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EXTENSIVITY AND ENTROPY CURRENT AT THERMODYNAMIC EQUILIBRIUM WITH ACCELERATION

INTRODUCTION

- What is the entropy current?
  - It is a vector current s^μ that makes the entropy S extensive:

$$S = -\operatorname{tr}(\hat{\rho}\log\hat{\rho}) = \int_\Sigma \mathrm{d}\Sigma\, n_\mu s^\mu.$$

- Why is it interesting?
  - It enters the local version of the second law of thermodynamics.
  - It is a postulated ingredient of Israel's relativistic hydrodynamics.
  - It is responsible for the constitutive equations of the conserved currents.
- What is the problem with it?
  - It is not the TEV of a current dependent on quantum fields, unlike charge currents.
  - In Israel's theory it is postulated but not *derived*.
- We put forward a method to derive it including quantum corrections.
- We perform a specific calculation at thermodynamic equilibrium with acceleration.

[Israel 1976]

MOTIVATIONS

- Relativistic hydrodynamics
  - Astrophysics and cosmology: expectation value of energy-momentum tensor at thermodynamic equilibrium with quantum corrections
  - Quark-Gluon Plasma as relativistic quantum fluid at local thermodynamic equilibrium with acceleration and vorticity
- Quantum Field Theory
  - Relativistic quantum effects at low temperature due to acceleration (Unruh effect)

OUTLINE

1. Relativistic quantum statistical mechanics
2. Global thermodynamic equilibrium with acceleration
3. Thermal expectation values and Unruh effect

[F. Becattini Phys.Rev. D97 (2018) no.8, 085013]

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4. Entropy current and extensivity
5. Entropy current at global equilibrium with acceleration
6. Entanglement entropy and Unruh effect

[F. Becattini and D.R. Phys.Rev. D99 (2019) no.12, 125011]

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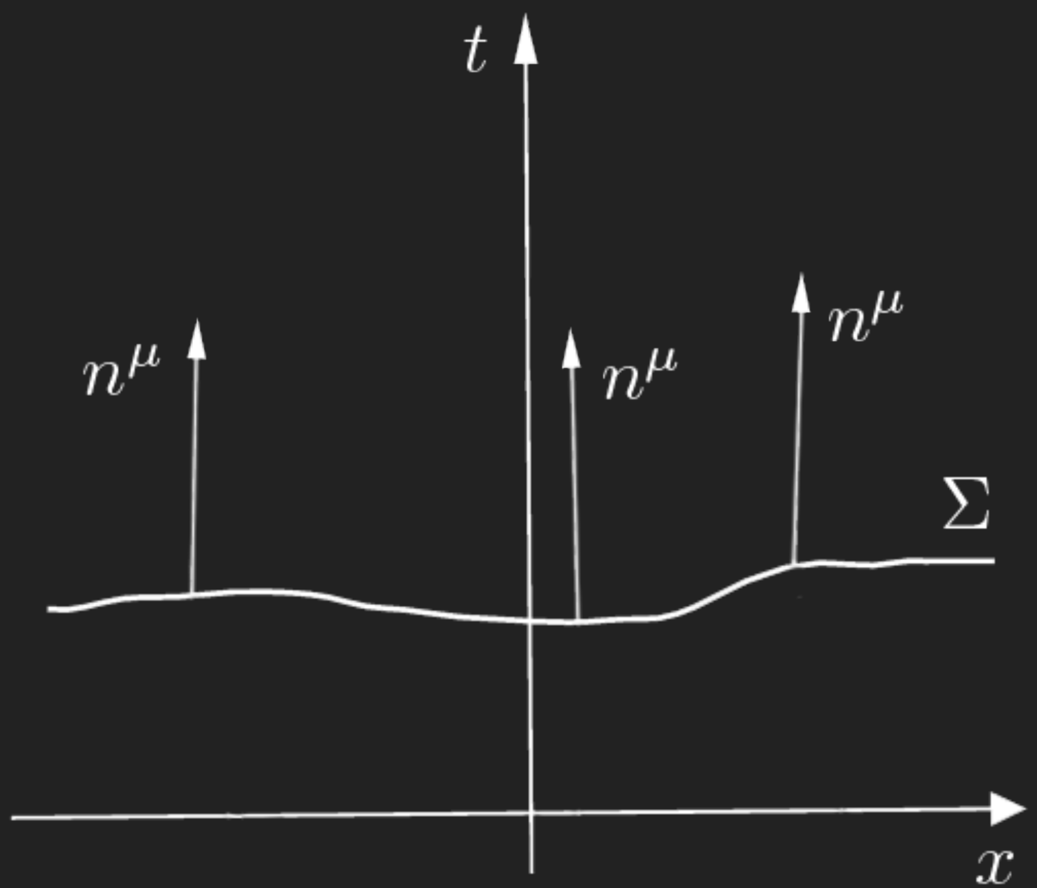
7. Summary

