# Fractionalization in spin systems

## An fRG perspective

Dietrich Roscher

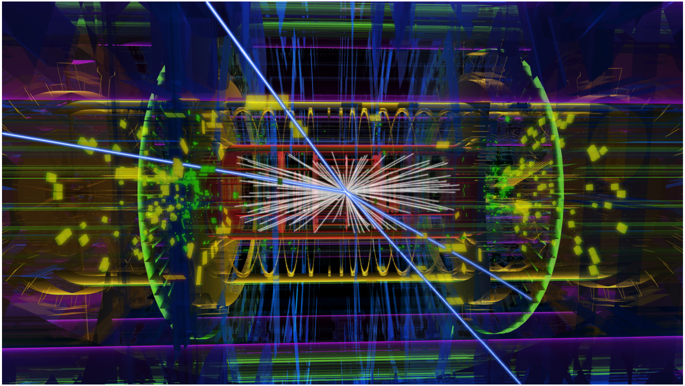
with Michael M. Scherer, Nico Gneist, Simon Trebst, Sebastian Diehl

arXiv:1905.01060

Institute for Theoretical Physics / Universität zu Köln

Cold Quantum Coffee, Heidelberg
May 7th, 2019

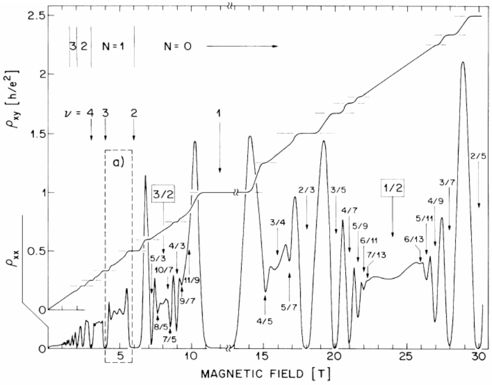
# High energy... "fractionalization"



ATLAS Experiment © 2016 CERN

Shattering bound states by brute force

# Fractionalization in solids

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Low-energy Collective Effect

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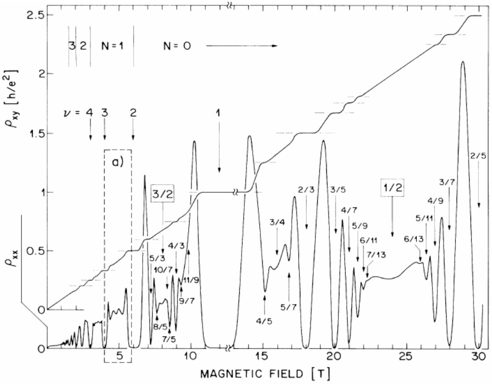


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## Low-energy Collective Effect

- Seems perfectly suited for field theory & (f)RG
- Extremely rich/confusing field: anyons, majoranas, gauge fields...
- Actual physical observables/interpretation?

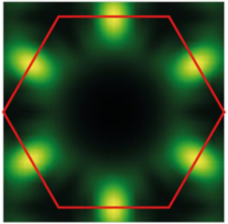
# Spin systems & Spin liquids

Heisenberg model:

$$\mathcal{H} = \sum_{\langle i,j \rangle} J_{ij} S_i^\mu \cdot S_j^\mu$$

Magnetic phases (SU(2) symmetry breaking):

$$\sum_i \langle S_i^\mu \rangle \neq 0$$



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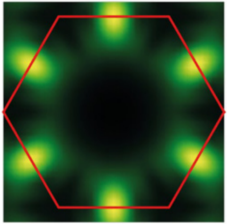

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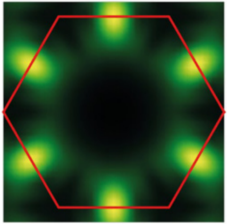

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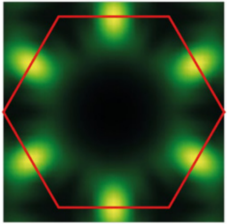

