

Quantum Gravity meets Dark Matter

Manuel Reichert

Cold Quantum Coffee, Heidelberg University, 23. October 2019

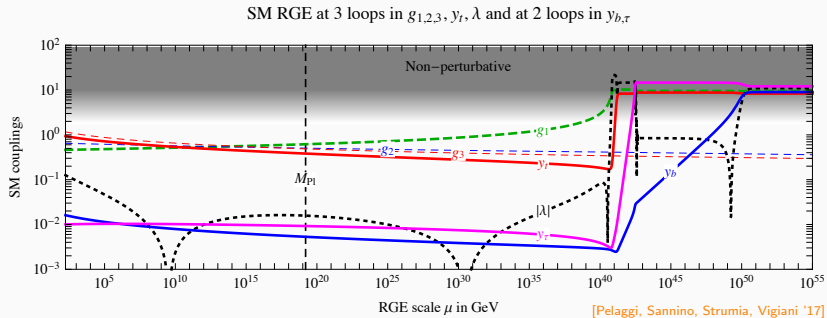
CP³-Origins, SDU Odense, Denmark

MR, Juri Smirnov: [arXiv:1910.xxxxx](#)

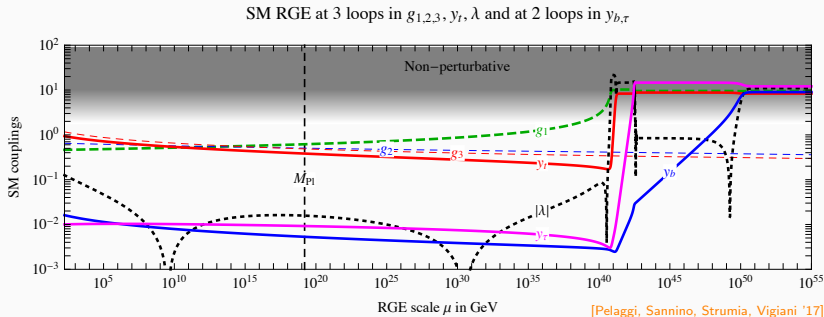
CP³ Origins
Cosmology & Particle Physics

CP³

Couplings of the Standard Model



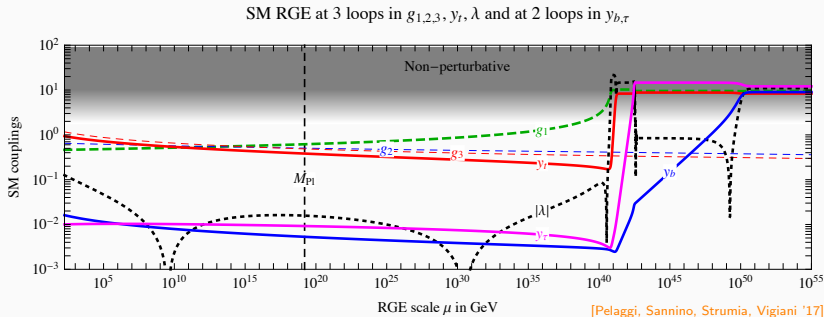
Couplings of the Standard Model



Potentially UV complete theories

- Asymptotically safe QG
- Large N gauge theories
- ...

Couplings of the Standard Model



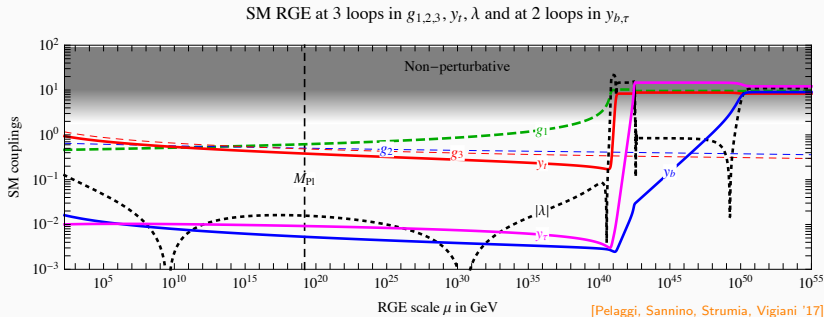
Phenomenology

- Baryogenesis
- Dark matter
- ...

Potentially UV complete theories

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Couplings of the Standard Model



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Asymptotically Safe Quantum Gravity

Quantum gravity in perturbation theory

Einstein-Hilbert action with $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G_N} h_{\mu\nu}$

$$S_{\text{EH}} = \frac{1}{16\pi G_N} \int_x \sqrt{\det g_{\mu\nu}} (2\Lambda - R(g_{\mu\nu}))$$

Perturbatively non-renormalisable

$$[G_N] = -2$$

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Perturbatively non-renormalisable

$$[G_N] = -2$$

Actual evidence by two-loop Goroff-Sagnotti counter term

$$S_{\text{GS}} \sim \int_x \sqrt{\det g_{\mu\nu}} C_{\mu\nu}{}^{\kappa\lambda} C_{\kappa\lambda}{}^{\rho\sigma} C_{\rho\sigma}{}^{\mu\nu}$$

[t Hooft, Veltmann '74; Goroff, Sagnotti '85]

Start of an infinite series of counter terms: No predictivity

Quantum gravity in perturbation theory

Higher-derivative gravity

$$S_{\text{HD}} = \int_x \sqrt{\det g_{\mu\nu}} \left(\frac{1}{2\lambda} C^2 - \frac{w}{3\lambda} R^2 \right) + S_{\text{EH}}$$

Perturbatively renormalisable

$$[w] = 0$$

$$[\lambda] = 0$$

Quantum gravity in perturbation theory

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Asymptotically free

$$\lambda^* = 0$$

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Perturbatively renormalisable

$$[w] = 0$$

$$[\lambda] = 0$$

Asymptotically free

$$\lambda^* = 0$$

But perturbatively non-unitary

$$G_{\text{graviton}} \sim \frac{1}{p^2 + p^4/M_{\text{Pl}}^2} = \frac{1}{p^2} - \frac{1}{M_{\text{Pl}}^2 + p^2}$$

