# Degeneracies between canonical and non-canonical inflation

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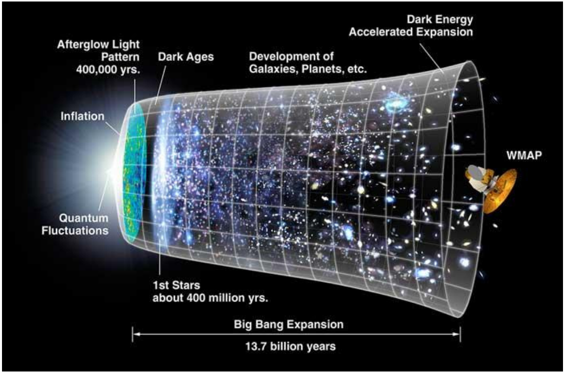
[1211.0070] Gwyn, Rummel and Westphal, “Resonant non-Gaussianity with equilateral properties,”

[1212.4135] Gwyn, Rummel and Westphal, “Relations between canonical and non-canonical inflation,”

# Outline

# Physics in the early universe




- What can we learn about UV physics from cosmology?
- Can we differentiate between models of inflation?

# Motivation

1. Planck: local $f_{NL}$ severely constrained, putting pressure on multifield models
2. $f_{NL}^{equil}$ (NC kinetic terms, varying $c_s$) relatively unconstrained
3. NC kinetic terms are also fairly generic in string theory models of inflation
4. However, there is degeneracy between canonical and noncanonical models *even* at the 3pt function level (Non gaussianities)
5. in [1211.0070] and [1212.4135] we try to understand this degeneracy better...

# UV sensitivity of inflation

The UV-complete theory in which inflation operates is unknown, so we take an EFT approach:

- Effective field theory: corrections from higher-dimensional operators should be suppressed by the cut-off $\Lambda$:

$$\mathcal{L}_{eff} = \mathcal{L}_{relevant} + \sum_n c_n \frac{\mathcal{O}_n}{\Lambda^{n-4}}$$

However, the EFT can be sensitive to the UV physics:

- Eta problem: Mass dimension 6 corrections can spoil the flatness of the potential: $\frac{\mathcal{O}_6}{M_p^2} \to \frac{\mathcal{O}_4}{M_p^2}\phi^2$

$$V_{eff} = V_0 + \frac{1}{2}m_0^2\phi^2 + \frac{\mathcal{O}_4}{M_p^2}\phi^2$$

$$< \mathcal{O}_4 > \sim V_0 \Rightarrow \eta = M_p^2 \frac{V''}{V} \sim \mathcal{O}(1).$$

# UV sensitivity of kinetic terms

In particular, non-canonical kinetic terms arise when massive degrees of freedom are integrated out:

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\rho)^2 + \frac{\rho}{M}(\partial\phi)^2 - \frac{1}{2}M^2\rho^2 \\ \Rightarrow \mathcal{L}_{eff} &= \frac{1}{2}(\partial\phi)^2 + \frac{(\partial\phi)^4}{M^4} \quad \text{for } H \ll M \end{aligned}$$

at energy scales $H << M$. E.g., the DBI action [Silverstein and Tong, 0310221]

$$\begin{aligned} \mathcal{L}_{DBI} &= -\Lambda^4\left[\sqrt{1-\frac{(\partial\phi)^2}{\Lambda^4}} - 1\right] - V(\phi) \\ &\approx \frac{1}{2}(\partial\phi)^2 + \frac{1}{8}\frac{(\partial\phi)^4}{\Lambda^4} + \ldots - V(\phi) \end{aligned}$$

# Non-canonical Lagrangian

A single scalar field coupled minimally to gravity ($X = \frac{1}{2}\dot{\phi}^2$):

$$S = \int d^4x \sqrt{-g_4} \left[ \frac{M_p^2}{2} \mathcal{R}_4 + p(X, \phi) \right]$$

For example,

$$p_{can} = X - V(\phi)$$

$$p_{DBI} = -\frac{1}{f(\phi)} \left( \sqrt{1 - 2f(\phi)X} - 1 \right) - V(\phi)$$

$$p_{Tach} = -V(\phi) \sqrt{1 - 2\frac{X}{\Lambda^4}}$$

$$p_K = K(\phi)X + \frac{X^2}{\Lambda^4}$$

# Non-canonical Inflation

Take separable action

$$p(X,\phi) = \Lambda^4 S(X) - V(\phi).$$

The inflationary solution is given by $X_{inf}(A)$ satisfying

$$\sqrt{\frac{2X}{\Lambda^4}\frac{dp}{dX}} = A$$

where $A = \frac{V'}{3H\lambda^2}$ is the noncanonicalness parameter.

