Prelude to the reference frame interpretation

Cold Quantum Coffee Seminar

Natalia Sánchez-Kuntz; Eduardo Nahmad-Achar

Institut für Theoretische Physik

Universität Heidelberg

The complementarity principle

In fact, it is only the mutual exclusion of any two experimental procedures, permitting the unambiguous definition of complementary physical quantities, which provides room for new physical laws, [...] which might at first sight appear irreconcilable with the basic principles of science.

The complementarity principle

In fact, it is only the **mutual exclusion of any two experimental procedures**, permitting the **unambiguous definition of complementary physical quantities**, which provides room for new physical laws, [...] which might at first sight appear irreconcilable with the basic principles of science.

Statement of the PBR theorem

Any model in which ψ represents mere information about an underlying physical state must make predictions that contradict those of quantum theory.

Premises

Hypothesis

- ψ represents mere information of the system it describes;

Assumptions

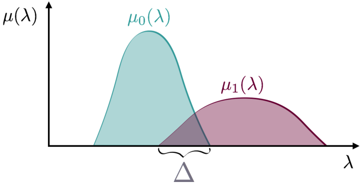
- There is an underlying physical state of the system;
- Systems that are prepared independently have independent physical states;

Characterisation of information

If λ is the phase space of physical states one can define the probability distribution of $|\psi_i\rangle$ over phase space,

$$\mu_i(\lambda)$$

If the distributions $\mu_0(\lambda)$ and $\mu_1(\lambda)$ of two non-orthogonal quantum states $|\psi_0\rangle$ and $|\psi_1\rangle$ overlap, then one can conclude that $|\psi_0\rangle$ and $|\psi_1\rangle$ represent mere information about the system they describe.



And vice versa.

Construction of the argument

Consider **two identical and independent preparation devices;** each device prepares a system in either the quantum state

$$|\psi_0\rangle = |0\rangle$$

or the quantum state

$$|\psi_1\rangle = |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Construction of the argument

Consider **two identical and independent preparation devices;** each device prepares a system in either the quantum state

$$|\psi_0\rangle = |0\rangle$$

or the quantum state

$$|\psi_1\rangle = |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

so that when **the two states are brought together,** the complete system can be prepared in any of the four quantum states

$$|0\rangle \otimes |0\rangle, \ |0\rangle \otimes |+\rangle, \ |+\rangle \otimes |0\rangle, \ \text{and} \ |+\rangle \otimes |+\rangle \tag{1}$$

Construction of the argument

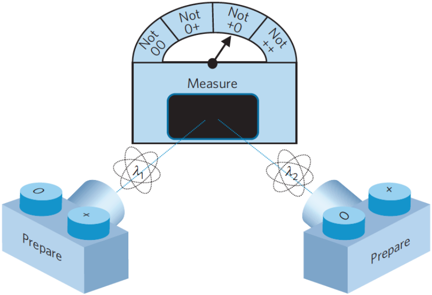


Figure given by PBR in Nat. Phys. 8, 475 (2012)

Construction of the argument

The complete system can be measured, and for this they propose an **entangled measurement** with the four possible outcomes:

$$|\xi_1\rangle = \frac{1}{\sqrt{2}}\Big[|0\rangle \otimes |1\rangle \,+\, |1\rangle \otimes |0\rangle\Big]$$

$$|\xi_2\rangle = \frac{1}{\sqrt{2}}\Big[|0\rangle \otimes |-\rangle \,+\, |1\rangle \otimes |+\rangle\Big]$$

$$|\xi_3\rangle = \frac{1}{\sqrt{2}}\Big[|+\rangle \otimes |1\rangle \,+\, |-\rangle \otimes |0\rangle\Big]$$

$$|\xi_4\rangle = \frac{1}{\sqrt{2}}\Big[|+\rangle \otimes |-\rangle \,+\, |-\rangle \otimes |+\rangle\Big]$$

Construction of the argument

The complete system can be measured, and for this they propose an **entangled measurement** with the four possible outcomes:

$$|\xi_1\rangle = \frac{1}{\sqrt{2}}\Big[|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle\Big]$$

$$|\xi_2\rangle = \frac{1}{\sqrt{2}}\Big[|0\rangle \otimes |-\rangle + |1\rangle \otimes |+\rangle\Big]$$

$$|\xi_3\rangle = \frac{1}{\sqrt{2}}\Big[|+\rangle \otimes |1\rangle + |-\rangle \otimes |0\rangle\Big]$$

$$|\xi_4\rangle = \frac{1}{\sqrt{2}}\Big[|+\rangle \otimes |-\rangle + |-\rangle \otimes |+\rangle\Big]$$

If $|\psi_0\rangle$ and $|\psi_1\rangle$ represent mere information, there is a probability $q^2 > 0$ that both systems result in physical states, λ_1 and λ_2, from the overlap region, Δ.

