The Magnetic and Electric Dipole Moments of the Muon

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Outline

• Introduction to the muon
• Magnetic ($a_\mu$) and electric ($d_\mu$) dipole moments
  - E821 result and the SM
  - (new) E821 EDM limit
• Limits on CPT/Lorentz Violation in muon spin precession
• Future improvements in $a_\mu$, $d_\mu$?
• Summary and conclusions.
First published observation of the muon came from cosmic rays:

Paul Kunze,

Z. Phys. 83, 1 (1933)

“a particle of uncertain nature”
Identified in 1936

Study of cosmic rays by Seth Neddermeyer and Carl Anderson

Note on the Nature of Cosmic-Ray Particles

Seth H. Neddermeyer and Carl D. Anderson
California Institute of Technology, Pasadena, California
(Received March 30, 1937)

Measurements\(^1\) of the energy loss of particles occurring in the cosmic-ray showers have shown that this loss is proportional to the energy of the particle. For massive particles, the energy loss is larger than for protons but smaller than for electrons obeying the Bethe-Heitler theory, we have taken about 6000 counter-tripped photo-
Muon properties:

- Lifetime $\sim 2.2 \mu s$, practically forever
- 2\textsuperscript{nd} generation lepton
- $m_\mu/m_e = 206.768\ 277(24)$
- Produced polarized
  - In-flight decay: both “forward” and “backward” muons are highly polarized
- Paul Scherrer Institut has $10^8$ low-energy $\mu/s$ in a beam
Death of the Muon

- Decay is self analyzing

The Muon Rest Frame

Highest energy $e^+$ are along muon spin
The positron carries the muon spin
The Quantum Theory of the Electron.

By P. A. M. Dirac, St. John’s College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received January 2, 1928.)

§ 4. The Hamiltonian for an Arbitrary Field.

To obtain the Hamiltonian for an electron in an electromagnetic field with scalar potential $A_0$ and vector potential $\mathbf{A}$, we adopt the usual procedure of substituting $p_0 + e/c \cdot A_0$ for $p_0$ and $\mathbf{p} + e/c \cdot \mathbf{A}$ for $\mathbf{p}$ in the Hamiltonian for no field. From equation (9) we thus obtain

$$\left[ p_0 + \frac{e}{c} A_0 + \rho_1 \left( \mathbf{\sigma} \cdot \mathbf{p} + \frac{e}{c} \mathbf{A} \right) + \rho_3 mc \right] \psi = 0. \quad (14)$$

This differs from (1) by the two extra terms

$$\frac{e\hbar}{c} (\mathbf{\sigma}, \mathbf{H}) + \frac{ie\hbar}{c} \rho_1 (\mathbf{\sigma}, \mathbf{E})$$

in $\mathbf{F}$. These two terms, when divided by the factor $2m$, can be regarded as the additional potential energy of the electron due to its new degree of freedom. The electron will therefore behave as though it has a magnetic moment $e\hbar/2mc \cdot \mathbf{\sigma}$ and an electric moment $ie\hbar/2mc \cdot \rho_1 \mathbf{\sigma}$. This magnetic moment is just that assumed in the spinning electron model. The electric moment, being a pure imaginary, we should not expect to appear in the model. It is doubtful whether
The magnetic dipole moment directed along spin.

\[ \overrightarrow{\mu}_s = g_s \left( \frac{e\hbar}{2m} \right) \vec{s} \]

Dirac Theory: \( g_s = 2 \)

\[ \mu = (1 + \alpha) \frac{e\hbar}{2m} \]

Dirac + Pauli moment

\[ \alpha = \frac{g - 2}{2} \]

For leptons, radiative corrections dominate the value of \( \alpha \approx 0.00116... \)

\[ e \text{ vrs. } \mu : \text{ relative contribution of heavier things} \]

\[ \left( \frac{m_\mu}{m_e} \right)^2 \approx 42,000 \]

B. Lee Roberts, Heidelberg – 11 June 2008
Modern Notation:

\[ \Gamma_\beta = e F_1 \bar{\psi}_R \gamma_\beta \psi_R + \frac{i e}{2m} F_2 \bar{\psi}_R \sigma_{\beta \delta} q^\delta \psi_L \]

