# Dark Side of the Universe and its Observational Signature.

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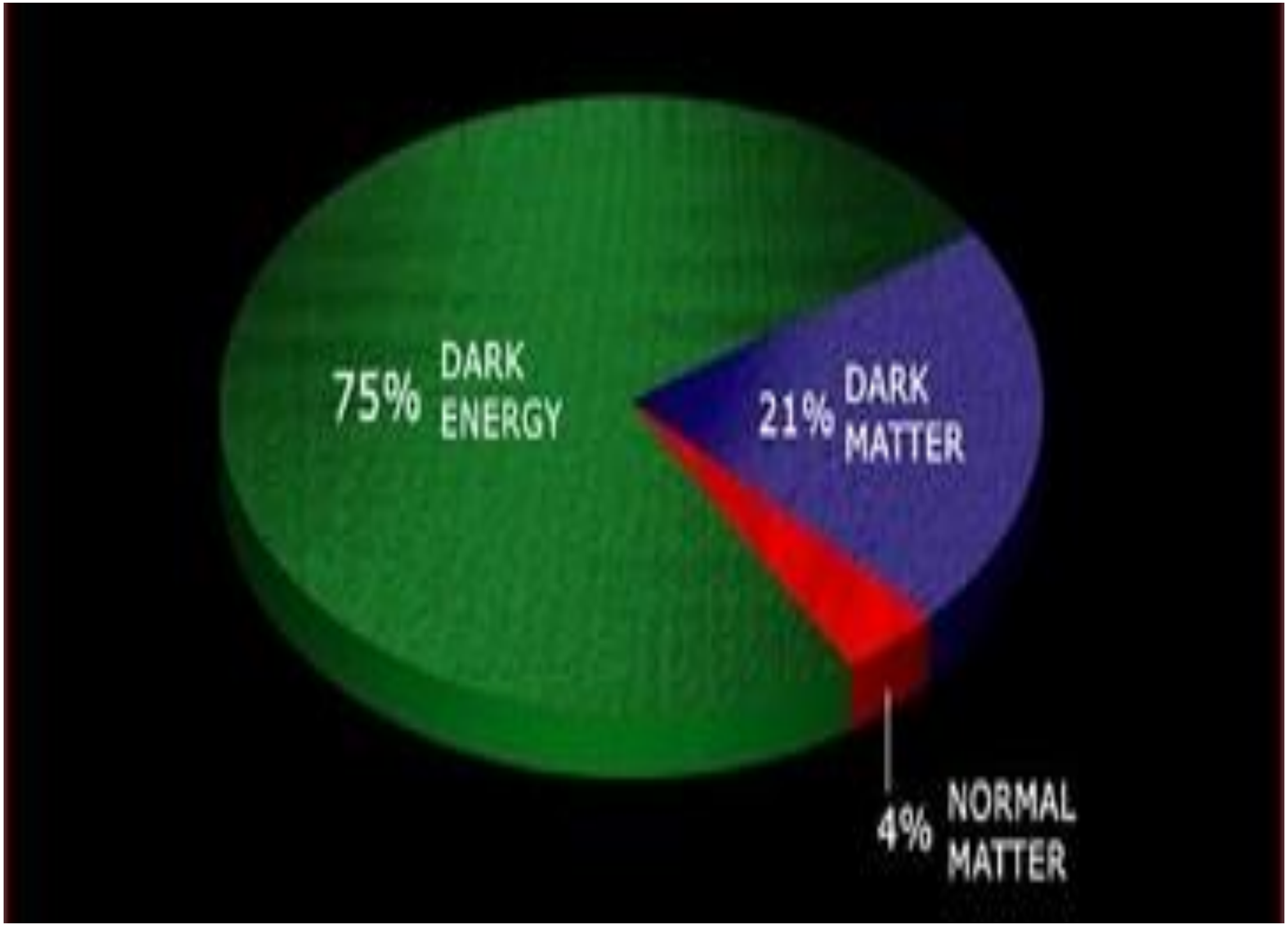
# References

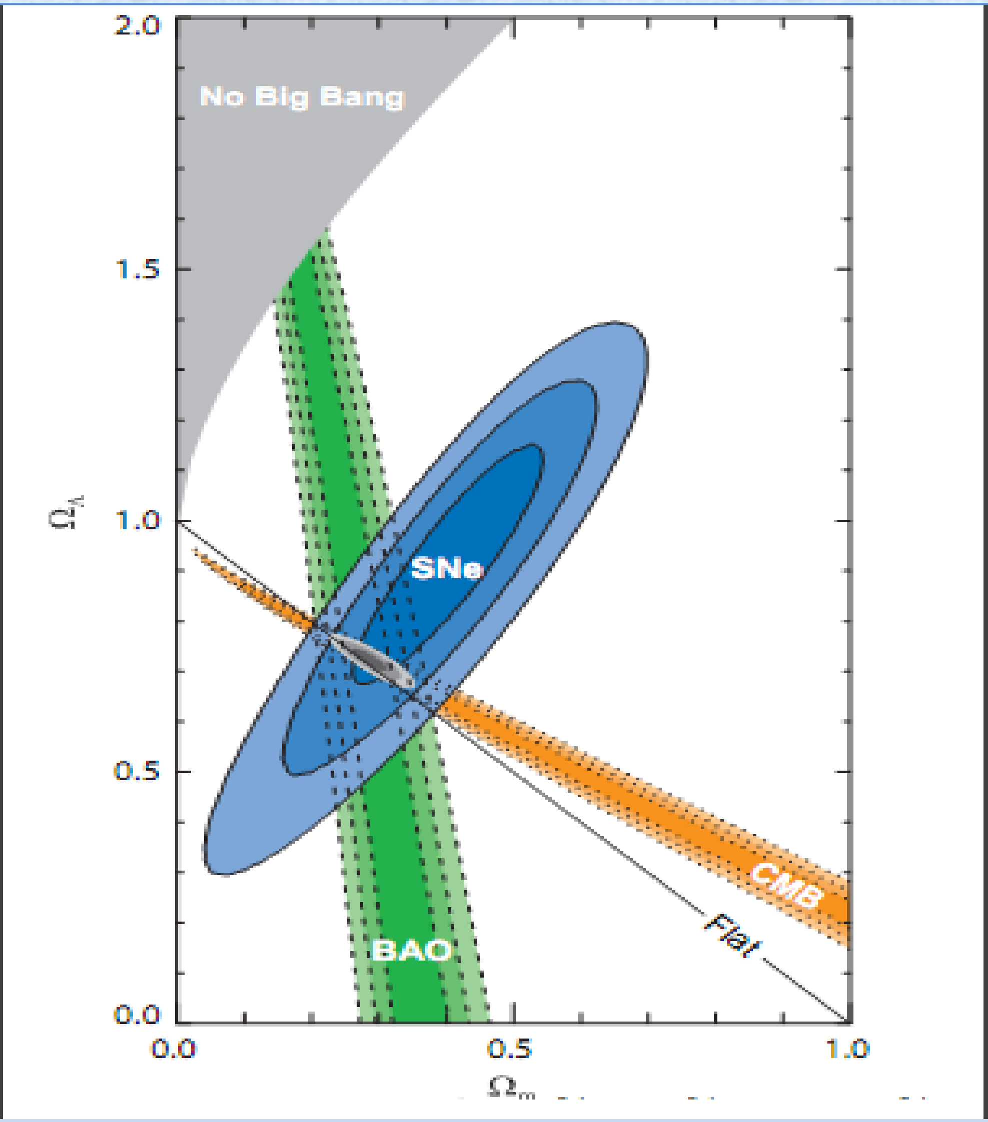
1. Constraining Thawing Quintessence.
Gaveshna Gupta, Subhabrata Majumdar and Anjan A Sen
**Mon. Not. Roy. Astron. Soc. 420, 13091316 (2012)**

2. GCG Parameterization for Growth Function and Current Constraints.
Gaveshna Gupta, Somasri Sen and Anjan A Sen
***JCAP*** **04, 028 (2012)**

# Constituents of the Universe



# Observational Evidence




Picture credit to M Kowalski et al. (Supernova Cosmology Project)

# The physics of accelerated expansion

General relativity and the Cosmological principle give the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}\left(\rho c^2 + 3p\right)$$

In conditions where negative pressure dominates, an accelerated period of expansion may be expected…. " *Dark energy* "

# Cosmological Constant

In the FRW background the modified Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

gives,

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} + \frac{\Lambda}{3}$$

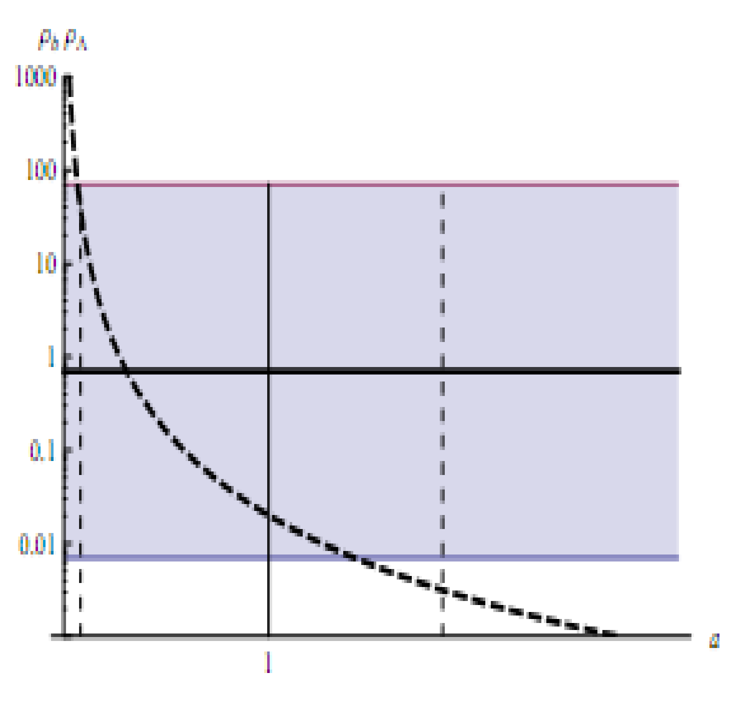
$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

# The Cosmological constant $\Lambda$ is the simplest candidate for Dark Energy.

Fine tuning problem.

$$\frac{\rho_{obs}}{\rho_{th}} \approx 10^{-120}$$

Cosmic coincidence problem.



# Alternative theories to explain the late time acceleration.

$$G_{\mu\upsilon} = T_{\mu\upsilon}$$

**Modify the gravity sector.**

- DGP model.
- Cardassian model.
- f(R) theories.
- Galileon models.

