A brief overview on the cosmology of bigravity

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Gravity at the Largest Scales

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Outline

- Why should (can) we modify GR?
- Building a consisted theory of massive gravity: some historical steps
- dRGT massive gravity
- beyond dRGT: bimetric gravity
- cosmology of bigravity

GR with Λ

Lovelock theorem (1971)

"The only second order, local, gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Enstein field equations with a cosmological constant"

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0$$

Despite the universal consensus that GR is beautiful and accurate, in recent years, a small industry of physicists has been working to modify it and test these modifications

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Why should we modify GR?

Modify GR: why? (1) "Solving" the cosmological constant problem

Supernovae data: universe has recently started accelerating in its expansion

Simplest interpretation:

- cosmological constant term in Einstein equation: $\rho_V \sim M_p^2 \Lambda$
- observed value of ρ_V for $\Lambda/M_p^2 \sim 10^{-65}$ **vs** QFT prediction $\Lambda/M_p^2 \sim 1$

Why this discrepancy?

- perhaps GR+Λ is not the correct answer . . .
- late acceleration is not given by (some form of) vacuum energy
- IR modification of gravity is responsible for late-time acceleration

One can "cook-up" many IR modifications which reproduce self-acceleration
ex. EH $\leftrightarrow$ F(R). . .

Technically natural IR modifications

In which sense IR modifications of GR can solve the cc problem?

Common critic vs modified gravity:

- small value of $\mathcal{H}_0$ with respect to M_p has to come from somewhere
- best that one can do is to shift the fine-tuning into other parameters

It is true!

. . . hope: this small value can be obtained in a "technically natural way"

Why technically naturalness is a good property

- no logical inconsistency in having small parameters (tech. natural or not)
- small & technically natural: hope $\exists$ classical mechanism driving value $\rightarrow 0$
- small & ~~technically natural~~: this mechanism is harder to find (quantum?)

Modify GR: why? (2) Understanding better GR

One of the best ways to understand about a structure: attempt to modify it

- slight modifications of a rigid structure (e.g. a car, a toy, GR)

$$\downarrow$$

things goes badly (?)

$$\downarrow$$

I understand why the structure has given properties

- deformations of a known structure $\rightarrow$ new structures

ex. massive gravity: Vainhstein mechanism restores GR at solar system scales
$\rightsquigarrow$ largely used by model builders (e.g. to shield moduli from extra dimensions)

Massive gravity: IR mod. of gravity where these points are nicely illustrated

(1) Technically natural acceleration

- force mediated by a massive graviton has Yukawa profile $\sim \frac{1}{r}e^{-mr}$
- choosing $m \simeq \mathcal{H}_0$, late time acceleration can be explained
- at low redshift, cosmological constant contribution $\simeq m/M_p$
- technically natural choice: $m \to 0$ diffeomorphism invariance is recovered

(2) Interesting lessons regarding continuity of physical predictions

- modifying IR often messes up UV
- new mechanisms come into play
  (e.g. extra dofs must decouple themselves in the limit $m \to$ 0)

Building a consistent theory of massive gravity is a non-trivial problem: some historical steps in this process...

Good achievements

- linear Fierz-Pauli MG (1939)
- Vainshtein screening (1970)
- dRGT potential (deRham et al. 2011)

Problems

- vDVZ discontinuity (van Dam et al. 1970)
- Boulware-Deser ghost (1972)
- cosmologically viable?

dRGT massive gravity in pills (I)

$$S = -\frac{M_g^2}{2}\int d^4x\sqrt{-g}\left[R(g) - 2m^2 V(g,f)\right] + \int d^4x\sqrt{-g}\mathcal{L}_m(g,\Phi)\,,$$

$$V(g,f) = \sum_{n=0}^{4}\beta_n e_n(X)\,, \qquad X = \sqrt{g^{-1}f}\,,$$

de Rham et al. [1107.0443]

where

$$e_0 = \mathbb{I}, \quad e_1 = [X], \quad e_2 = \frac{1}{2}([X]^2 - [X^2])\,, \quad e_3 = \frac{1}{6}([X]^3 - 3[X][X^2] + 2[X^3]\,,$$

$$e_4 = \frac{1}{24}([X]^4 - 6[X]^2[X^2] + 8[X][X^3] + 3[X^2]^2 - 6[X^4]) = \det X\,.$$

- 5 dofs around every backgrounds $\rightsquigarrow$ good candidate for ghost-free MG!
- Unsatiphactory aspects: f is an external element, cosmology not viable. . .

