Quantum Electrodynamics with Ultracold Atoms

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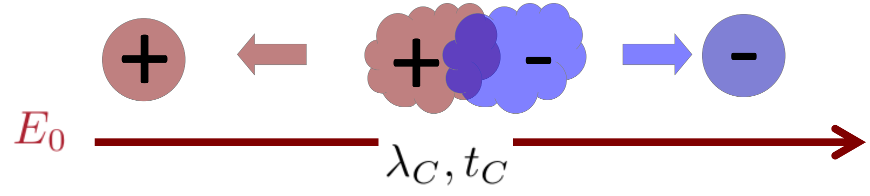
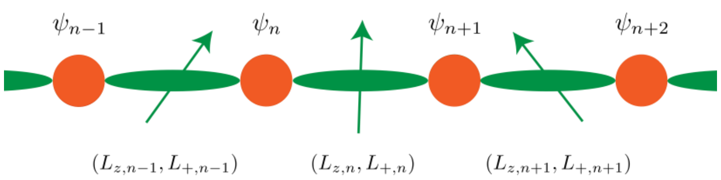
Motivation for QED (1+1)

Talk by Florian Hebenstreit
Talk by Fred Jendrzejewski



- Theoretical Motivation
  - Confinement
  - Chiral symmetry breaking
  - Schwinger Effect
  - Non-equilibrium physics
- Experimental Motivation
  - Observation of Schwinger Effect } Strong Field Physics
  - One dimensional system
  - No plaquette terms } AMO



Aim of this Talk

Can we observe the Schwinger Effect
in an ultracold atom setup?

Outline

- Experimental Realization of QED in (1+1)
- Schwinger Effect in a cold atom setup
- Conclusion

Ways to engineer gauge theories

- Different Strategies
  - Global Symmetry to Local Symmetry

    Zohar, Cirac, Reznik PRA 88.2 (2013): 023617

  - Energy Penalty

    Banerjee, D., et al, PRL 109.17 (2012): 175302.

  - Dissipative Driving

    Stannigel, PRL 112.12 (2014): 120406

  - Digital Quantum Simulation

    Zohar, et al. *arXiv:1607.08121* (2016).
    Tagliacozzo, L., et al. , Nature communications 4 (2013)

Kogut et al., PRD, vol 11, 2

Hamiltonian of QED (1+1)

Quantum Electrodynamics

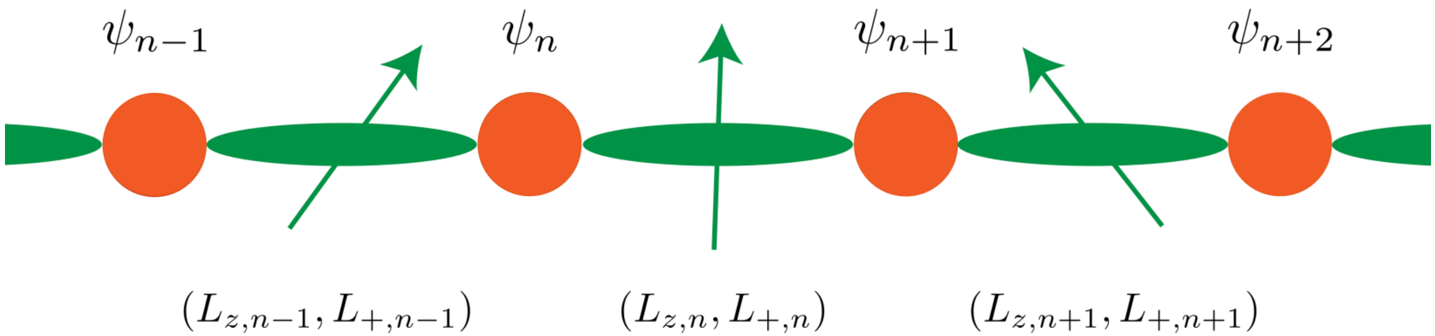
$$H_{KS} = \frac{a_L}{2}\sum_n E_n^2 + M\sum_n (-1)^n \psi_n^\dagger \psi_n - \frac{i}{2a_L}\sum_n [\psi_n^\dagger U_n \psi_{n+1} - h.c.]$$

Electric Field | Mass Term | Interaction Term

Gauss's Law

$$G_n \left|\text{phys}\right\rangle = 0 \text{ with } G_n = E_n - E_{n-1} - g\psi_n^\dagger \psi_n$$

Mapping on atomic systems



$$H_{KS} = \frac{g^2 a_L}{2} \sum_n L_{z,n}^2 + M \sum_n (-1)^n \psi_n^\dagger \psi_n + \frac{i}{2a_L\sqrt{\ell(\ell+1)}} \sum_n [\psi_n^\dagger L_{+,n} \psi_{n+1} - h.c.]$$