- NCI is attractive
- overshoot/ICFTP reduced when the NC regime is relevant

[Franche, RG, Underwood and Wissanji: 0912.1857 & 1002.2639]

# Observational signatures of NCI

NCI models $P(X,\phi)$ can lead to an observable amount of nongaussianity, of the equilateral type:[Chen, Huang, Kachru, Shiu: 0605045]

$$f_{NL}^{equil} \sim c_s^{-2}$$

where

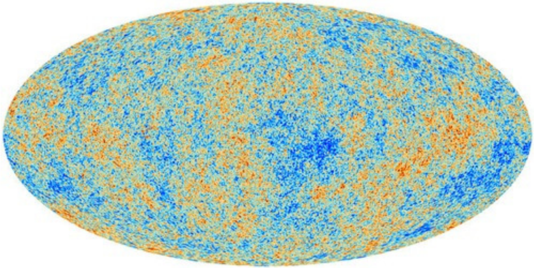
$$c_s^2 = \left(1 + 2X\frac{p_{XX}}{p_X}\right)^{-1}$$

- Potentially clear observational signature of NCI!
- Not yet ruled out by data....

# Constraints on NCI

New Planck bounds: [1303.5082 etc]

$$
\begin{aligned}
f_{NL}^{local} &= 2.7 \pm 5.8 \\
f_{NL}^{equil} &= -42 \pm 75 \\
f_{NL}^{ortho} &= -25 \pm 39 \\
c_s &\geq 0.02
\end{aligned}
$$



# Degeneracy

This looks like a clear signal should be possible, but we can still have degeneracy with canonical models....

# Field redefinitions

- for simple Lagrangians $p(X,\phi)$ can transform to a canonical action via a field redef eg $p(X,\phi) = -\frac{1}{2\phi^2}(\partial_\mu\phi)^2 - V(\phi)$ using $\psi = \ln\phi$.
- for more general $p(X,\phi)$ can always transform a canonical theory to a noncanonical one via canonical transformations [Bean et al, 0801.0742]:

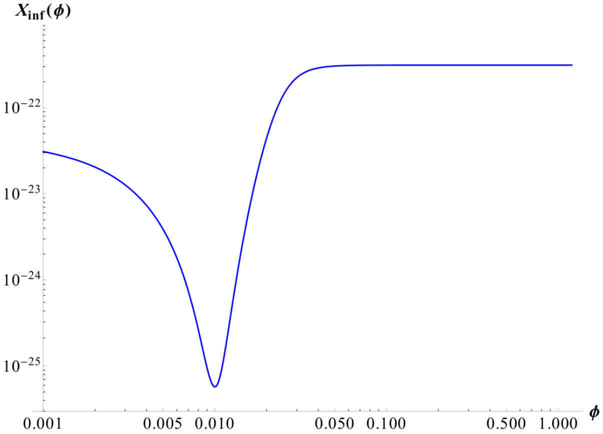
$$p = \frac{\partial F}{\partial \phi}, \ \tilde{p} = -\frac{\partial F}{\partial \tilde{\phi}}$$

for a generating functional $F(\phi, \tilde{\phi})$.

- However only separable NC theories with quadratic potentials can be transformed to canonical theories this way (AFAIK...). [RG, Rummel and Westphal, 1212.4135]

# Onshell transformation

**Can we construct a potential $V_{can}(\phi)$ which gives rise (in a canonical theory) to the same trajectory $X_{inf}(\phi)$ as in the noncanonical theory?**



# Onshell transformation

**Noncanonical theory:**

$$\Pi_{inf}(\phi) \approx \frac{\partial p}{\partial \phi}\frac{1}{3H}$$

$$\Pi = -\sqrt{2X}\frac{\partial p}{\partial X}$$

$$H^2 = \frac{\rho}{3M_p^2} = \frac{2Xp_X - p}{3M_p^2}$$

**Canonical theory:**

$$\dot{\phi} = -\frac{V'_{can}(\phi)}{3H(\phi)}$$

$$H^2(\phi) = \frac{V_{can}(\phi)}{3}$$

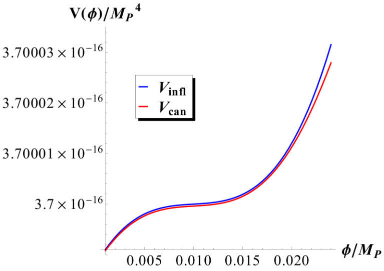
Given some $X(\phi) = X_{inf}$, integrate