$f_{\mu\nu} = \eta_{\mu\nu}$ no flat, nor close FRW solutions. Open solutions unstable

D'Amico et al. [1108.5231]

$f_{\mu\nu} =$FRW/d S ok FRW flat solutions. Instabilities (Higuchi ghosts)

De Felice et al. [1206.2080]

How to overcome the problem?

We modify the theory adding additional degrees of freedom:

- scalar dofs (quasi dilation, mass varying . . .) D'Amico et al. [1304.0723]
- tensor dofs (bigravity)

Hassan-Rosen bigravity

$$S = -\int d^4x\sqrt{-g}\left[\frac{M_g^2}{2}(R(g) - 2m^2V(g,f)) + \mathcal{L}_m(g,\Phi)\right] - \int d^4x\sqrt{-f}\,\frac{M_f^2}{2}R(f)\,,$$

$$V(g,f) = \sum_{n=0}^{4}\beta_n e_n(X)\,, \qquad X = \sqrt{g^{-1}f}\,, \qquad \text{dRGT potential}$$

The action is invariant under the following rescaling Hassan et al. [1109.3515]

$$f_{\mu\nu} \to \Omega^2 f_{\mu\nu}\,, \quad \beta_n \to \Omega^{-n}\beta_n\,, \quad M_f \to \Omega M_f$$

$\rightsquigarrow$ one parameter is redundant. We can choose $M_* = M_f/M_g = 1$

It overcomes all the unsatisfactory features of massive gravity:

- the metric $f_{\mu\nu}$ is now a dynamical object
- improved cosmology

Background solutions

- Homogeneous and isotropic background solutions Comelli et al. [1111.1983]

$$ds_g^2 = a^2(\tau)\left(-d\tau^2 + dx_i dx^i\right)\,, \quad ds_f^2 = b^2(\tau)\left(-c^2(\tau)d\tau^2 + dx_i dx^i\right)\,,$$

$$H = \frac{\mathcal{H}}{a} = \frac{a'}{a^2}\,, \qquad H_f = \frac{\mathcal{H}_f}{b} = \frac{b'}{b^2\, c}\,, \qquad r = \frac{b}{a}\,.$$

- Energy-momentum tensor of a perfect fluid coupled with $g_{\mu\nu}$

- Late-time effective Λ coming from the coupling between g and f

$$H^2 = \frac{8\pi G}{3}\left(\rho + \rho_g\right)\,, \qquad \rho_g = \frac{m^2}{8\pi G}\left(\beta_3\, r^3 + 3\beta_2\, r^2 + 3\beta_1\, r + \beta_0\right)\,.$$

- Bianchi constraint can be realized in two ways: two branches

$$m^2\left(\beta_3 r^2 + 2\beta_2 r + \beta_1\right)\left(\mathcal{H} - \mathcal{H}_f\right) = 0$$

Algebraic branch

Algebraic branch: Bianchi constraint implemented as

$$\left(\beta_3 r^2 + 2\beta_2 r + \beta_1\right) = 0$$

Background cosmology

- $r = \bar{r} =$cnst
- GR with effective cosmological constant, $\Lambda_{\text{eff}} = m^2\left(\beta_0 - 2\beta_3\bar{r}^3 - 3\beta_2\bar{r}^2\right)$

Cosmology of perturbations

- vector and scalar dofs have vanishing kinetic term and non-vanishing mass term
- non-dynamical or strongly coupled? $\rightsquigarrow$ it depends on the non-linear behavior ... $\rightsquigarrow$ non perturbative methods needed!