- Muon Magnetic Dipole Moment \( a_\mu \) chiral changing

\[ \bar{u}_\mu \left[ e F_1 (q^2) \gamma_\beta + \frac{i e}{2m_\mu} F_2 (q^2) \sigma_{\beta \delta} q^\delta \right] u_\mu \]

\[ F_1(0) = 1 \quad F_2(0) = a_\mu \]

- Muon EDM

\[ \bar{u}_\mu \left[ \frac{i e}{2m_\mu} F_2 (q^2) - F_3 (q^2) \gamma_5 \right] \sigma_{\beta \delta} q^\delta u_\mu \]

\[ F_2(0) = a_\mu \quad F_3(0) = d_\mu; \text{ EDM} \]
The SM Value for the muon anomaly ($10^{-10}$)

\[ 11 \ 658 \ 471.809 \ (0.016) \]

\[ + \ C_5 \left( \frac{\alpha}{\pi} \right)^5 \]

\[ 690.1 \ (4.7) \]

\[ - 9.79 \ (0.09) \]

\[ 11 \ (4) \]

\[ 15.4 \ (0.2) \]

\[ = 11 \ 659 \ 208.0(6.3) \]

B. Lee Roberts, Heidelberg – 11 June 2008
Since $a_\mu$ represents a sum over all physics, it is sensitive to a wide range of potential new physics.
\( a_\mu \) is sensitive to a wide range of new physics

- substructure

\[
\delta a_\mu (\Lambda_\mu) \approx \frac{m_\mu^2}{\Lambda^2_\mu}
\]
$a_\mu$ is sensitive to a wide range of new physics

- substructure
- SUSY (with large tan$\beta$)
- many other things (extra dimensions, etc.)

$$a_\mu (\text{SUSY}) \simeq \frac{\alpha(M_Z)}{8\pi \sin^2 \theta_W} \frac{m_\mu^2}{\tilde{m}^2} \tan \beta \left(1 - \frac{4\alpha}{\pi} \ln \frac{\tilde{m}}{m_\mu}\right)$$

$$\simeq (\text{sgn}_\mu) \times 10^{-10} \tan \beta \left(\frac{100 \text{ GeV}}{\tilde{m}}\right)^2$$

- many other things (extra dimensions, etc.)
Spin Motion in a Magnetic Field

Momentum turns with $\omega_C$, cyclotron frequency

Spin turns with $\omega_S$

$$\omega_C = \frac{eB}{mc\gamma}$$

$$\omega_S = \frac{geB}{2mc} + (1 - \gamma)\frac{eB}{\gamma mc}$$

Spin turns relative to the momentum with $\omega_a$

$$\omega_a = \omega_S - \omega_C = \left(\frac{g - 2}{2}\right)\frac{eB}{mc} = a\frac{eB}{mc}$$
Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon*  

Richard L. Garwin,† Leon M. Lederman, and Marcel Weinrich  
Physics Department, Nevis Cyclotron Laboratories, Columbia University, Irvington-on-Hudson, New York, New York  
(Received January 15, 1957)

Lee and Yangt–s have proposed that the long held space-time principles of invariance under charge conjugation, time reversal, and space reflection (parity) are violated by the “weak” interactions responsible for decay of nuclei, mesons, and strange particles. Their hypothesis, born out of the $\tau - \theta$ puzzle, was accompanied by the suggestion that confirmation should be sought (among other places) in the study of the successive reactions

$$\pi^+ \rightarrow \mu^+ + \nu, \quad (1)$$

$$\mu^+ \rightarrow e^+ + 2\nu. \quad (2)$$

They have pointed out that parity nonconservation implies a polarization of the spin of the muon emitted from stopped pions in (1) along the direction of motion and that furthermore, the angular distribution of electrons in (2) should serve as an analyzer for the muon polarization. They also point out that the longitudinal

VIII. Negative muons stopped in carbon show an asymmetry (also leaked backwards) of $a \approx -1/20$, i.e., about 15% of that for $\mu^+$.  
IX. The magnetic moment of the $\mu^-$, bound in carbon, is found to be negative and agrees within limited accuracy with that of the $\mu^+$.  
X. Large asymmetries are found for the $e^+$ from polarized $\mu^+$ beams stopped in polyethylene and calcium. Nuclear emulsion (as a target in Fig. 1) yields an asymmetry of about half that observed in carbon.