RELATIVISTIC QUANTUM STATISTICAL MECHANICS

- Thermal QFT: calculate thermal expectation values (TEVs) of operators $\langle \mathcal{O} \rangle = \mathrm{tr}(\hat{\rho}\mathcal{O})$.
- Need covariant expression for $\hat{\rho}$.
- Maximum entropy principle.
- Foliate spacetime with family $\Sigma(\tau)$ of spacelike hypersurfaces.
- Give energy-momentum and (possible) charge densities on $\Sigma(\tau)$

$$n_\mu T^{\mu\nu}, \qquad n_\mu j^\mu.$$



LOCAL THERMODYNAMIC EQUILIBRIUM

- Maximize $-\mathrm{tr}(\hat{\rho}_{\mathrm{LE}} \log \hat{\rho}_{\mathrm{LE}})$ with constraints on $\Sigma(\tau)$

$$n_\mu \langle \hat{T}^{\mu\nu} \rangle_{\mathrm{LE}} = n_\mu T^{\mu\nu}, \qquad n_\mu \langle \hat{j}^{\mu} \rangle_{\mathrm{LE}} = n_\mu j^{\mu}.$$

- Solution: *Local Thermodynamic Equilibrium* (*LTE*) operator

$$\hat{\rho}_{\mathrm{LE}} = \frac{1}{Z_{\mathrm{LE}}} \exp\left[-\int_{\Sigma(\tau)} \mathrm{d}\Sigma \, n_\mu \left(\hat{T}^{\mu\nu}\beta_\nu - \zeta \hat{j}^{\mu} \right) \right].$$

- β^μ *four-temperature* (timelike) such that:
  - $u^\mu = \beta^\mu / \sqrt{\beta^2}$ four-velocity
  - $T = 1/\sqrt{\beta^2}$ *proper temperature*
  - $\zeta = \mu / T$ with μ chemical potential

[Zubarev et al. 1979, Van Weert 1982]
[Becattini et al. 2015, Hayata et al. 2015]

GLOBAL THERMODYNAMIC EQUILIBRIUM

Require $\hat{\rho}_{\mathrm{LE}}$ to be τ-independent:
Global Thermodynamic Equilibrium (*GTE*) state

$$\hat{\rho} = \frac{1}{Z} \exp \left[- \int_{\Sigma} \mathrm{d}\Sigma \, n_{\mu} \left(\hat{T}^{\mu\nu} \beta_{\nu} - \zeta \hat{j}^{\mu} \right) \right]$$