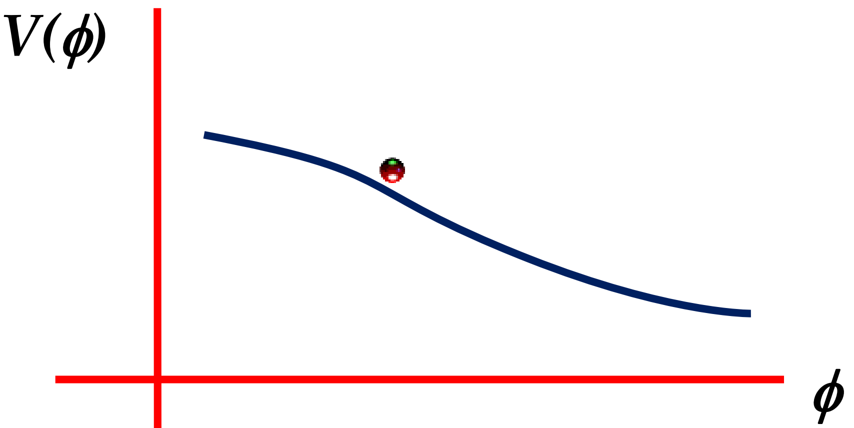
**Modify the matter sector.**

- Quintessence.
- Tachyon.
- Phantom.
- K- essence… etc.

# Dynamical Dark Energy (Quintessence)

$$\rho = \dot{\phi}^2 / 2 + V(\phi)$$

$$p = \dot{\phi}^2 / 2 - V(\phi)$$




$$w = p / \rho = \frac{\dot{\phi}^2 / 2 - V(\phi)}{\dot{\phi}^2 / 2 + V(\phi)} \geq -1$$

[Wetterich; Peebles & Ratra;
Caldwell, Dave & Steinhardt; etc.]

# Dynamics of Quintessence

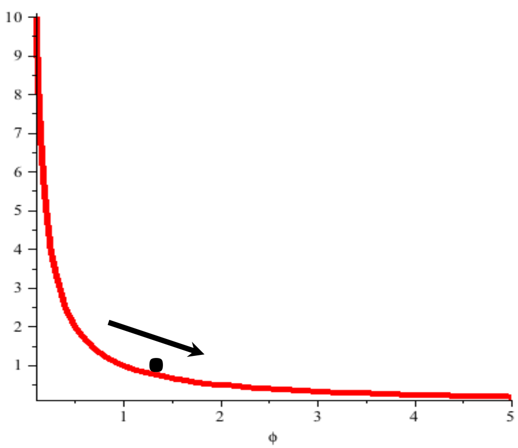
**Equation of motion of scalar field**

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

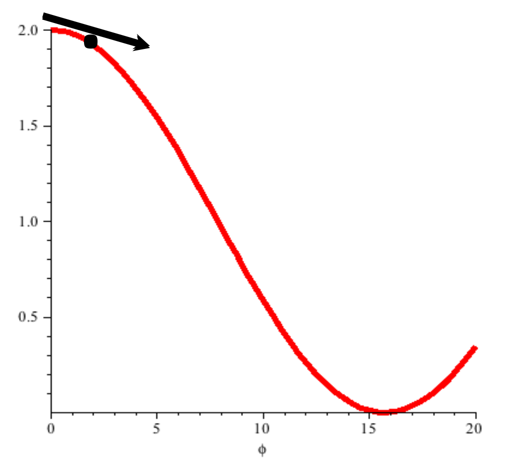
- Driven by steepness of potential.
- Slowed by Hubble friction.

**Broad classification**

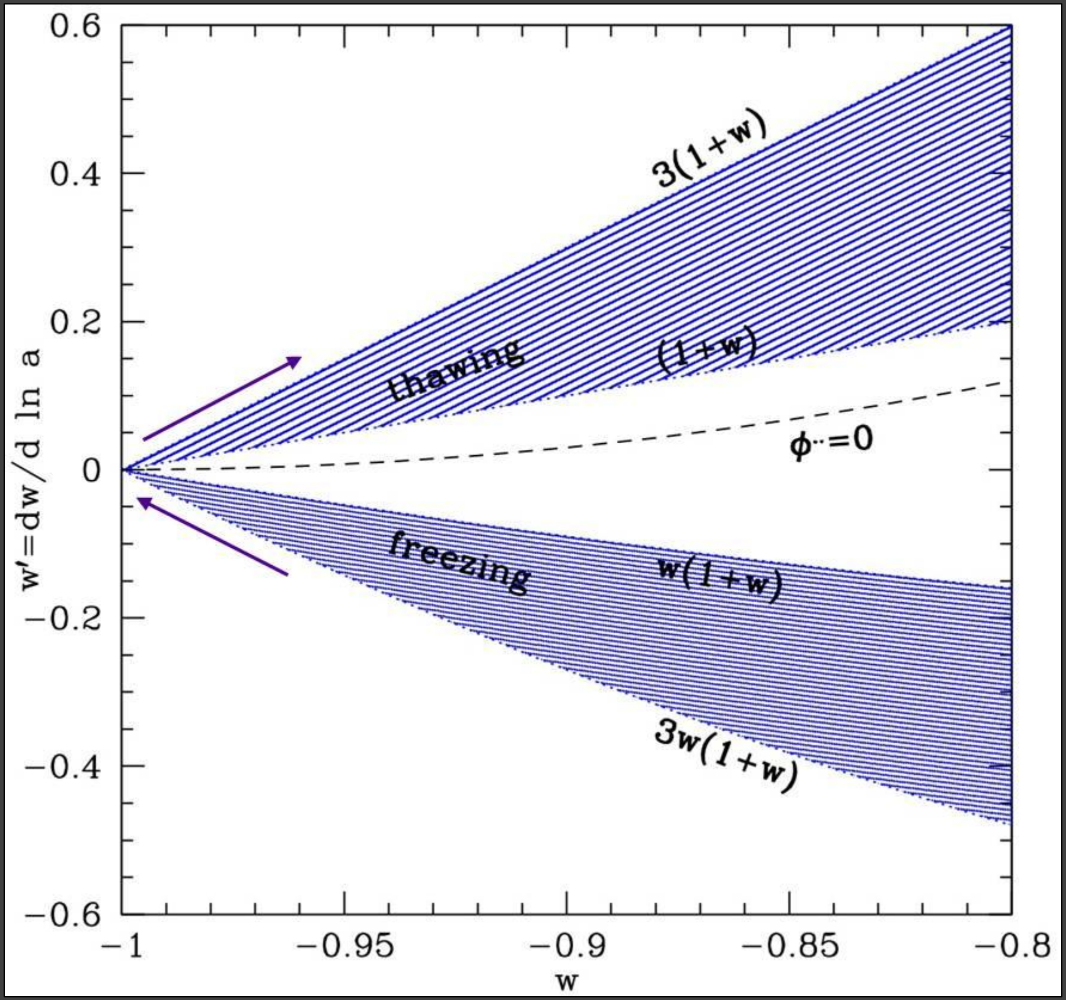
- Field rolls but decelerates as dominates energy.
- Field starts frozen by Hubble drag and then rolls.




Freezers Vs. Thawers



# Limits of Quintessence




[Caldwell & Linder, Phys.Rev.Lett.95:141301,2005 ]

# Thawing scalar field models

Equation of motion.

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Einstein equation.

$$3H^2 = \rho_m + \rho_\phi$$

Assume spatially flat Universe so that

$$\Omega_m + \Omega_\phi = 1$$

# Autonomous system

$$\gamma' = -3\gamma(2-\gamma) + \lambda(2-\gamma)\sqrt{3\gamma\Omega_\phi}$$
$$\Omega_\phi{}' = 3(1-\gamma)\Omega_\phi(1-\Omega_\phi)$$
$$\lambda' = -\sqrt{3}\lambda^2(\Gamma-1)\sqrt{\gamma\Omega_\phi}$$

Here,

$$\gamma = 1 + w_\phi$$

$$\lambda = -\frac{dV}{d\phi}\Big/V$$

$$\Gamma = V\frac{d^2V}{d\phi^2}\Big/\left(\frac{dV}{d\phi}\right)^2$$

[Scherrer & Sen Phys.Rev. D 77 083515]

The Scalar field is **initially frozen.**

$\dot{\phi}_i = 0 \qquad w_\phi \approx -1$

The parameter $\lambda_i$ determines deviation from **initial frozen state.**

## Key Features

Various power law potentials considered for which **$\Gamma$ is a constant.**

**No slow roll conditions** are assumed for the potential.

# Statefinder Hierarchy

Taylor expand the scale factor

$$a(t) = a(t_0) + a(t_0)\sum_{n=1}^{\infty} \frac{\alpha_n(t_0)}{n!}\left[H_0(t - t_0)\right]^n$$

Where,

$$\alpha_n = \frac{d^n a}{dt^n} \Big/ \left(aH^n\right)$$

$$S_2 = \alpha_2 + \frac{3}{2}\Omega_m$$

$$S_3 = \alpha_3$$

$$S_4 = \alpha_4 + \frac{3^2}{2}\Omega_m$$

[Arabsalmani & Sahni Phys. Rev D 83 ,043501]

And so on ….

