Recent Progress in Testing Alternate Theories of Gravity

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If it ain’t broke, don’t fix it
Why modify gravity?
The universe is accelerating, and we don’t know why!
Can’t GR accelerate?

GR can accelerate with a cosmological constant:

\[ G_{\mu\nu} = 8\pi G \left( T_{\mu\nu} + \Lambda g_{\mu\nu} \right) \]

Get acceleration if \( \Lambda \sim 10^{-12} \text{ eV}^4 \)
So, what’s the problem?

Naturalness!

E.g. electron decoupling

\[ \Lambda_{\text{eff}} = \Lambda_{\text{bare}} + m_e^4 \]

\[ \Lambda = -m_e^4 + 10^{-12} \]
Okay, so what happens next?

We have to do this for every particle:

$$\Lambda = - \underbrace{m_e^4}_{10^8 \text{ eV}^4} - \underbrace{m_\mu^4}_{10^{28} \text{ eV}^4} - \underbrace{m_\tau^4}_{10^{36} \text{ eV}^4} - \cdots - \underbrace{m_H^4}_{10^{56} \text{ eV}^4} + 10^{-12} \text{ eV}^4$$

Have to fine-tune a classical number against a quantum correction to 56 decimal places!

This is not natural
Technical naturalness to the rescue

E.g. fermion masses are natural because of chiral symmetry

\[ \Psi \rightarrow e^{i\alpha \gamma_5} \Psi \]

\[ \mathcal{L} = \underbrace{\bar{\Psi} i\gamma^\mu \partial_\mu \Psi} \quad \text{invariant} - \quad m \bar{\Psi} \Psi \quad \text{breaks symmetry} \]

Symmetry restored when \[ m \rightarrow 0 \]

\[ \Delta m^2 = m^2 + O(1)m^2 \]

A small fermion mass is natural.
How about gravity?

Can we find a theory of gravity with a technically natural vacuum?

- No naturalness problem
- No fine-tuning
- Self acceleration?
But wait? Isn’t gravity already constrained?

Yes and no!

Gravity tested in:

- Solar system - Newtonian and post-Newtonian
- Binary Pulsars - post-Newtonian

The fully relativistic structure has not been probed!
Local tests

E.g. Cassini measures light bending by the Sun

\[ ds^2 = (-1 + \frac{GM}{r}) \, dx^2 \]

“How much space is curved by a unit rest mass?”
What do local tests mean?

E.g. scalar-tensor theory - new scalar graviton

\[ \nabla^2 \Phi_N = 4\pi G \rho \quad F_N = -\nabla \Phi_N \]

\[ \nabla^2 \phi = 8\pi \alpha G \rho \quad F_5 = -\alpha \nabla \phi \]

\[ \phi = 2\alpha \Phi_N \Rightarrow \frac{F_5}{F_N} = 2\alpha^2 \]

Cassini: \[ \alpha < 10^{-5} \Rightarrow \text{Theory is GR on all scales} \]
Screening mechanisms to the rescue

Non-linear effects decouple cosmological scales from the solar system

solar system  astrophysics  cosmology

screened  partially screened  unscreened
This talk

1. Vainshtein mechanism

• Astrophysics only partially screened
• Identify novel probes in stars and galaxies
• Place new constraints
This talk

2. Disformal Gravity

• Can constrain cosmology using solar system tests
• Interesting cosmological phenomenology
• Still many unsolved mysteries
The Vainshtein mechanism

Recall the problem:

\[ \mathcal{L} = -\frac{1}{16\pi G} \partial_\mu \phi \partial^\mu \phi + \alpha \phi T \]

\[ \nabla^2 \phi = 8\alpha \pi G \rho \]
The Vainshtein mechanism

Try to fix this by adding new kinetic terms

E.g. cubic galileon:

\[ \mathcal{L} = - \frac{1}{16\pi G} \partial_\mu \phi \partial^\mu \phi - \frac{1}{16\pi G \Lambda^3} \partial_\mu \phi \partial^\mu \Box \phi + \alpha \phi T \]

\[ \nabla^2 \phi + \frac{1}{\Lambda^3 r^2} \frac{d}{dr} \left( r \phi' \right)^2 = 8\alpha \pi G \rho \]
Vainshtein Mechanism