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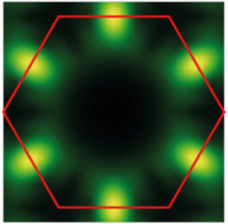

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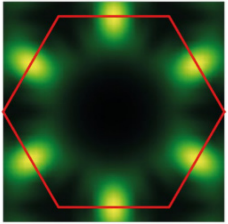

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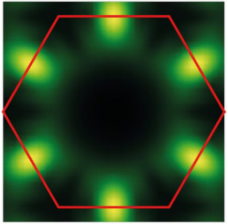

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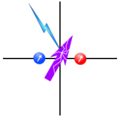
Spin systems with non-magnetic but non-trivial ground states

- Long-range entanglement
- Topological order
- Fractionalization

# Pseudofermion representation

Spin decomposition [A.A. Abrikosov, '65]:

$$S_i^\mu \equiv \frac{1}{2} f_{i\alpha}^\dagger \sigma_{\alpha\beta}^\mu f_{i\beta}$$



GIVEN: "Microscopic" Spin model:

$$\mathcal{H}^{\mathrm{UV}} = J \sum_{\langle i,j \rangle} S_i^\mu \cdot S_j^\mu \simeq -\frac{J}{2} \sum_{\langle i,j \rangle} f_{i\alpha}^\dagger f_{j\alpha} f_{j\beta}^\dagger f_{i\beta}$$

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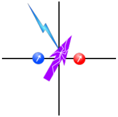

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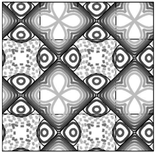
WANTED: Low-energy Spin liquid model [X.-G. Wen, '02]

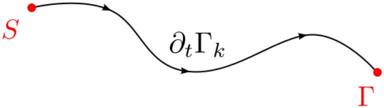
$$\mathcal{H}^{\mathrm{IR}} \sim \sum_{\langle i,j \rangle} \left[ Q_{ij} f_{i\alpha}^\dagger f_{j\alpha} + \Delta_{ij} \epsilon_{\alpha\beta} f_{i\alpha}^\dagger f_{j\beta}^\dagger + h.c. + \dots \right]$$

- 246 different classes (symmetries of $Q, \Delta$)
- Fractionalization, "Topological order"
- Ad hoc postulated...



# Pseudofermion functional RG

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\left[\frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}\right]$$

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$$\Gamma_k = \int_\tau \left[\sum_i f_{i\alpha}^\dagger (i\partial_\tau) f_{i\alpha} - \frac{J_k}{2}\sum_{\langle i,j\rangle} f_{i\alpha}^\dagger f_{j\alpha} f_{j\beta}^\dagger f_{i\beta}\right]$$

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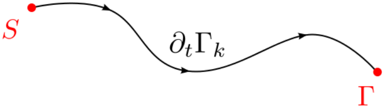


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Let's consider:

- SU(2) $\rightarrow$ SU($N$) "spins"
- on a 2D square lattice
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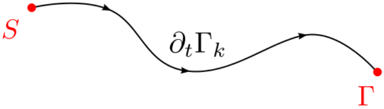


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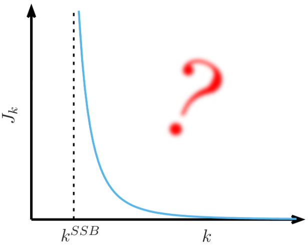
Result: $J_k \rightarrow \infty$



# Suppose we would bosonize...

$$\frac{J_k}{2} f^\dagger_{i\alpha} f_{j\alpha} f^\dagger_{j\beta} f_{i\beta} \rightsquigarrow m_Q Q^\dagger_{ij} Q_{ij} + Q_{ij} f^\dagger_{\alpha j} f_{\alpha i} \overset{\mathbf{RG}}{\rightsquigarrow} m_{Q,k} Q^\dagger_{ij} Q_{ij} + U_k^{n>2} \left[ \left( Q^\dagger Q \right)^n \right]$$

