COMPATIBLE ORDERS IN DIRAC MATERIALS: SYMMETRIES AND PHASE DIAGRAMS

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CONTENTS

MOTIVATION

LANDAU-GINZBURG-WILSON THEORY

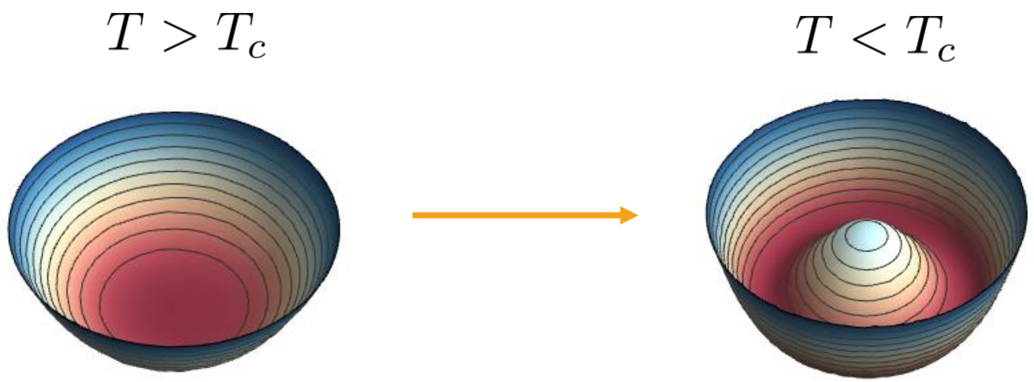
- Local order parameters $\Phi(\mathbf{r})$ describe/distinguish phases.
- Transitions described fully in terms of (only!) order parameter + fluctuations

$$S[\Phi] = \int \mathrm{d}^D\mathbf{r} \left(\frac{1}{2}(\nabla\Phi)^2 + V(\Phi) \right)$$

for analytic $V(\Phi)$

- Onset of **long range order** quantified by divergence of correlation length

$$\xi \sim |X - X_c|^{-\nu}$$



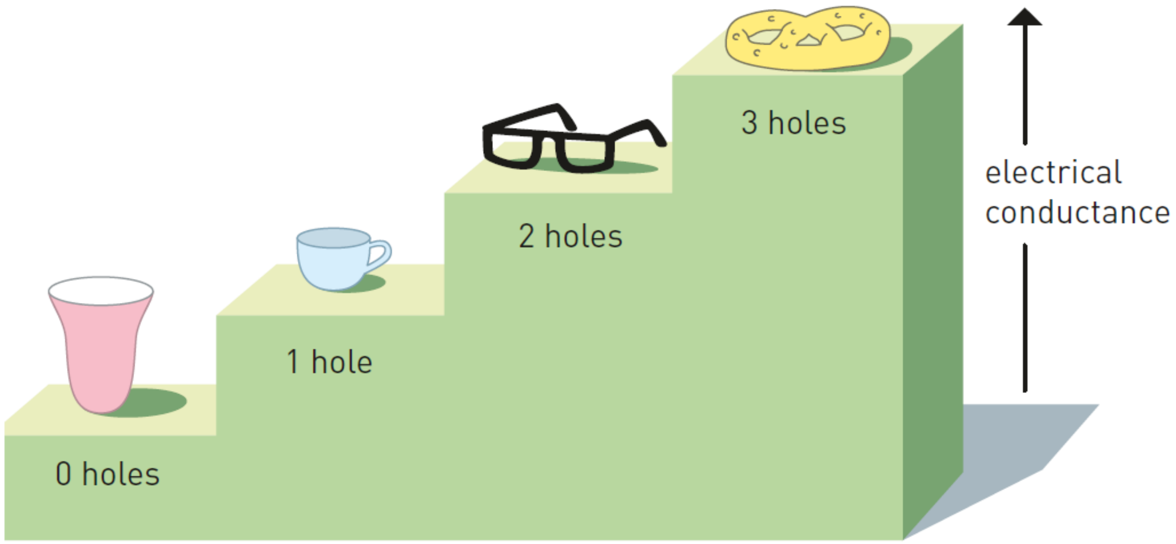
QUANTUM CRITICALITY BEYOND LGW - I

1. QPTs in which the basic assumptions of an LG description are not met

- Non local order parameters (e.g. TIs)
- Topological order

Features can include

- Key role of entanglement (e.g. QSL)
- Fractionalized excitations (e.g. FQH)



Johan Jarnestad - The Nobel prize in physics 2016

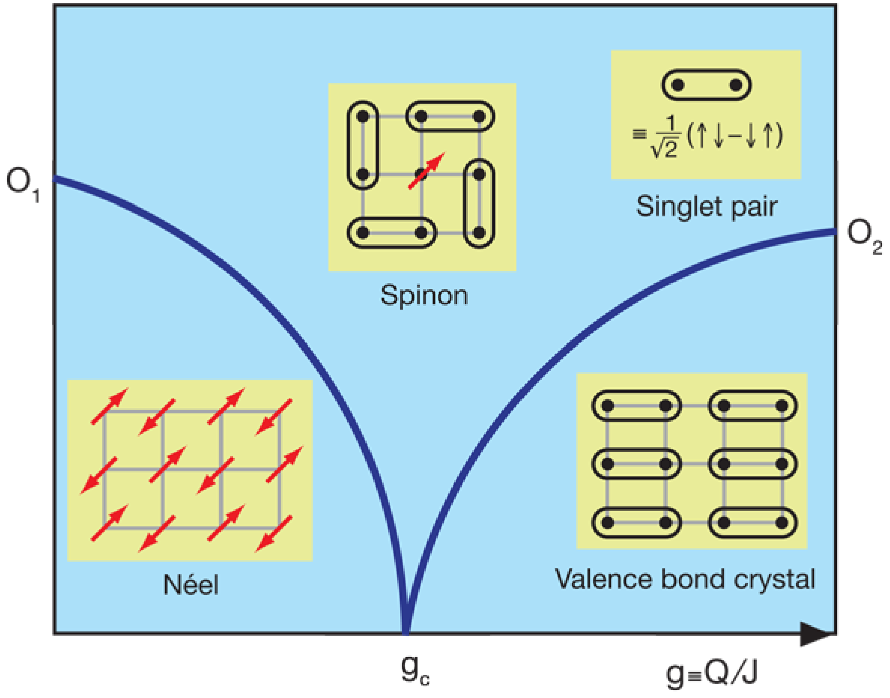
QUANTUM CRITICALITY BEYOND LG(W) - II

2. QPTs in which the basic assumptions of a LG description ARE met

- Fluctuation induced criticality (e.g. 1st order turns to 2nd order)
- Order-to-order transitions

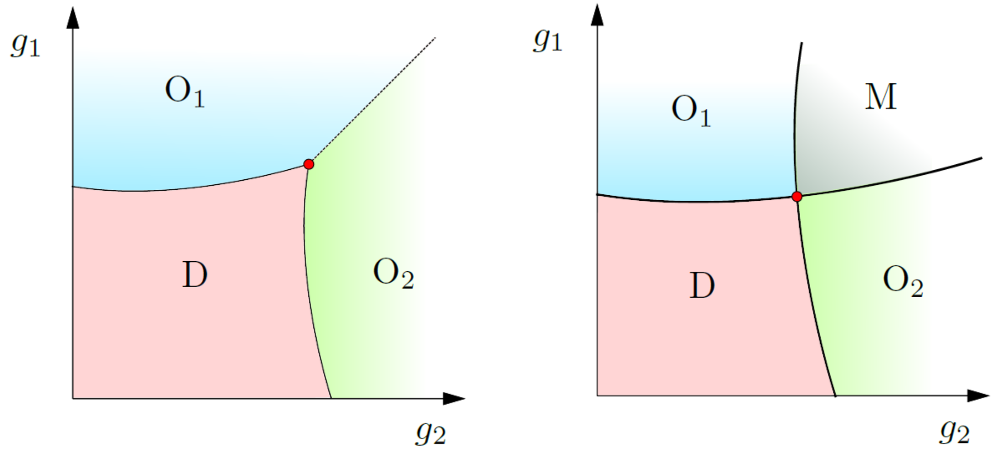
Features can include

- Large anomalous dimensions
- Emergent $O(N)$ symmetry at the critical point



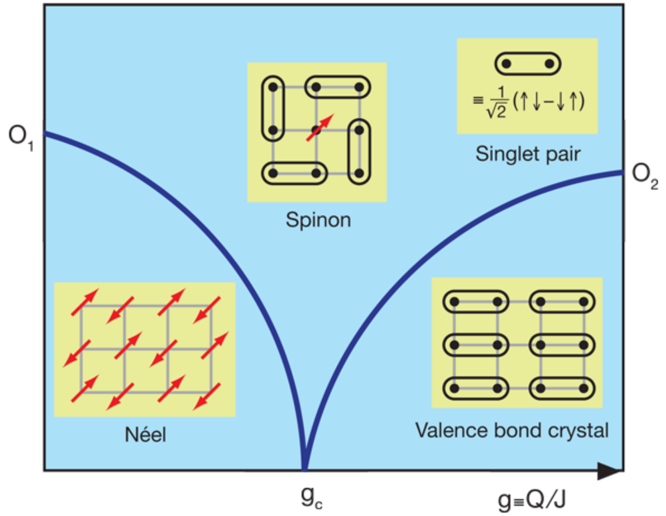
Singh, *Physics* **3**, 35 (2010)