Fig. 1. Experimental arrangement. The magnetizing field of 79 gauss per ampere.
First muon spin rotation experiment

\[ g = 2.0 \times 0.1 \]
Accurate Determination of the $\mu^+$ Magnetic Moment

R. L. Garwin,† D. P. Hutchinson, S. Penman,‡ and G. Shapiro§

Columbia University, New York, New York
(Received August 4, 1959)

Note added in proof.—Experiments which have recently been reported to us [J. Lathrop, et al. and A. Bearden et al., Phys. Rev. Letters (to be published)] indicate a mass value of $M_\mu = 206.76 \pm 0.09 \pm 0.08M_e$. This yields a value of $g_\mu = 2(1.00113_{-0.00012}^{+0.00016})$. Although the assigned errors are now slightly greater than above, it is to be noted that the new result represents a direct measurement, rather than a lower limit. The agreement

$$a = \frac{\alpha}{2\pi} = 0.001161$$
Subsequent (g-2) experiments measured the difference frequency, $\omega_a$, between the spin and momentum precession with an electric quadrupole field for vertical focusing:

$$\vec{\omega}_a = -\frac{e}{m} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

$$B \Rightarrow \langle B \rangle_{\mu-\text{dist}}$$

$\gamma_{\text{magic}} = 29.3$

$p_{\text{magic}} = 3.09 \text{ GeV/c}$
**Experimental Technique**

25ns bunch of $5 \times 10^{12}$ protons from AGS

• Muon polarization
• Muon storage ring
• Injection & kicking
• Focus with Electric Quadrupoles
• 24 electron calorimeters

$$\vec{\omega}_a = -\frac{e}{m} \alpha_{\mu} \vec{B}$$

$\mu^{-} \overline{\nu}_{\mu}$

Inflector

$\pi^-$

Target

Pions

$p=3.1\text{GeV/c}$

$x_c \approx 77\text{ mm}$

$b \approx 10\text{ mrad}$

$B \cdot \delta \lambda \approx 0.1\text{ Tm}$

(Thanks to Q. Peng)
muon (g-2) storage ring

Muon lifetime $t_m = 64.4 \, ms$
(g-2) period $t_a = 4.37 \, ms$
Cyclotron period $t_c = 149 \, ns$
To measure $\omega_a$, we used Pb-scintillating fiber calorimeters.

Count number of $e^-$ with $E_e \geq 1.8$ GeV
We count high-energy electrons as a function of time.

\[ 4 \times 10^9 \ e, \ E_{e^-} \geq 1.8 \ \text{GeV} \]

\[ f(t) \simeq N_0 e^{-\lambda t} [1 + A \cos \omega_d t + \phi] \]

electron time spectrum (2001)
The ± 1 ppm uniformity in the average field is obtained with special shimming tools.

We can shim the dipole, quadrupole, sextupole independently.
The ± 1 ppm uniformity in the average field is obtained with special shimming tools.

\[ \langle B \rangle_{\text{azimuth}} \]

\[ \sigma_{\text{syst on } \langle B \rangle_{\mu-\text{dist}}} = \pm 0.03 \text{ ppm} \]
The magnetic field is measured and controlled using pulsed NMR and the free-induction decay.

- Calibration to a spherical water sample that ties the field to the Larmor frequency of the free proton $\omega_p$.
- So we measure $\omega_a$ and $\omega_p$.

$$a\mu = \frac{\omega_a}{\omega_p} = \frac{\mu}{\mu + \frac{\mu}{\omega_a} - \frac{\omega_a}{\omega_p}}$$
When we started in 1983, theory and experiment were known to about 10 ppm.

Theory uncertainty was ~ 9 ppm

Experimental uncertainty was 7.3 ppm
E821 achieved 0.5 ppm and the $e^+e^-$ based theory is also at the 0.6 ppm level. Difference is $3.4\sigma$

$\Delta a_\mu^{(today)} = (29.5 \pm 8.8) \times 10^{-10}$

If the electroweak contribution is left out of the standard-model value, we get a 5.1 $\sigma$ difference.

$$\alpha_{\mu}^{EW} = 15.4(1)(.2) \times 10^{-10}$$

$$\Delta\text{(no EW)} = 44.9(8.8) \times 10^{-10}$$
In a constrained minimal supersymmetric model, \((g-2)_\mu\) provides an independent constraint on the SUSY LSP (lightest supersymmetric partner) being the dark matter candidate.