Construction of the argument

But the probability that the quantum state

$$|0\rangle \otimes |0\rangle$$

results in

$$|\xi_1\rangle$$

is zero,

Construction of the argument

But the probability that the quantum state

$$|0\rangle \otimes |0\rangle$$

results in

$$|\xi_1\rangle$$

is zero, same for $|0\rangle \otimes |+\rangle$ resulting in $|\xi_2\rangle$, for $|+\rangle \otimes |0\rangle$ resulting in $|\xi_3\rangle$, and for $|+\rangle \otimes |+\rangle$ resulting in $|\xi_4\rangle$.

Construction of the argument

But the probability that the quantum state

$$|0\rangle \otimes |0\rangle$$

results in

$$|\xi_1\rangle$$

is zero, same for $|0\rangle \otimes |+\rangle$ resulting in $|\xi_2\rangle$, for $|+\rangle \otimes |0\rangle$ resulting in $|\xi_3\rangle$, and for $|+\rangle \otimes |+\rangle$ resulting in $|\xi_4\rangle$.
This takes them to the conclusion that **if the state $\boldsymbol{\lambda_1 \otimes \lambda_2}$ that arrives to the detector is compatible with the four quantum states (1),** then **the measuring device could give a result that should,** following simple QM, **occur with zero probability.**

Hypotheses + Assumptions

ψ merely information;
Physical state for systems;
System independence;

Hypotheses + Assumptions

$$\left.\begin{array}{l}\psi \text{ merely information;}\\ \text{Physical state for systems;}\\ \text{System independence;}\end{array}\right\} \text{Contradiction}$$

Hypotheses + Assumptions

~~ψ merely information;~~
Physical state for systems;
System independence;

} Contradiction

Hypotheses + Assumptions

ψ merely information;
Physical state for systems;
System independence;
+ Measurement at the preparation stage;

} Contradiction

No measurement assumption

In the case where there is no distinguishability between the preparation of $|0\rangle$ and the preparation of $|+\rangle$, the state that would arrive at the detector would be

$$|\Psi\rangle = |\psi\rangle \otimes |\psi\rangle = N^2 \Big[|0\rangle + |+\rangle \Big] \otimes \Big[|0\rangle + |+\rangle \Big],$$

and not one of the states (1) assumed by PBR.

No measurement assumption

In the case where there is no distinguishability between the preparation of $|0\rangle$ and the preparation of $|+\rangle$, the state that would arrive at the detector would be

$$|\Psi\rangle = |\psi\rangle \otimes |\psi\rangle = N^2 \Big[|0\rangle + |+\rangle\Big] \otimes \Big[|0\rangle + |+\rangle\Big],$$

and not one of the states (1) assumed by PBR.
This state $|\Psi\rangle$ that arrives at the detector is compatible with the measurement basis used in the PBR theorem, in the sense that it may result in any of its elements $(|\xi_i\rangle)$ with non-zero probability.
Following the logic of PBR, no contradiction arises when regarding $|0\rangle$ and $|+\rangle$ as mere information.

Hypotheses + Assumptions

ψ merely information;
Physical state for systems;
System independence;
+ Measurement at the preparation stage;

} Contradiction

Hypotheses + Assumptions

ψ merely information;
Physical state for systems;
System independence;
~~+ Measurement at the preparation stage;~~

} Contradiction

Statement of the EPR theorem

Either Quantum Mechanics is not a complete theory or two quantities associated with non-commuting operators cannot have a simultaneous reality. Negation of the first statement leads to negation of the second one. Then Quantum Mechanics must be an incomplete theory.

Statement of the EPR theorem

Either Quantum Mechanics is not a complete theory or two quantities associated with non-commuting operators cannot have a simultaneous reality. **Negation of the first** statement leads to **negation of the second one**. Then Quantum Mechanics must be an incomplete theory.

