Lighthouses in the dark

Supernovae Ia \( m - M = 5 \log d_L + 25 \)

\[
d_L(z) = (1 + z) \int \frac{dz}{H(z)}
\]

\[
d^2 = \frac{L}{4\pi f}
\]
Bug or feature?

Conclusion: SNIa are dimmer than expected in a matter universe!

**BUT:**

- Dependence on progenitors?
- Contamination?
- Environment?
- Host galaxy?
- Dust?
- Lensing?

Ordinary matter

\[ d_L(z) = (1 + z) \int \frac{dz}{H(z)} \]
Cosmological explanation

There is however a simple cosmological solution

Local Hubble law

\[ r(z) = \frac{z}{H_0} \]

Global Hubble law

\[ r(z) = \int \frac{dz}{H(z)} \]

If \( H(z) \) in the past is smaller (i.e. acceleration), then \( r(z) \) is larger: larger distances (for a fixed redshift) make dimmer supernovae

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Cosmological constant

acceleration, in GR, can only occur if pressure is large and negative

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \]

\[ p_\Lambda = -\rho_\Lambda \]

Einstein 1917
We know so little about the evolution of the universe!
Prolegomena zu einer jeden künftigen Dark Energy physik

©Kant

Isotropy

Abundance

Observational requirements

Slow evolution

Weak clustering
Classifying the unknown

1. Cosmological constant
2. Dark energy $w=\text{const}$
3. Dark energy $w=w(z)$
4. quintessence
5. scalar-tensor models
6. coupled quintessence
7. mass varying neutrinos
8. k-essence
9. Chaplygin gas
10. Cardassian
11. quartessence
12. quiessence
13. phantoms
14. $f(R)$
15. Gauss-Bonnet
16. anisotropic dark energy
17. brane dark energy
18. backreaction
19. degravitation
20. TeVeS
21. oops....did I forget your model?

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The past ten years of dark energy models

\[
\int dx^4 \sqrt{-g} \left[ R + \frac{1}{2} \phi_{,\mu} \phi^{,\mu} + V(\phi) + L_{\text{matter}} \right]
\]

\[
\int dx^4 \sqrt{-g} \left[ f(\phi)R + \frac{1}{2} \phi_{,\mu} \phi^{,\mu} + V(\phi) + L_{\text{matter}} \right]
\]

\[
\int dx^4 \sqrt{-g} \left[ f(\phi)R + K(\frac{1}{2} \phi_{,\mu} \phi^{,\mu}) + V(\phi) + L_{\text{matter}} \right]
\]

\[
\int dx^4 \sqrt{-g} \left[ f(\phi, \frac{1}{2} \phi_{,\mu} \phi^{,\mu})R + G_{\mu\nu} \phi^{,\nu} \phi^{,\mu} + K(\frac{1}{2} \phi_{,\mu} \phi^{,\mu}) + V(\phi) + L_{\text{matter}} \right]
\]
The Horndeski Lagrangian

The most general 4D scalar field theory with second order equation of motion

\[ \int dx^4 \sqrt{-g} \left[ \sum_i L_i + L_{\text{matter}} \right] \]

\[ \mathcal{L}_2 = K(\phi, X), \]
\[ \mathcal{L}_3 = -G_3(\phi, X) \Box \phi, \]
\[ \mathcal{L}_4 = G_4(\phi, X) R + G_{4,X} \left[ (\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi) (\nabla^\mu \nabla^\nu \phi) \right], \]
\[ \mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu} (\nabla^\mu \nabla^\nu \phi) - \frac{1}{6} G_{5,X} \left[ (\Box \phi)^3 - 3(\Box \phi) (\nabla_\mu \nabla_\nu \phi) (\nabla^\mu \nabla^\nu \phi) + 2(\nabla^\mu \nabla_\alpha \phi) (\nabla^\alpha \nabla_\beta \phi) (\nabla^\beta \nabla_\mu \phi) \right]. \]

✓ First found by Horndeski in 1975
✓ rediscovered by Deffayet et al. in 2011
✓ no ghosts, no classical instabilities
✓ it modifies gravity!
✓ it includes f(R), Brans-Dicke, k-essence, Galileons, etc etc etc

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Massive gravity

Pauli-Fierz (1939) action: the only ghost-free quadratic action for a massive spin two field

\[ \int d^4x \sqrt{g} R_g + m^2 \int d^4x h_{\mu\nu} h_{\alpha\beta} (\eta^{\mu\alpha} \eta^{\nu\beta} - \eta^{\mu\nu} \eta^{\alpha\beta}) \]

The three capital sins of Pauli-Fierz theory:

- It does not reduce to massless gravity for \( m \to 0 \) (vDVZ disc.)
- It is linear
- It contains a ghost when extended to non-linear level (Boulware-Deser ghost)
The first problem was partially solved by Vainshtein (1972): there exists a radius below which the linear theory cannot be applied; For the Sun, this radius is larger than the solar system!