Historically muon \((g-2)\) has played an important role in restricting models of new physics. It provides constraints that are independent and complementary to high-energy experiments.

\[
\tan \beta = 10, \ \mu > 0
\]

\[
\begin{align*}
m_{h} &= 114 \text{ GeV} \\
m_{\chi^\pm} &= 104 \text{ GeV}
\end{align*}
\]

\(\alpha_\mu\) helps constrain new physics

CMSSM calculation Following Ellis, Olive, Santoso, Spanos, provided by K. Olive
The **Snowmass Points and Slopes** give reasonable benchmarks to test observables with model predictions.

Muon $g-2$ is a powerful discriminator ... **no matter where the final value lands!**
$\alpha_\mu$ will help constrain the interpretation of LHC data, e.g. $\tan \beta$ and $\text{sgn} \, \mu$ parameter

MSSM reference point SPS1a

With these SUSY parameters, LHC gets $\tan \beta$ of $10.22 \pm 9.1$.


Even with no improvement, $\alpha_\mu$ will provide the best value for $\tan \beta$ and show $\mu > 0$ to $> 3 \, \sigma$

$\Delta \alpha_\mu^{(\text{today})} = (29.5 \pm 8.8) \times 10^{-10}$
Improved experiment and theory for $a_\mu$ is important

MSSM reference point SPS1a

With these SUSY parameters, LHC gets $\tan \beta$ of $10.22 \pm 9.1$.


$\mu > 0$ by $> 6 \sigma$

$\tan \beta$ to $< 20\%$

$\sigma^{E821} = (6.3 \rightarrow 2.5) \times 10^{-10}$

$\sigma^{SM} = (6.1 \rightarrow 3.0) \times 10^{-10}$

$\Delta a_\mu^{(\text{future})} = (29.5 \pm 3.9) \times 10^{-10}$
Search for a Muon EDM
Electric Dipole Moment: \( P \neq T \)

\[
\mathcal{H} = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E}
\]

\( \vec{\mu}, \vec{d} \parallel \) to \( \vec{\sigma} \)

\( E, B, \vec{\mu} \text{ or } \vec{d} \)

\( P \quad - \quad + \quad + \quad \) Transformation Properties

\( C \quad - \quad - \quad - \quad \)

\( T \quad + \quad - \quad - \quad \)

If CPT is valid, an EDM would imply non-standard model \( CP \).
Purcell and Ramsey: EDM would violate Parity
Proposed to search for an EDM of the neutron

"raises directly the question of parity."

LETTERS TO THE EDITOR

On the Possibility of Electric Dipole Moments for Elementary Particles and Nuclei
E. M. Purcell and N. F. Ramsey
Department of Physics, Harvard University, Cambridge, Massachusetts
April 27, 1950

It is generally assumed on the basis of some suggestive theoretical symmetry arguments that nuclei and elementary particles can have no electric dipole moments. It is the purpose of this note to point out that although these theoretical arguments are valid when applied to molecular and atomic moments whose electromagnetic origin is well understood, their extension to nuclei and elementary particles rests on assumptions not yet tested.

One form of the argument against the possibility of an electric dipole moment of a nucleon or similar particle is that the dipole's orientation must be completely specified by the orientation of the angular momentum which, however, is an axial vector specifying a direction of circulation, not a direction of displacement as would be required to obtain an electric dipole moment from electrical charges. On the other hand, if the nucleon should spend part of its time asymmetrically dissociated into opposite magnetic poles of the type that Dirac has shown to be theoretically possible, a circulation of these magnetic poles could give rise to an electric dipole moment. To forestall a possible objection we may remark that this electric dipole would be a polar vector, being the product of the angular momentum (an axial vector) and the magnetic pole strength, which is a pseudoscalar in conformity with the usual convention that electric charges are polar vectors.

