Dynamical topological transitions in the massive Schwinger model

Torsten V. Zache

Cold quantum coffee

May 21, 2019, Heidelberg

Based on TVZ et al, *Phys. Rev. Lett.*, *122*(5), 050403 (2019)

Motivation: quantum simulation/computing

Quantum systems are exponentially complex: $\dim\left(\mathcal{H}^{\otimes p}\right) = (\dim \mathcal{H})^p$

Quantum speed up for computationally hard problems, e.g.

- Searching a large database („Grover‘s search“) $O(\sqrt{N})$ vs. $O(N)$

  Grover, ar*Xiv:9605043* (1996)

- Factorizing prime numbers („Shor‘s algorithm“)

  Shor, *SIAM review*, *41*(2), 303-332 (1999)

- Quantum Fourier transform $O(n^2)$ vs. $O(n2^n)$

Quantum computation/simulation:

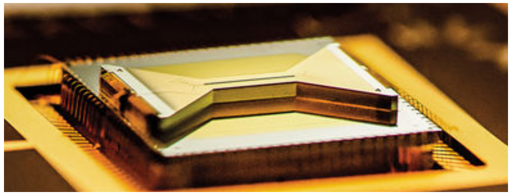
Manin, *Sovetskoye Radio, Moscow*, *128* (1980)
Feynman, *Int. J. Theor. Phys. 21*(6), 467-488 (1982)

simulate quantum physics with a quantum system!

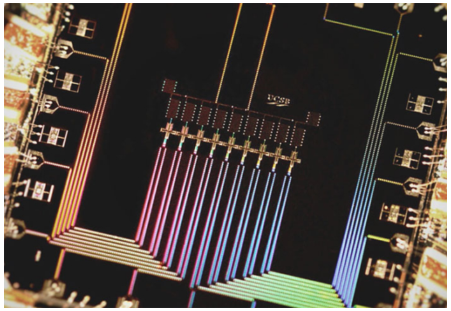
Outline

- Quantum simulation of lattice gauge theories
- The strong CP problem in QCD
- Dynamical quantum phase transitions
- Dynamical topological transitions in the massive Schwinger model

Quantum simulation of high-energy physics

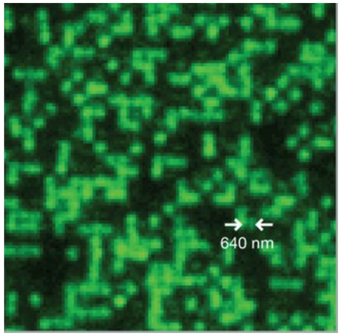


Many proposals with different approaches on various platforms:



trapped ions, superconducting qubits,.. (*digital*)

Byrnes & Yamamoto, *Physical Review A*, *73*(2), 022328 (2006)
Jordan, Lee & Preskill, *Science*, *336*(6085), 1130-1133 (2012)
Marcos et al., *Physical review letters*, *111*(11), 110504 (2013)
Zohar et al., *Physical review letters*, *118*(7), 070501 (2017)
...



ultracold gases,.. (*analog*)