# Concordance Cosmology

$$\Omega_m = \Omega_{m0}\frac{(1+z)^3}{h^2(z)}$$

Also,

$$\Omega_m = \frac{2}{3}(1+q)$$

$$\alpha_2 = 1 - \frac{3}{2}\Omega_m$$

$$\alpha_3 = 1$$

$$\alpha_4 = 1 - \frac{3^2}{2}\Omega_m$$

$$S_2 = \alpha_2 + \frac{3}{2}\Omega_m$$

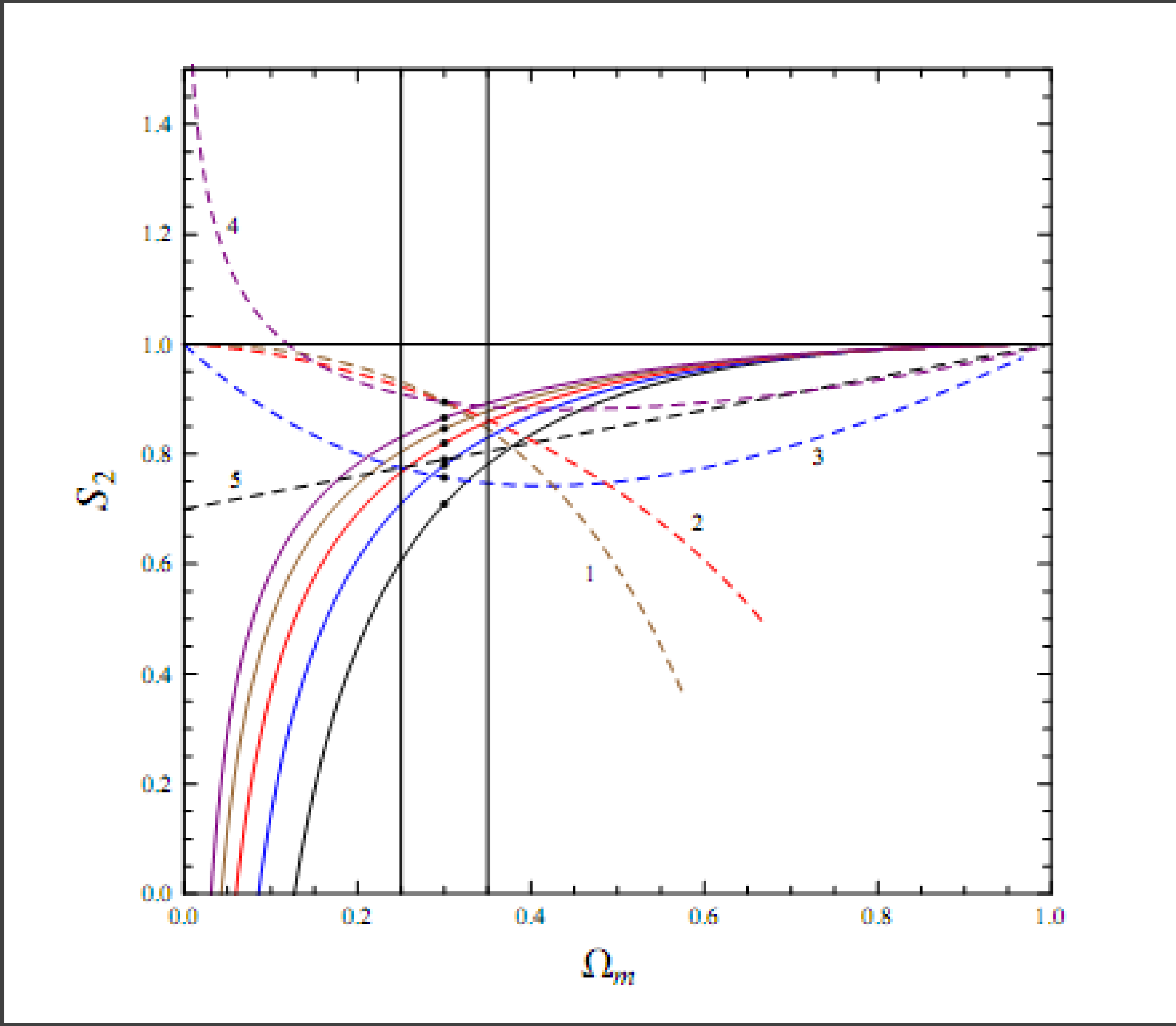
$$S_3 = \alpha_3$$

$$S_4 = \alpha_4 + \frac{3^2}{2}\Omega_m$$

Thus,

$$S_n \big|_{\Lambda CDM} = 1$$

# Plot of $S_2$ and $\Omega_m$



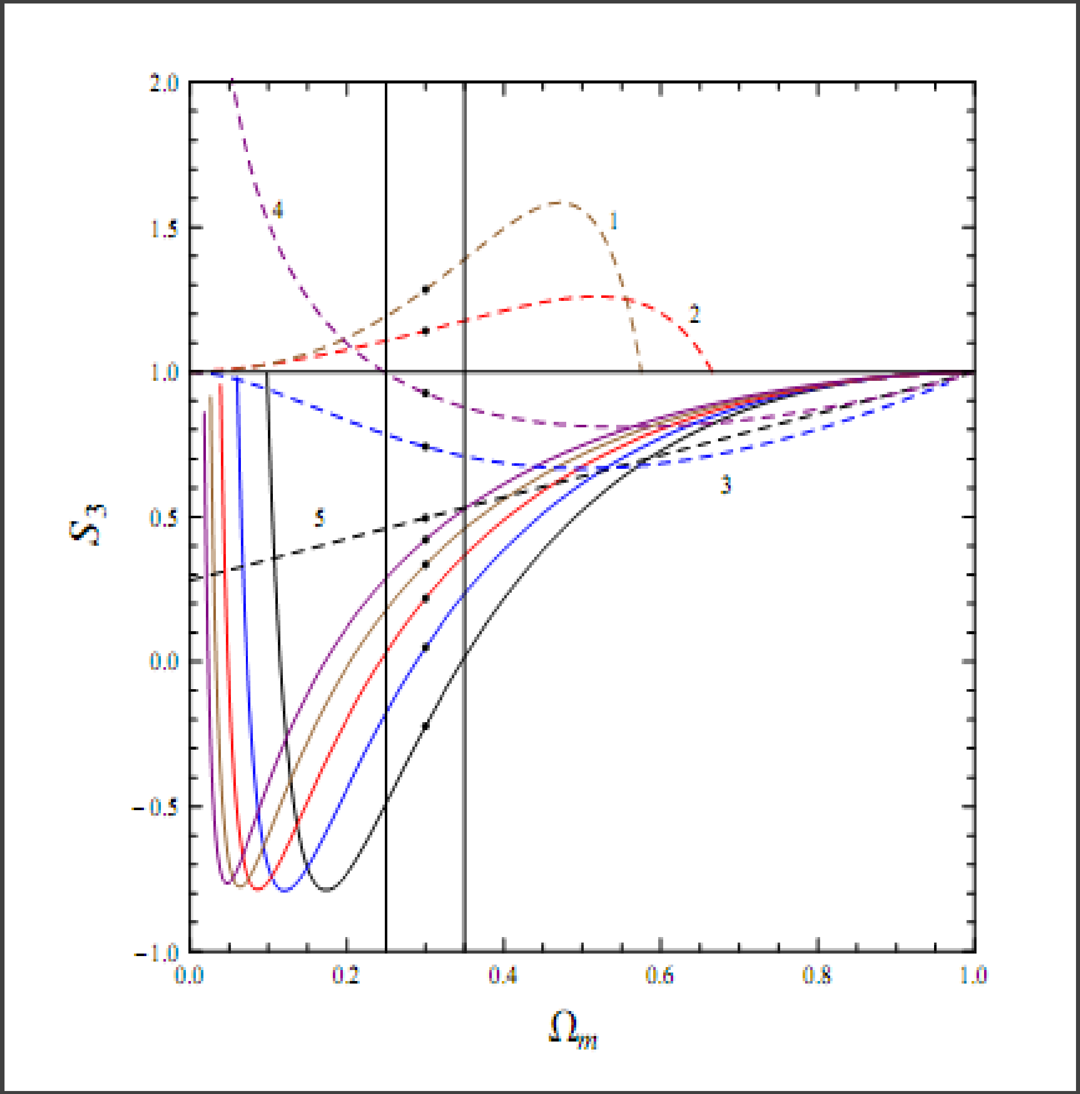

$\Omega_{m0} = 0.3$

**Solid lines -Thawing models** (for Γ = 0, 0.5, 1, 1.5 and 2)

**Dashed lines** - 1» CG, 2»GCG, 3»DGP, 4»CPL, 5»Const w.

**Dots** – Present day value

# Plot of $S_3$ and $\Omega_m$

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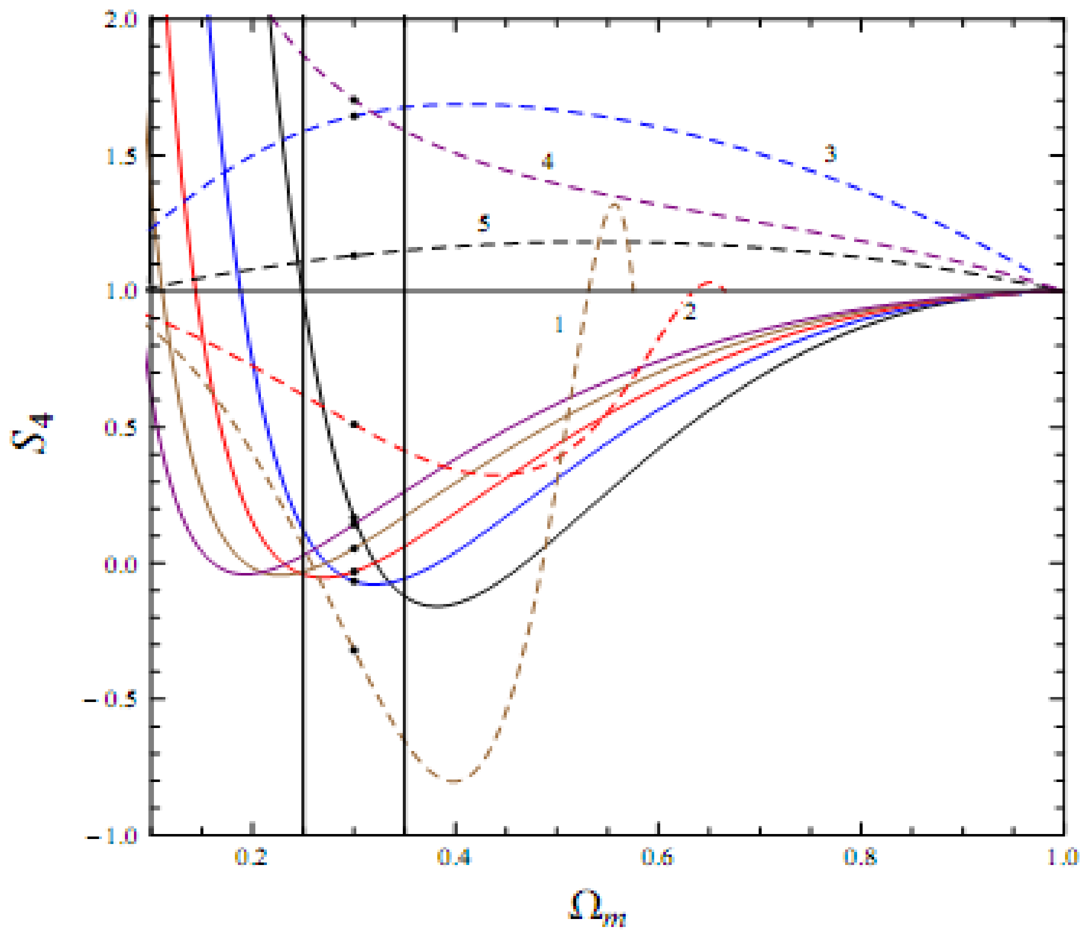
**Dots** – Present day value