The argument against electric dipoles, in another form, raises directly the question of parity. A nucleon with an electric dipole moment would show an asymmetry between left- and righthanded coordinate systems; in one system the dipole moment

Phys. Rev. 78 (1950)
Spin Frequencies: $\mu$ in B field with MDM & EDM

$$\vec{\omega} = -\frac{e}{m} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

$$\gamma_{\text{magic}} = 29.3$$

The motional E-field, $\beta \times B$, is ($\sim$GV/m).

$$d_\mu = \frac{\eta}{2} \left( \frac{e\hbar}{2mc} \right) \approx \eta \times 4.7 \times 10^{-14} \text{ e cm}$$

and

$$a_\mu = \left( \frac{g - 2}{2} \right)$$
Spin Frequencies: $\mu$ in $B$ field with MDM & EDM

$$\ddot{\omega} = -\frac{e}{m} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

$\gamma_{\text{magic}} = 29.3$

The motional $E$ - field, $\beta \times B$, is ($\sim$GV/m).

The EDM causes the spin to precess out of plane.
Total frequency \[ \omega = \sqrt{\omega_a^2 + \omega_\eta^2} \]

\[ \vec{d} = \eta \frac{q}{2mc} \vec{s} \quad \vec{\mu} = g \frac{q}{2m} \vec{s} \]

Plane of the spin precession tipped by the angle \( \delta \)

\[ \delta = \tan^{-1} \left( \frac{\eta \beta}{2a_\mu} \right) \]

\[ \omega = \omega_a \sqrt{1 + \tan^2 \delta} \]

Number above (+) and below (-) the midplane will vary as:

\[ N^\pm(t) \propto [1 \mp A_{EDM} \sin(\omega t + \phi) + A_\mu \cos(\omega t + \phi)] \]
We have looked for this vertical oscillation in 3 ways:

- 5-piece vertical hododscope in front of the calorimeters called an FSD
  - 14 detector stations
- Much finer x-y hododscope called a PSD
  - 5 detector stations
- Traceback straw tube array
  - 1 station
- No significant oscillation was found

\[ d_\mu < 2 \times 10^{-19} \text{ 95\% CL*} \]

- The observed \( \Delta a_\mu \) is not from an EDM at the 2.2 \( \sigma \) level

*Coming soon to a preprint server near you
The present EDM limits are orders of magnitude from the standard-model value

<table>
<thead>
<tr>
<th>Particle</th>
<th>Present EDM limit (e-cm)</th>
<th>SM value (e-cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>$2.9 \times 10^{-26}$</td>
<td>$10^{-32} - 10^{-31}$</td>
</tr>
<tr>
<td>$e^-$</td>
<td>$\sim 1.6 \times 10^{-27}$</td>
<td>$&lt; 10^{-41}$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$2 \times 10^{-19} * ($E821$)</td>
<td>$&lt; 10^{-38}$</td>
</tr>
<tr>
<td>future $\mu$ exp</td>
<td>$10^{-24}$ to $10^{-25}$</td>
<td></td>
</tr>
</tbody>
</table>
The SUSY CP problem!

The strong CP problem!

E. Hinds’ e-EDM experiment at Imperial College with YbF molecules is starting to explore this region.

Excluded region (Tl atomic beam)

Commins (2002)

d_e < 1.6 x 10^{-27} e.cm

10^{-22}
10^{-24}
10^{-26}
10^{-28}
10^{-30}
10^{-32}
10^{-34}
10^{-36}

e EDM (e.cm)

MSSM

\phi \sim 1

\phi \sim \alpha/\pi

Left - Right

Multi Higgs

Standard Model

with thanks to Ed Hinds
Dedicated EDM Experiment \( \rightarrow 10^{-24} - 10^{-25} \)

\[
\vec{\omega} = -\frac{e}{m} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]
\]

Use a radial E-field to turn off the \( \omega_a \) precession

\[
+ \frac{e}{m} \left[ \frac{\eta}{2} \left( \frac{\vec{E}}{c} + \vec{\beta} \times \vec{B} \right) \right]
\]

With \( \omega_a = 0 \), the EDM causes the spin to steadily precess out of the plane.