The second and third problems have been reconsidered very recently:

\[ S = -\frac{M_g^2}{2} \int d^4x \sqrt{-\det g} R(g) - \frac{M_f^2}{2} \int d^4x \sqrt{-\det f} R(f) + m^2 M_g^2 \int d^4x \sqrt{-\det g} \sum_{n=0}^{4} \beta_n e_n (\sqrt{g} \alpha^\beta f_{\beta\gamma}) + \int d^4x \sqrt{-\det g} L_m(g, \Phi) \]

The only ghost-free local non-linear massive gravity theory! deRham, Gabadadze, Tolley 2010
Hassan & Rosen, 2011

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Why testing gravity?

we only directly test gravity within the solar system, at the present time, and with “baryons”
Combine observations of background, linear and non-linear perturbations to reconstruct as much as possible the Horndeski/bimetric model … or rule it out!
Charles L. Bennett
Nature 440, 1126-1131(27 April 2006)
\[ D(z) = \frac{R}{\theta} = \frac{1}{H_0 \sqrt{-\Omega_{k0}}} \sinh \left( H_0 \sqrt{-\Omega_{k0}} \int \frac{dz}{H(z)} \right) \]
$H(z) = \frac{dz}{R}$
Then we can measure $H(z)$ and

$$D(z) = \frac{1}{H_0 \sqrt{-\Omega_{k0}}} \sinh(H_0 \sqrt{-\Omega_{k0}} \int \frac{dz}{H(z)})$$

and therefore we can reconstruct the full FRW metric

$$ds^2 = dt^2 - \frac{a(t)^2}{\left(1 - \frac{\Omega_k H^2}{4} r^2 \right)^2} (dx^2 + dy^2 + dz^2)$$
Two free functions

\[ ds^2 = a^2[(1 + 2\Psi)dt^2 - (1 + 2\Phi)(dx^2 + dy^2 + dz^2)] \]

At linear order we can write:

- Poisson equation

\[ \nabla^2 \Psi = 4\pi G a^2 \rho_m \delta_m \]

- zero anisotropic stress

\[ 1 = -\frac{\Phi}{\Psi} \]
Two free functions

\[ ds^2 = a^2[(1 + 2\Psi)dt^2 - (1 + 2\Phi)(dx^2 + dy^2 + dz^2)] \]

At linear order we can write:

- modified Poisson equation

\[ \nabla^2 \Psi = 4\pi G a^2 Y(k, a) \rho_m \delta_m \]

- non-zero anisotropic stress

\[ \eta(k, a) = -\frac{\Phi}{\Psi} \]
Modified Gravity at the linear level

- standard gravity
  \[ Y(k, a) = 1 \]
  \[ \eta(k, a) = 1 \]

- scalar-tensor models
  \[ Y(a) = \frac{G^*}{FG_{\text{cav},0}} \frac{2(F + F'^2)}{2F + 3F'^2} \]
  \[ \eta(a) = 1 + \frac{F'^2}{F + F'^2} \]

- f(R)
  \[ Y(a) = \frac{G^*}{FG_{\text{cav},0}} \frac{1 + 4m\frac{k^2}{a^2R}}{1 + 3m\frac{k^2}{a^2R}} \]
  \[ \eta(a) = 1 + \frac{m\frac{k^2}{a^2R}}{1 + 2m\frac{k^2}{a^2R}} \]

- DGP
  \[ Y(a) = 1 - \frac{1}{3\beta} \]
  \[ \beta = 1 + 2Hr_{c}w_{DE} \]
  \[ \eta(a) = 1 + \frac{2}{3\beta - 1} \]

- massive bi-gravity
  \[ Y(a) = \ldots \]
  \[ \eta(a) = \ldots \]