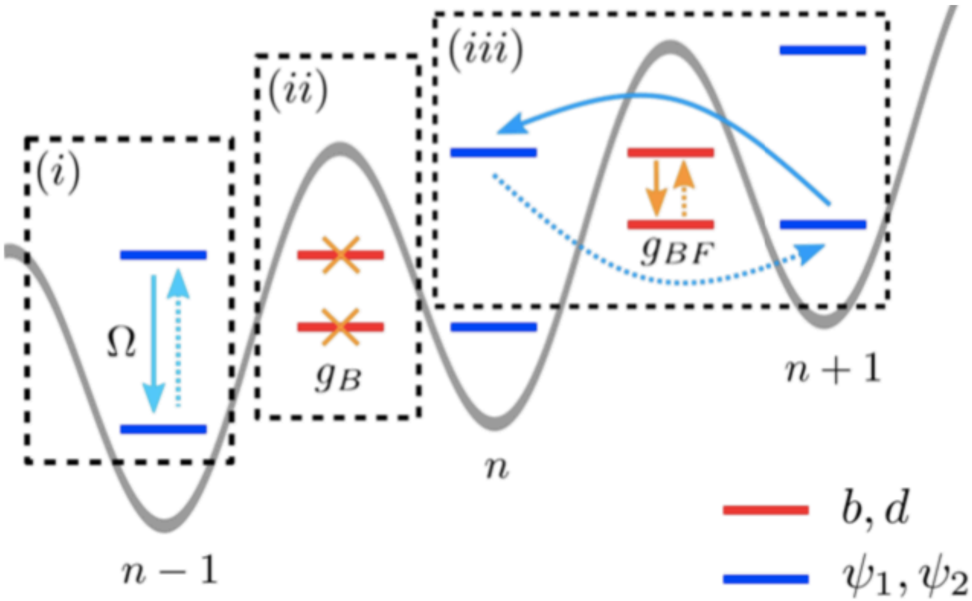
Büchler et al., *Physical review letters*, *95*(4), 040402 (2005)
Wiese, *Annalen der Physik*, *525*(10-11), 777-796 (2013)
Tagliacozzo et al., *Nature communications*, *4*, 2615 (2013)
Zohar, Cirac & Reznik, *Reports on Progress in Physics*, *79*(1), 014401 (2015)
...

Ion chip trap, IONQ, picture from www.sciencemag.org (Jan. 10 2018)
Google's superconducting chip, picture from physicsworlds.com (Jun. 10 2016)
quantum gas mircoscope picture from Bakr et al. *Nature*, *462*(7269), 74 (2009)

Implementation of lattice gauge theories

Limited realizations so far:
one-dimensional systems, few lattice sites, ground state or short time dynamics, ..

Martinez et al., *Nature*, *534*(7608), 516 (2016)
…
Klco et al., *Phys. Rev. A*, *98*(3), 032331 (2018)
Kokail et al., *arXiv:1810.03421* (2018)
Schweizer et al., *arXiv:1901.07103* (2019)
…



TVZ et al., *Quantum science and technology*, *3*(3), 034010 (2018)

Real-world phenomena (higher dimensions, non-abelian gauge groups) lie in the far future –
what can we do *now*?

The strong CP problem

Violates combined charge (C) and parity (P) symmetry!

QCD with a 'vacuum angle':

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}^a \left(i\gamma^\mu D_\mu - m\right)\psi^a - \frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} \boxed{- \theta \frac{g^2 N_f}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{a,\mu\nu}}$$

experimental fact:
QCD conserves CP

$$|\theta| \lesssim 10^{-10}$$

Chupp et al *Reviews of Modern Physics 91*(1), 015001 (2019)

… but why???

Peccei & Quinn:
The angle could be a dynamical field (the 'axion') …

$$\theta = \langle\hat{\theta}\rangle$$

Peccei & Quinn, *Physical Review Letters*, *38*(25), 1440 (1977)

Setup: quench dynamics in QED2

Vacuum structure of QED2:

$$\mathbb{R} \sim S^1 \to U(1) \sim S^1 \qquad \nu = \frac{e}{4\pi}\int d^2x\, \epsilon_{\mu\nu}F^{\mu\nu}$$

Hamiltonian with a topological term:

Coleman, Annals Phys. 101, 239 (1976)

$$H_\theta = \int_x \left\{ \bar{\psi}_x \left[i\gamma^1 \left(\partial_x - ieA_x\right) + m \right] \psi_x + \frac{1}{2}E_x^2 + \frac{e\theta}{2\pi}E_x \right\}$$

Use the axial anomaly to rewrite it:

$$H_\theta = \int_x \left\{ \bar{\psi}_x \left[i\gamma^1 \left(\partial_x - ieA_x\right) + m\, e^{i\theta\gamma_5} \right] \psi_x + \frac{1}{2}E_x^2 \right\}$$

Consider a general quench and calculate real-time dynamics:

$$\theta \to \theta' \qquad |\psi(t)\rangle = \mathrm{e}^{-iH_{\theta'}t}\,|\Omega(\theta)\rangle$$

Dynamical quantum phase transitions

Partition function in thermal equilibrium:

$$Z(\beta) = \mathrm{Tr}\left[e^{-\beta H}\right]$$
$$= e^{-V f(\beta)}$$

Loschmidt amplitude out of equilibrium:

$$L(t) = \left\langle \psi(0) | e^{-iHt} | \psi(0) \right\rangle$$
$$= e^{-V\Gamma(t)}$$