# Plot of $S_4$ and $\Omega_m$




$\Omega_{m0} = 0.3$

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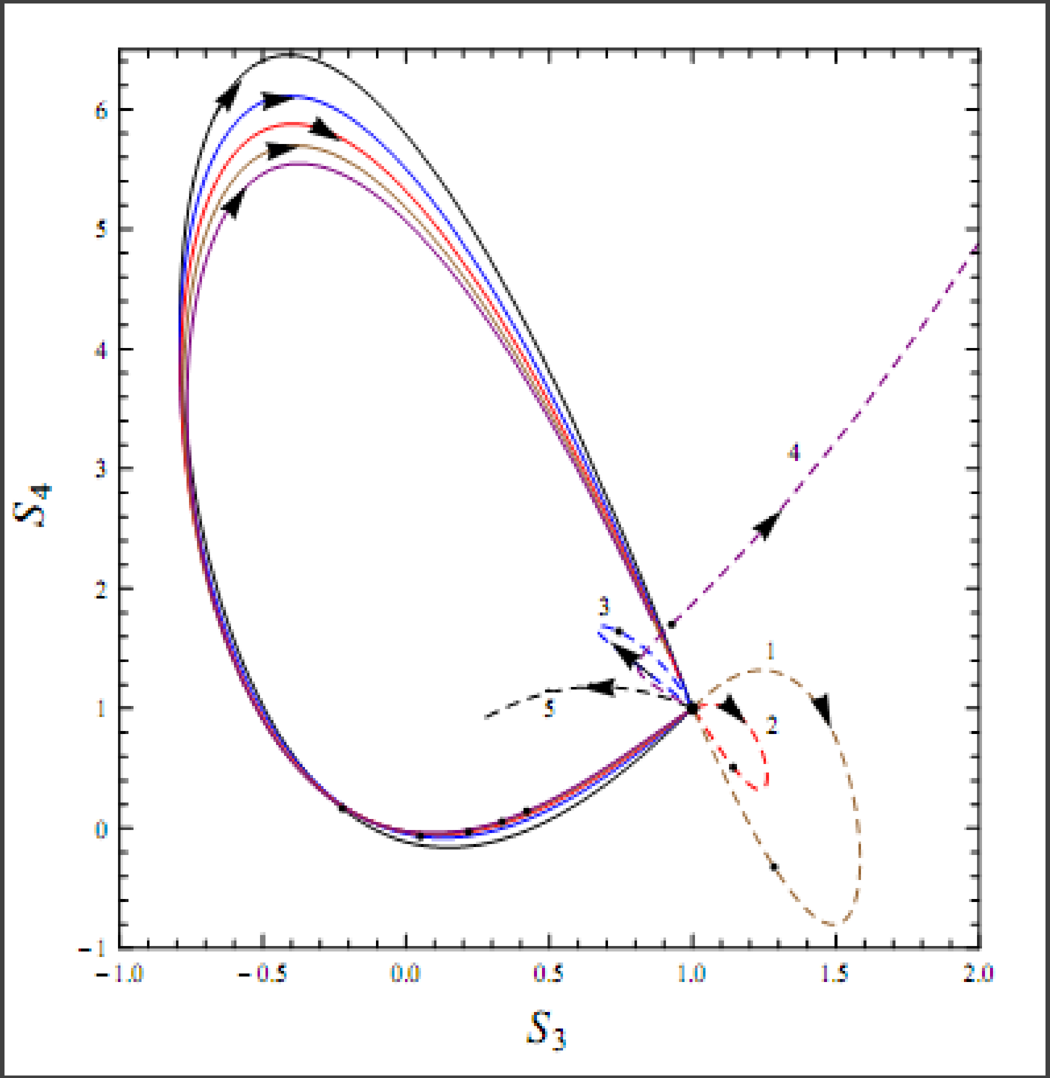
**Dashed lines** - 1» CG, 2»GCG, 3»DGP, 4»CPL, 5»Const w.

**Dots** – Present day value

# Plot of $S_4$ and $S_3$




$\Omega_{m0} = 0.3$

**Solid lines -Thawing models** (for Γ = 0, 0.5, 1, 1.5 and 2)

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**Dots** – Present day value

# W-W’ Phase Plane for Thawing Models.

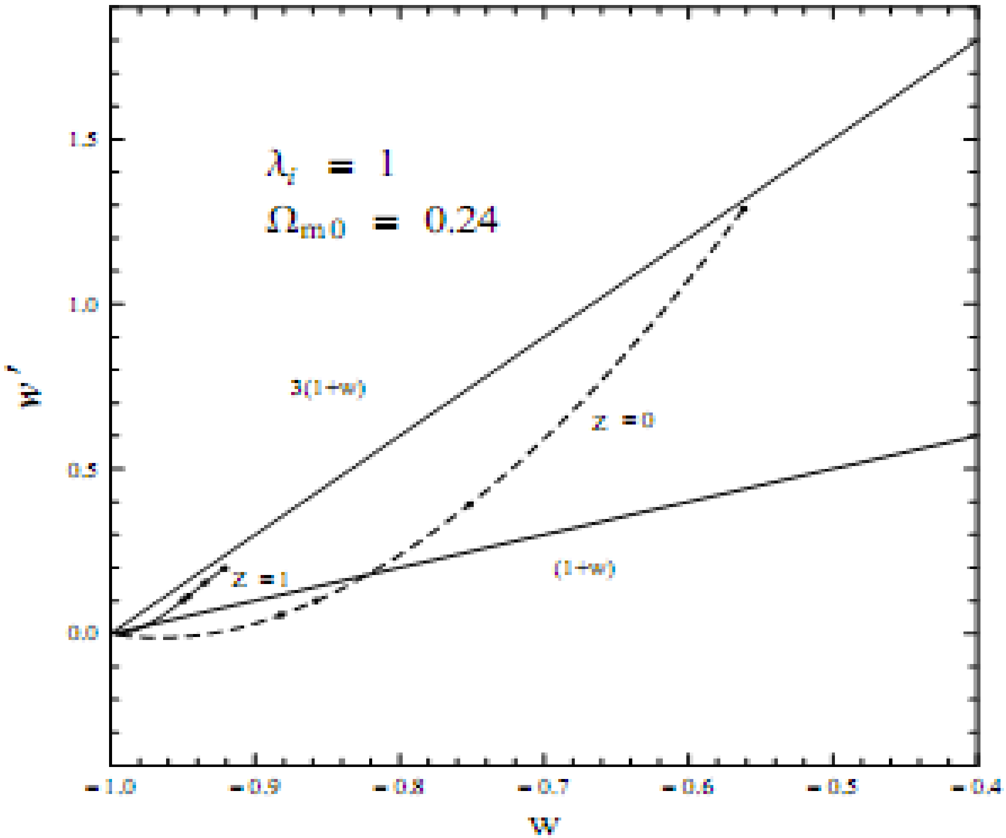
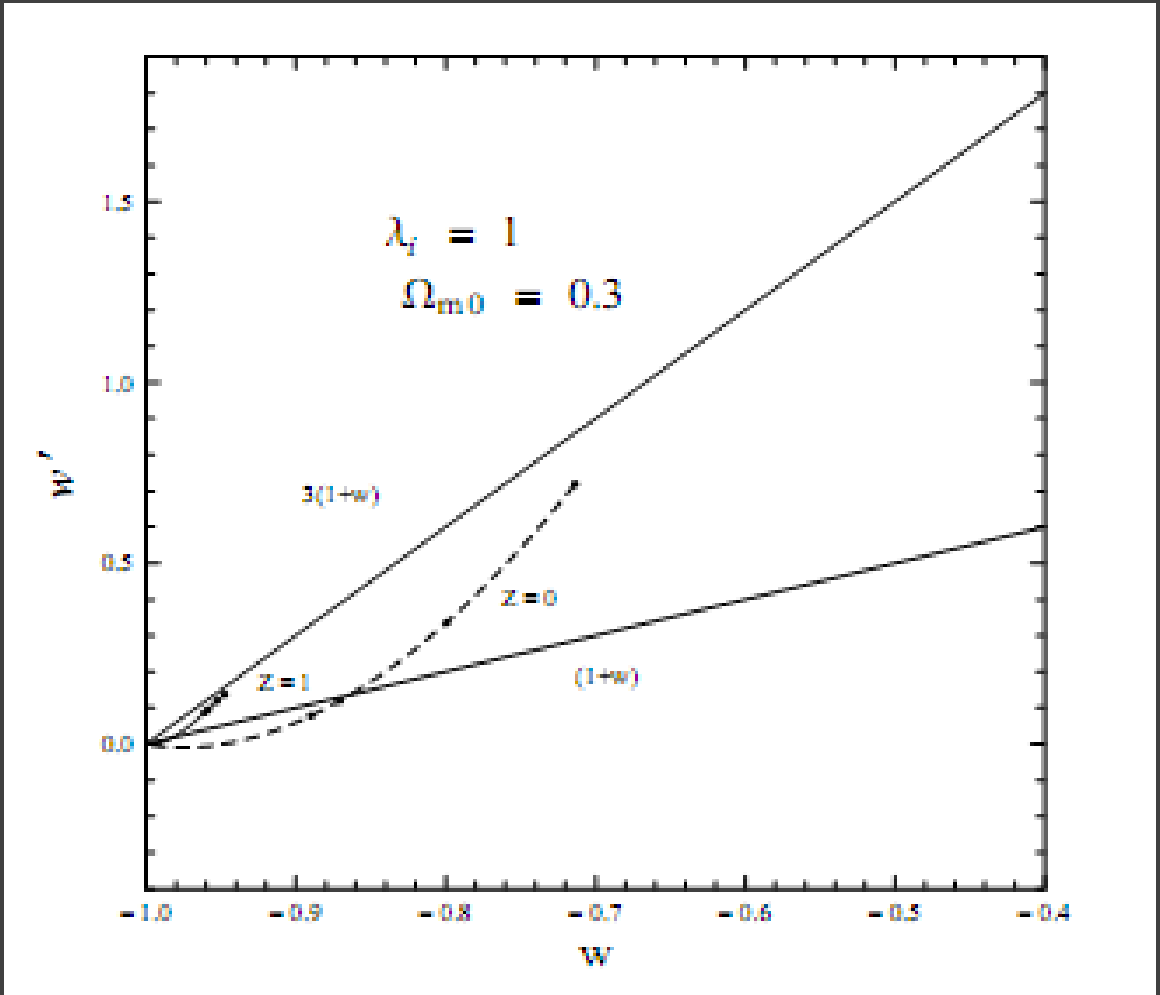



**Figure 5.** The panel shows the dynamics of scalar field in $w - w'$ plane at redshifts $z = 0$ *(solid)*and $z = 1$ *(dashed)*.The dots *(black)* represents the potentials $V = \phi, \phi^2, \phi^{-2}, \phi^{-1}$ (from top to bottom, for each z)respectively.

# W-W’ Phase Plane for Thawing Models.



# Observational Constraints

- With the assumption of a flat Universe we use the latest observational data.
- **Type IA Supernovae** Union 2 compilation.
- **BAO** measurement from SDSS.
- **CMBR** measurement by WMAP 7.
- **H(z)** data from HST key project and Stern et.al
- **Simulated dataset** based on upcoming JDEM SN survey.