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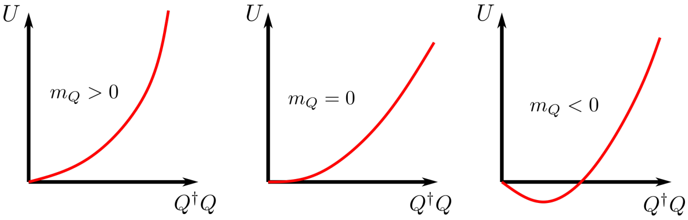
Second order (well...) phase transition ($m_Q \sim J_k^{-1}$):

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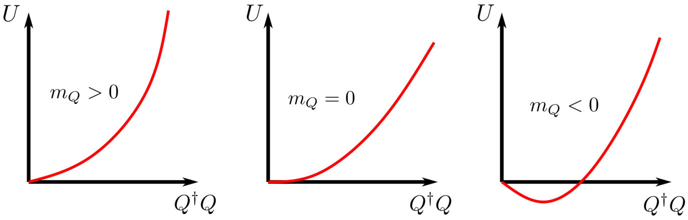


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Drawbacks of bosonization:

- Bias by choice of channel *and/or* massive cost
- Fierz ambiguity
- Spatially inhomogeneous phases?

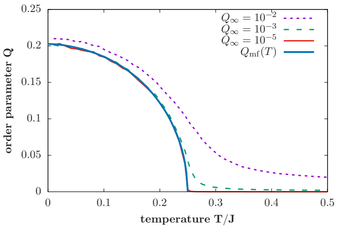
# Infinitesimal explicit symmetry breaking

[M. Salmhofer, C. Honerkamp, W. Metzner, O. Lauscher, '04]

$$\Gamma_k = \int_\tau \left[ \sum_i f^\dagger_{i\alpha}(i\partial_\tau) f_{i\alpha} - \frac{J_k}{2} \sum_{\langle i,j \rangle} f^\dagger_{i\alpha} f_{j\alpha} f^\dagger_{j\beta} f_{i\beta} + \sum_{\langle i,j \rangle} Q_{ij,k} f^\dagger_{i\alpha} f_{j\alpha} + \ldots \right]$$

- Minimal bias as $Q_\infty \to 0$
- New vertices (Fierz-completeness!)
- Exact for SU($N \to \infty$)

[DR, F.L. Buessen, M.M. Scherer, S. Trebst, S. Diehl, '18]



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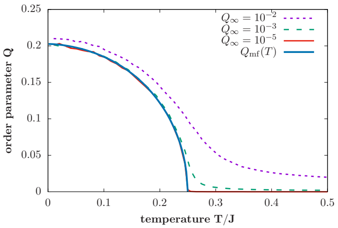


Wait a minute... symmetry breaking??

# Symmetries... what's real?

$$\mathcal{H}^{\rm UV} = J \sum_{\langle i,j \rangle} S_i^\mu \cdot S_j^\mu, \qquad S_i^\mu \equiv \frac{1}{2} f_{i\alpha}^\dagger \sigma_{\alpha\beta}^\mu f_{i\beta}$$

The obvious:

- *Global* **SU(N)**: better not be broken for spin liquid

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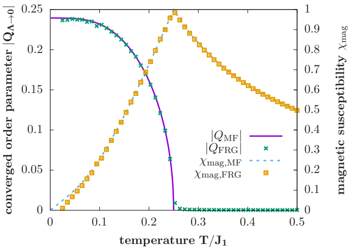

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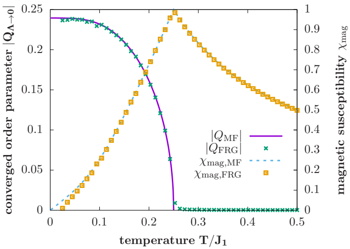

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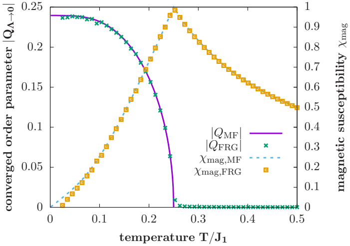