Statement of the EPR theorem

Either Quantum Mechanics is not a complete theory or two quantities associated with non-commuting operators cannot have a simultaneous reality. Negation of the first statement leads to negation of the second one. **Then Quantum Mechanics must be an incomplete theory.**

Premises

Hypothesis

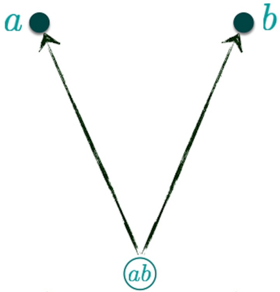
- Completeness;

Assumptions

- Local realism;
- Counterfactuality;

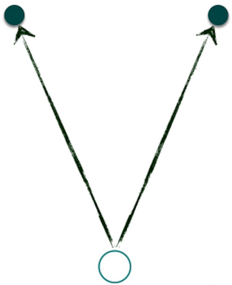
Characterisation of reality

Construction of the argument



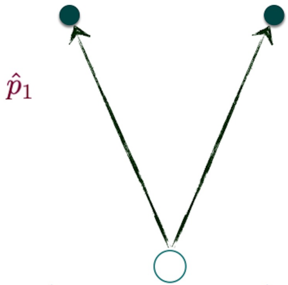
$$\Psi(a,b) = \int v_p(a)u_p(b)dp = \int \varphi_x(a)\phi_x(b)dx$$

Construction of the argument



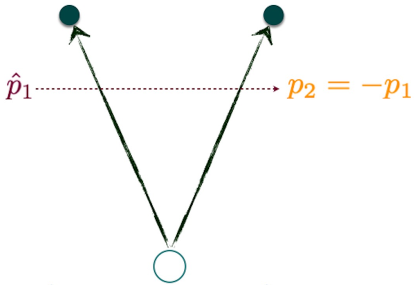
$$\Psi(x_1, x_2) = \int v_p(x_1) u_p(x_2) dp = \int \varphi_x(x_1) \phi_x(x_2) dx$$

Construction of the argument



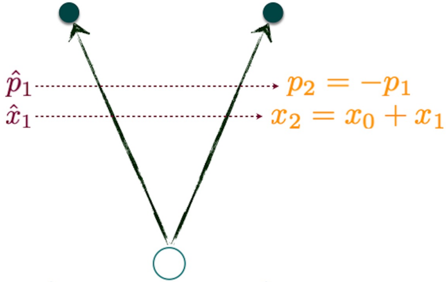
$$\Psi(x_1, x_2) = \int v_p(x_1) u_p(x_2) dp = \int \varphi_x(x_1) \phi_x(x_2) dx$$

Construction of the argument



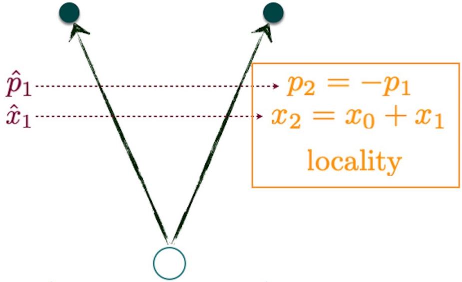
$$\Psi(x_1, x_2) = \int v_p(x_1) u_p(x_2) dp = \int \varphi_x(x_1) \phi_x(x_2) dx$$

Construction of the argument



$$\Psi(x_1, x_2) = \int v_p(x_1) u_p(x_2) dp = \int \varphi_x(x_1) \phi_x(x_2) dx$$

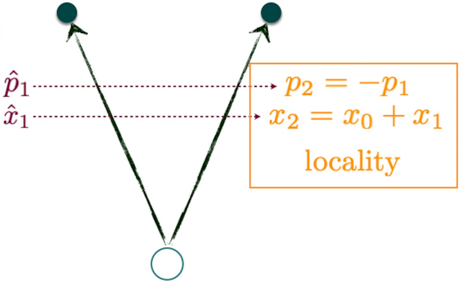
Construction of the argument



$$\Psi(x_1, x_2) = \int v_p(x_1) u_p(x_2) dp = \int \varphi_x(x_1) \phi_x(x_2) dx$$