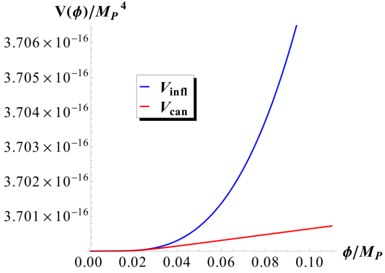
$$\sqrt{6X}d\phi = \frac{dV_{can}}{\sqrt{V_{can}}}$$

$$\Rightarrow V_{can}(\phi) = \left(\sqrt{V_{can}} + \int_{\phi_0}^{\phi} d\phi' \sqrt{\frac{3}{2}X_{inf}(\phi')}\right)^2$$

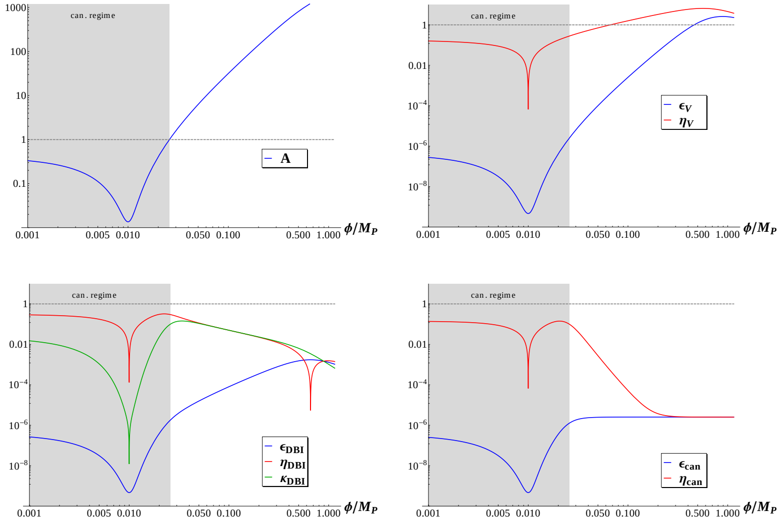
# DBI + Inflection point potential

$$V_{inf}(\phi) = V_0 + \lambda(\phi = \phi_0) + \beta(\phi - \phi_0)^3$$



# Canon vs Noncanon



# Observables

**(Non)Canonical theory:**

$$\Delta_s^2(k) = \left.\frac{1}{8\pi^2}\frac{H^2}{M_p^2}\frac{1}{c_s\epsilon}\right|_{c_s k=aH}$$

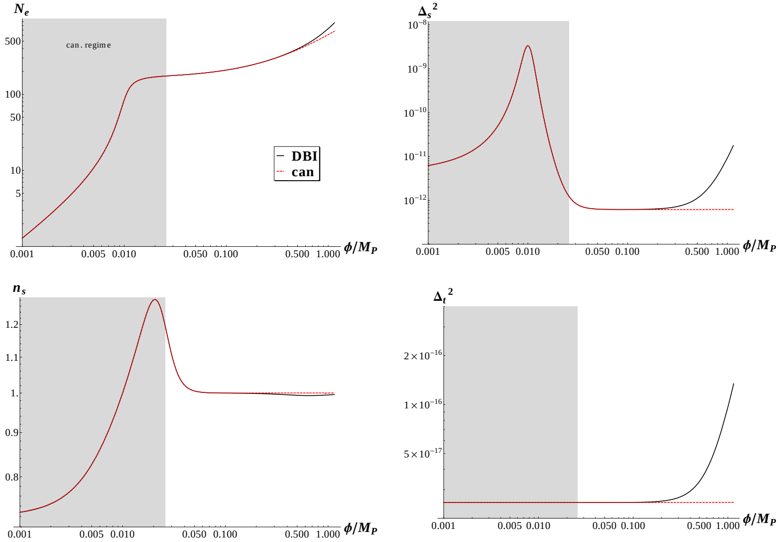
$$\Delta_t^2(k) = \left.\frac{2}{\pi^2}\frac{H^2}{M_p^2}\right|_{k=aH}$$