$f(\beta_c)$ Non-analyticity $\Gamma(t_c)$

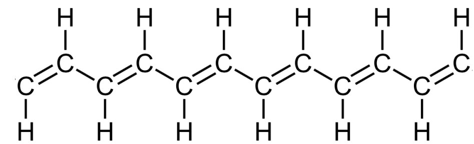
Heyl, Polkovnikov & Kehrein, *Physical review letters*, *110*(13), 135704 (2013).

Thermal phase transition at β_c

Dynamical quantum phase transition (DQPT) at t_c

Topological insulators: the SSH model

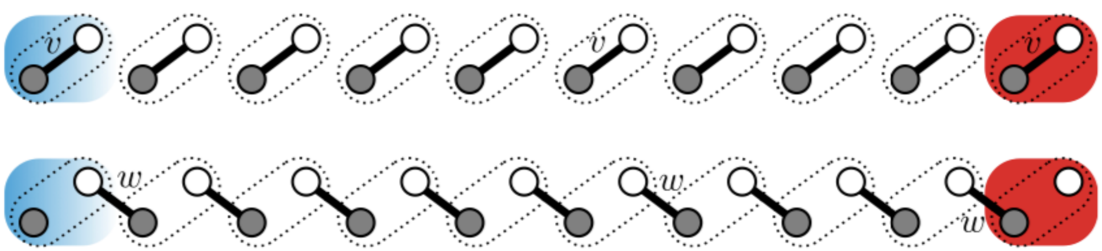
Spinless fermions with staggered hopping
as a model for polyacetylene:



$$H = \sum_n \left[v \left(c^\dagger_{n,A} c_{n,B} + \text{h.c.} \right) + w \left(c^\dagger_{n,B} c_{n+1,A} + \text{h.c.} \right) \right]$$

Su, Schrieffer, Heeger.
Phys. Rev. Lett. *42*(25), 1698 (1979)

Two gapped (insulating) phases distinguished by their topology:



Asboth et al.
A short course on topological insulators
arXiv:1509.02295

effective field theory: free Dirac fermions

$$H \to \int dx \, \left[\psi_x \left(\sigma^x m - i \sigma^y \partial_x \right) \psi_x \right]$$