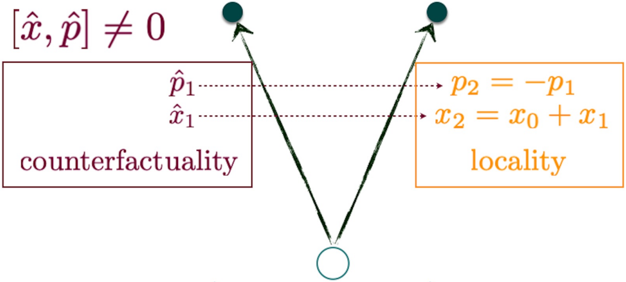
Construction of the argument

$$[\hat{x}, \hat{p}] \neq 0$$



$$\Psi(x_1, x_2) = \int v_p(x_1) u_p(x_2) dp = \int \varphi_x(x_1) \phi_x(x_2) dx$$

Construction of the argument



$$\Psi(x_1, x_2) = \int v_p(x_1) u_p(x_2) dp = \int \varphi_x(x_1) \phi_x(x_2) dx$$

Hypotheses + Assumptions

Completeness;
Local realism;
Counterfactuality;

Hypotheses + Assumptions

$$\left.\begin{array}{l}\text{Completeness;}\\ \text{Local realism;}\\ \text{Counterfactuality;}\end{array}\right\}\text{Contradiction}$$

Hypotheses + Assumptions

~~Completeness;~~ Local realism; Counterfactuality;	} Contradiction

Statement of Bell's theorem

Any mathematical description which "completes" Quantum Mechanics with local hidden variables has to satisfy an inequality. Since this inequality is violated by the predictions of Quantum Mechanics, the latter cannot be completed by means of local hidden variables.

Premises

Hypothesis

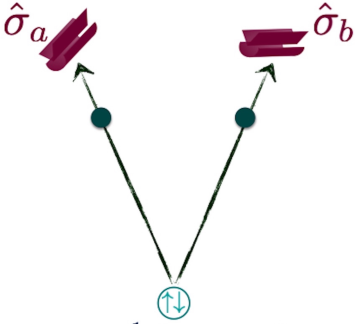
- Existence of local hidden variables that determine the state of a system *before* a measurement is made (local realism);

Assumptions

- Counterfactuality;
- Correct predictions of QM;

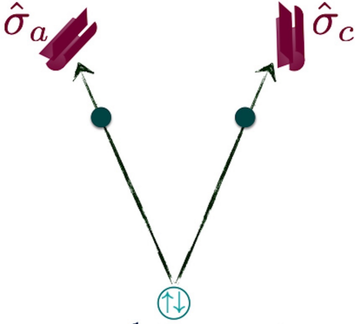
Characterisation of local hidden variables

Construction of the argument



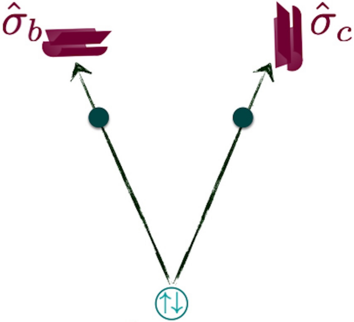
$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right]$$

Construction of the argument



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right]$$

Construction of the argument



$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\Big]$$

Construction of the argument

$$A(\hat{z}, \lambda) = \pm 1 \quad B(\hat{z}, \lambda) = \pm 1$$

$$E(\hat{\sigma}_a, \hat{\sigma}_b) = \int_{\Lambda} A(\hat{a}, \lambda) B(\hat{b}, \lambda) \rho(\lambda) d\lambda$$

Construction of the argument

$$|E(\hat{\sigma}_a, \hat{\sigma}_b) - E(\hat{\sigma}_a, \hat{\sigma}_c)| =$$

$$\left| \int_\Lambda A_1(\hat{a}, \lambda) B_1(\hat{b}, \lambda) \rho(\lambda) d\lambda - \int_\Lambda A_2(\hat{a}, \lambda) B_2(\hat{c}, \lambda) \rho(\lambda) d\lambda \right|$$

Construction of the argument

$$|E(\hat{\sigma}_a, \hat{\sigma}_b) - E(\hat{\sigma}_a, \hat{\sigma}_c)| =$$

$$\left| \int_\Lambda A_1(\hat{a}, \lambda) B_1(\hat{b}, \lambda) \rho(\lambda) d\lambda - \int_\Lambda A_2(\hat{a}, \lambda) B_2(\hat{c}, \lambda) \rho(\lambda) d\lambda \right|$$

$$\left[A_1(\hat{a}, \lambda) = A_2(\hat{a}, \lambda) \right] \quad ...(*1)$$

$$\stackrel{*1}{=} \left| \int_\Lambda A_1(\hat{a}, \lambda) B_1(\hat{b}, \lambda) \left[1 - B_1(\hat{b}, \lambda) B_2(\hat{c}, \lambda) \right] \rho(\lambda) d\lambda \right|$$