$$n_s(k)-1 = -\left.2\epsilon-\eta\right|_{c_s k=aH}$$

$$n_t(k) = -2\left.\epsilon\right|_{k=aH}$$

- $\epsilon = -\frac{\dot{H}}{H^2};\ \eta = \frac{\dot{\epsilon}}{H\epsilon}$ so in the canonical limit $\epsilon \to \epsilon_V;\ \eta \to 4\epsilon_V - 2\eta_V$
- Recall that $c_s^{-2} = 1 + 2X\frac{p_{XX}}{p_X}$
- note that time of horizon crossing is different for scalar modes in NCI

# Comparison of Observables



# Analytic understanding?

For theories with (1) a canonical limit where $V = V_{can}$ and (2) a speed limit st $X_{inf} = \Lambda^4 R$ when $A$ is large (from finite convergence radius), $\Delta_s^2(k), \Delta_t^2(k), N_e$ match when

$$V_{can} \approx V \; ; \; c_s = \frac{\sqrt{2R}}{A} \text{ for } A \gg 1$$

.

- can have $V \approx V_{can}$ and $V' >> V''_{can}$ in some intermediate regime for A
- $c_s^2(A) = \frac{A \frac{\partial X_{inf}}{\partial A}}{2X_{inf}} \approx \frac{1}{A^n}$ for $X_{inf} = X_{inf}(A^n)$. Then we get the matching condition for DBI:

$$X_{inf}^{DBI} = \frac{\Lambda^4}{2} \frac{A^2}{1 + A^2}$$

No other working examples.... DBI special?

# What about nongaussianities??

# Resonant NG

Axionic shift symmetry will receive small periodic modulations from NP effects [Chen, Easther, Lim 0801.3295 & Flauger and Pajer 1002.0833]

$$V(\phi) = V_0(\phi) + \Lambda^4 \cos(\frac{\phi}{f})$$

$$\Rightarrow \frac{\mathcal{G}(k_1, k_2.k_3)}{k_1 k_2 k_3} = f^{res}\left[\sin(\frac{\sqrt{2\epsilon_\star}}{f}\ln\frac{K}{k_\star}) + \sum \cos() + ...\right]$$

where

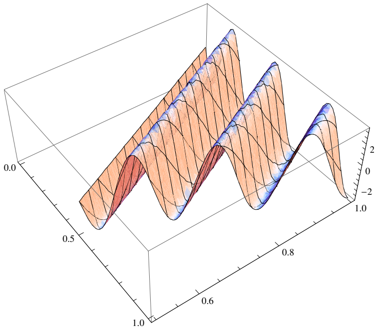
$$f^{res} = \frac{3b_\star\sqrt{2\pi}}{8}(\frac{\sqrt{2\epsilon_\star}}{f})^{3/2}$$

$$b_\star = \frac{\Lambda^4}{V_0'(\phi_\star)f}$$

$$K = k_1 + k_2 + k_3.$$

NG comes from $\dot{\delta}$ where $\delta = \frac{\ddot{H}}{2H\dot{H}}$ in interaction term. $f$ is the axion decay constant.

# Resonant NG




Less than 10 % overlap with the other shapes (local, equilateral, orthogonal)

# Multiple sources

$$V(\phi) = V_0(\phi) + \sum_i A_i \cos(\frac{\phi + c_i}{f_i})$$

$$\frac{\dot{\delta}}{H} = \sum_i \frac{\sqrt{2\epsilon_\star}}{f_i} 3 b_i^\star \cos(\frac{\phi_0 + c_i}{f_i})$$

$$\frac{\mathcal{G}(k_1, k_2, k_3)}{k_1 k_2 k_3} = \sum_i \frac{3\sqrt{2\pi} b_i^\star}{8} \left(\frac{\sqrt{2\epsilon_\star}}{f_i}\right)^{3/2} \sin(\frac{\sqrt{2\epsilon_\star}}{f_i} \ln \frac{K}{k_\star} + \frac{c_i}{f_i})$$

Can choose $b_i^\star, f_i, c_i$ to get an overlap with a periodic equilateral shape for $N = \mathcal{O}(10)$ terms!