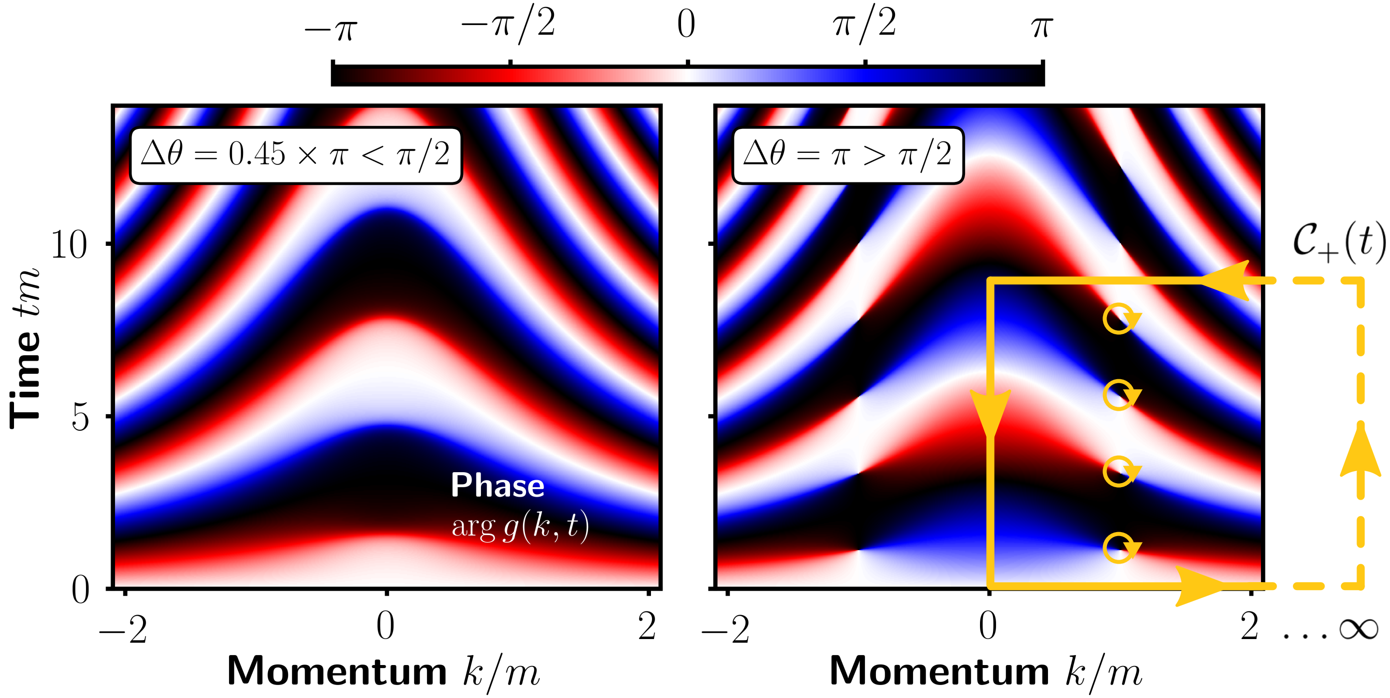
$$m = v - w \,, \quad a = \frac{1}{w}$$

$$\psi_x \leftarrow \frac{1}{\sqrt{a}} \begin{pmatrix} c_{n,A} \\ c_{n,B} \end{pmatrix} \quad (a \to 0)$$

Results

Real-time topology without interactions

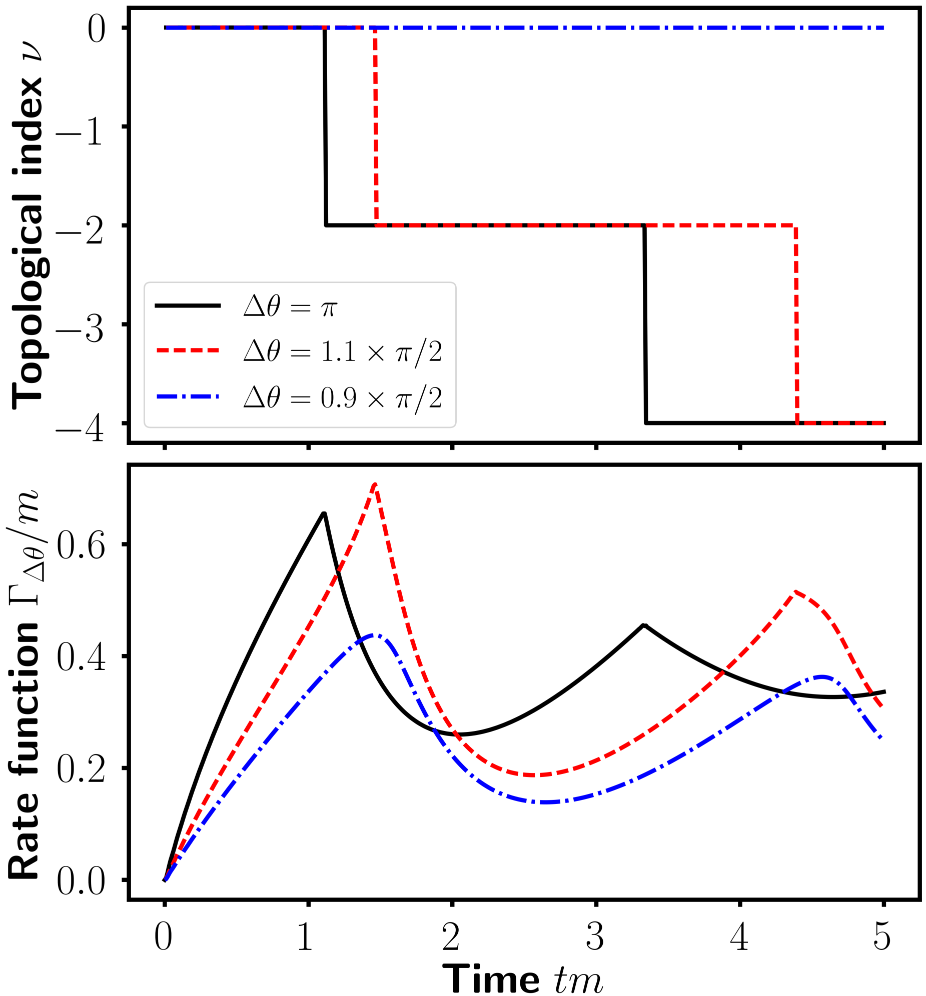
Time-ordered correlation function: $g(x,t) = \langle \psi_x^\dagger(t)\, \mathrm{e}^{-ie\int_0^x dx' A_{x'}(t)}\, \psi_0(0)\rangle$