Boisseau et al. 2000
Acquaviva et al. 2004
Schimd et al. 2004
L.A., Kunz & Sapone 2007
Bean et al. 2006
Hu et al. 2006
Tsujikawa 2007
Lue et al. 2004;
Koyama et al. 2006

see F. Koennig and L. A. 2014
Cambridge 2014
In the quasi-static limit, every Horndeski and massive bigravity model is characterized at linear scales by the two functions

\[ \eta(k, a) = h_2 \left( \frac{1 + k^2 h_4}{1 + k^2 h_5} \right) \]

\[ Y(k, a) = h_1 \left( \frac{1 + k^2 h_5}{1 + k^2 h_3} \right) \]

\( k = \text{wavenumber} \)

\( h_i = \text{time-dependent functions} \)


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Modified Gravity at the linear level

\[ h_1 = \frac{w_4}{w_1}, \quad h_2 = \frac{w_1}{w_4}, \quad h_3 = \frac{H^2 2w_1^2 w_2 w_4 - w_3^2 w_4 + 4w_1 w_2 w_1 - 2w_1^2 (w_2 + \rho_m)}{2XM^2}, \]
\[ h_4 = \frac{H^2 2w_1^2 w_2 - w_2 w_4 H + 2w_1 w_1 H + w_2 w_1 - w_1 (w_2 + \rho_m)}{w_1}, \]
\[ h_5 = \frac{H^2 2w_1^2 w_2 - w_2 w_4 H + 4w_1 w_1 H + 2w_1^2 - w_4 (w_2 + \rho_m)}{w_4}, \]

\[ w_1 = 1 + 2 (G_4 - 2XG_{4,x} + XG_{5,\phi} - \dot{\phi}XHG_{5,x}), \]
\[ w_2 = -2\tilde{\phi} (XG_{3,x} - G_{4,\phi} - 2XG_{4,\phi x}) + 2H(w_1 - 4X(G_{4,x} + 2XG_{4,xx} - G_{5,\phi} - XG_{5,\phi x})) - 2\tilde{\phi} XH^2 (3G_{5,x} + 2XG_{5,xx}), \]
\[ w_3 = 3X (K_{,x} + 2XK_{,xx} - 2G_{3,\phi} - 2XG_{3,\phi x}) + 18\tilde{\phi} XH (2G_{3,x} + XG_{3,xx}) - 18\tilde{\phi}H (G_{4,\phi} + 5XG_{4,\phi x} + 2X^2 G_{4,\phi xx}) - 18H^2 (1 + G_4 - 7XG_{4,x} - 16X^2 G_{4,xx} - 4X^3 G_{4,xxx}) - 18XH^2 \left(6G_{5,\phi} + 9XG_{5,\phi x} + 2X^2 G_{5,\phi xx}\right) + 6\tilde{\phi} XH^3 \left(15G_{5,x} + 13XG_{5,xx} + 2X^2 G_{5,xxx}\right), \]
\[ w_4 = 1 + 2 (G_4 - XG_{5,\phi} - XG_{5,x} \tilde{\phi}). \]

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Every Horndeski model induces at linear level, on sub-Hubble scales, a Newton-Yukawa potential

\[ \Psi(r) = -\frac{G_{\text{eff}} M}{r} (1 + \beta e^{-r/\lambda}) \]

where \( G_{\text{eff}} \), \( \beta \) and \( \lambda \) depend on space and time
Reconstruction of the metric

\[ ds^2 = \alpha^2[(1 + 2\Psi)dt^2 - (1 + 2\Phi)(dx^2 + dy^2 + dz^2)] \]

Non-relativistic particles respond to \( \Psi \)

\[ \delta_m'' + (1 + \frac{\mathcal{H}'}{\mathcal{H}})\delta_m' = -k^2\Psi \]

Relativistic particles respond to \( \Phi - \Psi \)

\[ \alpha = \int \nabla_{\text{perp}} (\Psi - \Phi) dz \]
All you can ever observe in linear Cosmology

Expansion rate
Amplitude of the power spectrum
Redshift distortion of the power spectrum
Weak lensing

as function of redshift and scale!

How to combine observations to test the theory?