Electric Field | Mass Term | Interaction Term

$$G_n = L_{z,n} - L_{z,n-1} - \psi_n^\dagger \psi_n$$

Zohar, Cirac, Reznik PRA 88.2 (2013): 023617

Starting Point of Ultracold Atom Setup

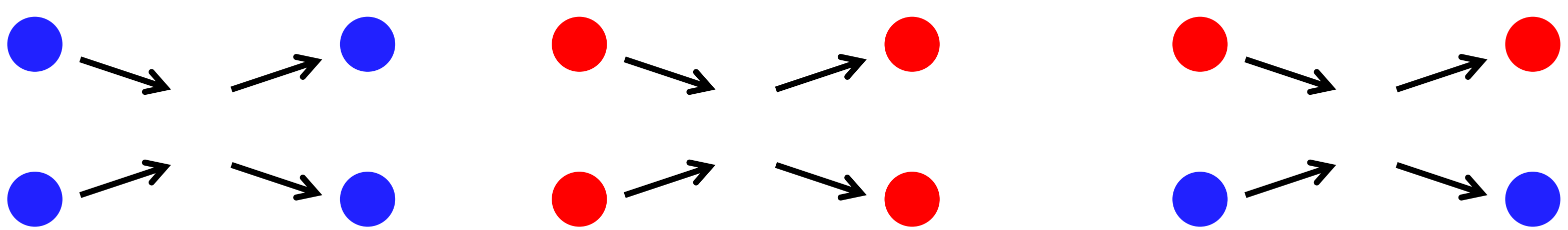
- Kinetic Energy

$$H_T = \frac{\hbar^2}{2M_b}\int d^3\mathbf{x}\sum_\alpha |\nabla\phi_\alpha(\mathbf{x})|^2 + \frac{\hbar^2}{2M_f}\int d^3\mathbf{x}\sum_\alpha |\nabla\psi_\alpha(\mathbf{x})|^2$$

- Potential Energy

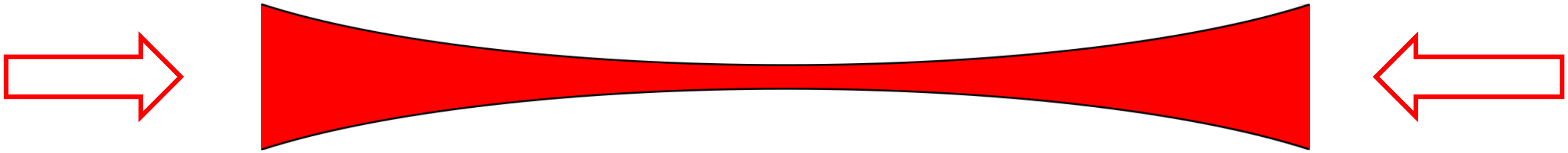
$$H_V = \int d^3\mathbf{x}\sum_\alpha V^b_\alpha(\mathbf{x})\phi^\dagger_\alpha(\mathbf{x})\phi_\alpha(\mathbf{x}) + \int d^3\mathbf{x}\sum_\alpha V^f_\alpha(\mathbf{x})\psi^\dagger_\alpha(\mathbf{x})\psi_\alpha(\mathbf{x})$$