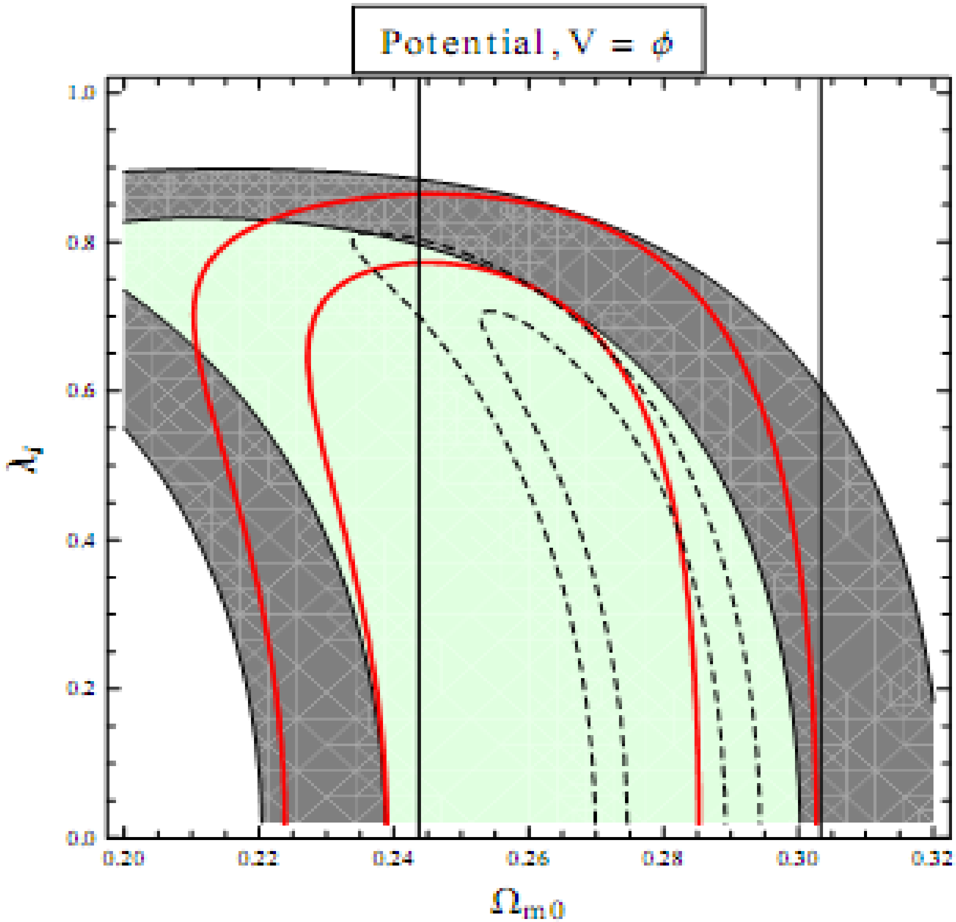



**Figure 7.** The $1\sigma$ and $2\sigma$ confidence contours in $\lambda i$ - $\Omega_{m0}$ plane for $\Gamma = 0$.The shaded regions are constraints from SN+BAO data while the thick lines are constraints from SN+BAO+CMB+H(z) data respectively. The dashed lines are for the simulated JDEM data. The two vertical lines represent the WMAP7 bound on $\Omega_{m0}$

**Shaded region** – Sn+Bao

**Solid lines** (red) - Sn+Bao+CMB+Hubble

**Dashed lines** – JDEM

**Vertical lines** – WMAP7 bound.

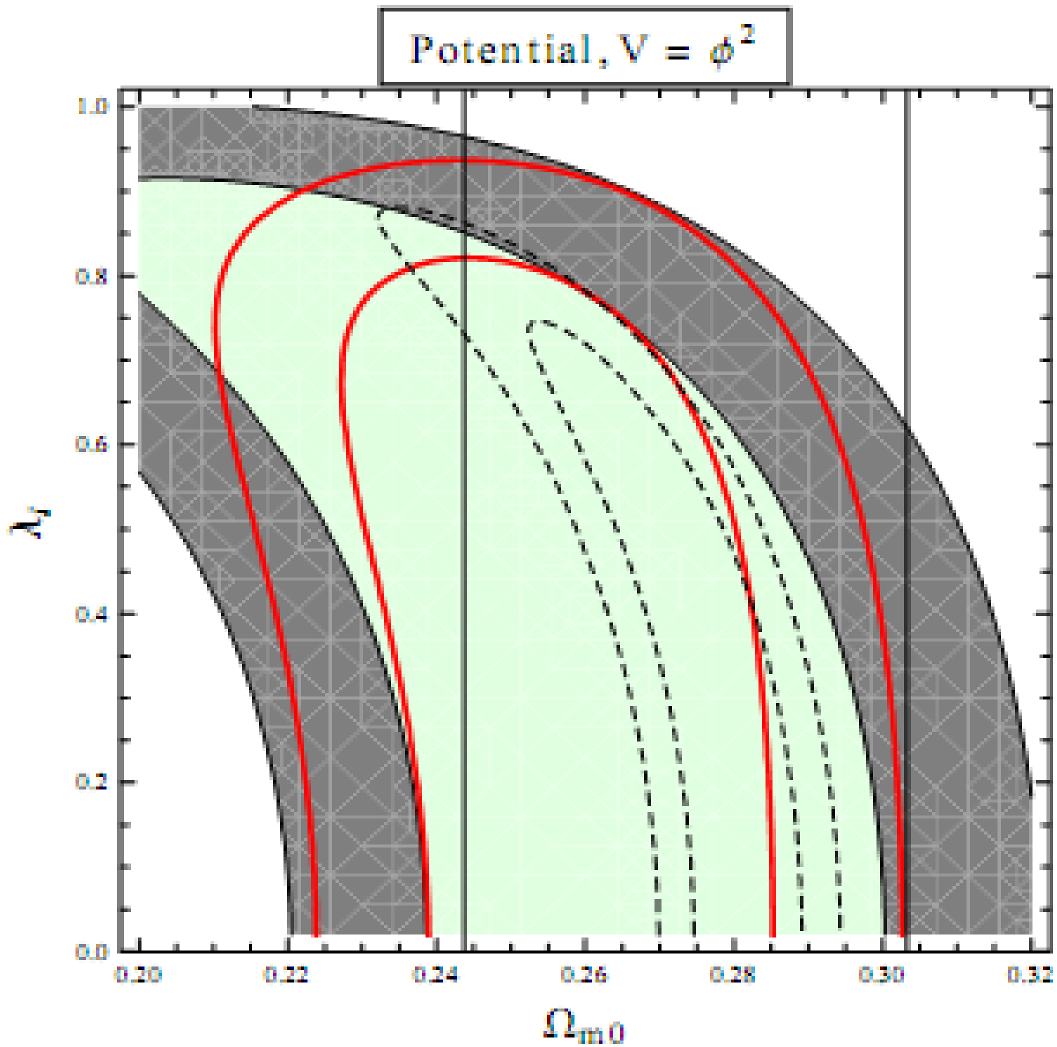


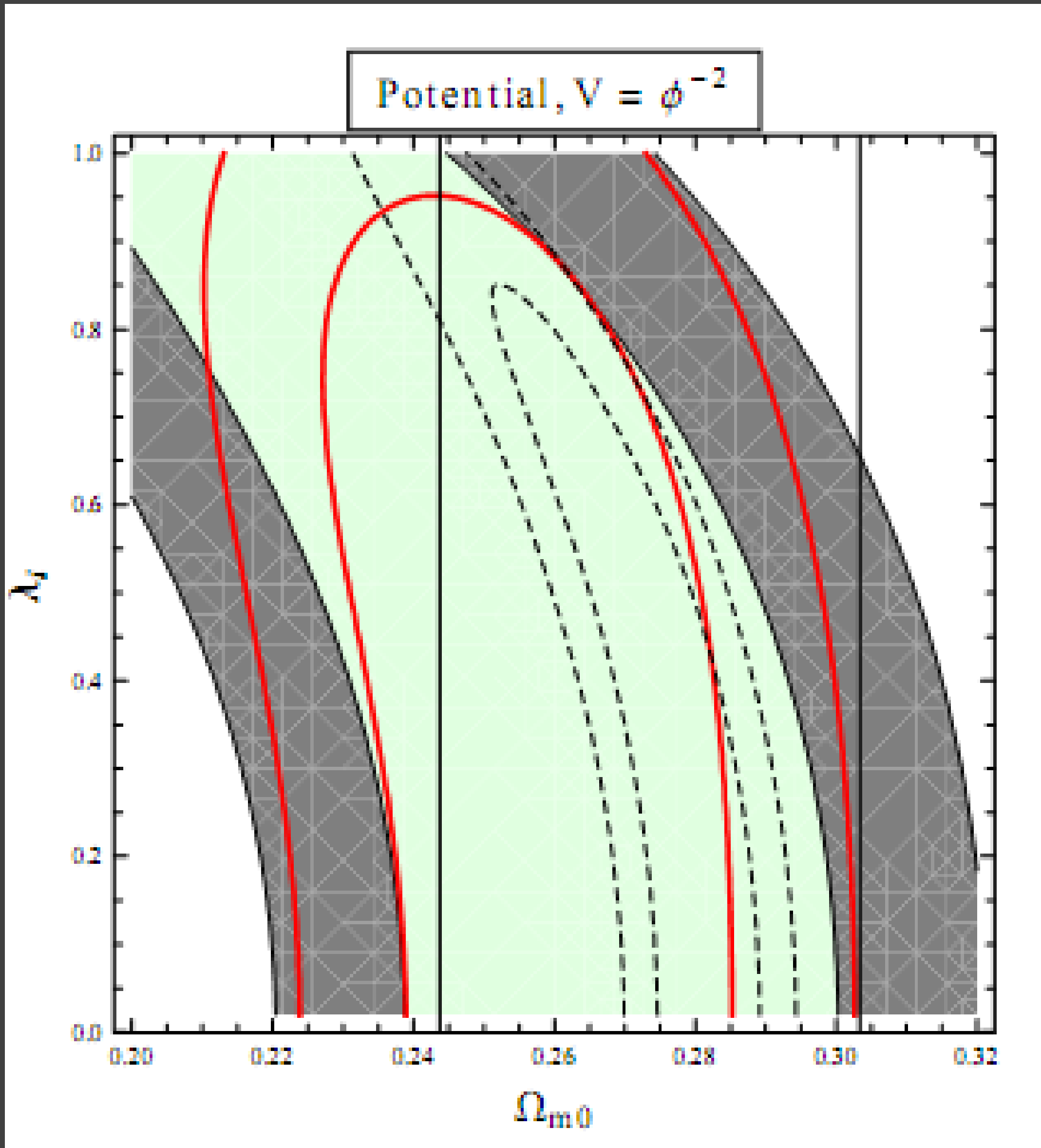

Figure 8. Same as Figure 7, but with $\Gamma = 0.5$.