MORE ON "NON-LANDAU-NESS" OF THE SECOND KIND



Landau

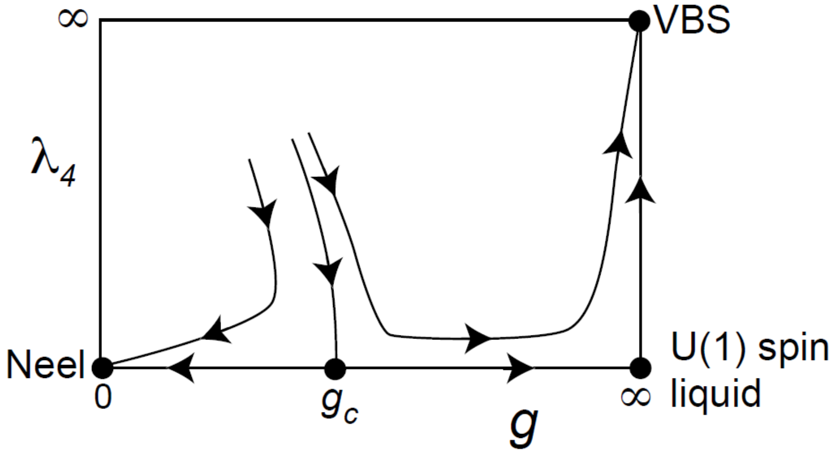
vs



Deconfined Quantum Criticality (DQCP)

Continuous order to order transitions

- E.g. Nèel to VBS transition in spin ½ quantum antiferromagnets.
- Effective theory is an NCCP1 for the spinons z , where $\vec{\varphi} = z^\dagger \vec{\sigma} z$ is the Nèel OP.
- Topological defects on each phase play a **key** role: skyrmions on AFM side and "vortices" of VBS.



Senthil et.al. JPSJ **74** (2005)

SYMMETRIES OF A DQCP

onset of Nèel order ⟷ $O(3)$ symmetry breaking

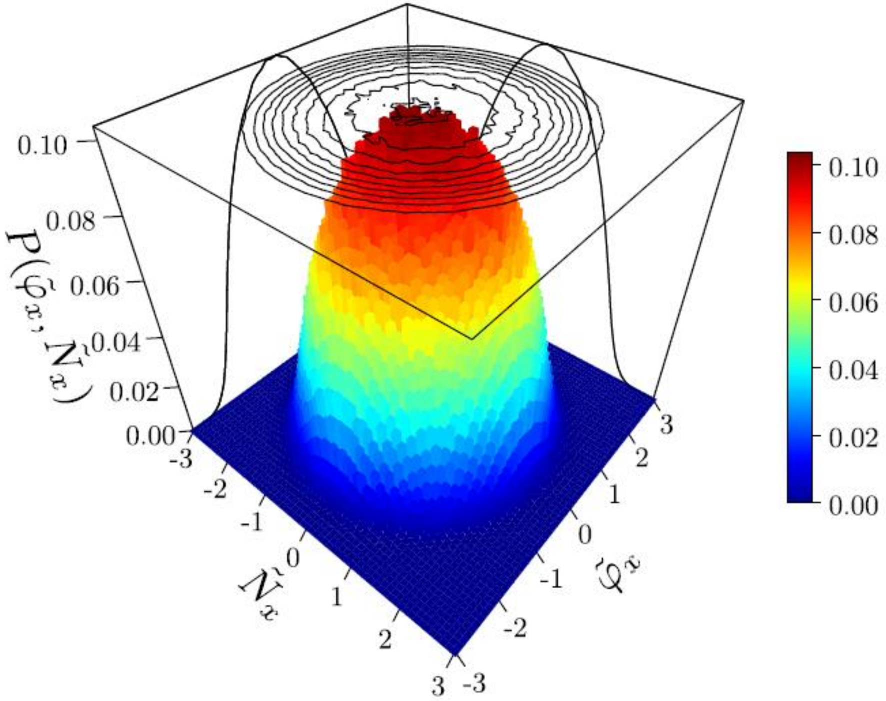
onset of VBS order ⟷ $O(2)$ symmetry breaking

What about the QCP itself?

Emergent $O(5)$ symmetry at the critical point!

- Expected from new(-ish) web of dualities*
- Seen in numerics of a "J/Q model" **

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \vec{S}_j - Q \sum_{\langle ijkl \rangle} \left(\vec{S}_i \vec{S}_j - 1/4 \right) \left(\vec{S}_k \vec{S}_l - 1/4 \right)$$

