Multipartite entanglement certification in quantum many-body systems using quench dynamics

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Cold Quantum Coffee - 19/11/2019

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$$\theta = ?$$

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Tools: measurements of a quantum state

$$\rho(\theta) = e^{-i\theta O} \rho_0 e^{+i\theta O}$$

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How precise can this estimation be?

Cramér-Rao Bound

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Conditional probability distribution:

$$f(\mu|\theta)$$

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Fisher information:

$$F = \sum_{\mu} f(\mu|\theta) \left(\partial_\theta \ln f(\mu|\theta)\right)$$

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Bound on the precision of any estimator:

$$\mathrm{Var}\,(\hat{\theta}) \geq F^{-1}$$

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$$\text{POVM}\{E_\mu\} \quad \Rightarrow \quad f(\mu|\theta) = Tr\left(\rho(\theta)E_\mu\right)$$

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Best precision achievable with ρ_0:

$$\mathrm{Var}\left(\hat{\theta}\right) \geq F_Q^{-1}$$

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$$F_Q = 2\sum_{\lambda,\lambda'} \frac{\rho_\lambda - \rho_{\lambda'}}{\rho_\lambda + \rho_{\lambda'}} (\rho_\lambda - \rho_{\lambda'}) |\langle\lambda| O |\lambda'\rangle|^2,$$

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- Entangled states $\neq$ product states
- k-partite entangled states $\neq$ k-producible states

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$$F_Q \leq kN \text{ for } O = \frac{1}{2} \sum_j \sigma_j^z$$

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k-partite entanglement certifcation:

$$F_Q > kN \Rightarrow \text{k-partite entanglement}$$

Background in Quantum Metrology

Multipartite Entanglement Certification

Quench Dynamics

1D Fermi-Hubbard Model

QFI in the thermal ensemble (BLACKBOARD)

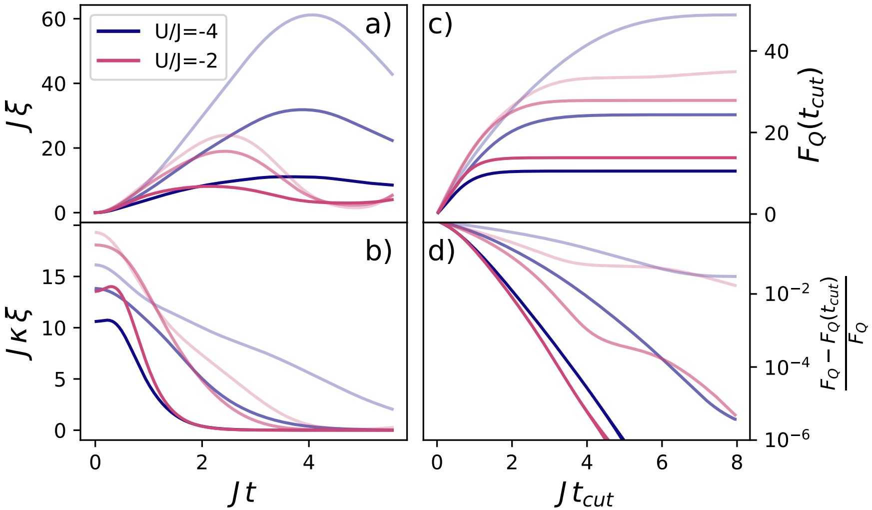
$$F_Q\left[\rho, O\right] = \frac{4}{\pi}\int_0^\infty d\omega \tanh\left(\frac{\omega}{2T}\right)\chi''(\omega, T)$$

$$F_Q\left[\rho, O\right] = 4T\int_0^\infty dt\,\frac{\chi(t, T)}{\sinh\left(\pi t T\right)}$$

QFI from quench dynamics

$$F_Q\left[\rho, O\right] = \frac{4\pi T^2}{q}\int_0^\infty dt\,\frac{\Delta O(t)_{\text{quench}}}{\sinh\left(\pi t T\right)\tanh\left(\pi t T\right)}$$

Quench Protocol



Model Overview

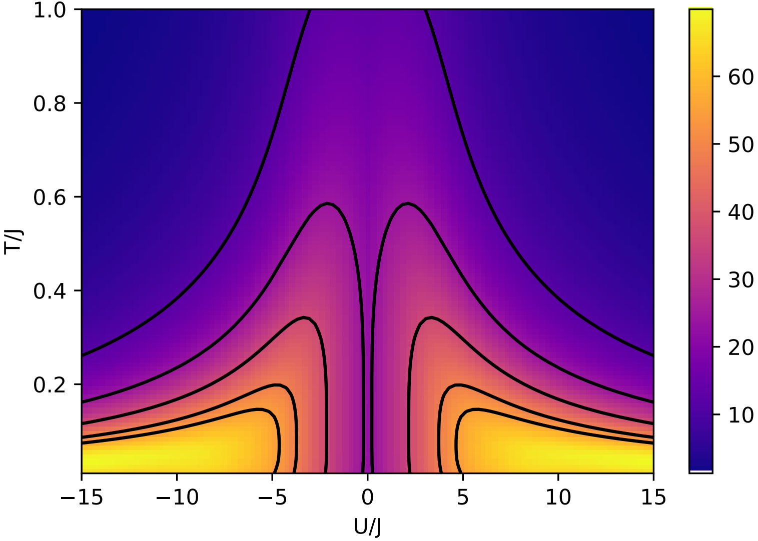
Two fermionic species with on-site interactions in a 1D chain:

$$H_0 = -J \sum_{x,\sigma} \left(c^\dagger_{\sigma x} c_{\sigma x+1} + h.c. \right) + U \sum_x \left(c^\dagger_{\downarrow x} c_{\downarrow x} c^\dagger_{\uparrow x} c_{\uparrow x} \right)$$

Staggered magnetization/density:

$$O_\pm = \sum_x (-1)^x \left(c^\dagger_{\uparrow x} c_{\uparrow x} \mp c^\dagger_{\downarrow x} c_{\downarrow x} \right)$$

Entanglement Certified with $O_{\pm}$



Outlook

- Implementation in ultra-cold atoms experiments
- Generalize to different ensembles
- Analogous results for local thermalization/ETH
- Probe entanglement in topological states of matter

Thank you