We can integrate this once:

\[ x + \left( \frac{r_V}{r} \right)^3 x^2 = 2\alpha^2 \]

\[ x = \frac{F_5}{F_N} \]

\[ r \ll r_V \Rightarrow \frac{F_5}{F_N} = 2\alpha^2 \left( \frac{r}{r_V} \right)^{\frac{3}{2}} \ll 1 \]

\[ r \gg r_V \Rightarrow \frac{F_5}{F_N} = 2\alpha^2 \sim O(1) \]

\[ r_V \text{ - Vainshtein radius} \]
Vainshtein Screening

\[
\frac{F_5}{F_N} \quad 2\alpha^2 \quad r \quad r_V
\]

Screened

Unscreened
Astrophysical Screening

\[ r_\odot^V \geq 10^2 \, \text{pc} \]

Exhibited in:

- DGP - tension with data
- Covariant galileons - too much ISW
- Massive gravity - no FRW solutions
- Massive bigravity - unstable (or is it?)
- Beyond Horndeski - new and unexplored

Mechanism is partially broken in beyond Horndeski
Vainshtein Breaking

\[ ds^2 = -(1 + 2\Phi) \, dt^2 + (1 - 2\Psi) \, \delta_{ij} \, dx^i \, dx^j \]

Motion of NR matter

Bending of Light

**GR:**

\[ \frac{d\Phi}{dr} = \frac{GM(r)}{r} \]

\[ \frac{d\Psi}{dr} = \frac{GM(r)}{r} \]
Vainshtein Breaking

Stars and satellites behave differently

\[ \frac{d\Psi}{dr} = \frac{GM(r)}{r} - \frac{5\gamma G}{4r} \frac{dM(r)}{dr} \]

\[ \frac{d\Phi}{dr} = \frac{GM(r)}{r} + \frac{\gamma G}{4} \frac{d^2M(r)}{dr^2} \]

Cosmological field

\[ \gamma = \left( \frac{\dot{\phi_0}}{\Lambda} \right)^4 \]

Light bent differently
Potential probes

- Stellar structure
- Galactic rotation curves
- Gravitational lensing
Stellar Structure Tests

Main idea:

- Stars burn fuel to stave off gravitational collapse
- Changing gravity changes the burning rate
- This alters the temperature, luminosity and life time
Vainshtein Stars

Gravity weaker

Slower burning rate

Dimmer and cooler stars that live longer
Polytropic stars

\[ P = K \rho^{\frac{4}{3}} \]

Balls of gas that collapse under gravity - no physics

- No nuclear burning, convection etc.
- Can isolate new effects of MG
- Not realistic enough to compare with data
Mass-G-Luminosity relation

\[ P_{\text{gas}} = \frac{\rho k_B T}{\mu m_H} \quad \quad P_{\text{rad}} = \frac{1}{3} a T^4 \]

Gas pressure - \( L \propto G^4 M^3 \)

Radiation pressure - \( L \propto GM \)

High-mass stars are more radiation pressure-supported
Vainshtein Polytropes

\[
\frac{L_{\text{MG}}}{L_{\text{GR}}} = \gamma
\]

\(\gamma = 0.1\)

\(\gamma = 0.3\)

\(\gamma = 0.5\)
Realistic stars

We have modified MESA to include MG:

• Fully consistent treatment of stellar structure

• No approximations

• Includes burning, convection, mass loss etc.