[Stelle '74]

Asymptotically safe quantum gravity

Weinberg's proposal '76

Non-perturbative UV fixed point of the renormalisation group flow

- Metric carries fundamental degrees of freedom
- Diffeomorphism invariance is the symmetry of the theory

Asymptotically safe quantum gravity

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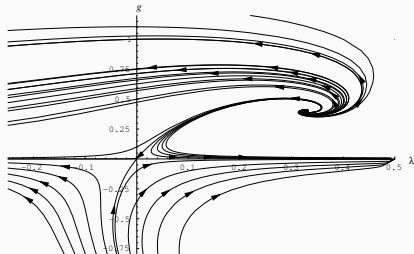
Non-perturbative UV fixed point of the renormalisation group flow

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$$S_{\text{EH}} = \frac{1}{16\pi G_{\text{N}}} \int \sqrt{g} (2\Lambda - R)$$

$$k\partial_k g \equiv \beta_g \xrightarrow{k \rightarrow \infty} 0$$

$$k\partial_k \lambda \equiv \beta_\lambda \xrightarrow{k \rightarrow \infty} 0$$



[Reuter '96; Reuter, Saueressig '01]

Asymptotically safe quantum gravity

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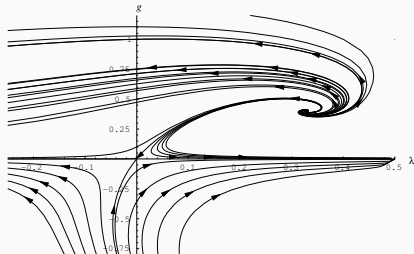
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[Reuter '96; Reuter, Saueressig '01]

Predictivity $\Leftrightarrow$ UV critical hypersurface is finite dimensional

Unitarity $\Leftrightarrow$ Properties of the spectral function

Expansion in curvature invariants

- $\mathcal{O}(R^0)$: Λ
- $\mathcal{O}(R^1)$: R
- $\mathcal{O}(R^2)$: $R^2, R_{\mu\nu}R^{\mu\nu}, C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$
- $\mathcal{O}(R^3)$: $R^3, R\Box R, C_{\mu\nu\rho\sigma}\Box C^{\mu\nu\rho\sigma}, RR_{\mu\nu}R^{\mu\nu}, R_{\mu}^{\nu}R_{\nu}^{\rho}R_{\rho}^{\mu},$
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- $\mathcal{O}(R^4)$: ...

Expansion in curvature invariants

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Expansion in curvature invariants

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[Codello, Percacci, Rahmede '07; Machado, Saueressig '07; ...]

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[Gies, Knorr, Lippoldt, Saueressig '16]

Expansion in curvature invariants

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[Bosma, Knorr, Saueressig '19; Knorr, Ripken, Saueressig '19]

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Aims at apparent convergence

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Let's zoom into EH truncation!

Aims at apparent convergence

Challenge: keeping track of diffeomorphism invariance

Metric split

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{Z_h G_N} h_{\mu\nu}$$

Gauge-fixed Einstein-Hilbert action with regulator

$$S = S_{\text{EH}}[g = \bar{g} + h] + S_{\text{gf}}[\bar{g}, h] + S_{\text{gh}}[\bar{g}, h, c, \bar{c}] + S_{\text{reg}}[\bar{g}, h]$$

Effective action depends separately on $\bar{g}_{\mu\nu}$ and $h_{\mu\nu}$

$$\Gamma_k \equiv \Gamma_k[\bar{g}, h]$$

The dynamics are carried by the correlators of $h_{\mu\nu}$

$$\frac{\delta^n \Gamma_k}{\delta h^n} \equiv \Gamma_k^{(nh)} \equiv \langle h_1 \dots h_n \rangle$$

Vertex expansion of the effective action

Infinite tower
of coupled functional
differential equations

$$\partial_t \Gamma_k = \frac{1}{2} \text{ (graviton loop) } - \text{ (ghost loop) }$$

$$\partial_t \Gamma_k^{(h)} = -\frac{1}{2} \text{ (graviton loop with 1 external line) } + \text{ (ghost loop with 1 external line) }$$

$$\partial_t \Gamma_k^{(2h)} = -\frac{1}{2} \text{ (graviton loop with 2 external lines) } + \text{ (graviton loop with 2 external lines) } - 2 \text{ (ghost loop with 2 external lines) }$$

$$\partial_t \Gamma_k^{(cc)} = \text{ (ghost loop with 2 external lines) } + \text{ (ghost loop with 2 external lines) }$$

$$\partial_t \Gamma_k^{(3h)} = -\frac{1}{2} \text{ (graviton loop with 3 external lines) } + 3 \text{ (graviton loop with 3 external lines) } - 3 \text{ (graviton loop with 3 external lines) } + 6 \text{ (ghost loop with 3 external lines) }$$

$$\begin{aligned} \partial_t \Gamma_k^{(4h)} = & -\frac{1}{2} \text{ (graviton loop with 4 external lines) } + 3 \text{ (graviton loop with 4 external lines) } + 4 \text{ (graviton loop with 4 external lines) } - 6 \text{ (ghost loop with 4 external lines) } \\ & - 12 \text{ (graviton loop with 4 external lines) } + 12 \text{ (graviton loop with 4 external lines) } - 24 \text{ (ghost loop with 4 external lines) } \end{aligned}$$

==== = graviton

....>.... = ghost

$\partial_t \Gamma_k^{(n)}$ depends on
 $\Gamma_k^{(2)}, \dots, \Gamma_k^{(n+2)}$

Vertex expansion of the effective action

[Reuter '96; ...]