Dynamical branch

Dynamical branch: Bianchi constraint implemented as

$$(\mathcal{H} - \mathcal{H}_f) = 0$$

Background cosmology

$$H^2 = \frac{8\pi G}{3}\,(\rho + \rho_g)\ , \qquad \rho_g = \frac{m^2}{8\pi G}\left(\beta_3\, r^3 + 3\beta_2\, r^2 + 3\beta_1\, r + \beta_0\right)\ .$$

$$H_f^2 = \frac{m^2}{3}\left(\frac{\beta_1}{r^3} + \frac{3\beta_2}{r^2} + \frac{3\beta_3}{r} + \beta_4\right)\ .$$

Cosmology of perturbations

- are cosmological perturbation stable? for which choice of parameters?
- "natural" choice is to consider $\beta_n \simeq 1$ and $M_* = 1$ (through rescaling)

Following set of independent equations for the background

$$c = \frac{\mathcal{H}r + r'}{\mathcal{H}r},$$

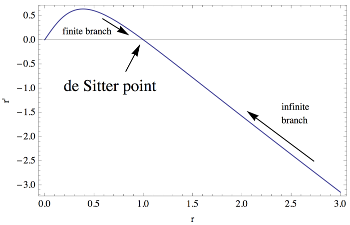
$$\rho_m = M_p^2 m^2 \left(\frac{\beta_1}{r} - 3\beta_1 r + \beta_4 r^2 \right) - \rho_r,$$

$$\frac{r'}{r} = \frac{-9\beta_1 r^2 + 3\beta_1 + 3\beta_4 r^3 + r M_p^{-2} m^{-2} \rho_r}{3\beta_1 r^2 + \beta_1 - 2\beta_4 r^3} \mathcal{H},$$

$$\mathcal{H}^2 = a^2 m^2 \frac{\beta_1 + \beta_4 r^3}{3r}$$

$\rightsquigarrow$ we can extract value $r(\tau_0)$

$M_* = 1$



finite branch: gradient exponential instabilities in the scalar sector $\forall \beta_i$

infinite branch: no exponential instabilities in the scalar sector for $\beta_1\beta_4$ model

Koennig et al. [1407.4331]

The most promising model: $\beta_1\beta_4$ model

Infinite branch

- $\beta_1\beta_4$ free of exponential instabilities
- from a first analysis, this model seems promising

Further investigations give G.C. et al. [1412.5979], Lagos et al. [1410.0207], G.C. et al. [1505.0109]

- violation of the Higuchi bound in the tensor sector (but not problematic)
- violation of the Higuchi bound in the scalar sector: big problem!

Primordial scalar ghost!

in the absence of a mechanism to modify the scalar sector in the UV the sub model is ruled out...

In the finite branch:

- Higuchi bound can be satisfied for proper choices of parameters
- gradient exponential instabilies

Is there a way to avoid gradient exponential instabilities?

Ways out

- $m^2 \gg m_{eff}^2 = m^2\,\lambda(\beta_n) \simeq \mathcal{H}_0^2$
- $M_* = M_f/M_g \to 0$

Features

- fine-tuned (bare) parameters
- technically natural acceleration
- Bigravity=GR+ $\mathcal{O}(...)$

$m^2 \gg 1$ with β_n tuned

de Felice et al. [1404.0008]

$$m^2_{eff} = m^2 \lambda(r, \beta_n)$$

$\mathcal{H}_0^2$ big fine-tuned

Main features

- m^2 coupling term parametrically large
- $m^2_{eff} = m^2\, \lambda(\beta_n, r) \simeq \mathcal{H}_0^2$
- constrained parameters in order to avoid singularities/Higuchi instabilities

Resulting cosmology

- effective cosmological constant (technically natural)
- in the finite branch, gradient instabilities pushed to unobservable scales
- model indistinguishable from ΛCDM (graviton oscillations?)