• Can compare with data
$M = 1M_\odot \quad Z = 0.02 \quad \bullet = \text{solar age}$

No change on red giant

Dimmer + cooler on main-sequence

GR, $Z = 0.03$
Vainshtein vs. GR

- Main-sequence cooler and dimmer
- No change to red giant phase
- MS degenerate with GR + more metals

May be detectable by comparing MS and RG fits to globular clusters.
Galactic rotation curves

Circular velocity:

\[ v_{\text{circ}}^2 = \frac{\mathrm{d}\Phi}{\mathrm{d}r} \]

New features in rotation curves?

\[ \frac{\mathrm{d}\Phi}{\mathrm{d}r} = \frac{GM(r)}{r} + \frac{\Upsilon G}{4} \frac{\mathrm{d}^2M(r)}{\mathrm{d}r^2} \]
Galactic rotation curves

NFW density profile:

$$\rho(r) = \frac{\rho_s}{\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2}$$

$$v_{\text{circ}}^2 = \frac{4\pi G r_s^3 \rho_s}{r} \left[ \ln \left(1 + \frac{r}{r_s}\right) - \left(1 + \frac{r_s}{r}\right)^{-1} + \frac{\chi}{4} \frac{(r_s/r - 1)}{(1 + r_s/r)^3} \right]$$
Vainshtein rotation curves

\[
v = \begin{cases} 
\text{GR} \\
\gamma = 0.5 \\
\gamma = 1 
\end{cases}
\]

Koyama & JS 2015
Vainshtein rotation curves

Measure using 21 cm
Measure using stellar motions

Deviations in 21 cm region compared with stellar prediction
Gravitational lensing

Lensing of light (relative to NR force): \( \frac{\Phi + \Psi}{2\Phi} \)

GR: \( \Phi + \Psi = 2\Phi \)

BH: \( \Phi + \Psi = \frac{8\pi G r_s^3 \rho_s}{r} \left[ \ln \left(1 + \frac{r}{r_s}\right) + \frac{\Upsilon}{8} \frac{6 + \frac{r_s}{r}}{(1 + \frac{r_s}{r})^2} \right] \)
Vainshtein lensing

Deviations from GR in the strong lensing regime

$\gamma = 1$

$\gamma = 0.5$

$\gamma = 0.2$

Koyama & JS 2015
Disformal Gravity

\[ S = \int d^4x \sqrt{-g} \frac{M_{pl}^2}{2} \left[ \frac{R}{2} - \frac{1}{2} \nabla_{\mu} \phi \nabla^{\mu} \phi - V(\phi) \right] + S_m[\tilde{g}_{\mu\nu}] \]

\[ \tilde{g}_{\mu\nu} = A^2(\phi) \begin{bmatrix} g_{\mu\nu} + \frac{B^2(\phi)}{\Lambda^2} \partial_{\mu} \phi \partial_{\nu} \phi \end{bmatrix} \]

Jordan frame \hspace{1cm} conformal \hspace{1cm} Einstein frame \hspace{1cm} disformal

Matter moves on geodesics of \( \tilde{g}_{\mu\nu} \) not \( g_{\mu\nu} \)

\( \Rightarrow \) fifth-force

Beckenstein 1992
Potential Problem

Jordan frame metric can become singular:

\[
\frac{\sqrt{-\tilde{g}}}{\sqrt{-g}} = A^4 \sqrt{1 + \frac{B^2 (\partial \phi)^2}{\Lambda^2}} = A^4 \sqrt{1 - \frac{\dot{\phi}_0}{\Lambda^2}}
\]

Solutions slow down to avoid this, but no one knows why.