$$\partial_t \Gamma_k = \frac{1}{2} \text{ (loop with } \otimes \text{)} - \text{ (dashed loop with } \otimes \text{ and red arrows)}$$

Background-field approximation: $\bar{g}, \bar{\lambda}$

$$\partial_t \Gamma_k^{(h)} = -\frac{1}{2} \text{ (loop with } \otimes \text{ and } h \text{ external lines)} + \text{ (dashed loop with } \otimes \text{ and } h \text{ external lines, red arrows)}$$

$$\partial_t \Gamma_k^{(2h)} = -\frac{1}{2} \text{ (loop with } \otimes \text{ and } 2h \text{ external lines)} + \text{ (loop with } \otimes \text{ and } 2h \text{ external lines)} - 2 \text{ (dashed loop with } \otimes \text{ and } 2h \text{ external lines, red arrows)}$$

$$\partial_t \Gamma_k^{(cc\bar{c})} = \text{ (dashed loop with } \otimes \text{ and } cc\bar{c} \text{ external lines, red arrows)} + \text{ (loop with } \otimes \text{ and } cc\bar{c} \text{ external lines, red arrows)}$$

$$\partial_t \Gamma_k^{(3h)} = -\frac{1}{2} \text{ (loop with } \otimes \text{ and } 3h \text{ external lines)} + 3 \text{ (loop with } \otimes \text{ and } 3h \text{ external lines)} - 3 \text{ (loop with } \otimes \text{ and } 3h \text{ external lines)} + 6 \text{ (dashed loop with } \otimes \text{ and } 3h \text{ external lines, red arrows)}$$

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Vertex expansion of the effective action

[Manrique, Reuter, Saueressig '11; ...]

$$\partial_t \Gamma_k = \frac{1}{2} \text{ (loop with cross) } - \text{ (dashed loop with cross and arrows) } \quad \bar{g}, \bar{\lambda}$$

$$\partial_t \Gamma_k^{(h)} = -\frac{1}{2} \text{ (loop with cross and external line) } + \text{ (dashed loop with cross and external line) }$$

Level-one approximation: g_1, λ_1

$$\partial_t \Gamma_k^{(2h)} = -\frac{1}{2} \text{ (loop with cross and two external lines) } + \text{ (loop with cross and two external lines) } - 2 \text{ (dashed loop with cross and two external lines) }$$

$$\partial_t \Gamma_k^{(c\bar{c})} = \text{ (dashed loop with cross and external lines) } + \text{ (dashed loop with cross and external lines) }$$

$$\partial_t \Gamma_k^{(3h)} = -\frac{1}{2} \text{ (loop with cross and three external lines) } + 3 \text{ (loop with cross and three external lines) } - 3 \text{ (loop with cross and three external lines) } + 6 \text{ (dashed loop with cross and three external lines) }$$

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Vertex expansion of the effective action

[Christiansen, Litim, Pawłowski, Rodigast '14; ...]

$$\begin{aligned}
 \partial_t \Gamma_k &= \frac{1}{2} \text{ (loop with } \otimes \text{)} - \text{ (dashed loop with } \otimes \text{ and red arrows)} & \bar{g}, \bar{\lambda} \\
 \partial_t \Gamma_k^{(h)} &= -\frac{1}{2} \text{ (loop with } \otimes \text{ and external line)} + \text{ (dashed loop with } \otimes \text{ and external line)} & g_1, \lambda_1 \\
 \partial_t \Gamma_k^{(2h)} &= -\frac{1}{2} \text{ (loop with } \otimes \text{ and two external lines)} + \text{ (loop with } \otimes \text{ and two external lines)} - 2 \text{ (dashed loop with } \otimes \text{ and two external lines)} & Z_h(p^2), \mu = -2\lambda_2 \\
 \partial_t \Gamma_k^{(c\bar{c})} &= \text{ (dashed loop with } \otimes \text{ and external lines)} + \text{ (dashed loop with } \otimes \text{ and external lines)} & Z_c(p^2)
 \end{aligned}$$

$$\begin{aligned}
 \partial_t \Gamma_k^{(3h)} &= -\frac{1}{2} \text{ (loop with } \otimes \text{ and three external lines)} + 3 \text{ (loop with } \otimes \text{ and three external lines)} - 3 \text{ (loop with } \otimes \text{ and three external lines)} + 6 \text{ (dashed loop with } \otimes \text{ and three external lines)} \\
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 \end{aligned}$$

Vertex expansion of the effective action

[Christiansen, Knorr, Meibohm, Pawłowski, MR '15; ...]

$$\begin{aligned}
 \partial_t \Gamma_k &= \frac{1}{2} \text{Diagram 1} - \text{Diagram 2} & \bar{g}, \bar{\lambda} \\
 \partial_t \Gamma_k^{(h)} &= -\frac{1}{2} \text{Diagram 3} + \text{Diagram 4} & g_1, \lambda_1 \\
 \partial_t \Gamma_k^{(2h)} &= -\frac{1}{2} \text{Diagram 5} + \text{Diagram 6} - 2 \text{Diagram 7} & Z_h(p^2), \mu = -2\lambda_2 \\
 \partial_t \Gamma_k^{(c\bar{c})} &= \text{Diagram 8} + \text{Diagram 9} & Z_c(p^2) \\
 \partial_t \Gamma_k^{(3h)} &= -\frac{1}{2} \text{Diagram 10} + 3 \text{Diagram 11} - 3 \text{Diagram 12} + 6 \text{Diagram 13} & g_3, \lambda_3 \\
 \partial_t \Gamma_k^{(4h)} &= -\frac{1}{2} \text{Diagram 14} + 3 \text{Diagram 15} + 4 \text{Diagram 16} - 6 \text{Diagram 17} \\
 &\quad - 12 \text{Diagram 18} + 12 \text{Diagram 19} - 24 \text{Diagram 20}
 \end{aligned}$$

The diagrams represent various Feynman diagrams for the vertex expansion of the effective action. They are organized into rows corresponding to different components of the effective action. The diagrams are drawn with blue lines and vertices, and some include red dashed lines and arrows indicating specific contributions or symmetries.