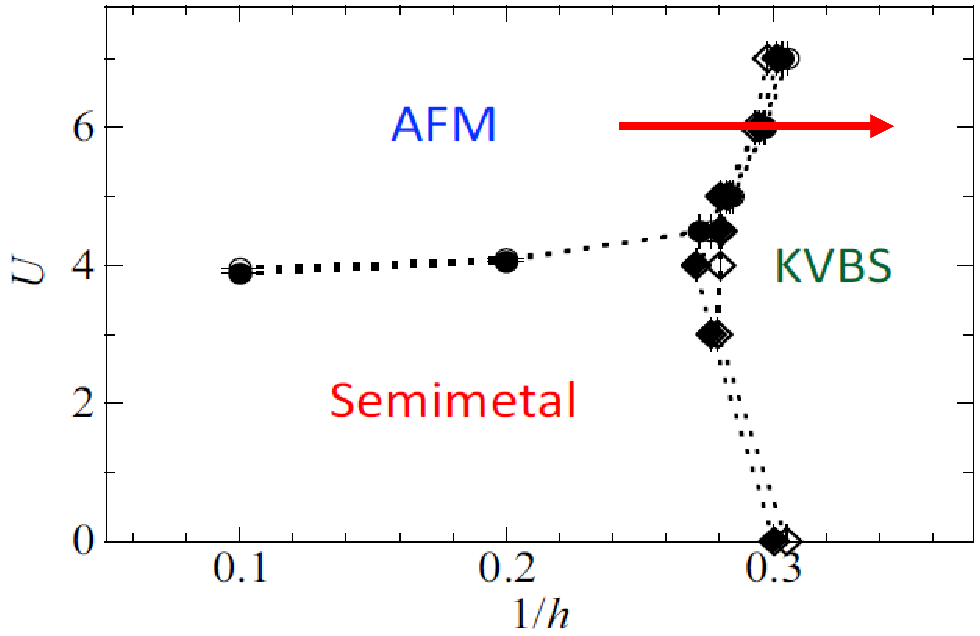


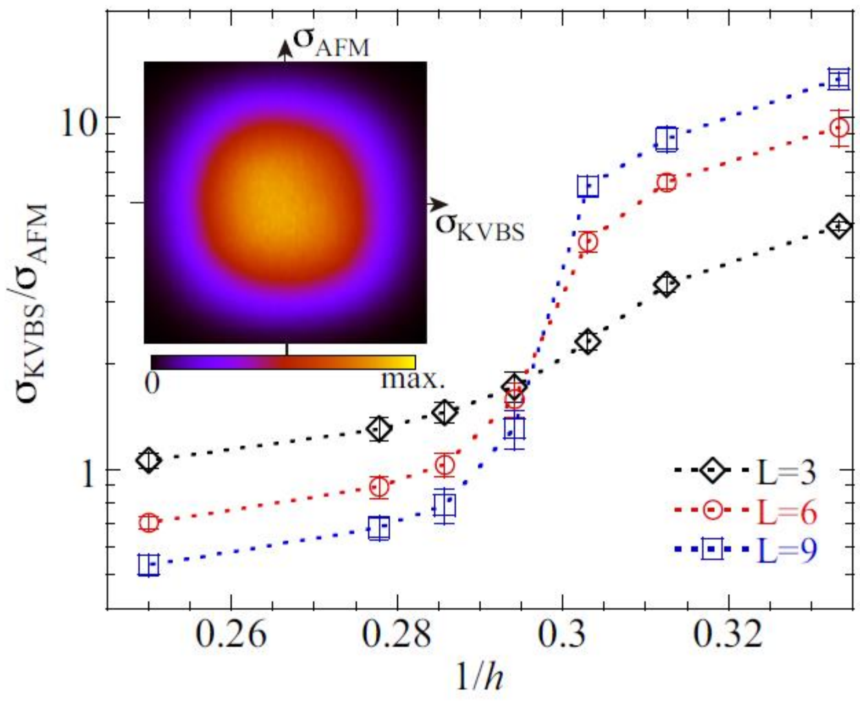
*Wang et. al. PRX **7** 031051 (2017)
** Nahum et. al. PRL **115** 267203 (2015)

DQCP ON THE HONEYCOMB LATTICE?

Evidence from QMC simulations of a model with $O(3) \oplus \mathbb{Z}_2$ symmetry

- Direct continuous transition between the phases
- Emergent $O(4)$ symmetry at the transition
- One particle gap at the Dirac points remains **open**





BUT

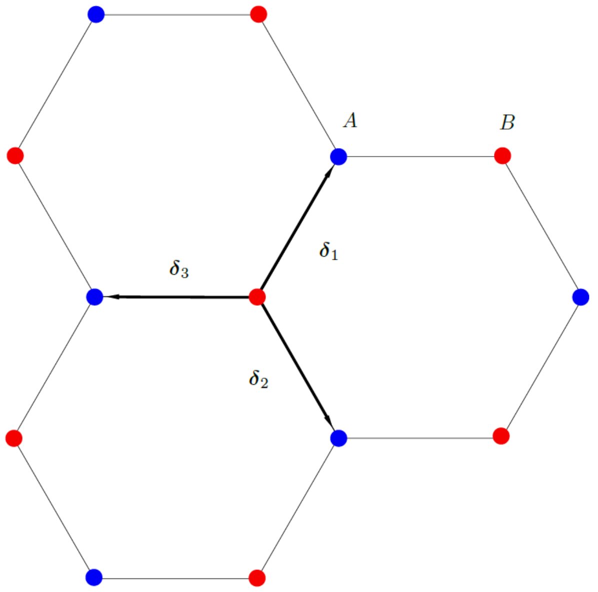
$\mathbb{Z}_2$ OPs do not support vortices!

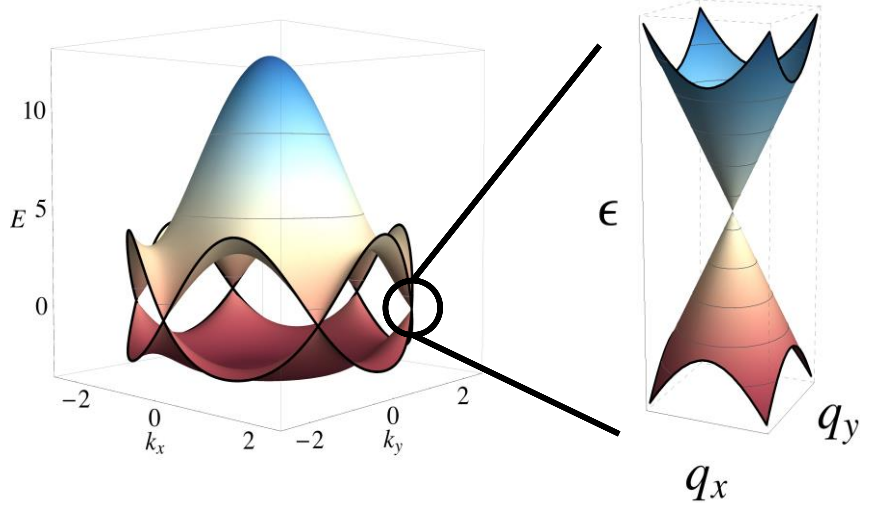
Different mechanism?

Sato et.al. PRL **119** 197203 (2017)

CHIRAL DIRAC FERMIONS AND WHERE TO FIND THEM

FERMIONS ON THE HONEYCOMB LATTICE





$$\hat{H}_0 = -t \sum_{\mathbf{R},i} \left[\hat{u}^\dagger(\mathbf{R})\hat{v}(\mathbf{R}+\boldsymbol{\delta}_i) + \text{ h.c.}\right]$$

$$E_\pm(\mathbf{q}) \approx \pm v_\mathrm{f}|\mathbf{q}| \qquad \frac{v_\mathrm{f}}{c} \ll 1$$

$$\widehat{\psi}(\mathbf{q}) := \left(\hat{u}(\mathbf{K}+\mathbf{q}), \hat{v}(\mathbf{K}+\mathbf{q}), \hat{v}(-\mathbf{K}+\mathbf{q}), \hat{u}(-\mathbf{K}+\mathbf{q})\right)^\mathsf{T},$$

$$\hat{H}_\mathrm{eff} = -iv_\mathrm{f}\int \mathrm{d}^2x\, \hat{\psi}^\dagger(x)\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}\hat{\psi}(x)$$

As seen also in e.g. high Tc superconductors, topological insulators and the physics of the half filled Landau level

INTERACTIONS AND ORDER PARAMETERS

Considering interactions (e.g.) $\hat{H}_{\rm int} = V_1 \sum_{\langle \mathbf{R},\mathbf{R}'\rangle} n(\mathbf{R}) n(\mathbf{R}') + V_2 \sum_{\langle\langle \mathbf{R},\mathbf{R}'\rangle\rangle} n(\mathbf{R}) n(\mathbf{R}')$

with $n(\mathbf{R}) = \hat{\psi}^\dagger(\mathbf{R})\hat{\psi}(\mathbf{R})$

Ordered phases: characterized by nonvanishing expectation values of OPs:

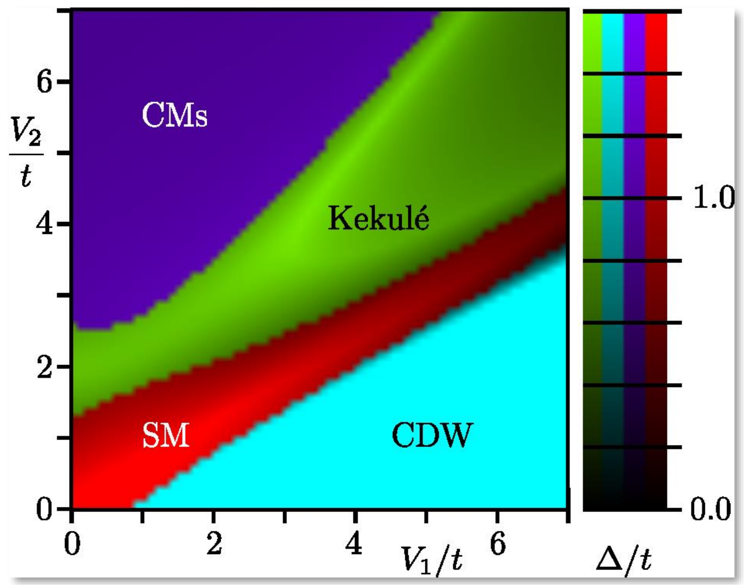
- **CDW:** staggered density means $\langle \chi \rangle \neq 0$ with $\chi \propto a_\sigma^\dagger a_\sigma - b_\sigma^\dagger b_\sigma$
- **SDW:** antiferromagnetic order means $\langle \boldsymbol{\phi} \rangle \neq \mathbf{0}$ with

$$\boldsymbol{\phi} \propto a_\sigma^\dagger \boldsymbol{\Sigma}_{\sigma\sigma'} a_{\sigma'} - b_\sigma^\dagger \boldsymbol{\Sigma}_{\sigma\sigma'} b_{\sigma'}$$