\( \omega_\eta \)
Muon EDM Limits: Present and Future

E821: G. Bennett, et al., (Muon g-2 collaboration) to be submitted to PRD 2008

\[ \sigma_\eta = \frac{\sqrt{2}}{\gamma \tau (e/m) \beta BA \sqrt{N}} \]

Need:
\[ N A^2 = 10^{16} \text{ for} \]
\[ d_\mu \approx 10^{-23} \text{ e·cm} \]

1 June 2008
Connection between MDM, EDM and the lepton flavor violating transition moment $\mu \rightarrow e$

SUSY $\Rightarrow$ slepton mixing

$$\begin{align*}
\Delta m_{\tilde{\mu} \tilde{e}}^2 &\quad \Delta m_{\tilde{\mu} \tilde{\tau}}^2 \\
\Delta m_{\tilde{e} \tilde{\mu}}^2 &\quad \Delta m_{\tilde{\tau} \tilde{\mu}}^2
\end{align*}$$
An intermezzo:

The search for CPT/Lorentz violation in muon spin precession
Search for Lorentz and CPT Violation Effects in Muon Spin Precession

G. W. Bennett,2 B. Bousquet,10 H. N. Brown,2 G. Bunce,2 R. M. Carey,1 P. Cushman,10 G. T. Danby,2 P. T. Debevec,8 M. Deile,13 H. Deng,13 W. Deninger,8 S. K. Dhawan,13 V. P. Druzhinin,3 L. Duong,10 E. Efstrathiadis,1 F. J. M. Farley,13 G. V. Fedotovitch,3 S. Giron,10 F. E. Gray,8 D. Grigoriev,3 M. Grosse-Perdekamp,13 A. Grossmann,7 M. F. Hare,1 D. W. Hertzog,8 X. Huang,1 X. W. Hughes,13,* M. Iwasaki,12 K. Jungmann,6,7 D. Kawall,13 M. Kawamura,12 B. I. Khazin,3 J. Kindem,10 F. Krienen,7 I. Kronkvist,10 A. Lam,1 R. Larsen,2 Y. Y. Lee,2 I. Logashenko,1,3 R. McNabb,8,10 W. Meng,2 J. Mi,2 J. P. Miller,1 Y. Mizumachi,9,11 W. M. Morse,2 D. Nikas,2 C. J. G. Onderwater,6,8 Y. Orlov,4 C. S. Özben,2,8 J. M. Paley,1 Q. Peng,1 C. C. Polly,8 J. Pretz,13 R. Prigl,2 G. zu Putlitz,7 T. Qian,10 S. I. Redin,3,13 O. Rind,1 B. L. Roberts,1 N. Ryskulov,3 S. Sedykh,8 Y. K. Semertzidis,2 P. Shagin,10 Yu. M. Shatunov,3 E. P. Sichtermann,13 E. Solodov,3 M. Soossong,8 A. Steinmetz,13 L. R. Sulak,1 C. Timmermans,10 A. Trofimov,1 D. Urner,8 P. von Walter,7 D. Warburton,2 D. Winn,5 A. Yamamoto,9 and D. Zimmerman10

(Muon g − 2 Collaboration)

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(Received 29 September 2007; published 4 March 2008)
What we measure that could show CPT/Lorentz violation

\[ \bar{\omega}_a = - \frac{e}{m} \left[ a_{\mu} \vec{B} - \left( a_{\mu} - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right] \]

\[ \omega_a = \omega_S - \omega_C; \text{ where } \omega_C \text{ is unaffected by CPT/Lorentz to lowest order.} \]

- **BUT** \( \omega_a = \omega_a(\vec{B}) \Rightarrow \omega_a(\omega_p) \)

- Instead we have to use \( \mathcal{R} = \frac{\omega_a}{\omega_p} \)

- \( \omega_p \) is not affected to our level of sensitivity
CPT/Lorentz violation in the Lagrangian*

\[ \mathcal{L}' = -a_\kappa \bar{\psi} \gamma^\kappa \psi - b_\kappa \bar{\psi} \gamma_5 \gamma^\kappa \psi - \frac{1}{2} H_{\kappa\lambda} \bar{\psi} \sigma^{\kappa\lambda} \psi + \frac{1}{2} i c_{\kappa\lambda} \bar{\psi} \gamma^\kappa D^\lambda \psi + \frac{1}{2} i d_{\kappa\lambda} \bar{\psi} \gamma_5 \gamma^\kappa D^\lambda \psi \]

- \( a_\kappa, b_\kappa \) are CPT odd, others CPT even
- All terms violate Lorentz invariance
- In lowest-order, \( a_\mu \) is insensitive to violating terms

