150 kHz range

$\rightarrow$ expected new limit on $\delta \alpha$ : $7 \times 10^{-8}$

An iodine line as reference at 565 nm has been calibrated using a self-referenced frequency comb
(collaboration with T. W. Hänsch, R. Holzwarth, T. Udem, M. Zimmermann, MPQ)
Experimental Result II


Transition frequency $\nu_0^{\text{exp}} = \sqrt{\nu_a \nu_0}$, derived in experiments on slow ($\beta = 0.03$) and fast ($\beta = 0.064$) ion beams:

\[ \nu_0^{(\text{slow})} = 546 \, 466 \, 918 \, 624 \pm 130 \, \text{kHz} \]
\[ \nu_0^{(\text{fast})} = 546 \, 466 \, 918 \, 511 \pm 102 \, \text{kHz} \]

Derive time dilation test parameter $\hat{\alpha}$ and rest frame frequency $\nu_0$ independently:

\[ |\delta\hat{\alpha}| < 8 \times 10^{-8} \]

$\nu_0 = 546 \, 466 \, 918 \, 656 \pm 169 \, \text{kHz}$

Best value so far: 400 kHz
[E.Riis et al., PRA 49, 207 (1994)]

This experiment is also sensitive to certain parameters in the Standard Model Extension [M. Tobar et al., Phys. Rev. D 2005; C. Lane, Phys. Rev. D 2005]:

- photon sector: $\hat{\kappa}_{tr} < 10^{-7}$
- fermion sector: Proton \[ |c^p_{XX} + c^p_{YY} - 2c^p_{ZZ}| < 10^{-11} \]
  \[ |c^p_{TJ} + c^p_{JT}| < 10^{-8} \]
  Electron \[ |c^p_{XX} + c^p_{YY} - 2c^p_{ZZ}| < 10^{-5} \]
  \[ |c^p_{TJ} + c^p_{JT}| < 10^{-2} \]
• Implementation of experiment on Li ions at $\beta = 0.34$ currently under way at GSI in Darmstadt. Goal: $\rightarrow$ push $\delta \alpha$ into $10^{-9}$ range.
First Results

C. Novotny, to be published

• Doppler-shifted frequencies measured to ±200 MHz (we think ± 1 MHz is possible)

\[ \frac{\nu_a \nu_p}{\nu_0^2} = 1 + 2\alpha \beta^2 \quad |\delta \tilde{\alpha}| < 1 \times 10^{-6} \]

still one order of magnitude short of best limit

• analyze higher-order terms:

\[ \frac{\nu_a \nu_p}{\nu_0^2} = 1 + 2\alpha \beta^2 + (\alpha + 2\alpha_2) \beta^4 \]

• use \( \delta \alpha < 8 \times 10^{-8} \) from TSR

\[ |\delta \tilde{\alpha}_2| < 9 \times 10^{-6} \]

30fold improvement over Hydrogen\@ \( \beta = 0.84 \) [MacArthur PRA 56, 282 (1986)]
Summary

CPT tests:
mainly from particle antiparticle comparison: e.g. proton
further limits from electron and muon magnetic moments and neutral meson masses

Lorentz tests (some of them are also CPT tests):
• baryonic matter:
  protons and neutrons: e.g. clock-comparison experiments
  further limits for electrons from spin-polarized torsion pendulum
• photons:
  interferometric tests: Michelson-Morley, Kennedy-Thorndike constrain tensor components of SME
  Ives-Stilwell test constrains scalar component of SME

not covered in this talk: many other constraints from neutrinos, non-baryonic matter, astrophysical observations, planned space-based tests …
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A team of 14 (!) people have done an experiment and have published their earth-shaking (sic :-) result on exactly one (yes 1 (!), i.e. a single) page, titled: "Experimental Test of Special Relativity" by G. Saatho, S. Karpuk, S. Reinhardt, U. Eisenbarth, I. Hocq, G. Huber, S. Krohn, R. Hu-noz-Horta, J. Lassen, D. Schwalm, M. Weidemüller, A. Wolf, S. Wricke and G. Gwinner

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I strongly doubt the correctness of this experiment!
Laser-Ion Angle Misalignment

assuming plane waves we expect:

$$\frac{\Delta \nu_a}{\nu_a} = -\theta^2 \beta^2$$

- Measurement (solid line) confirms expectation (dashed)
- Laser-ion angle uncertainty (vertical dashed lines) leads to frequency error of 10 kHz