We take our OPs to be of the form $\Phi \sim \psi^\dagger \beta_\Phi \psi$

If, additionally, they satisfy $\beta_\Phi^2 = \mathbb{1}$ and $\{\alpha_j, \beta_\Phi\} = 0$ for $j = 1, \ldots, d$ they act as **chirality breaking masses**

$$E(\mathbf{q}) \sim |\mathbf{q}| \longrightarrow \sqrt{\mathbf{q}^2 + m^2}$$



García-Martínez et.al. PRB **88** 245123 (2013)

FIELD THEORY SETUP

Effective low energy theories:

Promoting the OPs to dynamical fields and considering arbitrary flavours of fermions , leads to **Gross Neveu Yukawa (GNY) field theories** i.e.

$$S = \int \mathrm{d}^3x \, \mathscr{L}$$

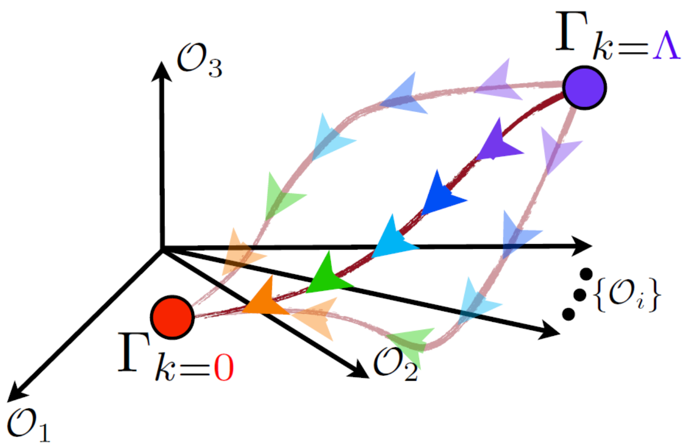
$$\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_b + \mathscr{L}_y$$

- $\mathscr{L}_0 = \overline{\psi}_\alpha \left(\gamma_0 \partial_\tau + v_\mathrm{f} \boldsymbol{\gamma} \cdot \boldsymbol{\nabla}\right) \psi_\alpha \qquad \alpha = 1, \cdots, N_\mathrm{f}$
- $\mathscr{L}_y = g \, \psi^\dagger_\alpha \beta^i_\Phi \Phi_i \psi_\alpha$ Yukawa coupling
- $\mathscr{L}_b = (\partial_\tau \Phi)^2 - v_\mathrm{b}^2 (\boldsymbol{\nabla}\Phi)^2 + V(\Phi)$ with potential including all terms allowed by symmetry

In general **no** reason to expect $v_\mathrm{f} = v_\mathrm{b}$, but this holds at criticality: (Pozo et. al. PRB **98** 115122 (2018), Roy et. al. JHEP 2016)

FUNCTIONAL RG AND TRUNCATIONS

- Study the flowing action Γ_k interpolating between the microscopic action $k \to \Lambda \quad \Gamma_k \to S$ and full effective action $k \to 0 \quad \Gamma_k \to \Gamma$



- Implement the succesive integrating out of degrees of freedom through the regulator R_k

- Flow equation given by $\partial_t \Gamma = \frac{1}{2}\mathrm{Str}\left(\frac{\partial_t R_k}{\Gamma^{(2)}_{0,k} + R_k}\right)$

- Nonperturbative regime ($D = 3$ and small N_f) as well as symmetry broken phases readily accesible

- Truncation: LPA'

$$\Gamma_k = \int d^D x \Big\{ \bar{\psi}_\nu \left(-iZ_{\psi,k}\partial\!\!\!/ + g_{i,k} M_{\Phi,i}\Phi_i\right)\psi_\nu - \frac{1}{2} Z_{\Phi_i,k}\Phi_i \partial^2 \Phi_i + V_k(\Phi) \Big\} \,.$$

A GENERAL MECHANISM FOR TRANSITIONS WITH EMERGENT SYMMETRY

COMPATIBLE MASSES

Two families of **masses** $\{\phi_i\}_{i=1}^{N_1}, \{\chi_j\}_{j=1}^{N_2}$ are compatible if

$$\{\beta_\phi^i, \beta_\chi^j\} = 0$$

Effective theory of **meeting of three phases** . Here:

$$O(N_1) \oplus O(N_2)$$

criticality.

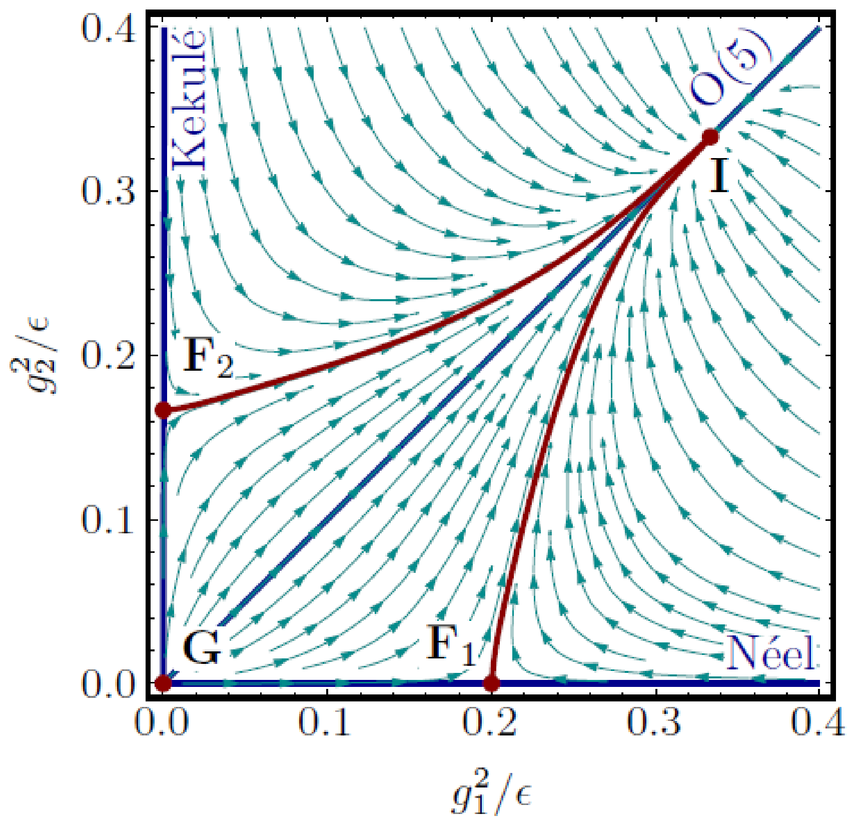
Known: there is always* a stable isotropic fixed point (IFP) i.e. a fixed point with **a larger, emergent**

$$O(N_1 + N_2)$$

symmetry.

BUT, the IFP is not the whole story!

$N_1 = 3, N_2 = 2$



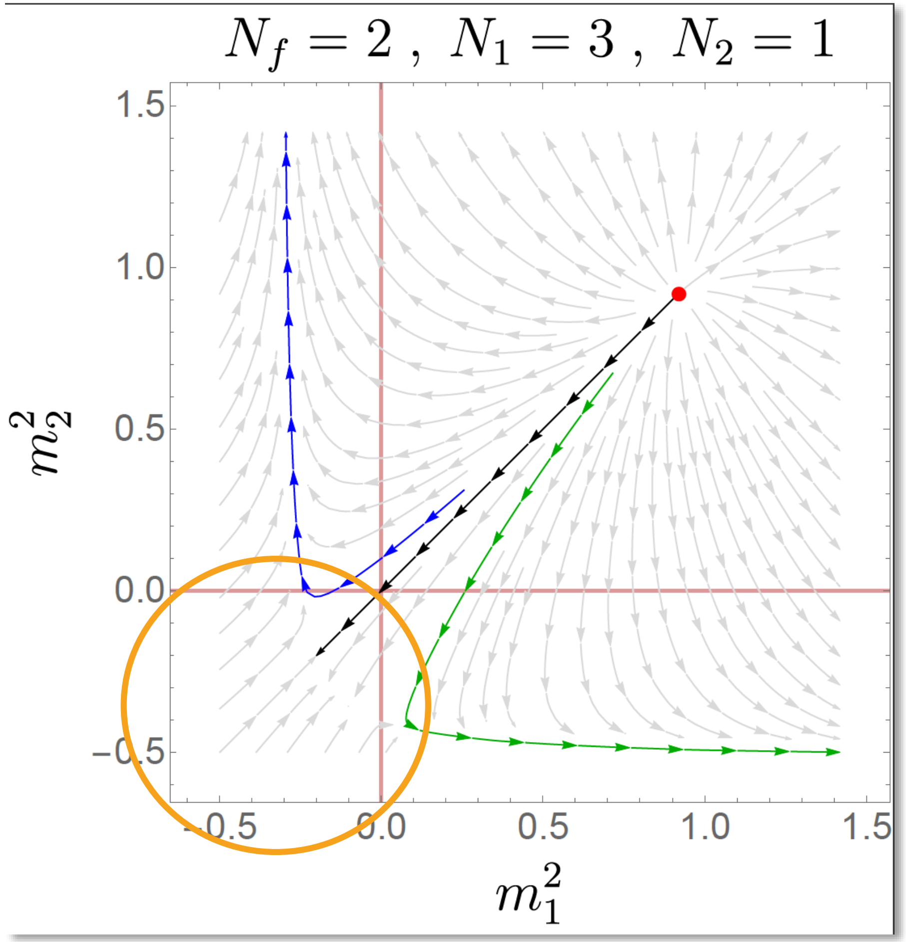
*Conditions apply!