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- *Local* **U(1)**: broken by $Q_{ij} \sim \langle f_{i\alpha}^\dagger f_{j\alpha} \rangle$
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# Artificial & Local

Pseudofermion Spin operator: $S_i^\mu = f_{i\alpha}^\dagger \sigma_{\alpha\beta}^\mu f_{i\alpha}$

Reformulate (for $N = 2$): $\psi_i \equiv \begin{pmatrix} f_{i\uparrow} & f_{i\downarrow} \\ f_{i\uparrow}^\dagger & -f_{i\downarrow}^\dagger \end{pmatrix}$

Heisenberg model:

$$\mathcal{H} = J \sum_{\langle i,j \rangle} S_i^\mu \cdot S_j^\mu = \frac{J}{16} \sum_{\langle i,j \rangle} \mathrm{Tr}\left[\psi_i^\dagger \psi_i \, \sigma^{\mu,T}\right] \cdot \mathrm{Tr}\left[\psi_j^\dagger \psi_j \, \sigma^{\mu,T}\right]$$

...invariant under $\psi_i \rightarrow h_i \psi_i$ with $h_i \in \mathrm{SU}(2)$.

# Hilbert spaces & Constraints

Spin operator: $\{|\uparrow\rangle, |\downarrow\rangle\}$ $\quad \xleftrightarrow{?} \quad$ *Pseudo*fermions: $\{|0,0\rangle, |0,1\rangle, |1,0\rangle, |1,1\rangle\}$

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However, *gauging* [I. Affleck, Z. Zou, T. Hsu, P.W. Anderson '88]

$$L = \frac{1}{2}\sum_i \mathrm{Tr}\left[\psi^\dagger_i (i\partial_\tau)\psi_i\right] - H : \quad (i\partial_t) \rightarrow (i\partial_t) + A^\mu \sigma^\mu$$

enforces constraint and restores full local SU(2).

*Emergent* gauge fields in spin liquids!

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# Implementing the constraint

Issues of the SU(N) gauge theory:

- Overcompleteness

$$f_\alpha^\dagger f_\alpha = 1$$

$$f_\uparrow^\dagger f_\downarrow^\dagger = 0$$

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Issues of the SU(N) gauge theory:

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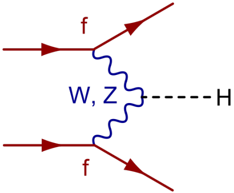
- ...on a lattice - monopoles?



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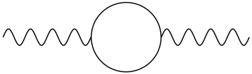

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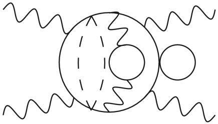

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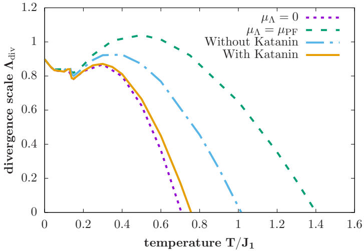
- Reliable truncation scheme?
- Gauge invariant regularization/mWTI?

# Implementing the constraint

Alternatively: add $\mu_{\mathrm{PF}} = \frac{i}{2}\pi T$

[V.N. Popov, S.A. Fedotov '88]

- No local SU($N$) to begin with
- Simple implementation
- Straightforward truncation
- Physical interpretation!?



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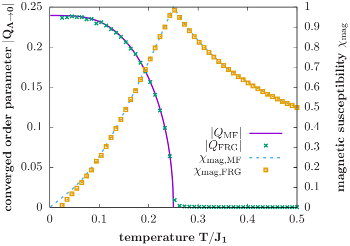