- Interaction Energy $H_I = H_{BB} + H_{FF} + H_{BF}$

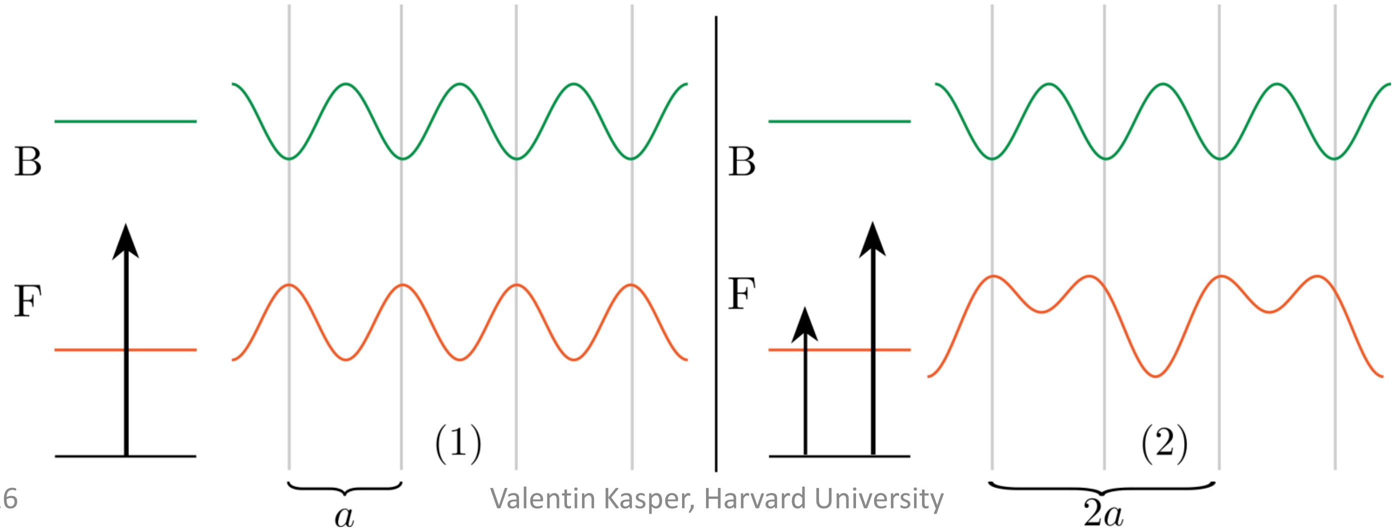


1D and Lattice Structure

- One dimensional via radial confinement

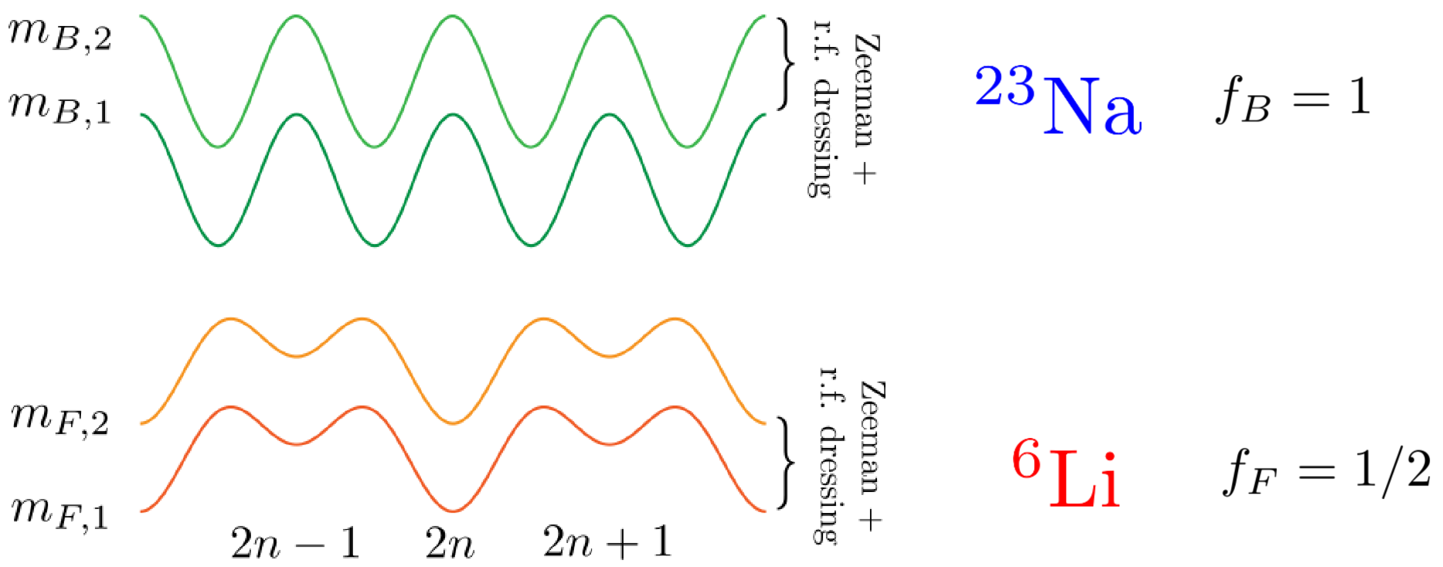


- Staggered structure via superlattice



Magnetic Field

- Hyperfine Splitting



- Why only two lines?

Starting Point of Ultracold Atom Setup

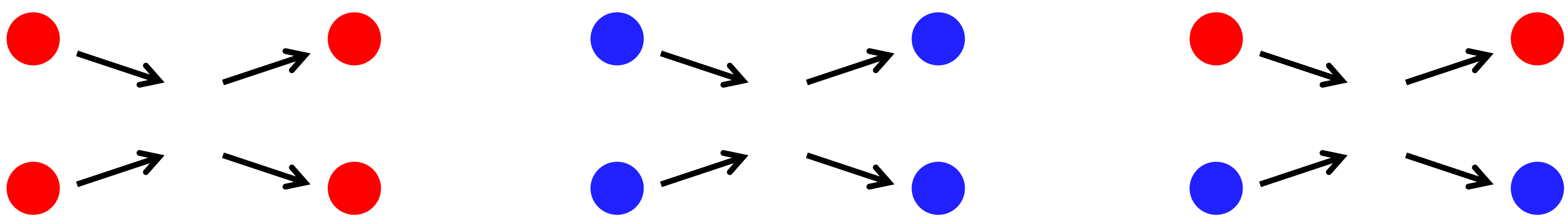
- Kinetic Energy

$$H_T = \frac{\hbar^2}{2M_b}\int d^3\mathbf{x}\sum_\alpha |\nabla\phi_\alpha(\mathbf{x})|^2 + \frac{\hbar^2}{2M_f}\int d^3\mathbf{x}\sum_\alpha |\nabla\psi_\alpha(\mathbf{x})|^2$$