count (anti-)vortices in $\mathbf{z} = (k,t)$ plane:

$$n_\pm(t) = \oint_{\mathcal{C}_\pm(t)} \frac{d\mathbf{z}}{2\pi} \left[\tilde{g}^\dagger(\mathbf{z})\nabla_\mathbf{z}\tilde{g}(\mathbf{z})\right] \qquad \tilde{g}(\mathbf{z}) = \frac{g(k,t)}{|g(k,t)|}$$

Dynamical topological quantum phase transitions



Topological invariant of the time-ordered correlator:

$$\nu(t) = n_+(t) - n_-(t)$$

Rate function of the Loschmidt amplitude:

$$|L_{\theta\to\theta'}(t)| = |\langle\psi(0)|\psi(t)\rangle| \\ = \mathrm{e}^{-V\Gamma_{\theta\to\theta'}(t)}$$

In the absence of interactions:

$$L_{\theta\to\theta'}(t) = \prod_k g_{\theta\to\theta'}(k,t)$$

Related work: Budich and Heyl, Phys. Rev. B 93, 085416 (2016)

Interpretation with equal-time correlations

- Loschmidt amplitude & two-time correlators are difficult to measure!

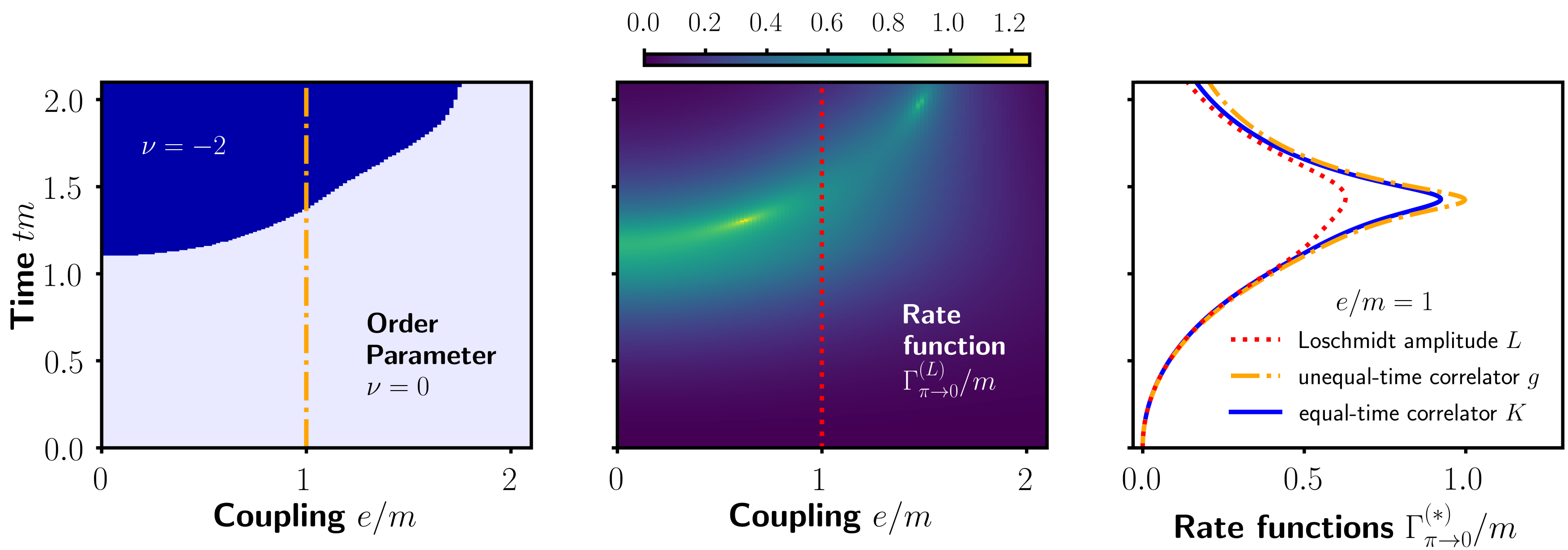
- gaussian state: $F_{xy}(t) = \langle \left[\psi_x(t), \bar{\psi}_y(t) \right] \rangle$