Vertex expansion of the effective action

[Denz, Pawłowski, MR '16]

$$\begin{aligned}
 \partial_t \Gamma_k &= \frac{1}{2} \text{[Diagram: circle with cross]} - \text{[Diagram: dashed circle with cross and red arrows]} & \bar{g}, \bar{\lambda} \\
 \partial_t \Gamma_k^{(h)} &= -\frac{1}{2} \text{[Diagram: circle with cross and one external line]} + \text{[Diagram: dashed circle with cross and one external line]} & g_1, \lambda_1 \\
 \partial_t \Gamma_k^{(2h)} &= -\frac{1}{2} \text{[Diagram: circle with cross and two external lines]} + \text{[Diagram: circle with cross and two external lines]} - 2 \text{[Diagram: dashed circle with cross and two external lines]} & Z_h(p^2), \mu = -2\lambda_2 \\
 \partial_t \Gamma_k^{(c\bar{c})} &= \text{[Diagram: dashed circle with cross and two external lines]} + \text{[Diagram: dashed circle with cross and two external lines]} & Z_c(p^2) \\
 \partial_t \Gamma_k^{(3h)} &= -\frac{1}{2} \text{[Diagram: circle with cross and three external lines]} + 3 \text{[Diagram: circle with cross and three external lines]} - 3 \text{[Diagram: circle with cross and three external lines]} + 6 \text{[Diagram: dashed circle with cross and three external lines]} & g_3, \lambda_3 \\
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 &\quad - 12 \text{[Diagram: circle with cross and four external lines]} + 12 \text{[Diagram: circle with cross and four external lines]} - 24 \text{[Diagram: dashed circle with cross and four external lines]} & g_4, \lambda_4
 \end{aligned}$$

Vertex expansion of the effective action

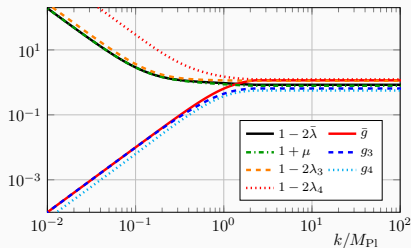
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 \partial_t \Gamma_k^{(2h)} &= -\frac{1}{2} \text{[Diagram: circle with cross and two external lines]} + \text{[Diagram: circle with cross and two external lines]} - 2 \text{[Diagram: dashed circle with cross and two external lines]} & Z_h(p^2), \mu = -2\lambda_2 \\
 \partial_t \Gamma_k^{(c\bar{c})} &= \text{[Diagram: dashed circle with cross and two external lines]} + \text{[Diagram: dashed circle with cross and two external lines]} & Z_c(p^2) \\
 \partial_t \Gamma_k^{(3h)} &= -\frac{1}{2} \text{[Diagram: circle with cross and three external lines]} + 3 \text{[Diagram: circle with cross and three external lines]} - 3 \text{[Diagram: circle with cross and three external lines]} + 6 \text{[Diagram: dashed circle with cross and three external lines]} & g_3, \lambda_3 \\
 \partial_t \Gamma_k^{(4h)} &= -\frac{1}{2} \text{[Diagram: circle with cross and four external lines]} + 3 \text{[Diagram: circle with cross and four external lines]} + 4 \text{[Diagram: circle with cross and four external lines]} - 6 \text{[Diagram: circle with cross and four external lines]} \\
 &\quad - 12 \text{[Diagram: circle with cross and four external lines]} + 12 \text{[Diagram: circle with cross and four external lines]} - 24 \text{[Diagram: dashed circle with cross and four external lines]} & g_4, \lambda_4 \\
 & & \approx 4 \cdot 10^{12} \text{ terms}
 \end{aligned}$$

UV fixed point

IR behaviour

General relativity



UV fixed point

Asymptotic safety

$$(\mu^*, \lambda_3^*, \lambda_4^*, g_3^*, g_4^*) = (-0.45, 0.12, 0.028, 0.83, 0.57)$$

$$\theta_i = (4.7, 2.0 \pm 3.1 i, -2.9, -8.0)$$

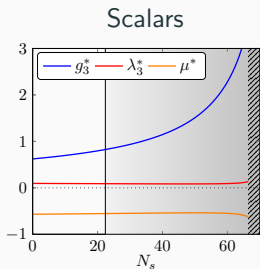
Three relevant directions: Λ , R and R^2

[Denz, Pawłowski, MR '16]

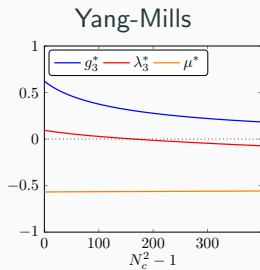
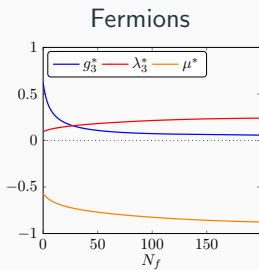
Towards a Standard-Model-gravity system

$$\begin{aligned} S = & \frac{1}{16\pi G_N} \int d^4x \sqrt{g} (2\Lambda - R) && \text{Gravity} \\ & + \frac{1}{2} \sum_{i=1}^{N_s} \int d^4x \sqrt{g} g^{\mu\nu} \partial_\mu \varphi^i \partial_\nu \varphi^i && \text{Scalars} \\ & + \sum_{j=1}^{N_f} \int d^4x \sqrt{g} \bar{\psi}_j \not{D} \psi_j && \text{Fermions} \\ & + \frac{1}{2} \int d^4x \sqrt{g} g^{\mu\nu} g^{\rho\sigma} \text{tr} F_{\mu\rho} F_{\nu\sigma} && \text{Yang-Mills} \end{aligned}$$