**Shaded region** – Sn+Bao

**Solid lines** (red) - Sn+Bao+CMB+Hubble

**Dashed lines** – JDEM

**Vertical lines** – WMAP7 bound.

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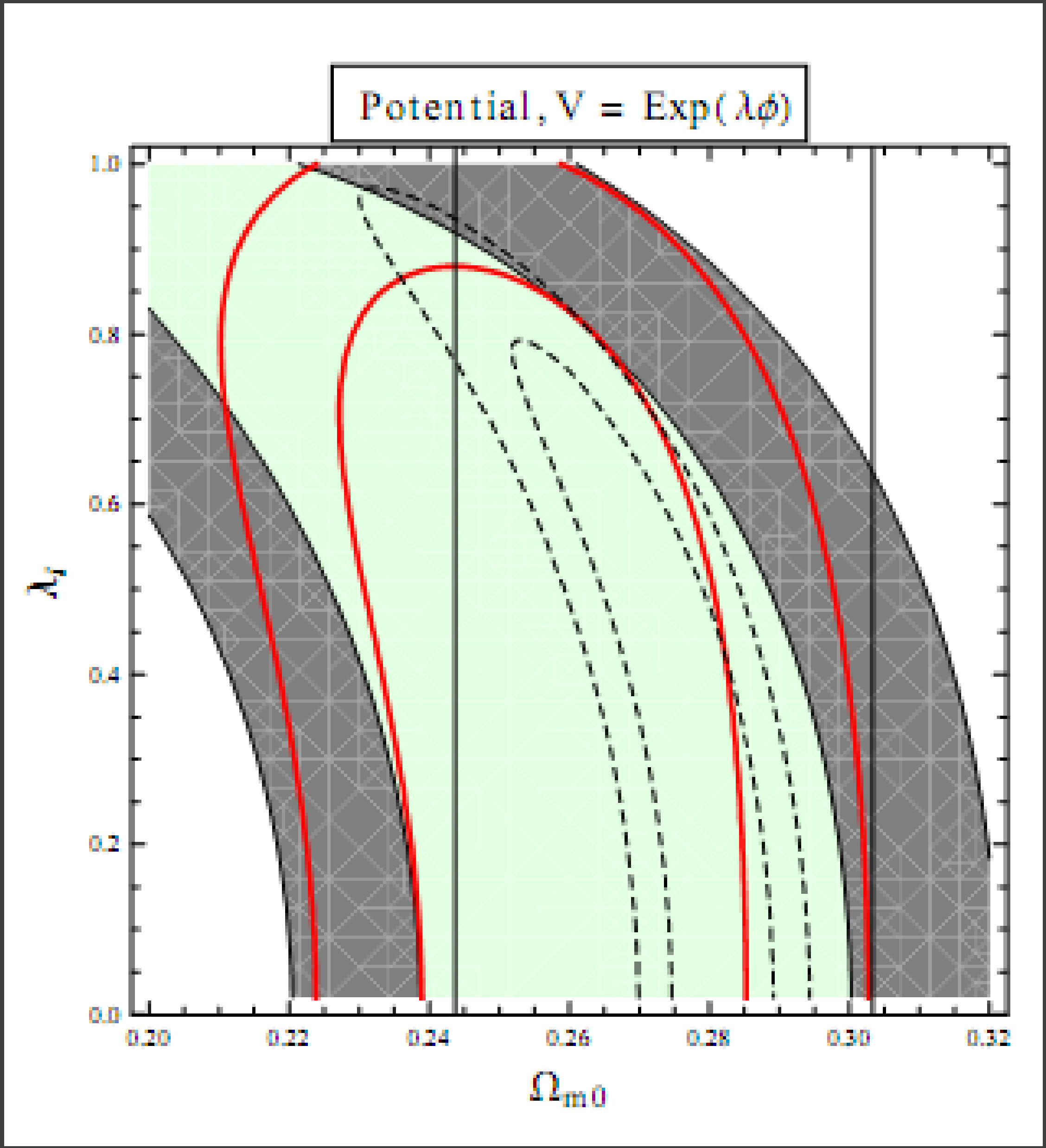


**Shaded region** – Sn+Bao

**Solid lines** (red) - Sn+Bao+CMB+Hubble

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**Vertical lines** – WMAP7 bound.

# Effectiveness of simulated data in distinguishing DE models using Statefinder Hierarchies.

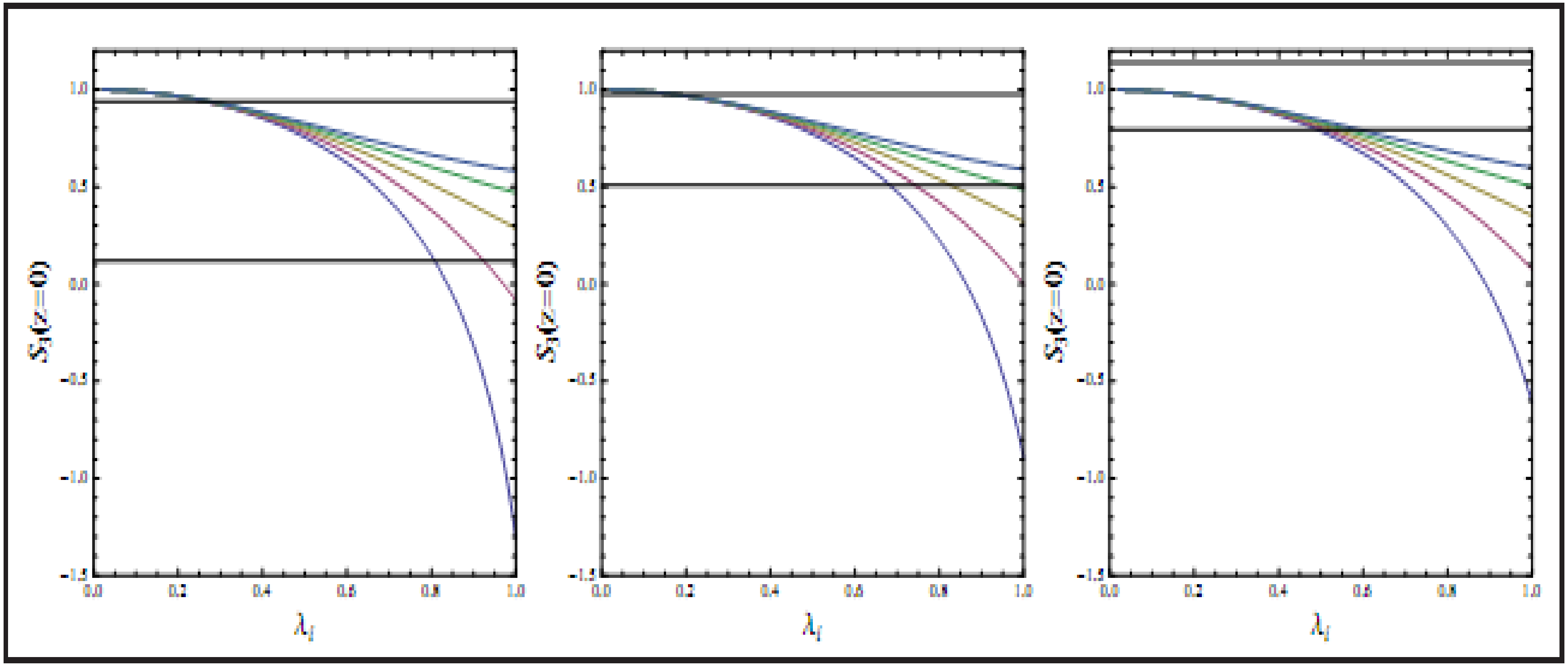



**Figure 11**: The behaviour of $S_3$ at present ($z = 0$) as a funtion of $\lambda_i$ for different thawing potentials. The lines represent $\Gamma = 0, 0.5, 1, 1.5, 2$ from bottom to top. $\Omega_{m0} = 0.24, 0.26, 0.28$ (from left to right) respectively. The horizontal lines represent the bound on $S_3$ at $z = 0$ due to the JDEM simulated data.

# Results

Thawing models described by 3 parameters.

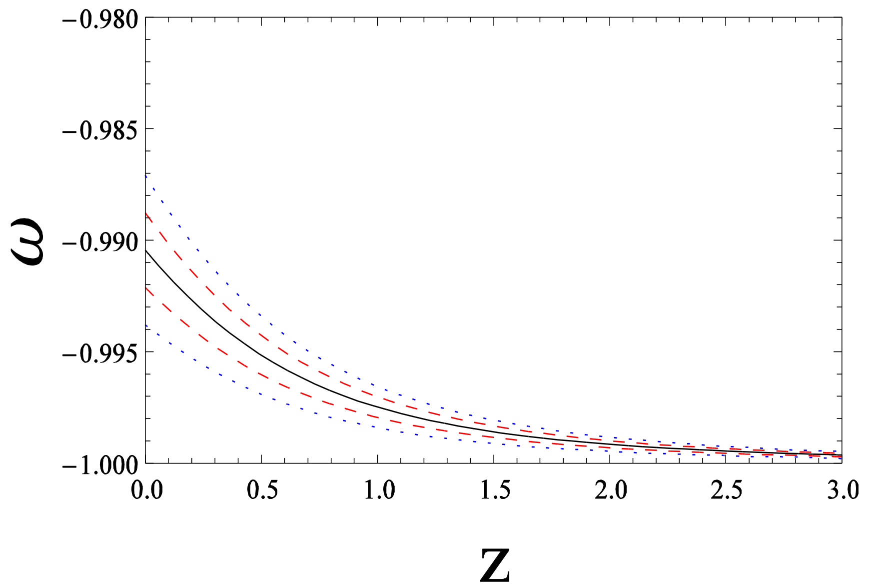
| $\lambda_i$ | $\Gamma$ | $\Omega_{m0}$ |
|---|---|---|
| • Describes deviation of thawing model from ΛCDM.<br>• We get its upper bound. | • Consider WMAP 7 bound current data always allow ΛCDM for all potentials.<br>$(\lambda_i = 0)$ | • Using simulated data like JDEM we get lower bound for $\lambda_i$ different from zero for some values of $\Omega_{m0}$ |

# Quintessence in String Theory

In a recent work by Panda, Sumitomo and Trivedi (**PRD, 83 (2011) 083506)** a model of quintessence is constructed in String Theory.

It is interesting to note that the potential in this set up comes out to be linear

$$V(\phi) = \frac{\mu^4}{f_a}\phi$$



# Alternative theories to explain the late time acceleration.

$$G_{\mu\upsilon} = T_{\mu\upsilon}$$

Modify the gravity sector.

- DGP model.
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- Galileon models.

Modify the matter sector.

- Quintessence.
- Tachyon.
- Phantom.
- K- essence… etc.

# K - Essence

Fields that have non canonical Kinetic term

$$S = \int d^4x \sqrt{-g}\, L(\phi, X)$$

$$X = \frac{1}{2}\partial^{\mu}\phi\partial_{\mu}\phi$$

The most used form of L(φ,X) is of Dirac-Born-Infeld (DBI) type

$$L(\phi, X) = -V(\phi)\sqrt{1-X}$$

The energy density and pressure for the Tachyon Field

$$\rho_{\phi} = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}$$

$$p_{\phi} = -V(\phi)\sqrt{1-\dot{\phi}^2}$$

The equation of motion for the field is

$$\frac{\ddot{\phi}}{\sqrt{1-\dot{\phi}^2}} + 3H\dot{\phi} + \frac{V'}{V} = 0$$

# Generalized Chaplygin Gas

The Action $L = -A\sqrt{1 - X}$ where $X = \frac{1}{2}\partial^{\mu}\phi\partial_{\mu}\phi$

In a FRW universe it gives EoS of a perfect fluid

$$p = \frac{-A}{\rho}$$

A more complicated k- essence action

$$L = -A^{\frac{1}{(1+\beta)}}\left[1 - X^{\frac{(1+\beta)}{2\beta}}\right]^{\frac{\beta}{(1+\beta)}}$$

EoS $p = \frac{-A}{\rho^{\beta}}$

β = 0 is ΛCDM

# The energy density and EoS of GCG are

$$\rho_{gcg} = \rho_{gcg0}\left[A_s + (1 - A_s)(1+z)^{3(1+\beta)}\right]^{1/(1+\beta)}$$

$$w_{gcg} = -\frac{A_s}{A_s + (1 - A_s)(1+z)^{3(1+\beta)}}$$

Where, $A_s = \dfrac{A}{\rho_{gcg0}^{1+\beta}}$

# Features of GCG

At present (z = 0)

$$A_s = -w_{gcg0}$$

$$for\ A_s = 1,\ w_{gcg0} = -1 \Rightarrow C.C$$

$$\beta = 0,\ \rho_{gcg} = \rho_{gcg0}[A_s + (1 - A_s)(1+z)^3] \Rightarrow \Lambda CDM$$

| | **Early time** | **Late time** |
|---|---|---|
| $0 < A_s \leq 1,\ (1+\beta) > 0$ | $\rho \propto a^{-3}$ | Const. |
| $0 < A_s \leq 1,\ (1+\beta) < 0$ | Const. | $\rho \propto a^{-3}$ |