- spin vector : $\mathbf{F} = (F_s, F_1, F_5)$

- Loschmidt echo: $|L_{\theta \to \theta'}(t)|^2 = K_{\theta \to \theta'}(t) = \prod_k \left[\mathbf{F}(k,t) + \mathbf{F}(k,0) \right]^2$

Numerics: Towards strong coupling

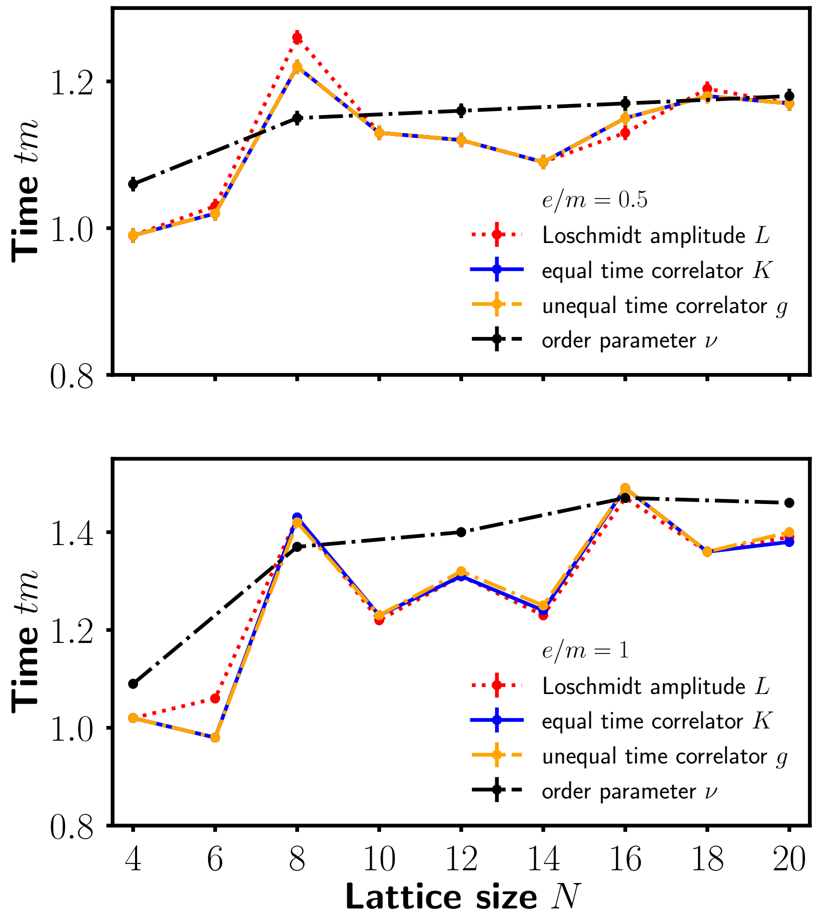
Results of a numerical simulation (exact diagonalization) on a lattice with $N = 8$ sites and lattice spacing $am = 0.8$



✓ order parameter robustly predicts a shift of the topological transition

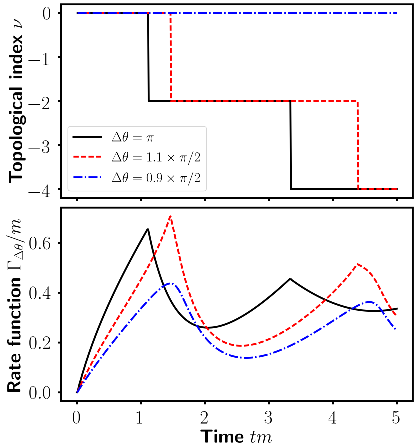
✓ good agreement of the transition times from different rate functions