Janssen et.al. (2018), Roy et.al. (2018)

SAME BUT DIFFERENT

Stable fixed point **still has two** relevant directions and fixed point info **not enough**

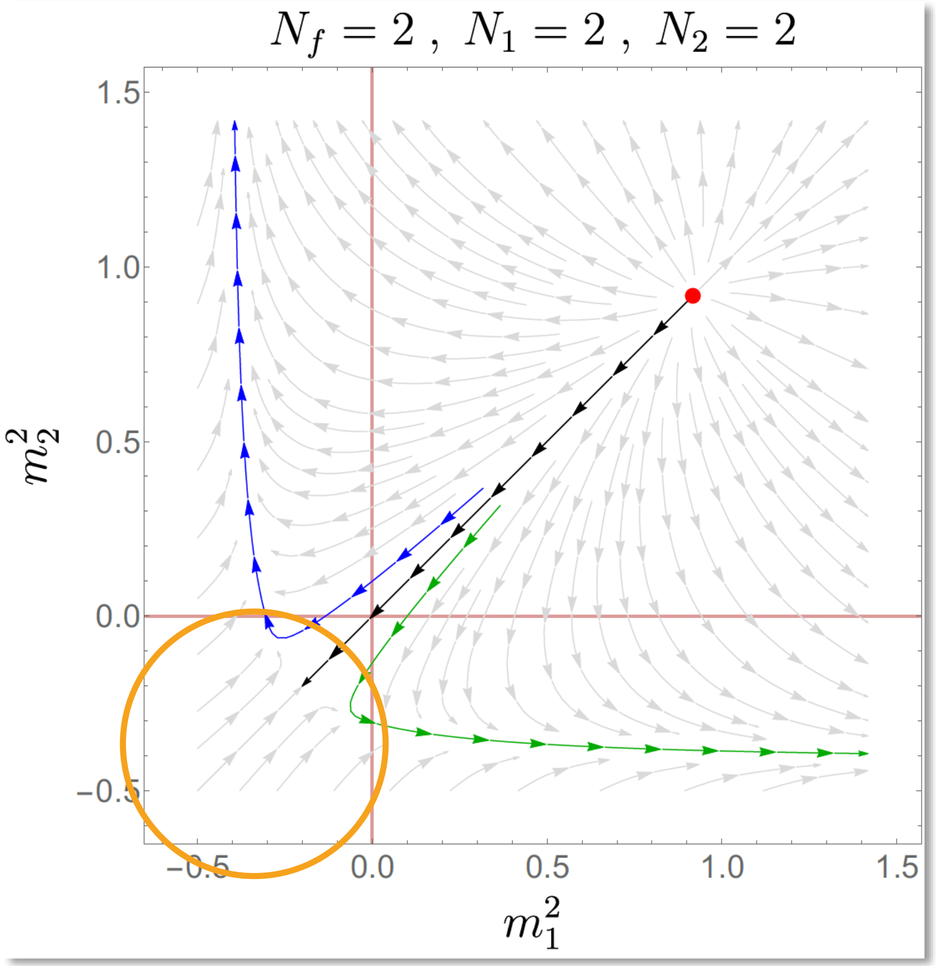
For, e.g. $N = 4$



Coexistence? (mixed phases)

Need to follow the evolution of the expectation values!

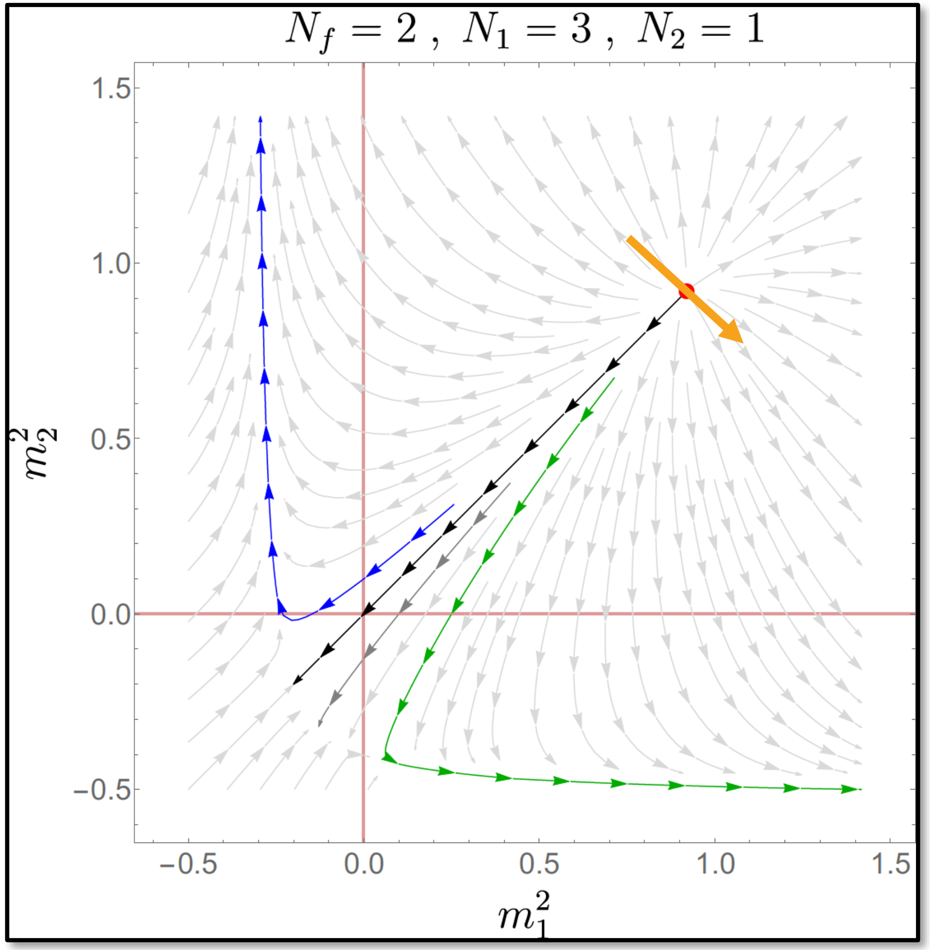
$$\kappa_\Phi := \langle \Phi^2/2 \rangle$$