Transient Acceleration

# Growth Function for GCG

The growth rate of large scale structures is given by matter density perturbation in linear regime, governed by equation

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\rho_m\delta = 0$$

$$\delta = \frac{\delta\rho_m}{\rho_m}$$

The background universe

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_{gcg}\right)$$

$$\Omega_m(a) = \frac{\rho_m}{\rho_m + \rho_{gcg}}$$

$$Growth\ Factor: \quad f \equiv \frac{d\ln\delta}{d\ln a}$$

$$3w_{gcg}\Omega_m(1-\Omega_m)\frac{df}{d\Omega_m} + f^2 + f\left[\frac{1}{2} - \frac{3}{2}w_{gcg}(1-\Omega_m(a))\right] = \frac{3}{2}\Omega_m(a).$$

# Effective parameterization for growth factor

$$f = \Omega_m^{\gamma(\Omega_m)}$$

Introduced by Wang & Steinhardt (APJ 1998)

For Dark energy with const. EoS

$$\gamma = \frac{3(w_0 - 1)}{6w_0 - 5} \quad For \;\; C.C, w = -1, \quad \gamma = \frac{6}{11}$$

Using parameterization

$$3w_{gcg}\Omega_m(1-\Omega_m)\ln\Omega_m \frac{d\gamma}{d\Omega_m} - 3w_{gcg}\Omega_m\left(\gamma - \frac{1}{2}\right) + \Omega_m^{\gamma} - \frac{3}{2}\Omega_m^{1-\gamma} + 3w_{gcg}\gamma - \frac{3}{2}w_{gcg} + \frac{1}{2} = 0$$

$$\gamma(\Omega_m) = \gamma\,\Big|_{(\Omega_m = 1)} + (\Omega_m - 1)\frac{d\gamma}{d\Omega_m}\,\Big|_{(\Omega_m = 1)} + O(\Omega_m - 1)^2$$

$$\gamma(\Omega_m) = 3\frac{(1-w_{gcg})}{(5-6w_{gcg})} + (1-\Omega_m)\frac{3(1-w_{gcg})\left(1-\frac{3w_{gcg}}{2}\right)}{125\left(1-\frac{6w_{gcg}}{5}\right)^3}$$

$$w_{gcg} = \frac{-A_s}{\left[A_s + (1-A_s)(1+z)^{3(1+\beta)}\right]}$$

normalized growth function "g" from the numerical solution

$$g(z) \equiv \frac{\delta(z)}{\delta(0)}$$

Approximate normalized growth function using parameterized form

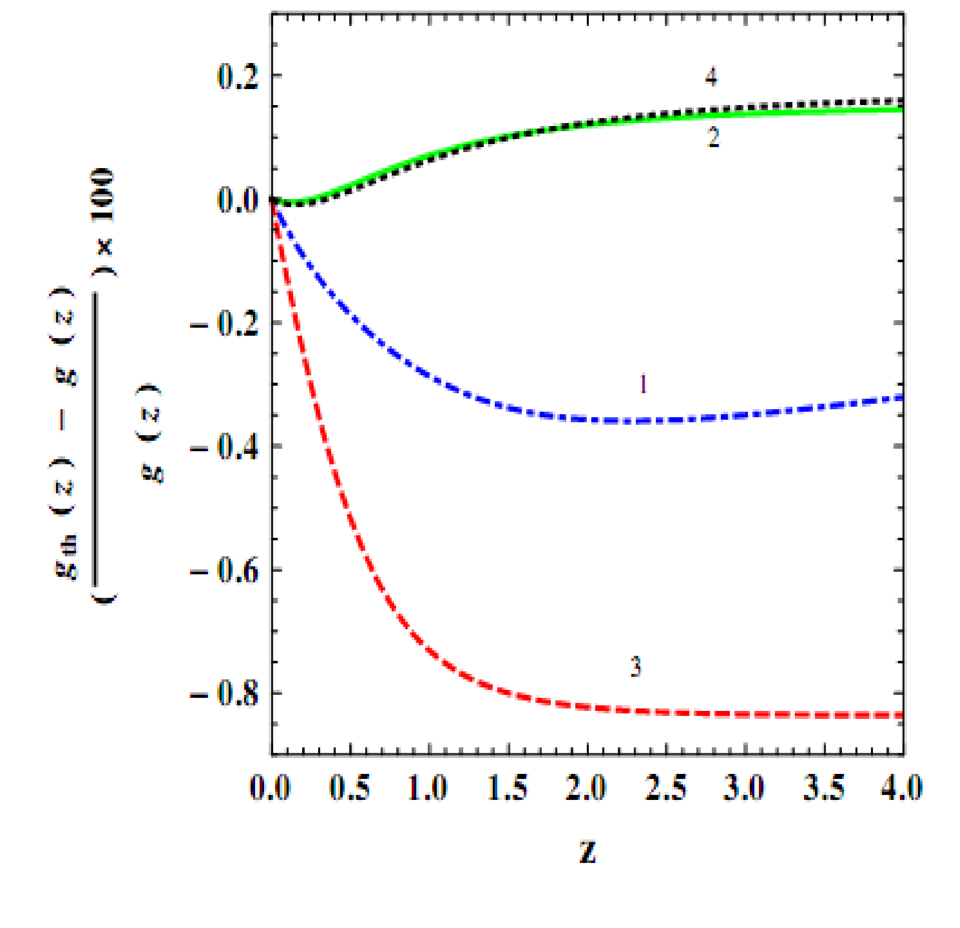
$$g_{th}(z) = \exp\int_1^{\frac{1}{1+z}} \Omega_m(a)^{\gamma}\frac{da}{a}$$

1. $A_s = 0.9, \beta = 0.1$

2. $A_s = 0.9, \beta = -1.05$

3. $A_s = 0.8, \beta = -0.6$

4. $A_s = 0.8, \beta = -1.02$



# Fitting GCG parameterization for $\gamma(\Omega_m)$ to other models

- Dark energy with CPL EoS

$$w_{de}(a) = w_0 + w_a(1-a)$$

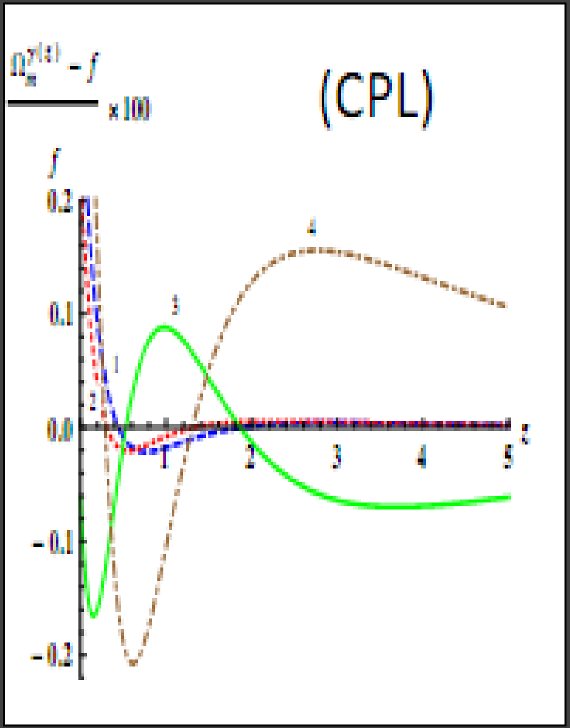
$$\begin{aligned} &w_0 = -1, w_a = 0 \rightarrow C.C \\ &w_0 = w, w_a = 0 \rightarrow \text{const.} EoS \\ &w_0 \neq 0, w_a \neq 0 \rightarrow \text{variable} EoS \end{aligned}$$

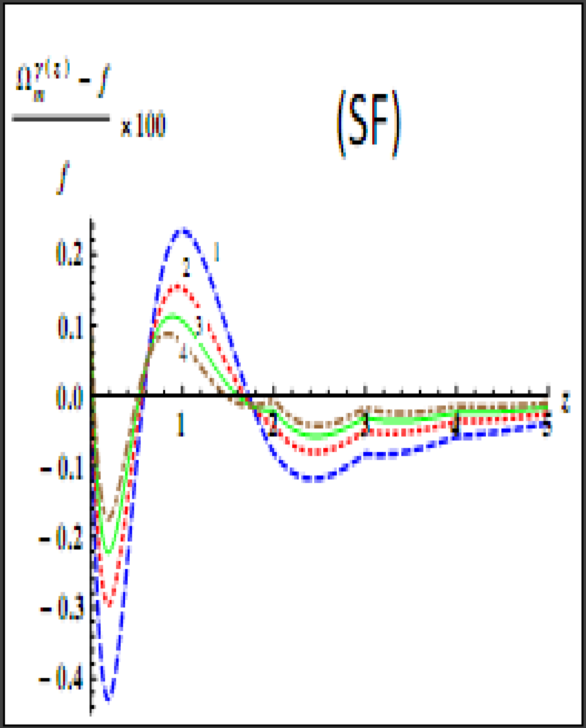
- Ordinary scalar field models

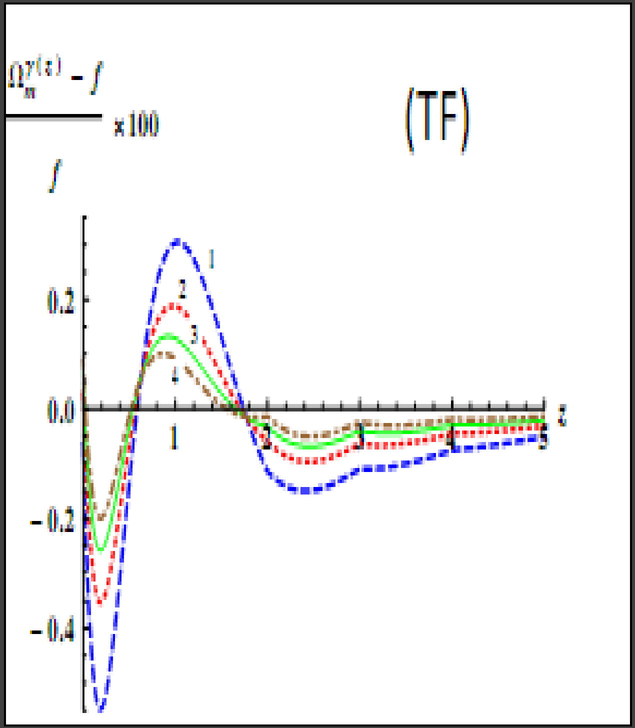
$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \qquad w_\varphi = \frac{\frac{\dot{\phi}^2}{2} - V(\phi)}{\frac{\dot{\phi}^2}{2} + V(\phi)}$$