- Potential Energy

$$H_V = \int d^3\mathbf{x}\sum_\alpha V_\alpha^b(\mathbf{x})\phi_\alpha^\dagger(\mathbf{x})\phi_\alpha(\mathbf{x}) + \int d^3\mathbf{x}\sum_\alpha V_\alpha^f(\mathbf{x})\psi_\alpha^\dagger(\mathbf{x})\psi_\alpha(\mathbf{x})$$

- Interaction Energy $H_I = H_{BB} + H_{FF} + H_{BF}$



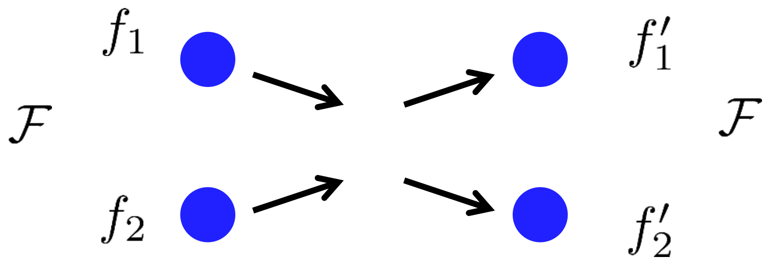
Interaction Terms (i)

- Local, F conserving interaction

$$V(\mathbf{x}_1, \mathbf{x}_2) = \delta(\mathbf{x}_1 - \mathbf{x}_2) \sum_{\mathcal{F}} g_{\mathcal{F}} \mathbf{P}_{\mathcal{F}}$$

- Coupling constant

$$g_{\mathcal{F}} = \frac{2\pi\hbar^2 a_{\mathcal{F}}}{M_r}$$



- Projection operator

$$\mathbf{P}_{\mathcal{F}} = \sum_{M} |f_1, f_2; \mathcal{F}, M\rangle \langle f_1, f_2; \mathcal{F}, M|$$

Interaction Terms (ii)

$$g_{\alpha\beta\gamma\delta} = \langle \alpha\beta | \sum_{\mathcal{F}} g_{\mathcal{F}} \mathbf{P}_{\mathcal{F}} | \gamma\delta \rangle$$

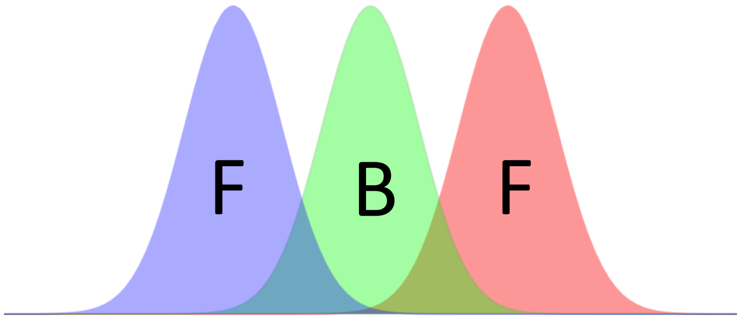
- Interaction Part

$$H_I = \int d^3\mathbf{x} \sum_{\alpha\beta\gamma\delta} \frac{g^{b}_{\alpha\beta\gamma\delta}}{2} \phi^\dagger_\alpha(\mathbf{x})\phi^\dagger_\beta(\mathbf{x})\phi_\delta(\mathbf{x})\phi_\gamma(\mathbf{x})$$

$$+ \int d^3\mathbf{x} \sum_{\alpha\beta\gamma\delta} \frac{g^{f}_{\alpha\beta\gamma\delta}}{2} \psi^\dagger_\alpha(\mathbf{x})\psi^\dagger_\beta(\mathbf{x})\psi_\delta(\mathbf{x})\psi_\gamma(\mathbf{x})$$

$$+ \int d^3\mathbf{x} \sum_{\alpha\beta\gamma\delta} \frac{g^{bf}_{\alpha\beta\gamma\delta}}{2} \psi^\dagger_\alpha(\mathbf{x})\phi^\dagger_\beta(\mathbf{x})\psi_\delta(\mathbf{x})\phi_\gamma(\mathbf{x}) \, .$$

Interaction Terms (iii)