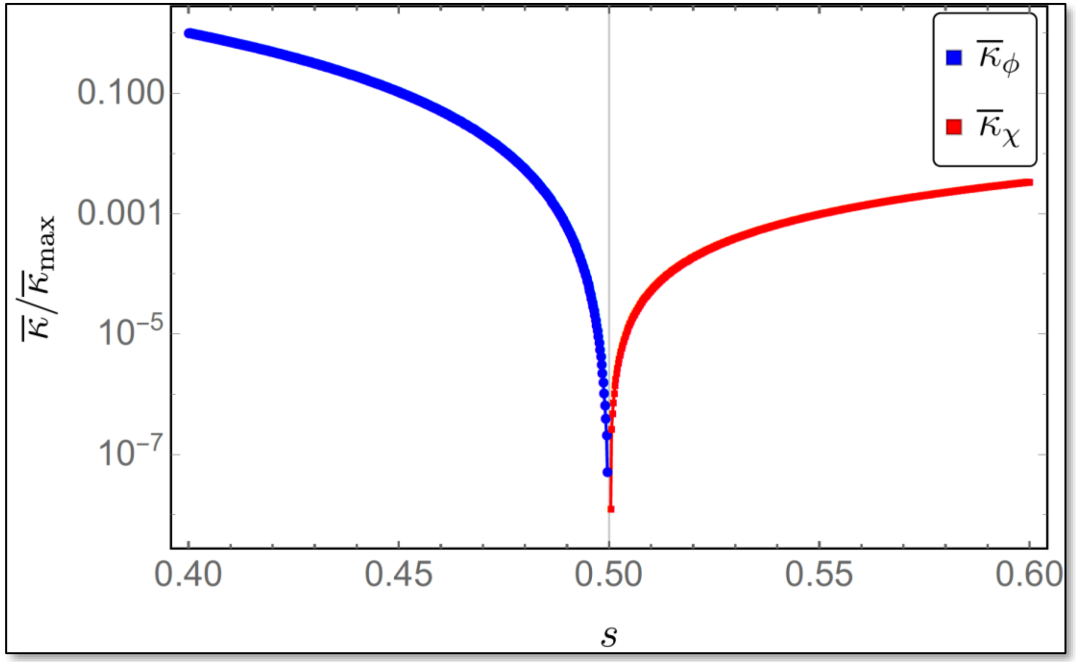
First order transitions?

ORDER-TO-ORDER TRANSITIONS?

Yes: possible by crossing *exactly* through the IFP (and closing the gap!)

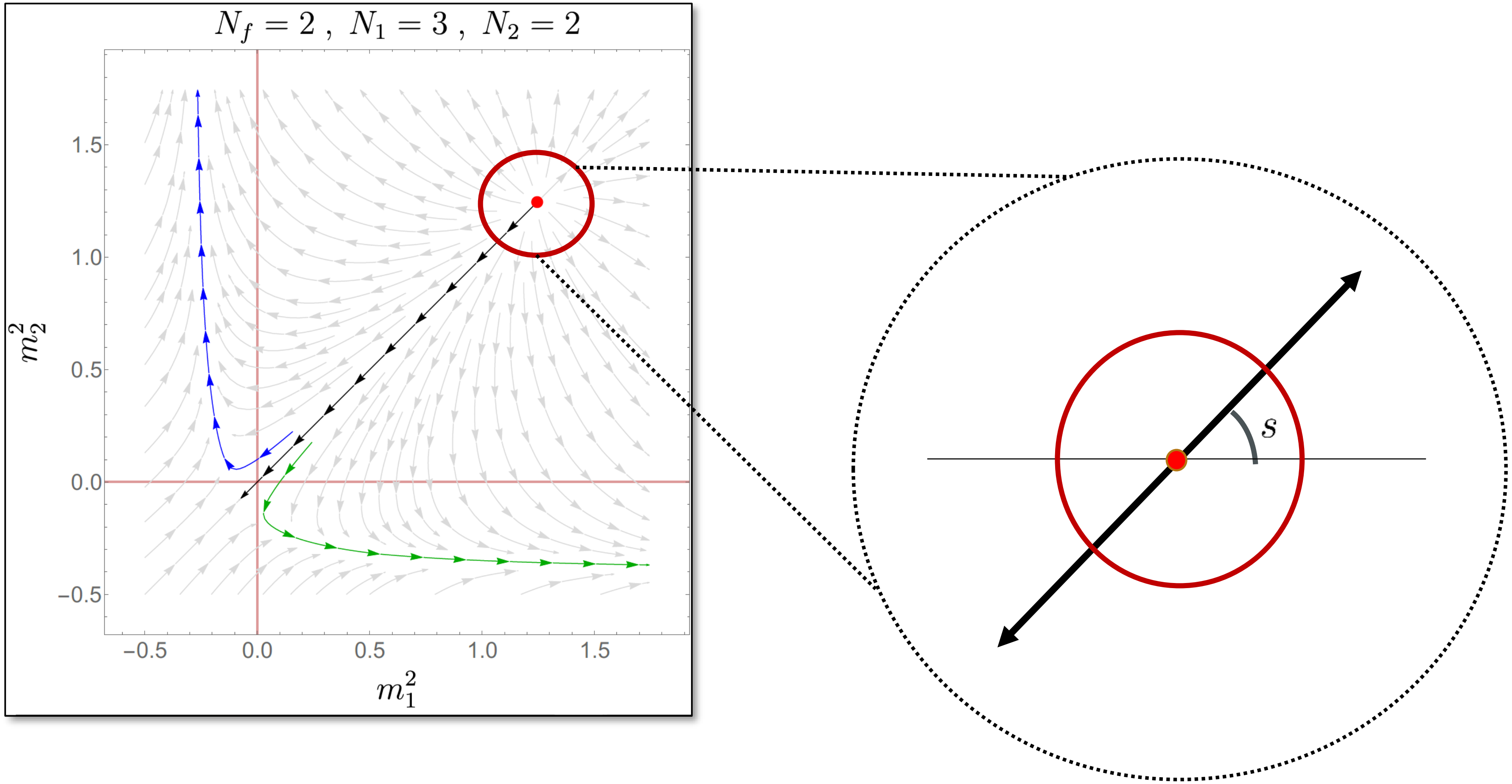


$$\overline{\kappa}_\Phi := \langle \Phi^2/2 \rangle$$



$$\Delta_{\rm sp} = 2\overline{g}_\phi^2 \overline{\kappa}_\phi + 2\overline{g}_\chi^2 \overline{\kappa}_\chi$$

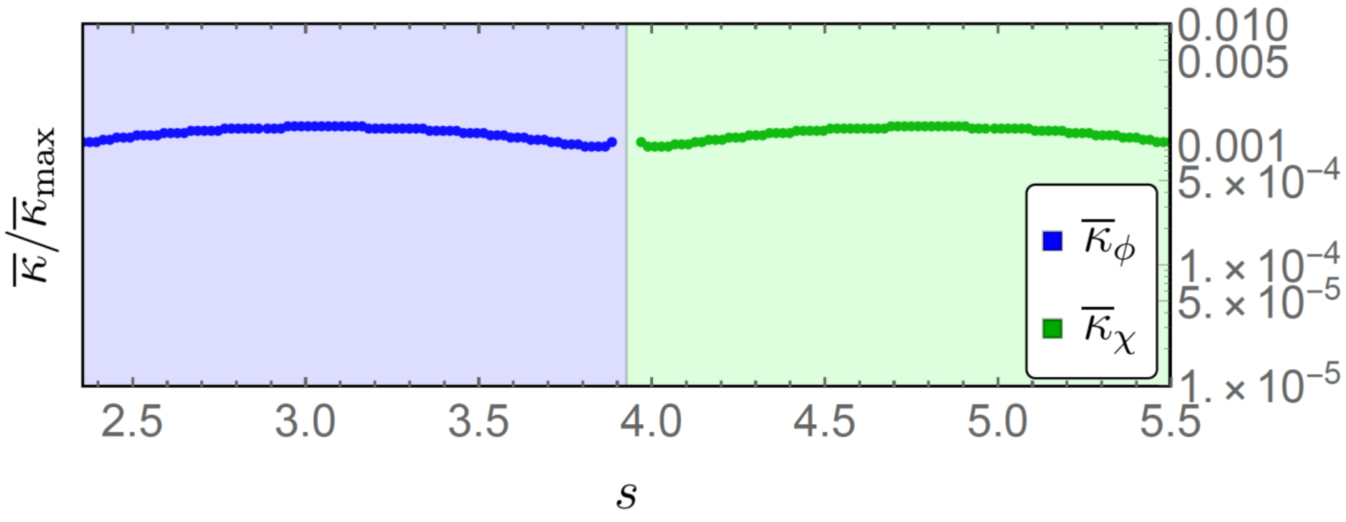
EXTRACTING THE PHASE DIAGRAM



PHENOMENOLOGY OF THE TRANSITION

- Is there a direct transition where the gap $\Delta_{\rm sp}$ remains open?