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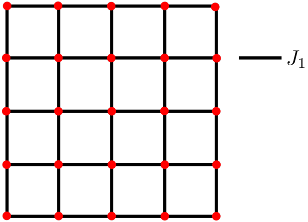
# Spatial Inhomogeneity - Staggered Flux Spin Liquid

$$\Gamma_k = \int_\tau \left[ \sum_i f^\dagger_{i\alpha}(i\partial_\tau) f_{i\alpha} - \frac{J_k}{2} \sum_{\langle i,j \rangle} f^\dagger_{i\alpha} f_{j\alpha} f^\dagger_{j\beta} f_{i\beta} + \sum_{\langle i,j \rangle} Q_{ij,k} f^\dagger_{i\alpha} f_{j\alpha} + \ldots \right]$$

Spin liquid states at large $N$:

- Naïve implementation: homogeneous “BZA” state



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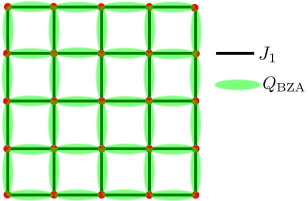

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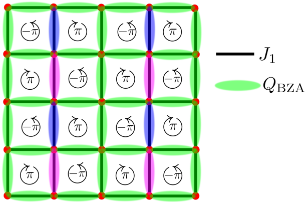

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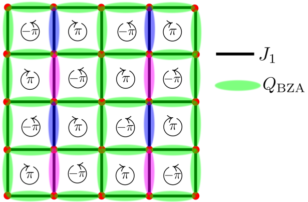



How to include spatially structured order parameters?

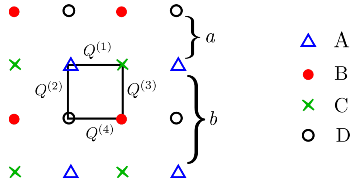
Brute force is *not* and option!

# Spatial Inhomogeneity - Staggered Flux Spin Liquid

Solution: Introduce of $M$ artificial sublattices $f_i \rightarrow \Psi = (f_i^A, f_i^B, f_i^C, ...)$

$$f^\dagger_{i\alpha} f_{j\alpha} f^\dagger_{j\beta} f_{i\beta} \longrightarrow (\Psi^\dagger_\alpha \eta^{AC} \Psi_\alpha)(\Psi^\dagger_\beta \eta^{CA} \Psi_\beta) + ...$$

- Finite-ranged OPs addressed by $M$-dimensional matrices
- Formally: mapping part of the translation group to an SU($M$)



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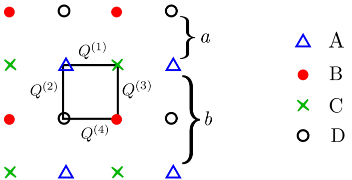
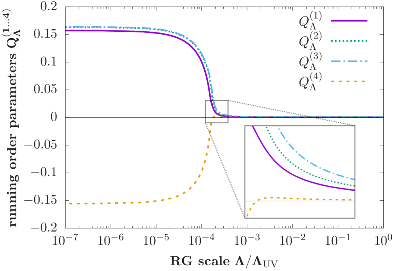
- Cost: $3 \to 24$ flow equations for 4-sublattice Heisenberg model



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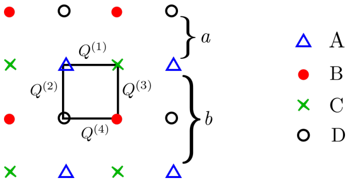
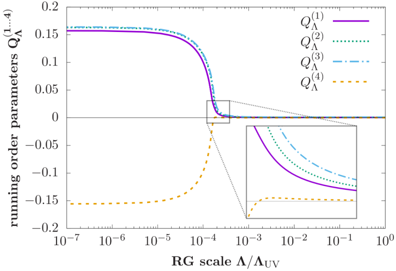


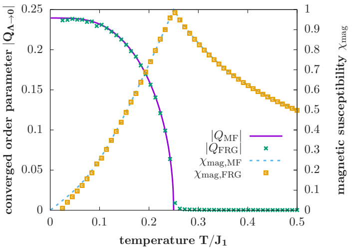

Bonus: Group theory analysis of symmetry breaking patterns

# Symmetries... what's real?

$$\mathcal{H}^{\mathrm{UV}} = J \sum_{\langle i,j \rangle} S_i^\mu \cdot S_j^\mu, \qquad S_i^\mu \equiv \frac{1}{2} f_{i\alpha}^\dagger \sigma_{\alpha\beta}^\mu f_{i\beta}$$