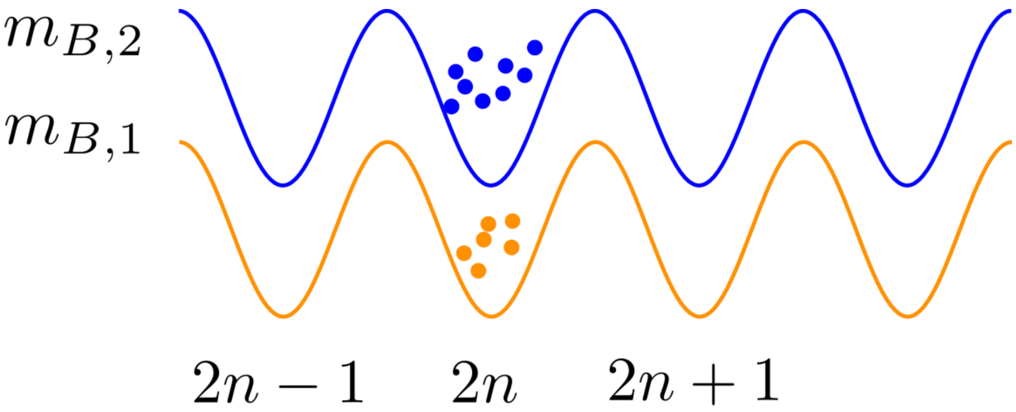
- Lattice sites and overlap integral

$$\boldsymbol{n} = (n_1, n_2, n_3, n_4)$$
$$\boldsymbol{\mu} = (\alpha, \beta, \gamma, \delta)$$

$$H_I = \frac{1}{2}\sum_{\mathbf{n},\boldsymbol{\mu}} U^b_{\mathbf{n}} g^b_{\boldsymbol{\mu}} \phi^\dagger_{n_1,\alpha}\phi^\dagger_{n_2,\beta}\phi_{n_4,\delta}\phi_{n_3,\gamma}$$
$$+ \frac{1}{2}\sum_{\mathbf{n},\boldsymbol{\mu}} U^f_{\mathbf{n}} g^f_{\boldsymbol{\mu}} \psi^\dagger_{n_1,\alpha}\psi^\dagger_{n_2,\beta}\psi_{n_4,\delta}\psi_{n_3,\gamma}$$
$$+ \frac{1}{2}\sum_{\mathbf{n},\boldsymbol{\mu}} U^{bf}_{\mathbf{n}} g^{bf}_{\boldsymbol{\mu}} \psi^\dagger_{n_1,\alpha}\phi^\dagger_{n_2,\beta}\phi_{n_4,\delta}\psi_{n_3,\gamma}\,.$$

Bosonic Interactions

- Staggered Structure of Bosons



$$\phi_{2n,-1} \equiv d_{2n}$$
$$\phi_{2n,0} \equiv b_{2n}$$

$$\phi_{2n+1,-1} \equiv b_{2n+1}$$
$$\phi_{2n+1,0} \equiv d_{2n+1}$$

$$H_I^b = \frac{g_{b,0} - g_{b,2}}{6} U_{\mathbf{n}}^b \sum_n L_{z,n}^2 + \Delta_{b,0} \sum_n (-1)^n L_{z,n}$$

Aim of this Talk

$$H_{KS} = \frac{g^2 a_L}{2} \sum_n L_{z,n}^2 + M \sum_n (-1)^n \psi_n^\dagger \psi_n - \frac{i}{2a_L \sqrt{\ell(\ell+1)}} \sum_n [\psi_n^\dagger L_{+,n} \psi_{n+1} - h.c.]$$

Electric Field

Mass Term

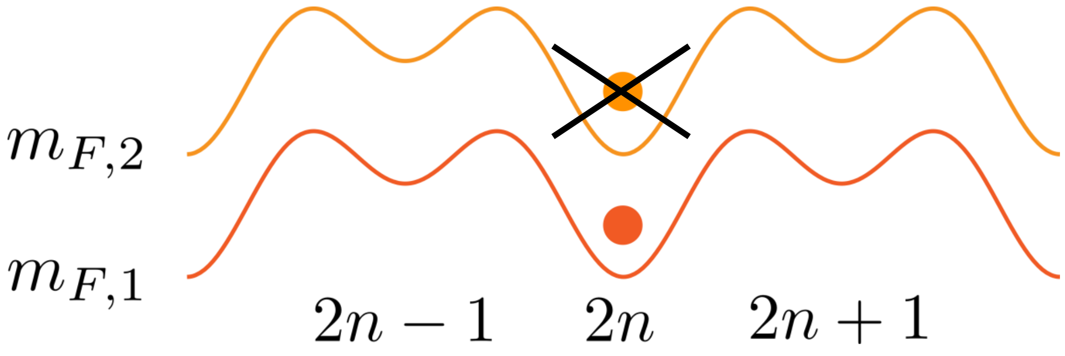
Interaction Term

Fermion Interactions

- Density-density Interactions

$$H_I^f = \sum_n U_{\mathbf{n}}^f g_{f,0} \psi^\dagger_{n,1/2} \psi^\dagger_{n,-1/2} \psi_{n,-1/2} \psi_{n,1/2}$$