$$N_{\rm f} = 2, N_1 = N_2 = 1$$

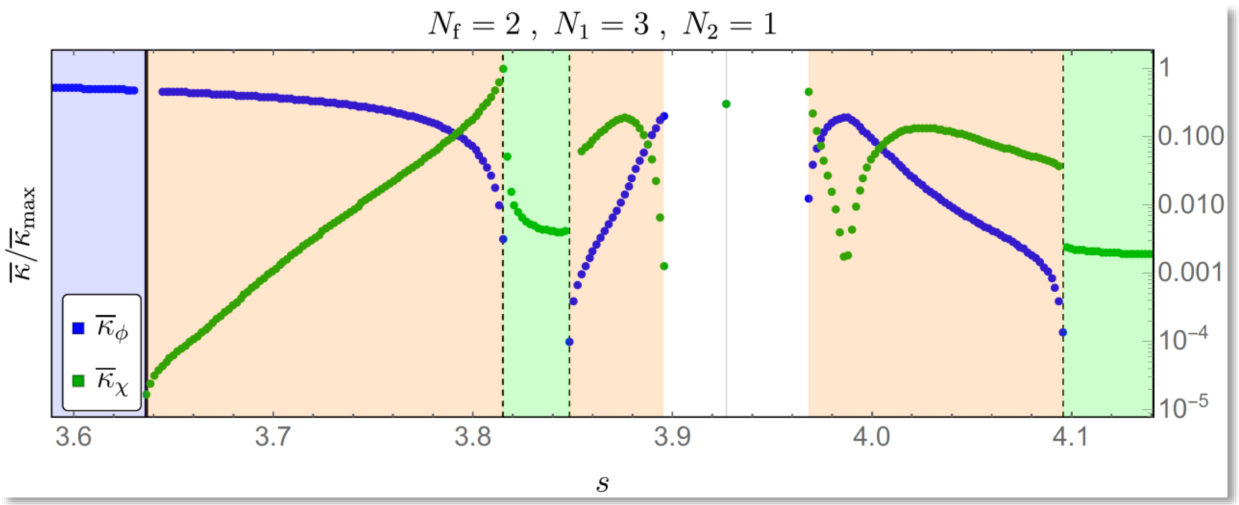


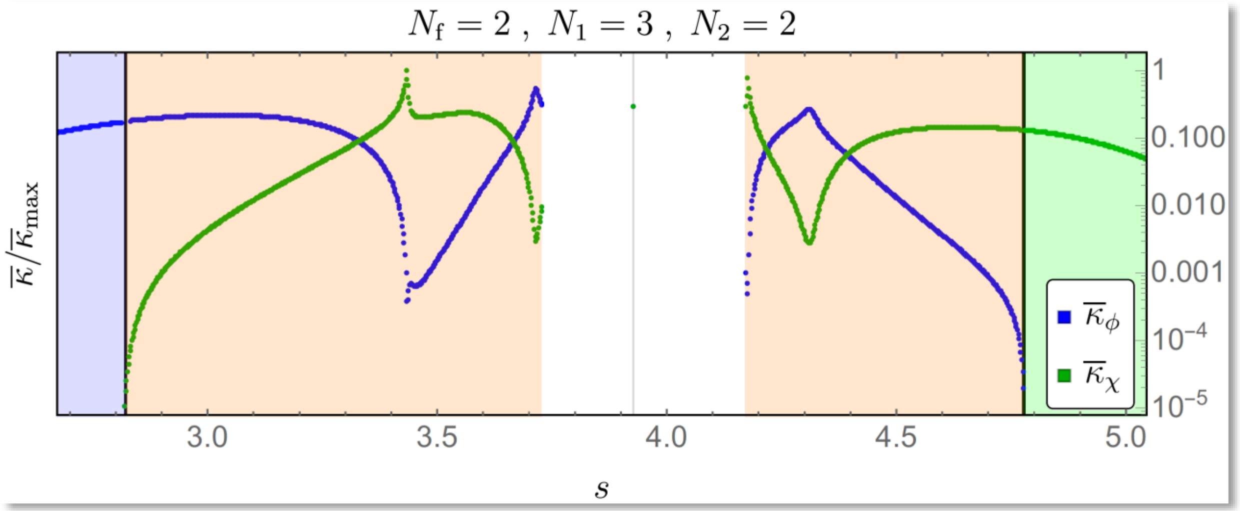
No evidence!

Instead either a first order transition…

PHENOMENOLOGY OF THE TRANSITION

- Is there a direct transition where the gap Δ_{sp} remains open?