- Non canonical (tachyon type) field models

$$\ddot{\phi} + 3H\dot{\phi}(1-\dot{\phi}^2) + \frac{V'}{V}(1-\dot{\phi}^2) = 0 \quad w_T = \dot{\phi}^2 - 1$$




1. $w_0 = -0.8,\ w_a = 0$
2. $w_0 = -1,\ w_a = 0$
3. $w_0 = -0.8,\ w_a = -0.3$
4. $w_0 = -1.1,\ w_a = 0.22$

1. $V(\phi) = \phi$
2. $V(\phi) = \phi^2$
3. $V(\phi) = e^{\phi}$
4. $V(\phi) = \phi^{-2}$

# Growth rate data

| z | $f_{obs}$ | σ | Ref. |
|---|---|---|---|
| 0.15 | 0.51 | 0.11 | 2df |
| 0.22 | 0.60 | 0.10 | Wiggle-z |
| 0.32 | 0.654 | 0.18 | 2df-SDS LRG |
| 0.35 | 0.70 | 0.18 | SDSS |
| 0.41 | 0.70 | 0.07 | Wiggle-z |
| 0.55 | 0.75 | 0.18 | 2df-SDSS LRG |
| 0.60 | 0.73 | 0.07 | Wiggle-z |
| 0.77 | 0.91 | 0.36 | GRS |
| 0.78 | 0.70 | 0.08 | Wiggle-z |
| 1.4 | 0.90 | 0.24 | XMM-Newton |
| 3.0 | 1.46 | 0.29 | Ly-alpha in SDSS |

# RMS Mass Fluctuation $\sigma_8(z)$

Another dependable observational probe for the growth function δ(z) is the redshift dependence of the rms mass fluctuation $\sigma_8(z)$. It is defined as ,

$$\sigma^2(R,z) = \int_0^\infty W^2(kR)\Delta^2(k,z)\frac{dk}{k} ,$$

With R = $8h^{-1}Mpc$ and $P_\delta(k,z)$ the mass power spectrum at redshift z.

The function $\sigma_8(z)$ is connected with δ(z) as

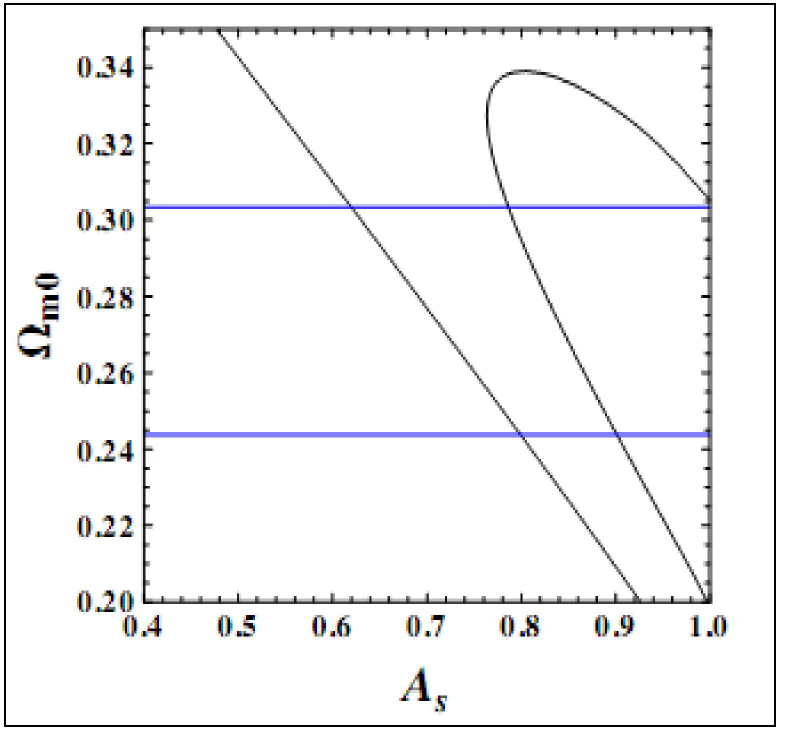
$$\sigma_8(z) = \frac{\delta(z)}{\delta(0)}\sigma_8(z=0)$$

This implies

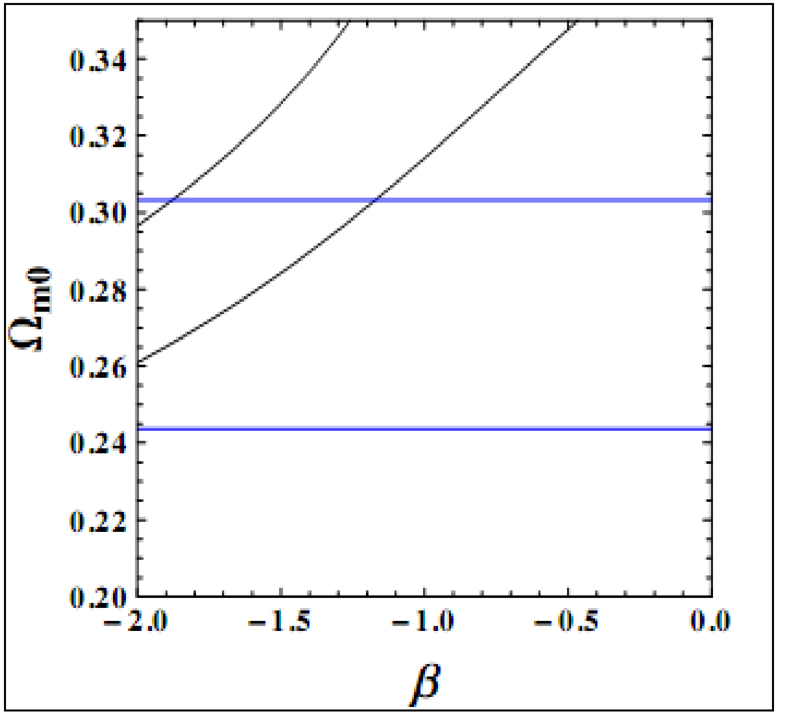
$$s_{th}(z_1, z_2) \equiv \frac{\sigma_8(z_1)}{\sigma_8(z2)} = \frac{\delta(z_1)}{\delta(z_2)} = \frac{e^{\int_1^{\frac{1}{1+z_1}} \Omega_m{}^\gamma(a)\frac{da}{a}}}{e^{\int_1^{\frac{1}{1+z_2}} \Omega_m{}^\gamma(a)\frac{da}{a}}}$$

# Observational Constraints

Marginalized over β




Marginalized over $A_s$

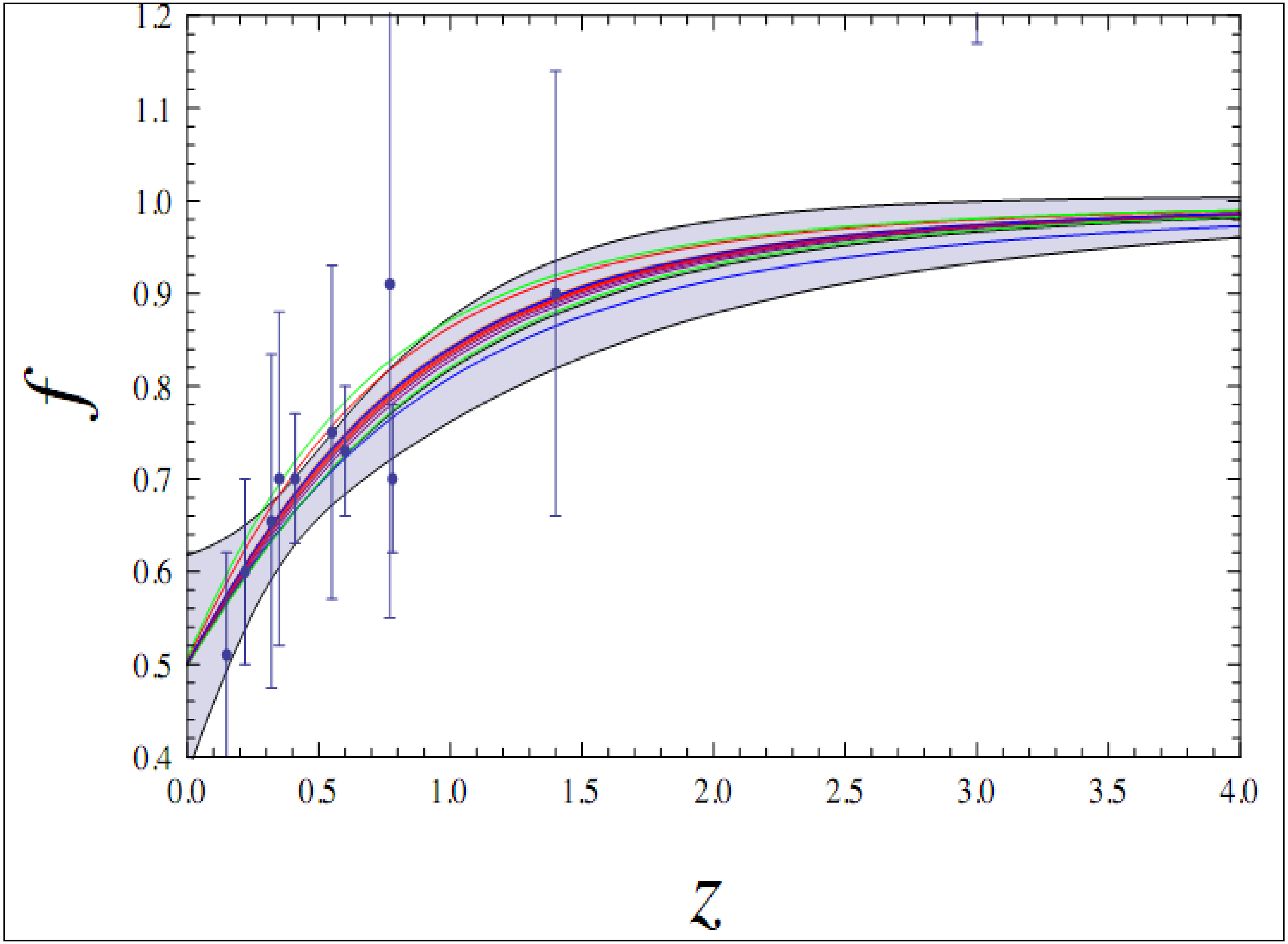



Horizontal lines WMAP 7 bound on $\Omega_{m0}$

At 2σ C.L β≤ -1.1

Transient Acceleration

# The reconstructed Growth function



# Summary

- Probing the higher order $S_n$ different thawing models not only become distinguishable among themselves they also behave significantly different from other dark energy models including ΛCDM.

- Using various observational data we constrain the deviation of the thawing model from ΛCDM.

- With current data the models cannot be distinguished from ΛCDM but with simulated data like JDEM we show for some values of $\Omega_{m0}$, the data may distinguish thawing models with ΛCDM.

- We study the growth of linear matter over density with generalized Chaplygin gas as dark energy candidate.

- We find that the parameterization for growth function fits the actual growth function with less than 1% difference.

- We show that the parameterization fits the growth function of other dark energy models with a great accuracy.

- We use the growth data to constrain the dark energy behaviour using the given parameterization and consequently, get a transient accelerating behaviour.

# Thanks