Construction of the argument

$$|E(\hat{\sigma}_a, \hat{\sigma}_b) - E(\hat{\sigma}_a, \hat{\sigma}_c)| =$$

$$\left| \int_\Lambda A_1(\hat{a}, \lambda) B_1(\hat{b}, \lambda) \left[1 - B_1(\hat{b}, \lambda) B_2(\hat{c}, \lambda) \right] \rho(\lambda) d\lambda \right|$$

Construction of the argument

$$|E(\hat{\sigma}_a, \hat{\sigma}_b) - E(\hat{\sigma}_a, \hat{\sigma}_c)| =$$

$$\left| \int_\Lambda A_1(\hat{a}, \lambda) B_1(\hat{b}, \lambda) \left[1 - B_1(\hat{b}, \lambda) B_2(\hat{c}, \lambda) \right] \rho(\lambda) d\lambda \right|$$

$$\left[B_1(\hat{b}, \lambda) = -A_3(\hat{b}, \lambda) \right] \quad ...(*2)$$

$$\stackrel{*2}{\leq} \int_\Lambda \left[1 + A_3(\hat{b}, \lambda) B_2(\hat{c}, \lambda) \right] \rho(\lambda) d\lambda$$

Construction of the argument

$$|E(\hat{\sigma}_a, \hat{\sigma}_b) - E(\hat{\sigma}_a, \hat{\sigma}_c)| \leq \int_\Lambda \left[1 + A_3(\hat{b}, \lambda) B_2(\hat{c}, \lambda)\right] \rho(\lambda) d\lambda$$

Construction of the argument

$$|E(\hat{\sigma}_a, \hat{\sigma}_b) - E(\hat{\sigma}_a, \hat{\sigma}_c)| \leq \int_\Lambda \left[1 + A_3(\hat{b}, \lambda) B_2(\hat{c}, \lambda)\right] \rho(\lambda) d\lambda$$

$$\left[B_2(\hat{c}, \lambda) = B_3(\hat{c}, \lambda)\right] \quad ...(*3)$$

$$\overset{*3}{=} \int_\Lambda \left[1 + A_3(\hat{b}, \lambda) B_3(\hat{c}, \lambda)\right] \rho(\lambda) d\lambda$$

$$= 1 + E(\hat{\sigma}_b, \hat{\sigma}_c)$$

Hypotheses + Assumptions

Local realism;
Counterfactuality;
Correct predictions of QM;

Hypotheses + Assumptions

$$\left.\begin{array}{l}\text{Local realism;}\\ \text{Counterfactuality;}\\ \text{Correct predictions of QM;}\end{array}\right\}\text{Contradiction}$$

Hypotheses + Assumptions

~~Local realism;~~ Counterfactuality; Correct predictions of QM;	} Contradiction

Statement of the KS theorem

The non-contextual assignment of simultaneous values to n observables defined on a system described by a state vector in a Hilbert space of dimension $d \geq 3$ is incompatible with the algebraic structure of QM.

Statement of the KS theorem

The non-contextual assignment of simultaneous values to n observables defined on a system described by a state vector in a **Hilbert space of dimension $d \geq 3$** is incompatible with the algebraic structure of QM.

Statement of the KS theorem

The non-contextual **assignment of simultaneous values to *n* observables** defined on a system described by a state vector in a Hilbert space of dimension $d \geq 3$ is incompatible with the algebraic structure of QM.

Statement of the KS theorem

The non-contextual assignment of simultaneous values to ***n*** observables defined on a system described by a state vector in a Hilbert space of dimension $d \geq 3$ is **incompatible with the algebraic structure of QM**.