“Natural pathology resistance”

Koivisto, Mota & Zumalacarregui 2012
Metric Singularity?

\[
\frac{\sqrt{-\tilde{g}}}{\sqrt{-g}} ^{10^{10}} \quad \frac{\sqrt{-g}}{\sqrt{-\tilde{g}}} ^{10^{10}}
\]

\[
N = \ln a
\]

Graph showing values of \( \frac{\sqrt{-\tilde{g}}}{\sqrt{-g}} \) and \( \frac{\sqrt{-g}}{\sqrt{-\tilde{g}}} \) against \( N = \ln a \) with specific values indicated on the graph.
Natural Pathology Resistance

Is this okay? No*, e.g. disformal only:

\[ \text{d} s^2 = -N^2 \text{d} t^2 + \cdots \quad N^2 = 1 - \frac{\phi^2}{\Lambda^2} \quad \text{d} T = N \text{d} t \]

Proper time \( \rightarrow 0 \)

Lapse \( \rightarrow 0 \) but so what? Two speeds:

\[ \frac{c^2_{\text{tensors}}}{c^2_{\text{light}}} = N^2 \]

Don’t know what the true non-relativistic limit is!
Non-relativistic limit

EOM is horrible:

\[ \chi \Box \phi - 8\pi G \frac{B^2}{\Lambda^2} T_{m}^{\mu \nu} \nabla_{\mu} \nabla_{\nu} \phi = \]

\[ - 8\pi \alpha G T_{m} - 8\pi G \frac{B^2}{\Lambda^2} (\alpha - \gamma) T_{m}^{\mu \nu} \partial_{\mu} \phi \partial_{\nu} \phi + \chi V(\phi) , \phi \]

\[ \chi = 1 + \frac{B^2 (\partial \phi)^2}{\Lambda^2} \quad \alpha = \frac{d \ln A}{d \phi} \quad \gamma = \frac{d \ln B}{d \phi} \]
Non-relativistic limit

But we have calculated the PPN parameters:

\[ \tilde{g}_{00} = 1 + 2U - 2\beta U^2 + \cdots \]

Preferred frame

\[ \tilde{g}_{0i} = -\frac{1}{2}(\cdots + \alpha_1 - \alpha_2 + \cdots) V_i - \frac{1}{2}(\cdots + \alpha_2 + \cdots) W_i \]

Light bending

\[ \tilde{g}_{ij} = (1 + 2\gamma_{PPN} U) \delta_{ij} \]
Non-relativistic limit

\[ \tilde{g}_{00} = 1 + 2U - 2\beta U^2 + \cdots \]

\[ \tilde{g}_{0i} = -\frac{1}{2} (\cdots + \alpha_1 - \alpha_2 + \cdots) V_i - \frac{1}{2} (\cdots + \alpha_2 + \cdots) W_i \]

\[ -\Upsilon \ (GR = 0) \]

\[ \tilde{g}_{ij} = (1 + 2\gamma_{PPN} U) \delta_{ij} \]

\[ \Upsilon = \left( \frac{B \dot{\phi}_0}{\Lambda} \right)^2 \]

\[ 1 - \Upsilon \ (GR = 1) \]

\[ -4\Upsilon \ (GR = 0) \]
Constraints

Strongest constraint comes from $\alpha_2$

$$\alpha_2 < 10^{-7}$$

Simple model: $V(\phi) \propto e^{-\lambda \phi}$, $B = 1$

$$\chi^2 \left( \frac{H_0}{\Lambda} \right)^2 < 4 \times 10^{-7}$$
Constraints
What does this mean?

\[ \chi^2 \left( \frac{H_0}{\Lambda} \right)^2 < 4 \times 10^{-7} \]

Need \[ \left( \frac{H_0}{\Lambda} \right)^2 \sim \mathcal{O}(1) \] for novel cosmology

Cosmology is identical to \( \Lambda \text{CDM} \)

Van de Bruck & Morrice 2015
Cosmology

Friedmann equations same as GR

Field equations are coupled:

\[
\dot{\rho} + 3H\rho = Q_0\dot{\phi}_\infty
\]

\[
\ddot{\phi}_\infty + 3H\dot{\phi}_\infty + V'(\phi_\infty) = -Q_0
\]

\[
Q_0 = 8\pi G\rho \frac{\alpha + \frac{B^2}{\Lambda^2} \left( [\gamma - \alpha] \phi^2_\infty - 3H\dot{\phi}_\infty - V,\phi \right)}{1 + \frac{B^2}{\Lambda^2} \left( 8\pi G\rho - \dot{\phi}^2_\infty \right)}
\]
Dynamical Systems

Different initial conditions, common late-time behaviour

\[ x = \frac{\phi'}{\sqrt{6}} \quad y = \frac{\sqrt{V}}{\sqrt{3H}} \]
Why is this useful?