UV fixed point with matter



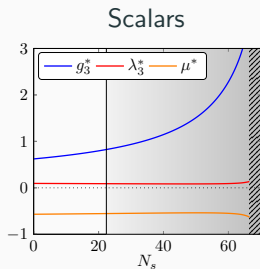
[Meibohm, Pawłowski, MR '16]



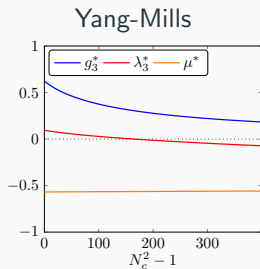
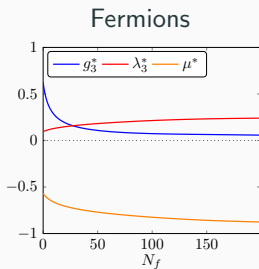
[Christiansen, Litim, Pawłowski, MR '17]

Gravity dominates the UV behaviour: $G_{\text{eff}} \sim \frac{g_3}{1+\mu}$

UV fixed point with matter



[Meibohm, Pawłowski, MR '16]



[Christiansen, Litim, Pawłowski, MR '17]

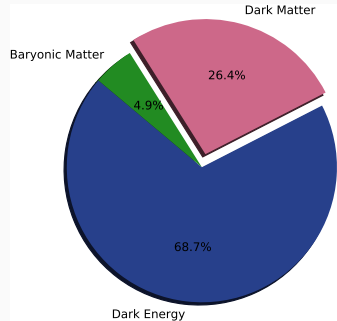
Gravity dominates the UV behaviour: $G_{\text{eff}} \sim \frac{g_3}{1+\mu}$

One force to rule them all

Dark Matter

Dark Matter evidence from

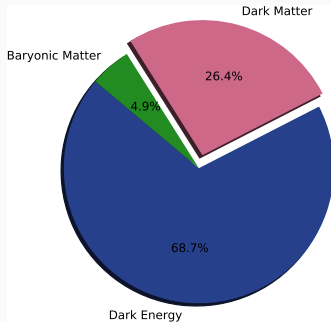
- Rotation curves
- CMB
- Structure formation
- ...



Dark Matter

Dark Matter can be

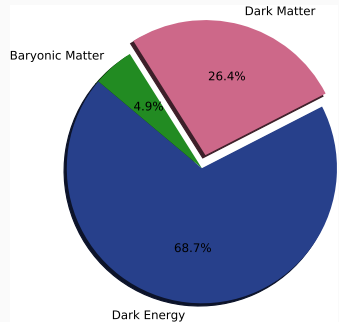
- a particle
- a modified gravitational interaction
- primordial black holes
- ...



Dark Matter

Dark Matter can be

- a particle
- a modified gravitational interaction
- primordial black holes
- ...

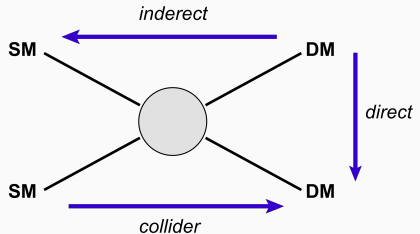


We want to use quantum gravity to constrain a given dark matter model

Dark Matter

A dark matter candidate

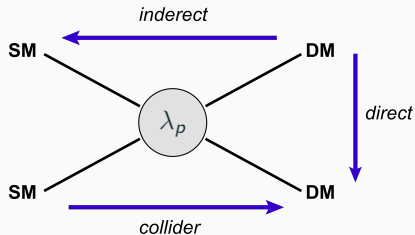
- is stable or long-lived on cosmic time scales
- has a portal interaction with the SM fields



Dark Matter

A dark matter candidate

- is stable or long-lived on cosmic time scales
- has a portal interaction with the SM fields

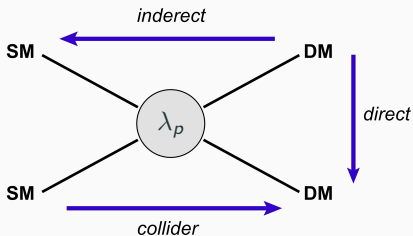


Example: Higgs portal $\lambda_p H^\dagger H S S^*$

Dark Matter

A dark matter candidate

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Example: Higgs portal $\lambda_p H^\dagger H S S^*$

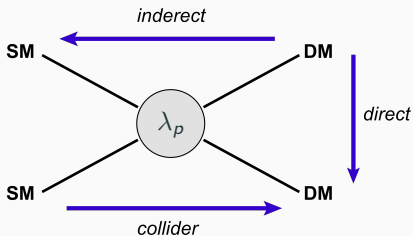
Various production mechanisms

- Thermal production (freeze out)
- Non-thermal production
(decay from heavier particle, during reheating)

Dark Matter

A dark matter candidate

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Example: Higgs portal $\lambda_p H^\dagger H S S^*$

Various production mechanisms

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Freeze out

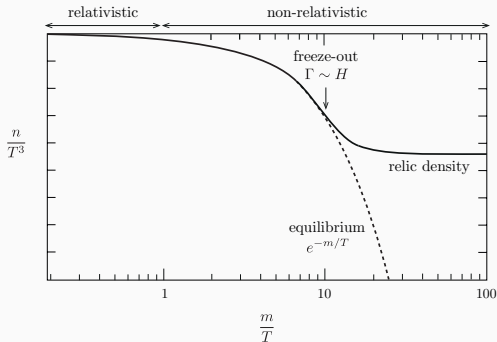
Number density of DM candidate is described by the Boltzmann equation

$$\frac{dn_\chi}{dt} + 3H n_\chi = -\langle \sigma(\chi\chi \rightarrow \text{SM}) v_{\text{rel.}} \rangle (n_\chi^2 - n_{\chi,\text{eq}}^2)$$