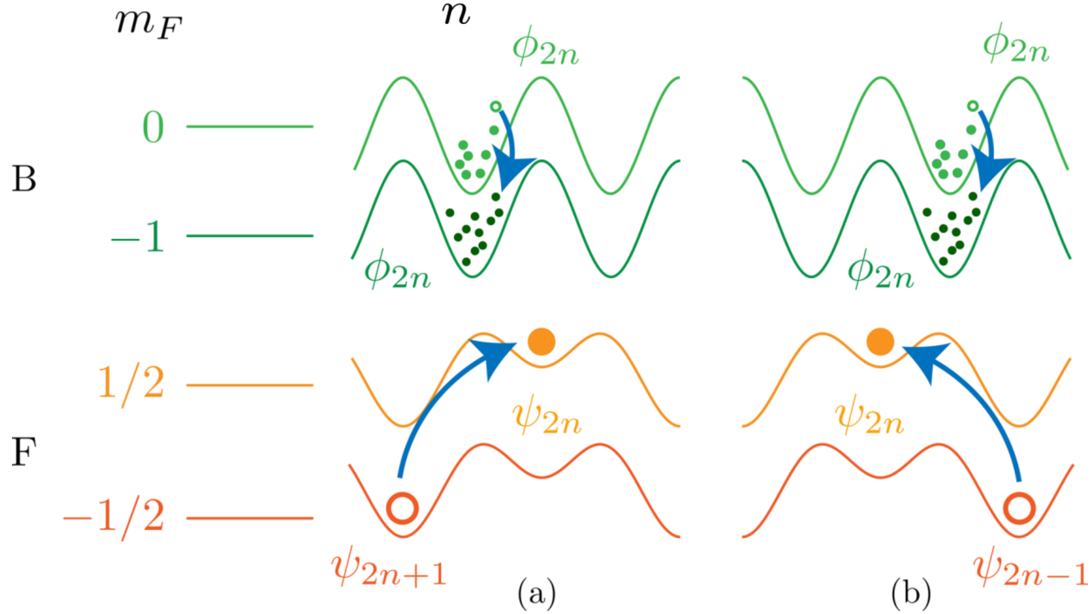
- Gauss's law eliminates this term $G_n \left|\text{phys}\right\rangle = 0$



Bose - Fermi Interaction

- Spin changing terms

$$H^{bf}_{I_{se}} = \frac{1}{2} U^{bf}_{\mathbf{n}} g^{bf}_{\boldsymbol{\mu}} \sum_{n} \left(\psi_n^{\dagger} L_{+,n} \psi_{n+1} + h.c. \right) \ .$$



- Gauss‘s Law eliminates elastic terms $G_n \left|\psi\right\rangle = 0$

Mass Term / Detuning

- Sum of three potential energies
- A) Potential Energy due to Lattice
- B) Elastic Scattering Contribution
- C) Detuning due magnetic field and rf

$$\Delta \sum_n (-1)^n \psi_n^\dagger \psi_n$$

Outline

- Experimental Realization of QED in (1+1)
- Schwinger Effect in a cold atom setup
- Conclusion

AMO and HEP connected

- Identification of coupling constants

$$H_{KS} = \frac{g^2 a_L}{2} \sum_n L_{z,n}^2 + M \sum_n (-1)^n \psi_n^\dagger \psi_n + \frac{i}{2a_L\sqrt{\ell(\ell+1)}} \sum_n [\psi_n^\dagger L_{+,n} \psi_{n+1} - h.c.]$$

$$\frac{g^2 a_L}{2M} = \frac{g_{b,0} - g_{b,2}}{6\Delta} U_{\mathbf{n}}^b$$

$$\frac{1}{2a_L\sqrt{\ell(\ell+1)}M} = \frac{\sqrt{2}(g_{bf,3/2} - g_{bf,1/2})}{6\Delta} U_{\mathbf{n}}^{bf}$$

Numbers

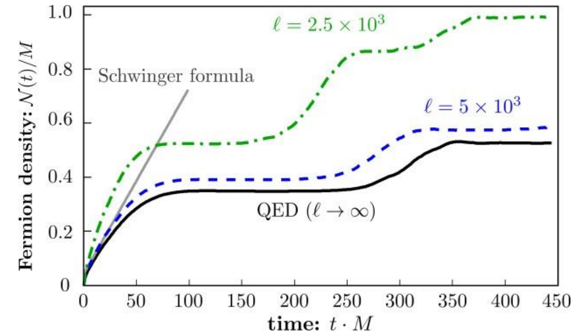
- Numbers of Bosons per Links $N_B \approx 100$
- Bose-Bose coupling $\chi_{BB}/h = 0.58\,\mathrm{Hz}$
- Bose-Fermi coupling $\chi_{BF}/h = 0.05\,\mathrm{Hz}$
- Detuning $\Delta/h \approx 10\,\mathrm{Hz}$
- Effective Bose-Fermi Interaction $\chi_{BF}N_B/h \approx 5\,\mathrm{Hz}$