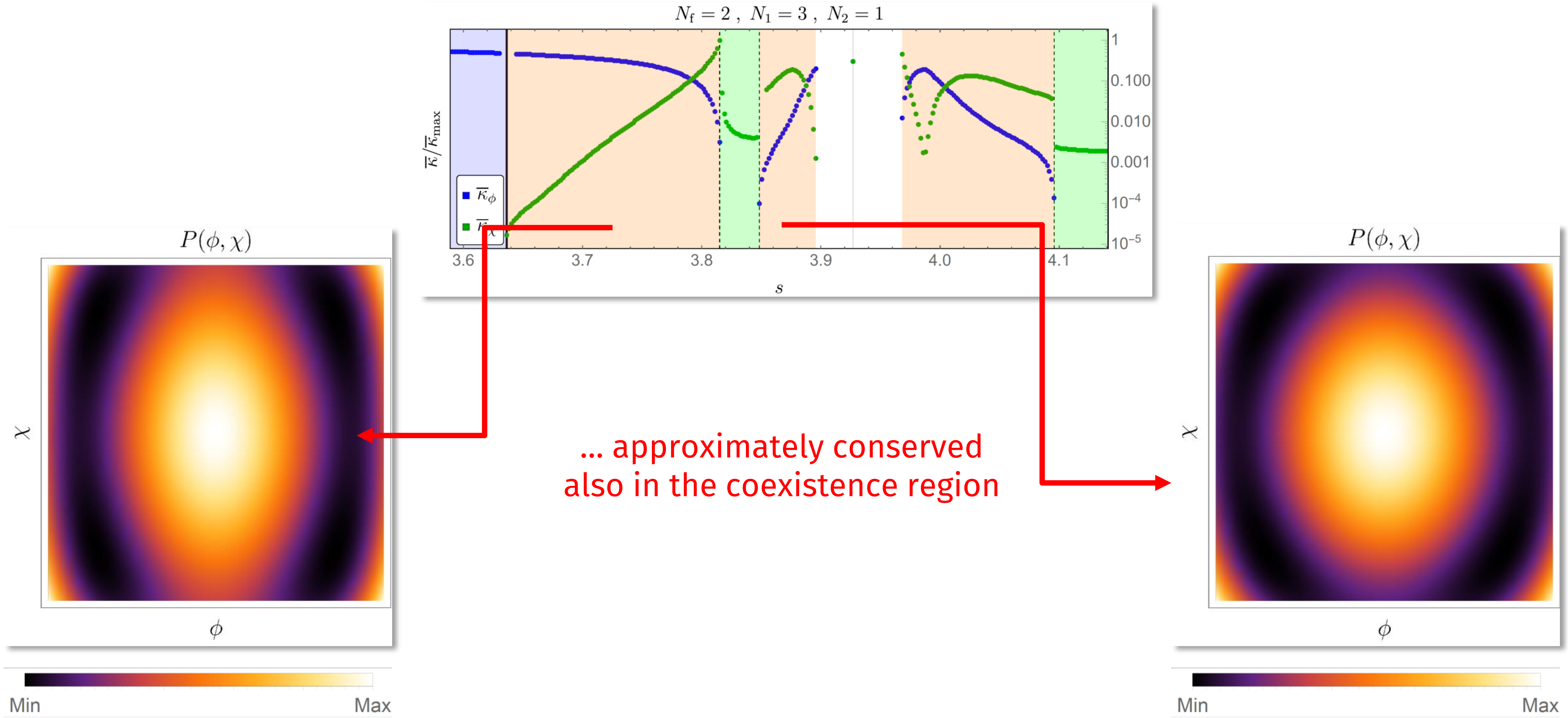




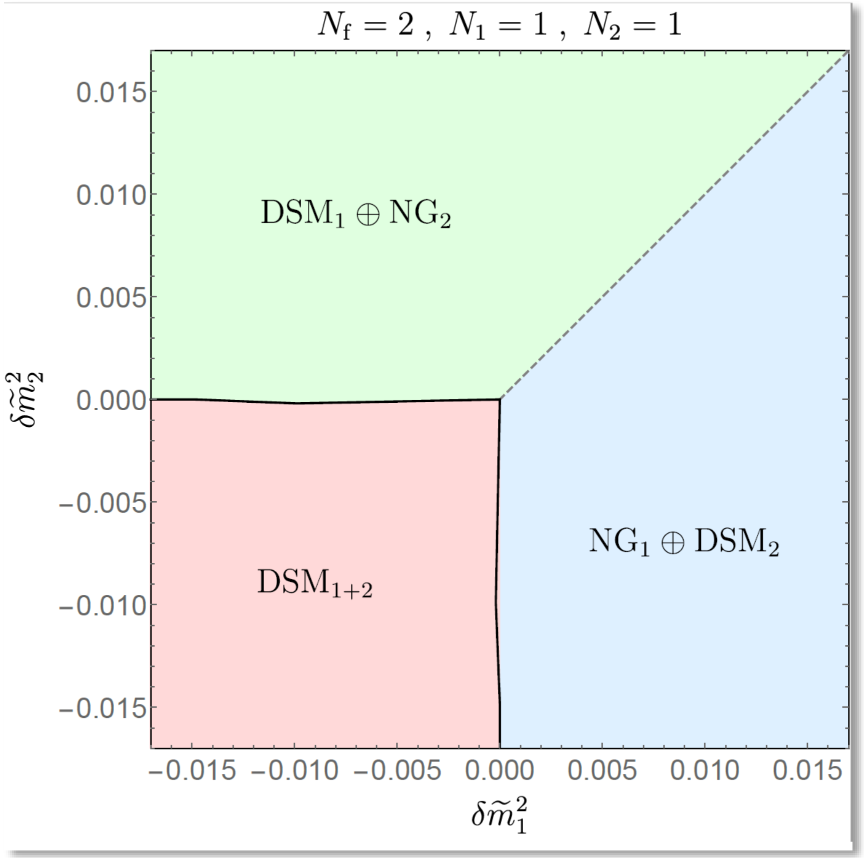
… or an extended region of coexistence

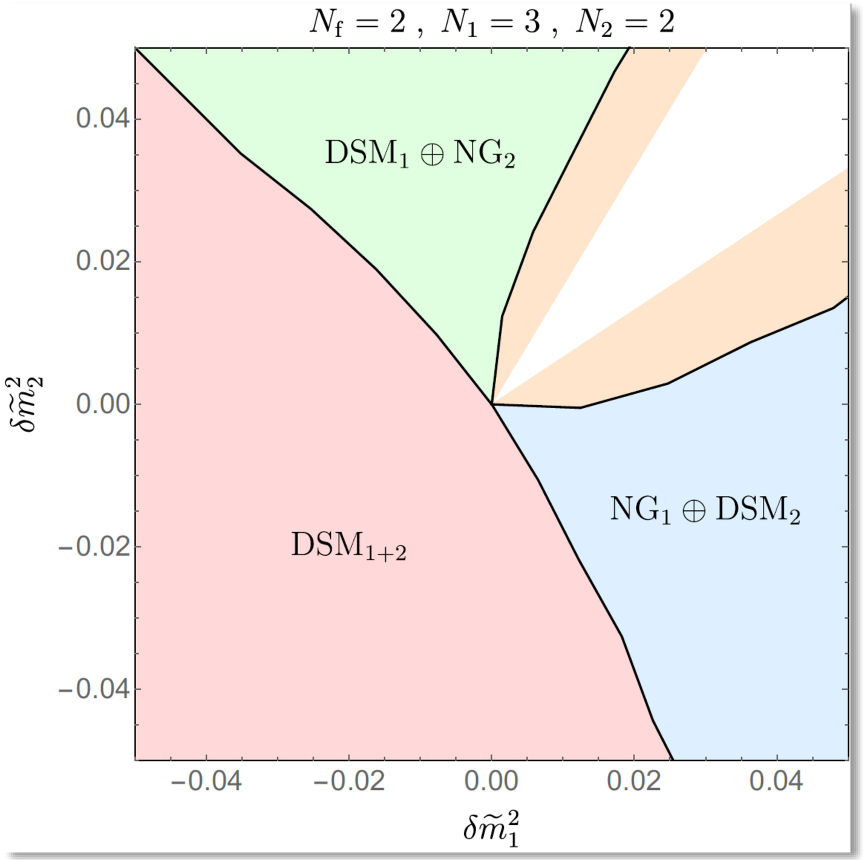
WHERE IS THE SYMMETRY OF THE IFP?

- Idea: follow the evolution of the potential in the coexistence region $P(\phi, \chi) := \lim_{k \to 0} V_k(\phi, \chi)$



PHASE DIAGRAMS



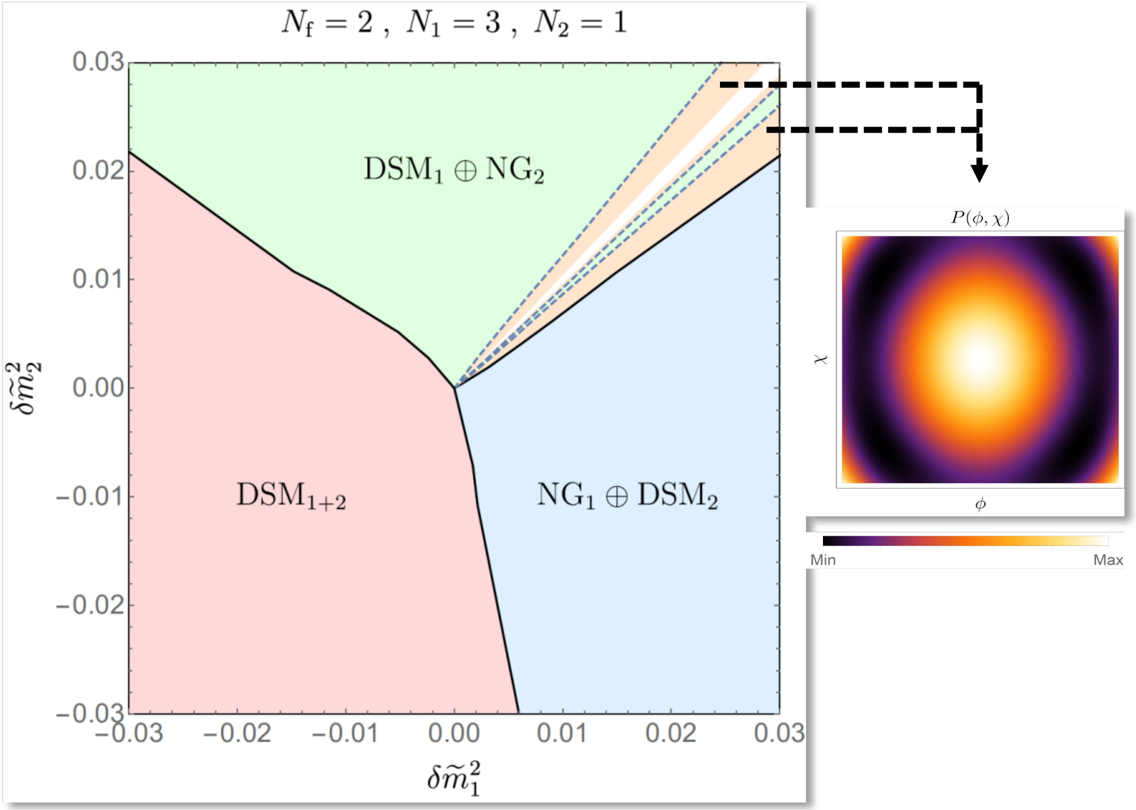


SUMMARY / OUTLOOK

- **Dirac materials** a rich playground for Non Landau criticality

- **Massless fermions as a "workaround"**
  - \+ discrete symmetry breaking = emergent length scales (not addressed here, but see: Torres et al, PRB **97**, 125137 (2018))
  - \+ compatible masses = enlarged symmetry and direct transitions
  - Amenable to fully analytical treatment

- **Numerical analysis of similar models?**
  - "Designer hamiltonians" easy to construct



THANKS FOR YOUR ATTENTION

Torres, Janssen, Scherer - arXiv:1906.XXXX?