# One-dimensional limit

$$x_2 = \frac{k_2}{k_1}, \quad x_3 = \frac{k_3}{k_1}, \quad x_\pm = x_2 \pm x_3$$

Resonant NG is to 1st order in $\frac{f_i}{\sqrt{2\epsilon_\star}}$ a fn of $x_+, k_1$ but not $x_-$:

$$\sin\left(\frac{\sqrt{2\epsilon_\star}}{f_i}\ln\frac{K}{k_\star}\right) = \sin\left(\frac{\sqrt{2\epsilon_\star}}{f_i}(\ln(1+x_+) + \ln\frac{k_1}{k_\star})\right)$$

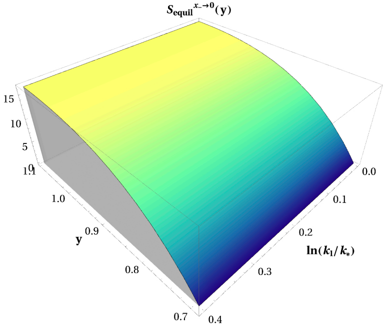
Can only reproduce NG which is predominantly a function of $x_+$, such as equil:

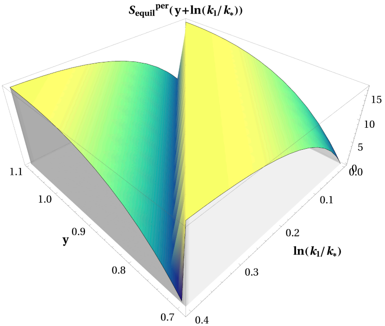
$$S_{eq}(k_1,k_2,k_3) = \frac{(k_1+k_2-k_3)(k_1+k_3-k_2)(k_3+k_2-k_1)}{k_1k_2k_3}$$

$$S_{eq}^{x_-\to 0}(x_+) = \frac{4(x_+-1)}{x_+^2}$$

$$C(S_{eq}, S_{eq}^{x_-\to 0}) = 0.93$$

# Periodic approximation



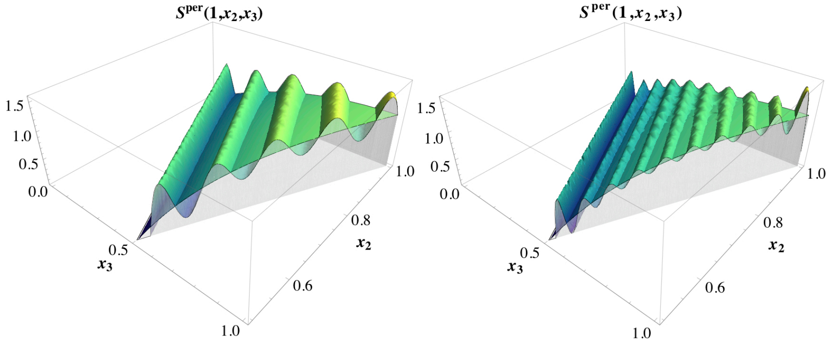




Need to approximate a scale-inv shape by a scale-dep shape: make a periodic generalisation of $S_{equil}^{x_-\to 0}$. The overlap is still considerable:

$$C(S_{equil}, S_{equil}^{per}) = 0.83$$

Can now fourier synthesize.

# Fourier series




Fourier expansion for $N = 5$(left) and $N = 10$(right)

# Observational constraints

$$f_i < 1; \quad b_i^\star < 1; \quad b_i^\star f_i < \frac{10^{-5}}{\sqrt{2\epsilon_\star}}$$

$$\Rightarrow \epsilon_\star f_{NL}^{eq} < 10^{-2}.$$

- as is, the power spectrum constraint implies a resonantly generated $f_{NL}^{equil} \leq \mathcal{O}(1)$
- if shift symmetry is collectively broken [Behbahani & Green, 1207.2779] (i.e. scale invariance is protected by several independent symmetries), N pt functions are no longer hierarchically suppressed with N.
- Then can have $f_{NL}$ up to 140 without implying a large oscillation in the power spectrum
- for small field models expect no NG (f too large to have fourier sum)

# Conclusions/Future Work

**Canonical/Noncanonical**

- We suspect the description in terms of a canonical theory may be special to the DBI case
- We don't know why this works (asking for fluctuations around the bg to match...)
- Might be able to match 3pt observables

**Resonant NG**

- Possible string theory/axion monodromy embedding? (series of instanton corrections expected)
- Applications elsewhere?