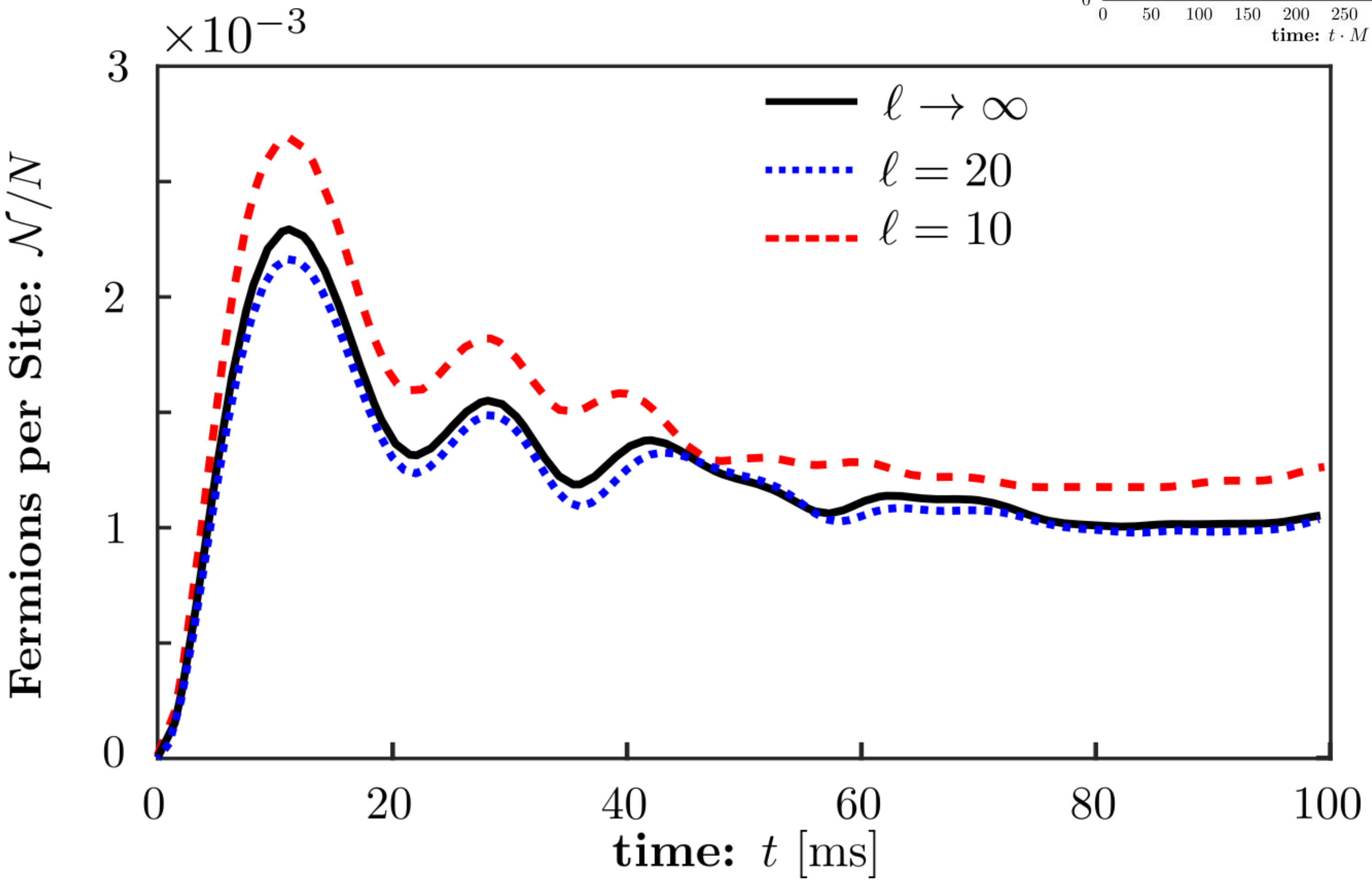
- Boson Tunneling $J_B/h \approx 0.25\,\mathrm{Hz}$
- Fermi Tunneling $J_F/h \approx 0.25\,\mathrm{Hz}$