Freeze out

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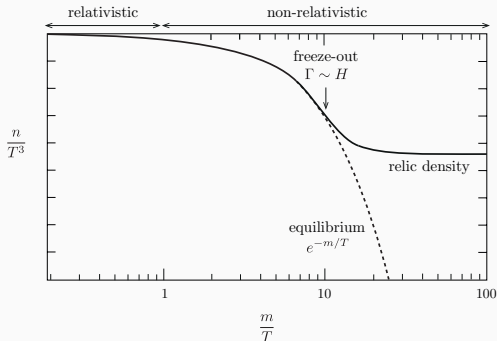


[Picture: Baumann '19]

Freeze out

Number density of DM candidate is described by the Boltzmann equation

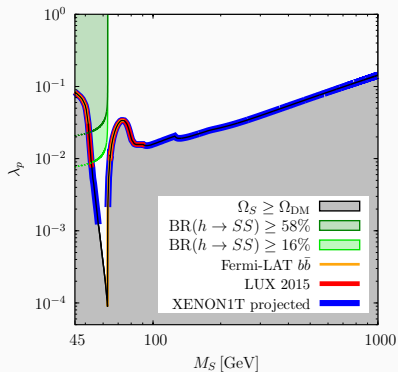
$$\frac{dn_\chi}{dt} + 3H n_\chi = -\langle\sigma(\chi\chi \rightarrow \text{SM})v_{\text{rel.}}\rangle(n_\chi^2 - n_{\chi,\text{eq}}^2)$$



[Picture: Baumann '19]

Smaller portal coupling $\rightarrow$ earlier decoupling $\rightarrow$ larger relic density

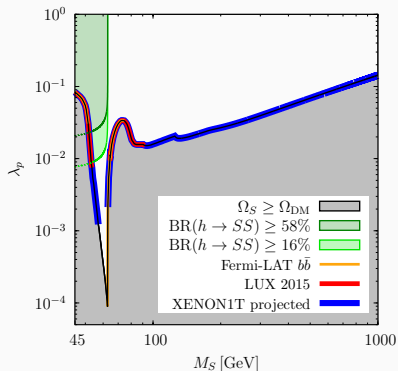
Scalar Higgs portal



[Duerr, Perez, Smirnov '15]

- Higgs resonance at $m_S \approx m_h/2$ allows for smaller coupling values

Scalar Higgs portal

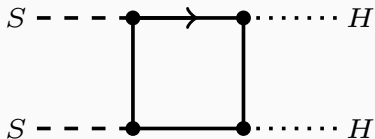


[Duerr, Perez, Smirnov '15]

- Higgs resonance at $m_S \approx m_h/2$ allows for smaller coupling values
- Quantum gravity prediction $\lambda_p(M_{\text{Pl}}) = 0$
- Portal coupling remains zero also below M_{Pl} [Eichhorn, Hamada, Lumma, Yamada '18]

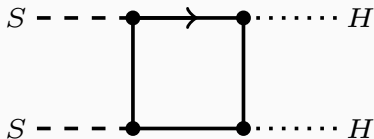
How can we generate the portal coupling?

Yukawa interaction can generate λ_p

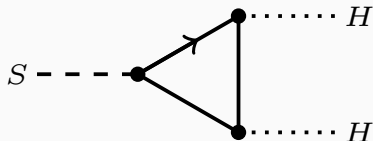


How can we generate the portal coupling?

Yukawa interaction can generate λ_p



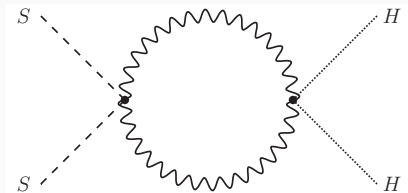
But also allows decay (breaks the stabilising Z_2 symmetry)



Yukawa interaction is fine as long as ψ cannot decay into light particles

How can we generate the portal coupling?

Gauge interaction $U(1)_X$



- Stability: interaction preserves Z_2 symmetry
- Kinetic mixing: no charge of Higgs boson under $U(1)_X$ needed

Lagrangian of $U(1)_X$ and $U(1)_Y$

$$\mathcal{L} \sim \frac{1}{4} F_{\mu\nu}^X F_X^{\mu\nu} + \frac{1}{4} F_{\mu\nu}^Y F_Y^{\mu\nu} + \frac{\epsilon}{2} F_{\mu\nu}^X F_Y^{\mu\nu}$$

Lagrangian of $U(1)_X$ and $U(1)_Y$

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- Eliminate $F_{\mu\nu}^X F_Y^{\mu\nu}$ by rotations and rescalings of the gauge fields
- Price to pay: non-diagonal covariant derivative

$$\mathcal{D}_\mu = \partial_\mu + i(g_Y n_Y) B_\mu + i(g_D n_X + g_\epsilon n_Y) Z'_\mu$$

- New gauge couplings g_D and g_ϵ

Lagrangian of the dark sector

$$\begin{aligned}\mathcal{L}_D &\sim \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{gauge}} \\ &\sim \frac{1}{2} D_\mu S D^\mu S^* + \lambda_p H^\dagger H S S^* + \lambda_S (S S^*)^2 + \frac{m_S^2}{2} S S^* \\ &\quad + i \bar{\psi} \not{D} \psi + M_\psi \bar{\psi} \psi + y_\psi S \bar{\psi} \psi^c \\ &\quad + \frac{1}{4} F_{\mu\nu}^X F_X^{\mu\nu} + \frac{\epsilon}{2} F_{\mu\nu}^Y F_X^{\mu\nu} + \frac{M_{Z'}^2}{2} (Z'_\mu - \partial_\mu \zeta)^2\end{aligned}$$