- Classify the entire solution space
- Know cosmological properties at the fixed points
- Identify models that have the properties we want
- Tells us which models to focus future efforts on
What we want

Fixed points with

• Dark energy domination - match observations

• Finite metric determinant - well-defined NR limit
Recap: Conformal Case

\[ V(\phi) = m_0^2 e^{-\lambda \phi} \quad A(\phi) = e^{\alpha \phi} \quad N = \ln a \]

Phase space is 2-dimensional

\[ x = \frac{\phi'}{\sqrt{6}} \quad y = \frac{\sqrt{V}}{\sqrt{3H}} \quad \lambda = -\frac{V'}{V} = \text{constant} \]

\[ \ddot{\phi}_\infty + 3H \dot{\phi}_\infty + V'(\phi) = 8\pi \alpha G \rho \]
Conformal phase space
Disformal System

What happens when we include a disformal coupling?

Old attractor is a saddle point - what’s going on?
Disformal System

\[ x = \frac{\phi'}{\sqrt{6}} \quad y = \frac{\sqrt{V}}{\sqrt{3H}} \quad \lambda = -\frac{V'}{V} = \text{constant} \]

\[ \ddot{\phi} + 3H \dot{\phi} - V'(\phi, \lambda) = -Q_0 x \quad \text{and} \quad y \]

Can't eliminate \( B(\phi, \lambda) \) in terms of \( x \) and \( y \)

Phase space is 3D

\[ Q_0 = 8\pi G \frac{B_H}{\Lambda} \left( \gamma - \alpha \right) \frac{\phi^2}{\Lambda^2} \left( \gamma \right) = \frac{Z}{\Lambda} = \frac{1}{Z + 1} + \frac{B^2}{\Lambda^2} \left( 8\pi G \phi \right) \]

\[ 0 \leq Z \leq 1 \quad \gamma \text{ constant} \]
Stability is altered

Perturbations in the z-direction are unstable

(3 eigenvalues instead of 2, one can be positive)

New dark energy dominated fixed point at

\[ x = 0, y = 1, Z = 1 \]

But: \[ \sqrt{-\tilde{g}} = 0 \]

Metric singularity at late times
Special Case

\[ \gamma = \lambda/2 \Rightarrow zy = \text{const} \]

New fixed point with interesting properties

\[ \Omega_{\text{DE}} = \Omega_{\text{DE}} \left( \lambda, \frac{\Lambda}{m_0} \right) \quad \omega_{\text{DE}} = \omega_{\text{DE}} \left( \frac{\Lambda}{m_0} \right) \]

Can always match any measurement without fine-tuning

BUT: \( \sqrt{-\tilde{g}} = 0 \)
What does this mean?

Only viable models are identical to conformal theories!*

* Possible loopholes are models whose dynamics are not described by this analysis
Summary

Vainshtein

- Vainshtein broken in beyond Horndeski
- Main-sequence stars are dimmer and cooler
- Circular velocity is lower
- Deviations from GR in strong lensing regime
Summary

Disformal

• Metric singularity not well-understood
• Solar system constraints give ΛCDM
• Cosmological solutions evolve towards singularity
• Only viable models have no disformal properties
• Still a lot to understand!
Thank you!
(and to my collaborators)

Vainshtein
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Papers
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Disformal
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Papers
15xx.xxxxx
1409.7296
1409.1734