$$E/E_c \gtrsim 1$$

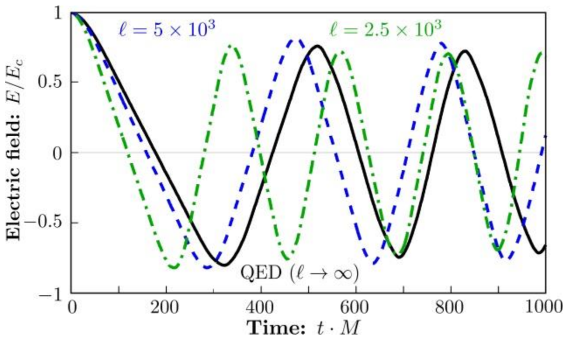
VK et al, arXiv:1608.03480

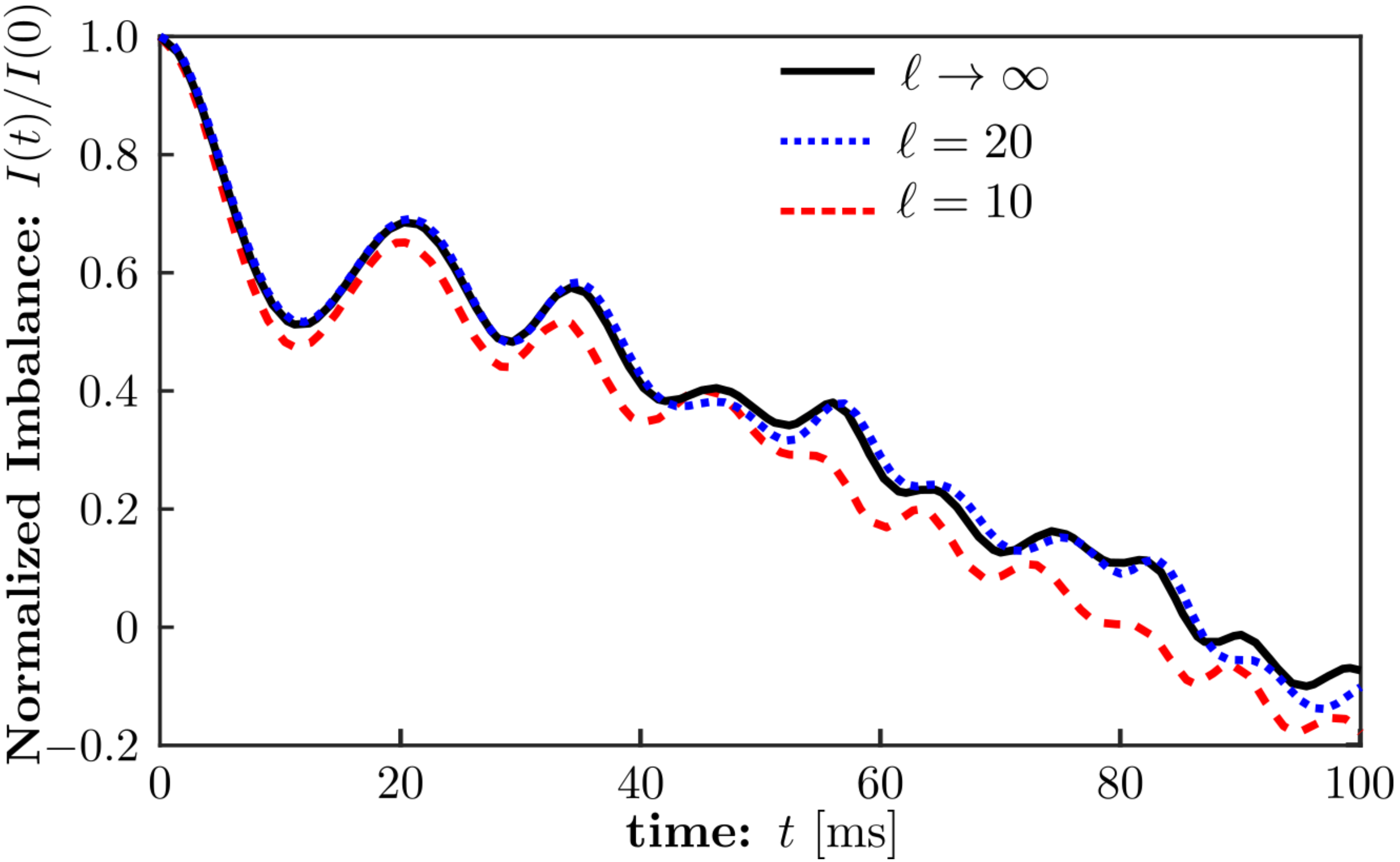
Schwinger Pair Production (i)





Schwinger Pair Production (ii)





Outline

- Experimental Realization of QED in (1+1)

- Schwinger Effect in a cold atom setup

- Conclusion

Conclusion

- Schwinger Effect feasible in cold atom system

To do:

- New Strategies to engineer gauge theories
- Simplify the proposals
- Theoretical predictions