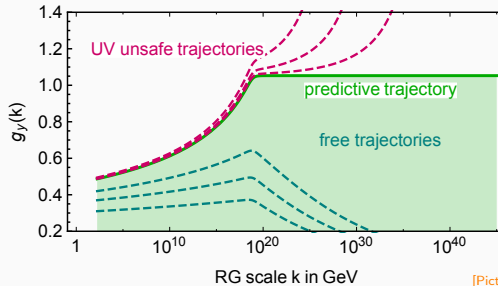
- Stueckelberg mechanism to give mass to Z'
- Vector-like fermion ψ for vacuum stability of S
- S or ψ is dark matter candidate depending on mass hierarchy

Dark Matter meets Quantum Gravity

- Standard Model extension that allows for a Dark Matter candidate
- Simple dark matter models preferred
- Demand that the model is UV complete with quantum gravity
- Assume no further particle content

Boundary conditions from asymptotically safety

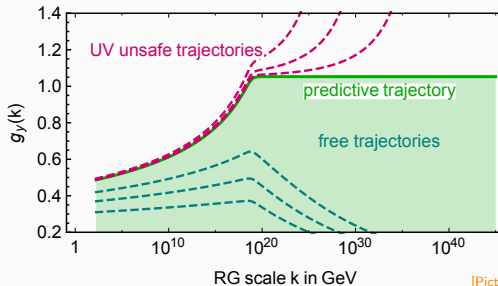
If matter couplings become too large, they run into a Landau pole



[Picture: Eichhorn, Versteegen '17]

Boundary conditions from asymptotically safety

If matter couplings become too large, they run into a Landau pole



[Picture: Eichhorn, Versteegen '17]

Notice difference between

- UV attractive (relevant) direction
- UV repulsive (irrelevant) direction

Quartic scalar coupling

Beta function of quartic scalar coupling

$$\beta_\lambda = \beta_{\lambda,\text{matter}} + f_\lambda$$

with *UV repulsive* fixed point $\lambda^* = 0$

[Pawlowski, MR, Wetterich, Yamada '18]

Quartic scalar coupling

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Boundary condition: $\lambda(M_{\text{Pl}}) \approx 0$

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[Pawlowski, MR, Wetterich, Yamada '18]

Boundary condition: $\lambda(M_{\text{Pl}}) \approx 0$

Application to Higgs mass

[Shaposhnikov, Wetterich '09]

$$m_h = 126 - 136 \text{ GeV}$$

$U(1)$ gauge coupling

$U(1)$ gauge beta function

$$\beta_g = \beta_{g,\text{matter}} - f_g g$$

$U(1)$ gauge coupling

$U(1)$ gauge beta function

$$\beta_g = \beta_{g,\text{matter}} - f_g g$$

f_g is positive

[Christiansen, Litim, Pawłowski, MR '17]

$$f_g = \frac{\tilde{G}}{16\pi} \left(\frac{8}{1 - 2\tilde{\Lambda}} - \frac{4}{(1 - 2\tilde{\Lambda})^2} \right)$$

$U(1)$ gauge coupling

$U(1)$ gauge beta function

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[Christiansen, Litim, Pawłowski, MR '17]

$$f_g = \frac{\tilde{G}}{16\pi} \left(\frac{8}{1 - 2\tilde{\Lambda}} - \frac{4}{(1 - 2\tilde{\Lambda})^2} \right)$$

Boundary condition at one loop ($\beta_{g,\text{matter}} = \beta_{g,1\text{-loop}} g^3$)

$$g(M_{\text{Pl}}) \leq \sqrt{\frac{f_g}{\beta_{g,1\text{-loop}}}}$$

We use $f_g \leq 0.04$

Yukawa beta function at one loop

$$\beta_y = \beta_{y,1\text{-loop-yukawa}} y^3 - \beta_{y,1\text{-loop-gauge}} y - f_y y$$

Boundary condition

$$y(M_{\text{Pl}}) \leq \sqrt{\frac{f_y + \beta_{y,1\text{-loop-gauge}}}{\beta_{y,1\text{-loop-yukawa}}}}$$

Application: top mass and difference between top & bottom mass

[Eichhorn, Held '17; '18]

When is the SM compatible with Asymptotic Safety?

- For $U(1)_Y$ we need $f_g \geq 9.8 \cdot 10^{-3}$
- For top and bottom mass we need $f_y \geq 10^{-4}$
- The Higgs mass is slightly wrong $m_h \approx 130 \text{ GeV}$

[Eichhorn, Held '18]

[Shaposhnikov, Wetterich '09]

Two perspectives on the Higgs mass

- Accept small difference
- Use freedom of SM extension to adjust Higgs mass

Scalar dark matter model

Properties

- $U(1)_X$ is identified with $U(1)_{B-L}$
- Right-handed neutrinos to make $B-L$ anomaly free
- Dark fermions for vacuum stability; decay via neutrino channel

Scalar dark matter model

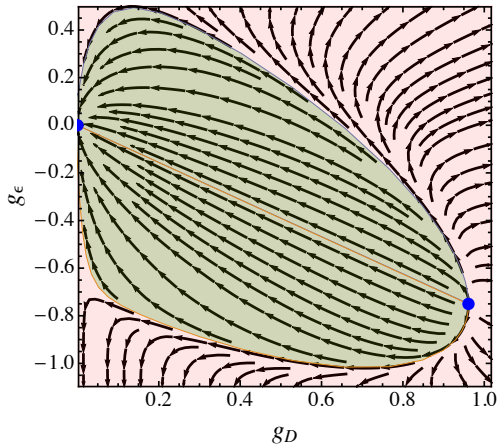
Properties

- $U(1)_X$ is identified with $U(1)_{B-L}$
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Predictivity from

- $\lambda_p(M_{\text{Pl}}) \approx 0$
- λ_p induced by g_D and g_ϵ , which are bounded as well
- Vacuum stability of S is crucial