Premises

Hypotheses

- Non-contextuality;
- Value definiteness;

Assumption

- Correct algebraic structure of QM;

Characterisation of non-contextual hidden variables

Construction of the argument

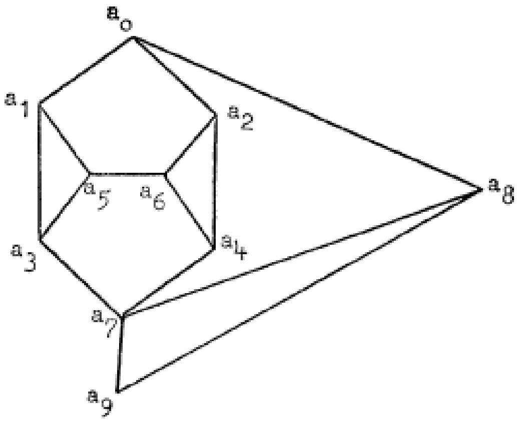
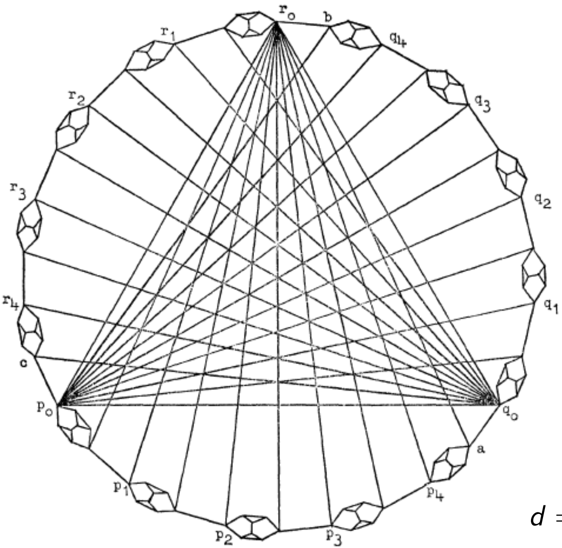


Figure given by KS in J. Math. Mech. 17, 59 (1967)

Construction of the argument



$d = 3; n = 117$

Figure given by KS in J. Math. Mech. 17, 59 (1967)

Construction of the argument

$\hat{\sigma}_x^1$	$\hat{\sigma}_x^2$	$\hat{\sigma}_x^1\hat{\sigma}_x^2$
$\hat{\sigma}_y^2$	$\hat{\sigma}_y^1$	$\hat{\sigma}_y^1\hat{\sigma}_y^2$
$\hat{\sigma}_x^1\hat{\sigma}_y^2$	$\hat{\sigma}_y^1\hat{\sigma}_x^2$	$\hat{\sigma}_z^1\hat{\sigma}_z^2$

$d = 4; n = 9$

Construction of the argument

$\hat{\sigma}^1{}_x$	$\hat{\sigma}^2{}_x$	$\hat{\sigma}^1{}_x\hat{\sigma}^2{}_x$
$\hat{\sigma}^2{}_y$	$\hat{\sigma}^1{}_y$	$\hat{\sigma}^1{}_y\hat{\sigma}^2{}_y$
$\hat{\sigma}^1{}_x\hat{\sigma}^2{}_y$	$\hat{\sigma}^1{}_y\hat{\sigma}^2{}_x$	$\hat{\sigma}^1{}_z\hat{\sigma}^2{}_z$

$d = 4; n = 9$

Construction of the argument

$\hat{\sigma}^1{}_x$	$\hat{\sigma}^2{}_x$	$\hat{\sigma}^1{}_x\hat{\sigma}^2{}_x$	$\longrightarrow \mathbb{I}_2$
$\hat{\sigma}^2{}_y$	$\hat{\sigma}^1{}_y$	$\hat{\sigma}^1{}_y\hat{\sigma}^2{}_y$	$\longrightarrow \mathbb{I}_2$
$\hat{\sigma}^1{}_x\hat{\sigma}^2{}_y$	$\hat{\sigma}^1{}_y\hat{\sigma}^2{}_x$	$\hat{\sigma}^1{}_z\hat{\sigma}^2{}_z$	$\longrightarrow \mathbb{I}_2$

Construction of the argument

$\hat{\sigma}^1{}_x$	$\hat{\sigma}^2{}_x$	$\hat{\sigma}^1{}_x\hat{\sigma}^2{}_x$	$\longrightarrow \mathbb{I}_2$
$\hat{\sigma}^2{}_y$	$\hat{\sigma}^1{}_y$	$\hat{\sigma}^1{}_y\hat{\sigma}^2{}_y$	$\longrightarrow \mathbb{I}_2$
$\hat{\sigma}^1{}_x\hat{\sigma}^2{}_y$	$\hat{\sigma}^1{}_y\hat{\sigma}^2{}_x$	$\hat{\sigma}^1{}_z\hat{\sigma}^2{}_z$	$\longrightarrow \mathbb{I}_2$
$\downarrow \mathbb{I}_2$	$\downarrow \mathbb{I}_2$	$\downarrow -\mathbb{I}_2$	