*Bluhm, Kostelecký, Lane, PRL 84,1098 (2000)
Two tests of CPT/Lorentz violation:

- Difference between $\omega_a$ for $\mu^+$ and $\mu^-$

$$\Delta R = -(3.6 \pm 3.7) \times 10^{-9}$$

Bennett, et al., Phys. Rev. D73, 072003-1

$$b_Z = -(8.7 \pm 8.9) \times 10^{-24} \, \text{GeV}$$

$$r \Delta \omega_a \equiv \frac{\Delta \omega_a}{m_\mu} = -(1.0 \pm 1.1) \times 10^{-23}$$

- Sidereal time oscillation in $\omega_a$ not seen:
The limits translate into 95% CL limits on parameters

\[
b^+_T \mu = \sqrt{(\bar{b}^+_X \mu)^2 + (\bar{b}^+_Y \mu)^2} \leq 1.4 \times 10^{-24} \text{ GeV}
\]

\[
\bar{b}^-_T \mu = \sqrt{(\bar{b}^-_X \mu)^2 + (\bar{b}^-_Y \mu)^2} \leq 2.6 \times 10^{-24} \text{ GeV}
\]

dividing by \(m_{\mu}\)

\[
\rho^+_{A\Omega} \mu \leq 2 \times 10^{-23}
\]

\[
\rho^-_{A\Omega} \mu \leq 3.8 \times 10^{-23}
\]

Muonium hyperfine structure \(\rho^+_{A\Omega} \mu \leq 5 \times 10^{-22}\)

electron in a penning trap \(\rho^-_{A\Omega} e \leq 1.6 \times 10^{-21}\)

Note that

\[
\frac{m_{\mu}}{M_P} = 8.7 \times 10^{-21}
\]
Future Improvements in $\alpha_\mu$?

$\sigma_{\text{stat}} = \pm 0.46 \text{ ppm}$  $\sigma_{\text{syst}} = \pm 0.28 \text{ ppm}$

- Theory (strong interaction part) will improve.
  - both lowest order, and light-by-light

- If money were no object, how well could the experiment be improved?
  - The limit of our technique is between $\sim 0.1$ and $0.06$ ppm.
The error budget for a new experiment represents a continuation of improvements already made during E821

| Systematic uncertainty (ppm) | 1998 | 1999 | 2000 | 2001 | E???
|-------------------------------|------|------|------|------|------
| Magnetic field - $w_p$        | 0.5  | 0.4  | 0.24 | 0.17 | ≤0.1
| Anomalous precession - $w_a$  | 0.8  | 0.3  | 0.31 | 0.21 | ≤0.1
| Statistical uncertainty (ppm)| 4.9  | 1.3  | 0.62 | 0.66 | ?    
| Total Uncertainty (ppm)      | 5.0  | 1.3  | 0.73 | 0.72 | ≃0.1

- **Field improvements**: better trolley calibrations, better tracking of the field with time, temperature stability of room, improvements in the hardware
- **Precession improvements** will involve new beam scraping scheme, lower thresholds, more complete digitization periods, better energy calibration
Possible Future Experiments?

- **Brookhaven**
  - E969 aimed for 0.2 ppm overall error
  - No funding, most unlikely

- **Fermilab**
  - The $\mu \rightarrow e$ conversion experiment is top priority in the recent P5 recommendations.
  - $g-2$ is mentioned as important, but with the three sites mentioned as possibilities. We would aim for 0.1 ppm total error. It could be done at FNAL, and we have received significant interest there.

- **J-PARC**
  - Significant interest in moving the ring there. goal is $\leq$ 0.1 total error
Summary

• The measurement of $e^-$ and $\mu^\pm$ magnetic dipole moments has been an important benchmark for the development of QED and the standard model of particle physics.

• The muon anomaly has been particularly valuable in restricting physics beyond the standard model, and will continue to do so in the LHC Era.

• There appears to be a difference between $a_\mu$ and the standard-model prediction at the 3.4 $\sigma$ level.

• Much activity continues on the theoretical front.

• A new limit on the EDM is now available.

• The experiment can certainly be improved...

and we look forward to discussions with FNAL and J-PARC.
THE END