Akrami et al. [1503.07521]

$$M_* = M_f/M_g \to 0$$

Main features

- $M_f \ll M_g$
- this condition after the rescaling writes $M_* = 1$, $\beta_n \gg 1$ and $\beta_{n+1} \gg \beta_n$

Resulting cosmology

- effective cosmological constant (technically natural)
- in the finite branch, gradient instabilities pushed to unobservable scales
- a small M_* increases the cut-off $\Lambda_3 \to \left(m^2 M_p M_*^{-1} \mathcal{O}(\beta_n)\right)^{1/3}$
- model indistinguishable from ΛCDM $\rightsquigarrow$ Bigravity=GR+$\mathcal{O}(M_*^2)$

Summary ...

Dynamical branch $(\mathcal{H}_f - \mathcal{H}) = 0$

Infinite branch

- $\beta_1\beta_4$ free from gradient instabilities
- ...but it is affected by scalar Higuchi ghost (primordial)
- can we modify the scalar sector in the UV to get rid of the ghost?

Finite branch is affected by gradient instabilities

- pushing instabilities at unobservable scales: $m^2 \gg m^2_{eff} \simeq \mathcal{H}_0^2$
- pushing instabilities at unobservable scales: $M_f \ll M_g$ $(\beta_n \gg 1,\ M_* = 1)$
- in both cases : Bigravity=GR+$\mathcal{O}(...)$

Algebraic branch $\left(\beta_3 r^2 + 2\beta_2 r + \beta_1\right) = 0$

- background ΛCDM-like
- perturbation problematic (strongly coupled dofs? non-dynamical dofs?)

Alternative approaches

Doubly coupled bigravity Akrami et al. [1306.0004] , Gumrukcuoglu et al. [1501.02790]

- dynamical branch: scalar ghost instabilities and vector gradient instabilities
- algebraic branch: no ghost instabilities, gradient instabilities?

Non-FRW background Nersisyan et al. [1502.03988]

Other modifications

- varying mass
- Lorentz violation
- . . .

Elegance is a matter of writing things properly (sometimes)

Sometimes a theory seems complicated only because we do not understand which is the proper way to write it

Classical example: Maxwell equations

$$\oiint_{\partial\Omega} E \cdot dS = \frac{1}{\epsilon_0} \iiint \rho \, dV$$
$$\oiint_{\partial\Omega} B \cdot dS = 0$$
$$\oint_{\partial\Sigma} E \cdot dl = -\frac{d}{dt} \iint B \cdot dS$$
$$\int_{\partial\Sigma} B \cdot dl = \mu_0 \iint_{\Sigma} j \cdot dS + \mu_0 \epsilon_0 \frac{d}{dt} \iint_{\Sigma} E \cdot dS$$

$$\partial_\mu F^{\mu\nu} = j^\nu \, , \ F_{[\alpha,\beta\gamma]} = 0$$

Modified gravity $\leftrightarrow$?

Thank you!

Sometimes a theory seems complicated only because we do not understand which is the proper way to write it

Classical example

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$
$$\nabla \cdot B = 0$$
$$\nabla \times E = -\frac{\partial B}{\partial t}$$
$$\nabla \times B = \mu_0 \left(j + \epsilon_0 \frac{\partial E}{\partial t} \right)$$

$$\partial_\mu F^{\mu\nu} = 0\,, \;\; F_{[\alpha,\beta\gamma]} = 0$$

Modified gravity $\leftrightarrow$?

$M_* = M_f/M_g \to 0$

Let us go back to the rescaling

$$f_{\mu\nu} \to \Omega^2 f_{\mu\nu}\,, \quad \beta_n \to \Omega^{-n}\beta_n\,, \quad M_f \to \Omega M_f$$

The choice $\Omega = M_f/M_g$ is completely meaningful, but it picks up a particular region of parameter space which may not capture all physically meaningful situations.

In particular $M_f/M_g \to 0$ will look extremely odd after the rescaling.

the region $M_* \to 1$ was not considered in the first scan of the parameter space. Is this viable cosmologically?