Hypotheses + Assumptions

Non-contextuality;
Value definiteness;
Correct algebraic structure of QM;

Hypotheses + Assumptions

Non-contextuality;
Value definiteness;
Correct algebraic structure of QM;

} Contradiction

Hypotheses + Assumptions

~~Non-contextuality;~~ Value definiteness; Correct algebraic structure of QM;	} Contradiction

Summary

~~Completeness;~~
Local realism;
Counterfactuality;
} EPR contradiction

~~Local realism;~~
Counterfactuality;
Predictions of QM;
} Bell contradiction

~~Non-contextuality;~~
Value definiteness (realism);
Algebra of QM;
} KS contradiction

Summary

~~Completeness;~~
Local realism;
Counterfactuality;
} EPR contradiction

~~Local realism;~~
Counterfactuality;
Predictions of QM;
} Bell contradiction

Summary

~~Completeness;~~
Local realism;
Counterfactuality;
} EPR contradiction

~~Local realism;~~
Counterfactuality;
Predictions of QM;
} Bell contradiction

} Completeness;
Local realism;
~~Counterfactuality;~~
Predictions of QM;

Summary

~~Completeness;~~
Local realism;
Counterfactuality;
} EPR contradiction

~~Local realism;~~
Counterfactuality;
Predictions of QM;
} Bell contradiction

} Completeness;
Local realism;
~~Counterfactuality;~~
Predictions of QM;

~~Non-contextuality;~~
Realism;
Algebra of QM;
} KS contradiction

Summary

~~Completeness;~~
Local realism;
Counterfactuality;
} EPR contradiction

~~Local realism;~~
Counterfactuality;
Predictions of QM;
} Bell contradiction

} Completeness;
Local realism;
~~Counterfactuality;~~
Predictions of QM;

~~Non-contextuality;~~
Realism;
Algebra of QM;
} KS contradiction } Complementarity principle

Summary

~~Completeness;~~
Local realism;
Counterfactuality;
} EPR contradiction

~~Local realism;~~
Counterfactuality;
Predictions of QM;
} Bell contradiction

} Completeness;
Local realism;
~~Counterfactuality;~~
Predictions of QM;

~~Non-contextuality;~~
Realism;
Algebra of QM;
} KS contradiction } Complementarity principle

Local, contextual, realism.

The complementarity principle

In fact, it is only the mutual exclusion of any two experimental procedures, permitting the unambiguous definition of complementary physical quantities, which provides room for new physical laws, [...] which might at first sight appear irreconcilable with the basic principles of science.

The complementarity principle

> *In fact, it is only the mutual exclusion of any two experimental procedures, permitting the unambiguous definition of complementary physical quantities, which provides room for new physical laws, [...] which might at first sight appear irreconcilable with the basic principles of science.*

Complementarity meant for Bohr an understanding of physical reality in regards to reference frames, the **defining objects of reference frames being the measuring apparatuses** and the quantities coming into being within these reference frames as complementary; meaning that **two or more complementary quantities cannot manifest in one and the same reference frame, and that each quantity must manifest in its corresponding reference frame.**

The reference frame interpretation

- A reference frame is determined by a complete set of commuting operators;
- Eigenstates of these complete set are **ontological states**;
- The states generated from linear combinations of different eigenstates of an observable are **quantum states**;

The reference frame interpretation

- A reference frame is determined by a complete set of commuting operators;
- Eigenstates of these complete set are **ontological states**;
- The states generated from linear combinations of different eigenstates of an observable are **quantum states**;
- The reference frame determines the ontological or informational character of ψ;

Complementary reference frames

Any non-commuting quantities define complementary reference frames.
So, the prototypical complementary quantities of position and momentum define complementary reference frames, since

$$[\hat{x}, \hat{p}] = -i\hbar$$

holds.

Complementary reference frames

Any non-commuting quantities define complementary reference frames.
So, the prototypical complementary quantities of position and momentum define complementary reference frames, since

$$[\hat{x}, \hat{p}] = -i\hbar$$

holds.
Time and energy also define complementary reference frames.