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The two problems of initial conditions

How do we know if the shape of the power spectrum we observe is due to dark energy or to initial conditions?

\[ P(k, z) = A P_{in}(k) T^2(k, z; DE, DM, etc) \]
Four model-independent observational quantities

Redshift distortion/Amplitude

\[ P_1 = \frac{\text{redshift distortion}}{\text{amplitude}} \]

Lensing/Redshift distortion

\[ P_2 = \frac{\text{lensing}}{\text{redshift distortion}} \]

Redshift distortion rate

\[ P_3 = \text{rate of redshift distortion} \]

Expansion rate

\[ E = \text{expansion rate} \]
Four model-independent observational quantities

Redshift distortion/Amplitude

$$P_1 = \frac{R}{A} = \frac{f}{b}$$

Lensing/Redshift distortion

$$P_2 = \frac{L}{R} = \frac{\Omega m_0 Y (1 + \eta)}{f}$$

Redshift distortion rate

$$P_3 = \frac{R'}{R} = \frac{f'}{f} + f$$

Expansion rate

$$E = \frac{H}{H_0}$$
Matter conservation equation independent of gravity theory

\[ \delta_m'' + \left(1 + \frac{H'}{H}\right)\delta_m' = -k^2\Psi = \frac{3}{2} H^2 \Omega_m Y \delta_m \]
Testing the entire Horndeski Lagrangian

A unique combination of model independent observables

\[
\frac{3 P_2 (1 + z)^3}{2 E^2 (P_3 + 2 + \frac{E'}{E})} - 1 = \eta = h_2 \left( \frac{1 + k^2 h_4}{1 + k^2 h_5} \right)
\]

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L.A. et al. 1210.0439
The importance of being eta

\[ \eta(\alpha_M, \alpha_T) \equiv -\frac{\Phi}{\Psi} \]

Grav. Wave equation

\[ \ddot{h} + 3H(1 + \alpha_M)\dot{h} + (1 + \alpha_T)k^2 h = 0 \]

It turns out that **iff** \( \eta \neq 1 \) then the GW equation is modified. CMB B-polarization can be a tool to detect modified gravity!

Saltas, I. Sawicki, L.A., M. Kunz
1406.7139, Phys Rev Lett 2014

Cambridge 2014
GW speed and lensing


Cambridge 2014
Constraints on GW speed at early times


Cambridge 2014
The proposed satellite LiteBIRD plans to measure $r$ at the reionization peak to 0.001!
Combine lensing and galaxy clustering!
Euclid Surveys

- Simultaneous (i) visible imaging (ii) NIR photometry (iii) NIR spectroscopy
- 15,000 square degrees
- 70 million redshifts, 2 billion images
- Median redshift $z = 1$
- PSF FWHM $\sim 0.18''$
- >1000 peoples, >10 countries

Euclid in a nutshell
History repeats itself…

Current CMB Quilt

Projected Satellite Errors

1998

2011

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Euclid's challenge

C. Di Porto & L.A. 2010

\[ s \equiv \frac{d \log \delta}{d \log a} \]

Growth of matter fluctuations

Euclid error forecast

Present error

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Results...

\[ \eta(k, a) = H_2 \left( \frac{1 + k^2 H_4}{1 + k^2 H_5} \right) \]

Model 1: \( \eta \) constant for all \( z, k \)
Error on \( \eta \) around 1%

Model 2: \( \eta \) varies in \( z \)
Error on \( \eta \)

<table>
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<th>( \bar{z} )</th>
<th>( \Delta P_1 )</th>
<th>( \Delta P_1(%) )</th>
<th>( P_1 )</th>
<th>( \Delta P_2 )</th>
<th>( \Delta P_2(%) )</th>
<th>( P_2 )</th>
<th>( \Delta P_3 )</th>
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<th>( (E'/E) )</th>
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<th>( \bar{\eta} )</th>
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</table>

TABLE X.  Fiducial values and errors for the parameters \( P_1, P_2, P_3, E'/E \) and \( \bar{\eta} \) for every bin. The last bin has been omitted since \( R' \) is not defined there.
Cosmological exclusion plot

\[ \Psi(r) = -\frac{GM}{r} \left(1 + Qe^{-r/\lambda}\right) \]

Caveats, caveats

1
Universal coupling?

2
unknown matter properties (sound speed)?

3
window between sound-horizon and non-linearity?

4
quasi static limit?

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Three Messages

1
If DE is not a Horndeski field or massive gravity, then…

2
k-binned data are crucial for model-indep tests!
e.g. growth factor, redshift distortion parameter

3
Only by combining galaxy clustering and lensing
can DE be constrained (or ruled out!) in a model-independent way

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