Allowed range for g_D and g_ϵ



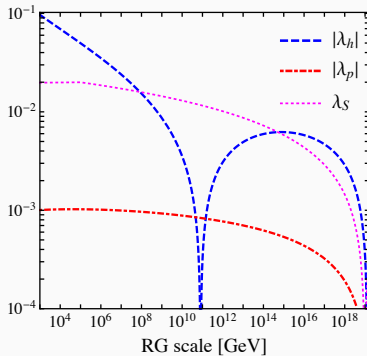
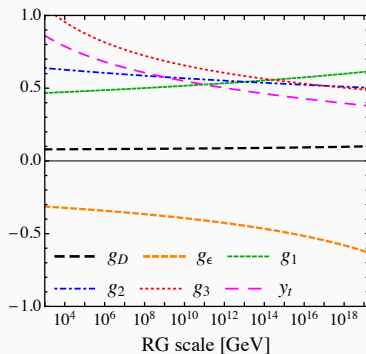
Asymptotically free couplings, if their values are in the green area at M_{Pl}

Example running

- Choose $g_D(M_{\text{Pl}})$
- Adjust $g_\epsilon(M_{\text{Pl}})$ to match Higgs mass
- Add fermions for vacuum stability

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Prediction for portal coupling

Use g_ϵ to fix Higgs mass and $f_g \leq 0.04$

$$|\lambda_p(\text{TeV})| \leq 0.08$$

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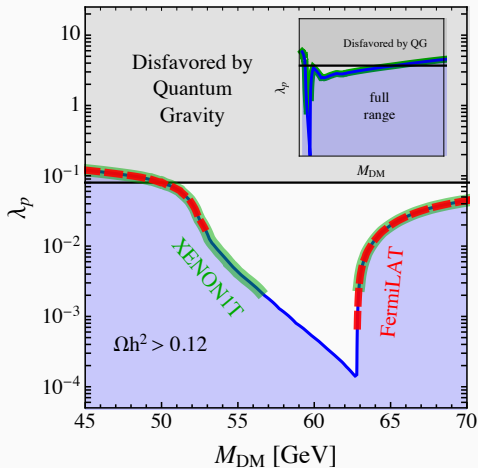
Accepting a small difference in the Higgs mass

$$|\lambda_p(\text{TeV})| \lesssim 1.1 f_g + 54 f_g^2$$

$$m_{h,\min} \approx (136 - 382 f_g) \text{ GeV}$$

For $f_g = 0.04$ we find $|\lambda_p| \leq 0.13$

Favoured mass range scalar dark matter



$$56 \text{ GeV} < M_{DM} < 63 \text{ GeV}$$

Fermionic dark matter model

Properties

- $U(1)_X$ with free quantum number n_ψ for fermion
- Scalar Higgs portal optional

Fermionic dark matter model

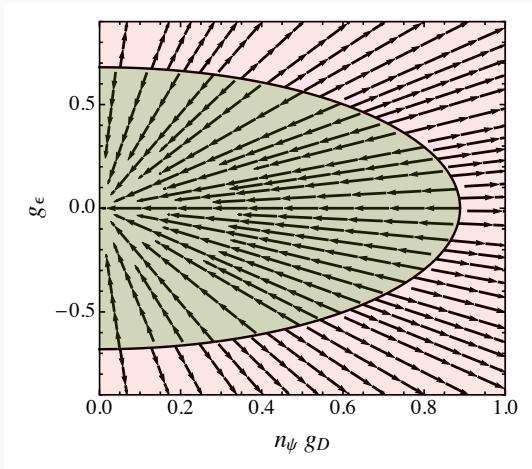
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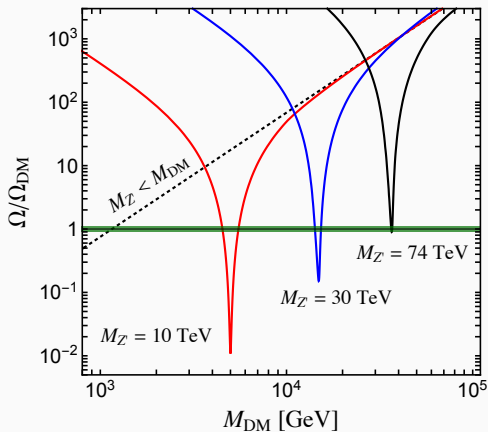
- Upper bound on $n_\psi g_D$
- Annihilation cross section $\sim n_\psi g_D$
- n_ψ drops out

Allowed range for g_D and g_ϵ

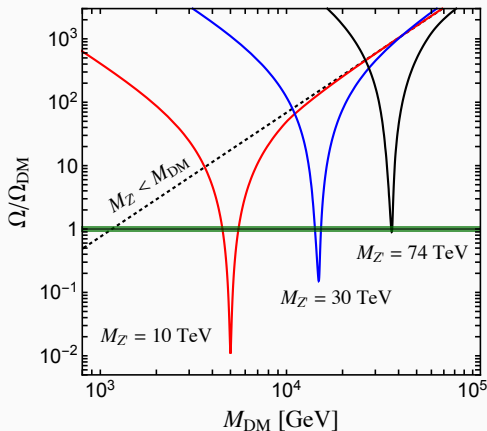


Asymptotically free couplings, if their values are in the green area at M_{Pl}

Favoured mass range fermionic dark matter



Favoured mass range fermionic dark matter



- non-resonant $M_{Z'} < M_{\text{DM}}$: $M_{\text{DM}} < 2 \text{ TeV}$
- resonant $M_{Z'} > M_{\text{DM}}$: $M_{\text{DM}} < 40 \text{ TeV}$

Summary

- Dark matter models guided by simplicity
- Demand asymptotic safety or freedom of all couplings
- Boundary conditions at M_{Pl} leads to constraints on the mass
- Scalar Higgs portal

$$56 \text{ GeV} < M_{\text{DM}} < 63 \text{ GeV}$$

- Fermionic dark matter

$$M_{\text{DM}} < 40 \text{ TeV}$$

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Thank you for your attention