Numerics: Lattice size dependence

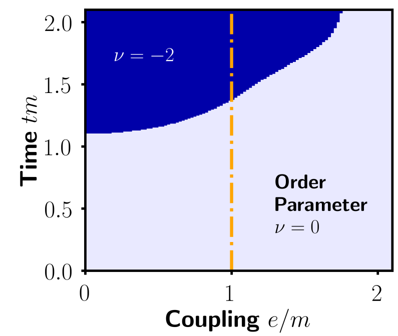


- all rate functions indicate similar transition times
- rate functions exhibit fluctuating finite size errors
- order parameter robust due to topological origin

Summary & outlook



- Noninteracting case: correlation functions and DQPT
  - ✓ Rigorous connection between correlators and DQPT
  - ✓ Physical interpretation of dynamical transitions
  - ✓ Definition of a dynamical topological invariant

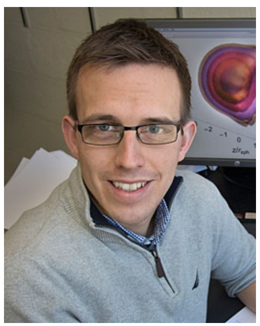


- Interacting case: dynamical topological order parameter
  - ✓ gauge invariant
  - ✓ integer-valued
  - ✓ accessible on small lattices

- To Do:
  - ➢ Experimental realization (cold atoms and/or trapped ions)
  - ➢ Application to other models (condensed matter)
  - ➢ Extension to more realistic scenarios (e.g. QCD with dynamical axions)

Presented work done in collaboration with

N. Mueller, J. T. Schneider, F. Jendrzejewski, J. Berges, and P. Hauke



Phys. Rev. Lett., *122*(5), 050403 (2019)

Supplementary material

The axial anomaly in QED

Fermions in a gauge field background:

$$S[A,\psi,\bar{\psi}] = \int d^4x\,\bar{\psi}\left[i\gamma^\mu\left(\partial_\mu - ieA_\mu\right) - m\right]\psi$$

Chiral rotation and axial vector current:

$$\psi \to e^{i\alpha\gamma_5}\psi \qquad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \qquad j_5^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi$$

(Non-)conservation law:

$$\partial_\mu j_5^\mu = 2im\,\bar{\psi}\gamma_5\psi + \frac{e^2}{8\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Adler, *Physical Review*, *177*(5), 2426-2438 (1969)
Bell & Jackiw, *Il Nuovo Cimento A*, *60*(1), 47-61 (1969)

Fermion path integral:

$$e^{iS[A]} = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\,e^{iS[A,\psi,\bar{\psi}]} \qquad \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

Topology of gauge fields

Yang-Mills Hamiltonian (in temporal-axial gauge):

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a,\mu\nu} \xrightarrow{A^a_0 = 0} H_{\mathrm{YM}} = \frac{1}{2}\left(\mathbf{E}^2 + \mathbf{B}^2\right)$$

(classical) zero-energy solutions are 'pure gauge':

$$\mathbf{A}_U(\mathbf{x}) = -\frac{i}{g} U(\mathbf{x}) \nabla U^\dagger(\mathbf{x}) \Rightarrow \mathbf{E} = 0\,,\ \mathbf{B} = 0\,,\ H_{\mathrm{YM}} = 0$$

Topological classification:

$$U(\mathbf{x}) : \mathbb{R}^3 \sim S^3 \to SU(2) \sim S^3 \qquad \nu = \frac{g^2}{32\pi^2} \int d^4x\, F^a_{\mu\nu} \tilde{F}^{a,\mu\nu} \in \mathbb{Z}$$

'Theta' vacua:

$$|\theta\rangle = \sum_{\nu \in \mathbb{Z}} e^{i\nu\theta} |\nu\rangle$$

Violates combined charge (C) and parity (P) symmetry!

Jackiw & Rebbi. in *Instantons In Gauge Theories* (25-28) (1994)