QFT I - Problem Set 2

(3) Two Point Function

(a) For the lattice with nearest neighbor interaction, we would like to compute the two point function

$$\langle 0|Q_j Q_{j+\Delta j}|0\rangle.$$

Hints: Express Q_j in terms of a and $a^\dagger$. Move to momentum space (you need two momenta, one for each Q). Then express a_q and $a_q^\dagger$ in terms of A_q and $A_q^\dagger$. Keep in mind that $\alpha = \alpha(q) = \alpha(-q)$ and $\beta = \beta(q) = \beta(-q)$ are momentum dependent! You do not need to evaluate the final sum over momentum q.

(b) Show that the correlator vanishes for $\Delta j \neq 0$ without nearest neighbor interactions (simple argument).

(4) Operator Gymnastics

(a) Compute the vacuum expectation value (vev)

$$\langle 0|\phi(\vec{x})\phi^\dagger(\vec{y})|0\rangle,$$

using the commutator $[\phi(\vec{x}), \phi^\dagger(\vec{y})]$. In our case, $\phi(\vec{x})$ annihilates the vacuum.

(b) Compute the vev

$$\langle 0|\phi(\vec{x}_1)\phi(\vec{x}_2)\phi^\dagger(\vec{x}_3)\phi^\dagger(\vec{x}_4)|0\rangle.$$

Hint: For operators A, B, C, we have $[A, BC] = [A, B]C + B[A, C]$.

(5) Matrix Elements

This exercise relies on (4).

(a) Compute and interpret the matrix element

$$\langle \vec{x}|\vec{q}\rangle.$$

(b) Compute the "static" correlation function for a single momentum mode

$$\lim_{\vec{q}\to\vec{p}} \langle \vec{q}|\phi(\vec{x})\phi^\dagger(\vec{y})|\vec{p}\rangle$$

for $\vec{x} \neq \vec{y}$. Compare to the result for the phonons.

Hint: $\lim_{\vec{q}\to\vec{p}} \int_{\vec{x}} \exp \mathrm{i}\vec{x}(\vec{q} - \vec{p}) = V_3$, where V_3 is the quantization volume.