Complementary reference frames

Any non-commuting quantities define complementary reference frames.
So, the prototypical complementary quantities of position and momentum define complementary reference frames, since

$$[\hat{x}, \hat{p}] = -i\hbar$$

holds.
Time and energy also define complementary reference frames.
Eigenstates of position are defined in space-time, while eigenstates of momentum are defined in, what we call, momentum-energy.

The complementary reference frame to position

A momentum measurement would force the system being measured to stand in the momentum reference frame. In the position representation,

$$\psi_{\vec{p}}(\vec{x}, t) = e^{\frac{i}{\hbar}(\vec{p}_0 \cdot \vec{x} - E_0 t)}$$

and, as we have seen, this state only depicts information of the particle's whereabouts in the position reference frame, while it is an ontological state in the momentum reference frame. Indeed,

$$\psi_{\vec{p}}(\vec{p}, E) = \delta^{(3)}(\vec{p} - \vec{p}_0)\delta(E - E_0)$$

The complementary reference frame to position

A momentum measurement would force the system being measured to stand in the momentum reference frame. In the position representation,

$$\psi_{\vec{p}}(\vec{x}, t) = e^{\frac{i}{\hbar}(\vec{p}_0 \cdot \vec{x} - E_0 t)}$$

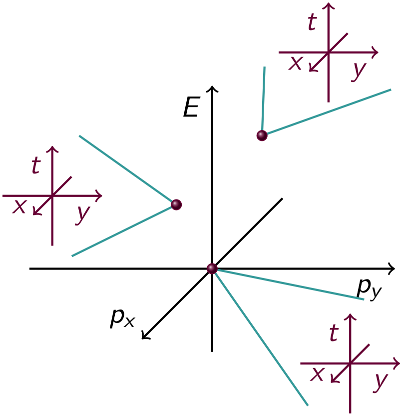
and, as we have seen, this state only depicts information of the particle's whereabouts in the position reference frame, while it is an ontological state in the momentum reference frame. Indeed,

$$\psi_{\vec{p}}(\vec{p}, E) = \delta^{(3)}(\vec{p} - \vec{p}_0)\delta(E - E_0)$$

For a free particle in a momentum eigenstate all of space-time is an equivalence class projected onto the location of the particle in momentum-energy.

The momentum-energy manifold

Four-dimensional space-time projected as an equivalence class to four-dimensional momentum-energy. As we know from SR, space and time are geometrically intertwined. We propose the same for energy and momentum.



Conclusions

- We can build an interpretation with the previous results, namely the reference frame interpretation;
- The character of ψ is more subtle than just the division between ontological/epistemological;
- This interpretation can be applied to explain in a local way the violation of Bell's inequality;
- It can also be used to explain *in a less paradoxical manner* the double slit experiment and the measurement problem;
- As an example, one can give a geometrical structure to momentum-energy, a manifold isomorphic to space-time;

All this is work in progress.

References

[1] Pusey, M. F., Barrett, J., Rudolph, T.: On the reality of the quantum state. Nat. Phys. 8, 475 (2012)

[2] Harrigan, N., Spekkens, R. W.: Einstein, incompleteness, and the epistemic view of quantum states. Found. Phys. 40, 125 (2010)

[3] Bohr, N.: Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. 48, 696 (1935)

[4] Faye, J.: Copenhagen interpretation of quantum mechanics. In *The Stanford Encyclopedia of Philosophy*, edited by E.N. Zalta, (2014)

[5] Bohr, N.: *Atomic Theory and the Description of Nature*, reprinted as *The Philosophical Writings of Niels Bohr, Vol. I*, (Ox Bow Press 1987)

[6] Dickson, M.: The EPR experiment: A prelude to Bohr's reply to EPR. In *History of Philosophy of Science, New Trends and Perspectives*, edited by M. Heidelberger and F. Stadler, 263 (Kluwer Academic Publishers 2002)

[7] Sánchez-Kuntz N., Nahmad-Achar E.: Quantum locality, rings a bell?: Bell's inequality meets local reality and true determinism. Found. Phys. 48, 27 (2018)

[8] Aharonov, Y.: Can we make sense out of the measurement process in relativistic quantum mechanics? Phys. Rev. D. 24, 359 (1981)

[9] Leifer, M.S.: Is the quantum state real? An extended review of ψ-ontology theorems. Quanta 3, 67 (2014)

[10] Sánchez-Kuntz N., Nahmad-Achar E.: The measure of PBR's reality arXiv:1810.11072 (2018)