The obvious:

- *Global* **SU(N)**: better not be broken for spin liquid **DONE**
- **Translation invariance**: Wen's classification **DONE**

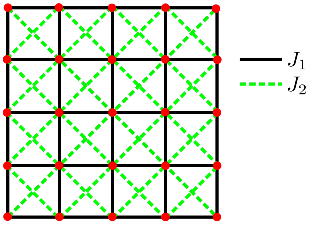


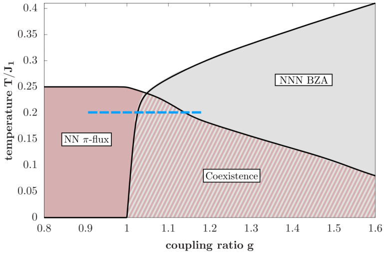

The not-so-obvious:

- *Local* **U(1)**: broken by $Q_{ij} \sim \langle f_{i\alpha}^\dagger f_{j\alpha} \rangle$ **DONE**
- *Artificial* symmetry breaking?
- Actually, that's not even all... **DONE**

# $\underbrace{\text{Real world}}_{\text{well...}}$ challenge: $J_1 - J_2$ Heisenberg model

arXiv:1905.01060




- SU($N$): Competition of SL ordering patterns

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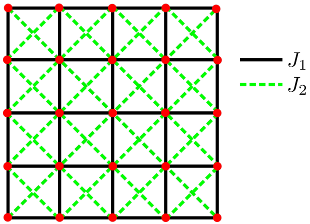


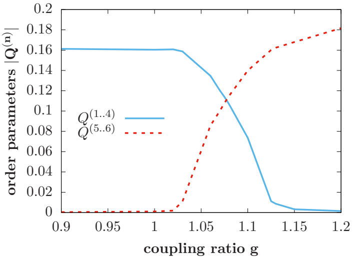


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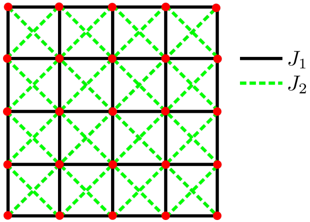


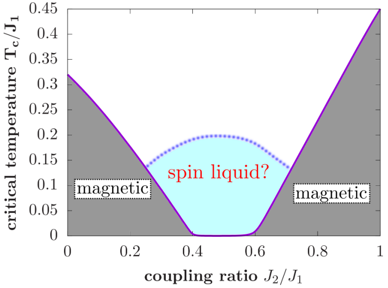


- SU($N$): Competition of SL ordering patterns
- SU(2): Localization of magnetic phases consistent with literature

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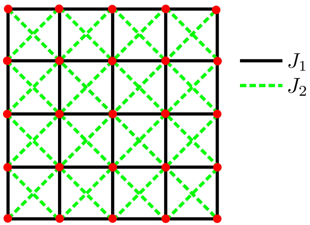


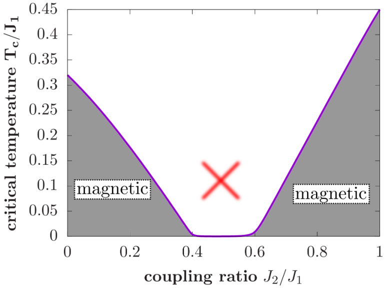


- SU($N$): Competition of SL ordering patterns
- SU(2): Localization of magnetic phases consistent with literature
- Non-magnetic regime is *not* a spin liquid of the type presented here
- fRG currently not sensitive to 4/6/8... fermion order parameters

# Conclusions & Outlook

Achievements of spin liquid fRG:

- Systematic emergence of spin liquids from microscopic spin models
- Clear picture of underlying ordering mechanisms
- Successful proof of principle
- Systematic inclusion of the fermion number constraint
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**THANK YOU!**