τ-independence

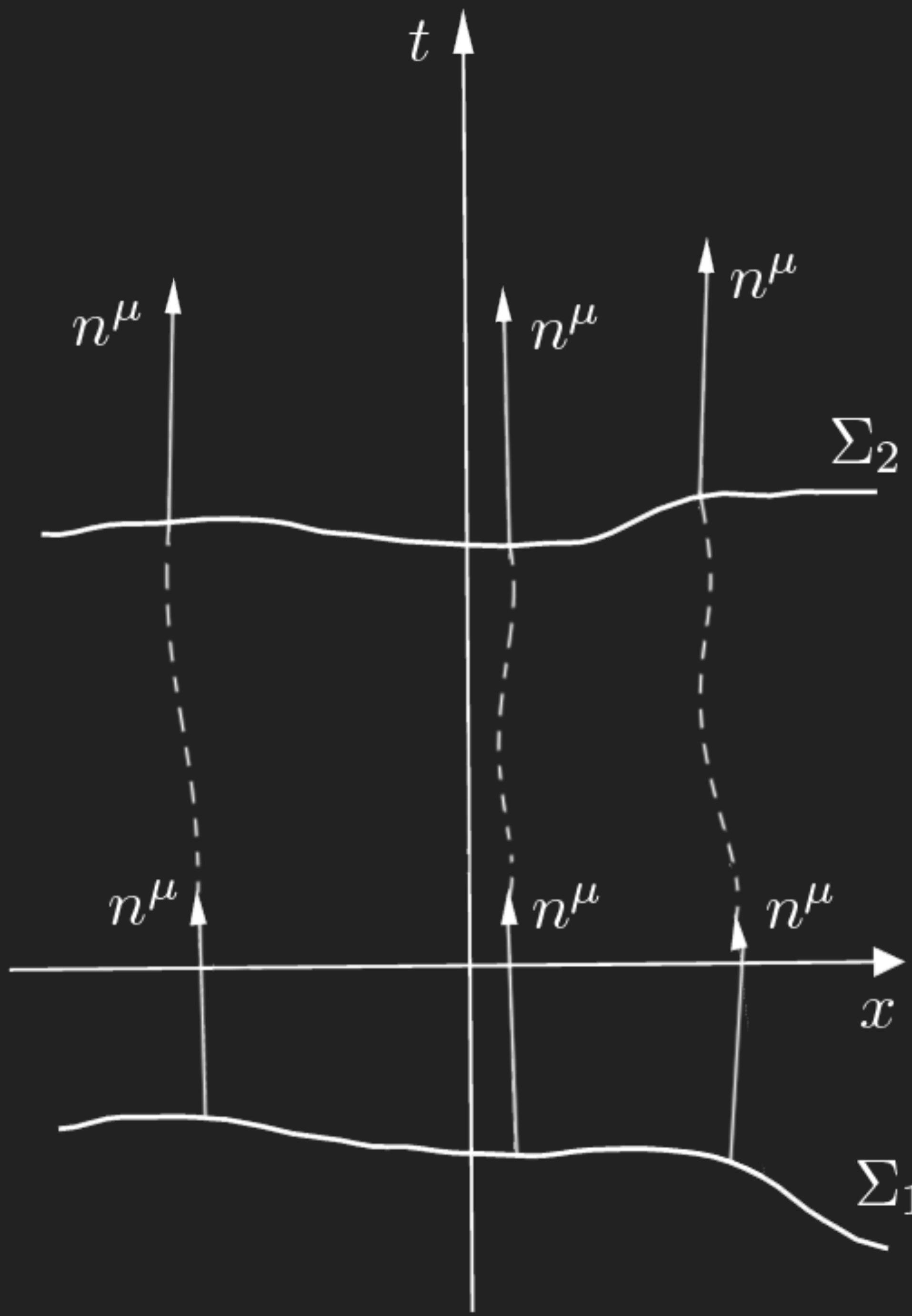
$\Updownarrow$

Σ-independence

$\Updownarrow$

$$\nabla_{\mu} \zeta = 0, \qquad \nabla_{\mu} \beta_{\nu} + \nabla_{\nu} \beta_{\mu} = 0$$

β^{μ} timelike Killing vector



GTE IN MINKOWSKI SPACETIME

In Minkowski spacetime:

$$\beta_\mu = b_\mu + \varpi_{\mu\nu} x^\nu$$

- b_μ constant
- $\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$
  constant *thermal vorticity*

Hence the GTE density operator:

$$\hat{\rho} = \frac{1}{Z} \exp\left[-b_\mu \hat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\hat{J}^{\mu\nu} + \zeta \hat{Q}\right]$$

- $(\hat{P}^\mu, \hat{J}^{\mu\nu})$ generators of Poincaré group

Different choices of $(b_\mu, \varpi_{\mu\nu})$ correspond to different GTEs. Set $\zeta = 0$ for simplicity.

- *Homogeneous GTE*:

$$b_\mu = \frac{1}{T_0}(1,0,0,0), \qquad \varpi_{\mu\nu} = 0$$

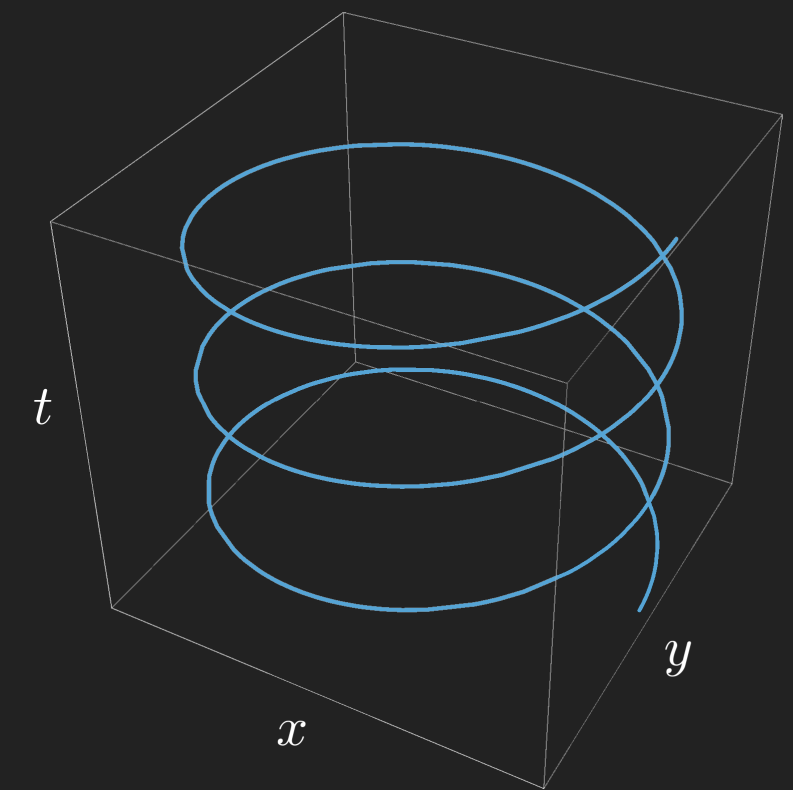
$$\beta_\mu = \frac{1}{T_0}(1,0,0,0), \qquad \hat{\rho} = \frac{1}{Z}\exp\left[-\frac{\hat{H}}{T_0}\right].$$

GTE IN MINKOWSKI SPACETIME

- *GTE with rotation*:

$$b_\mu = \frac{1}{T_0}(1,0,0,0), \qquad \varpi_{\mu\nu} = \frac{\omega}{T_0}\left(g_{1\mu}g_{2\nu} - g_{1\nu}g_{2\mu}\right)$$

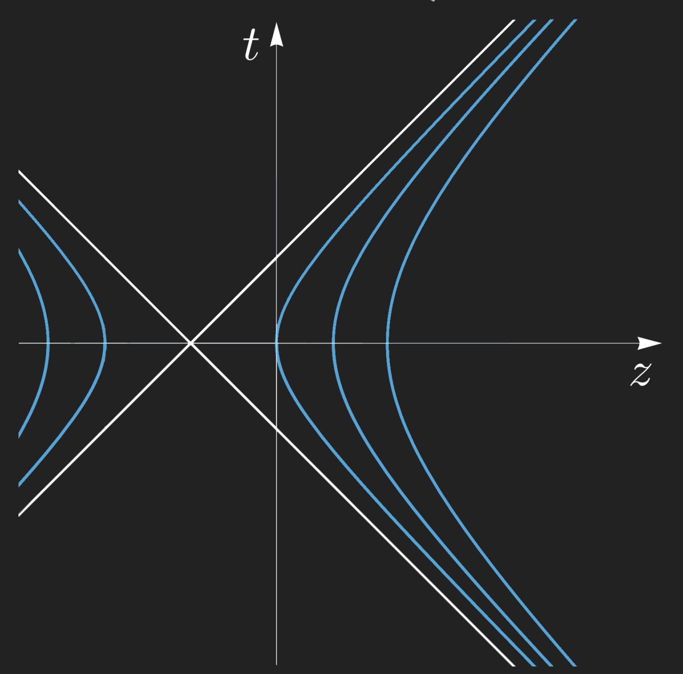
$$\beta_\mu = \frac{1}{T_0}(1,\boldsymbol{\omega}\times\mathbf{x}), \qquad \hat{\rho} = \frac{1}{Z}\exp\left[-\frac{\hat{H}}{T_0} + \frac{\omega}{T_0}\hat{J}_z\right]$$



- *GTE with acceleration*:

$$b_\mu = \frac{1}{T_0}(1,0,0,0), \qquad \varpi_{\mu\nu} = \frac{a}{T_0}\left(g_{0\nu}g_{3\mu} - g_{3\nu}g_{0\mu}\right)$$

$$\beta^\mu = \frac{a}{T_0}\left(\frac{1}{a} + z,0,0,t\right), \qquad \hat{\rho} = \frac{1}{Z}\exp\left[-\frac{\hat{H}}{T_0} + \frac{a}{T_0}\hat{K}_z\right]$$



GTE WITH ACCELERATION IN MINKOWSKI SPACETIME

- Shift $z' = z + 1/a$:

$$\beta^\mu = \frac{a}{T_0}\left(\frac{1}{a} + z, 0, 0, t\right) = \frac{a}{T_0}(z', 0, 0, t).$$

- Flow lines are hyperbolae with constant $z'^2 - t^2$:

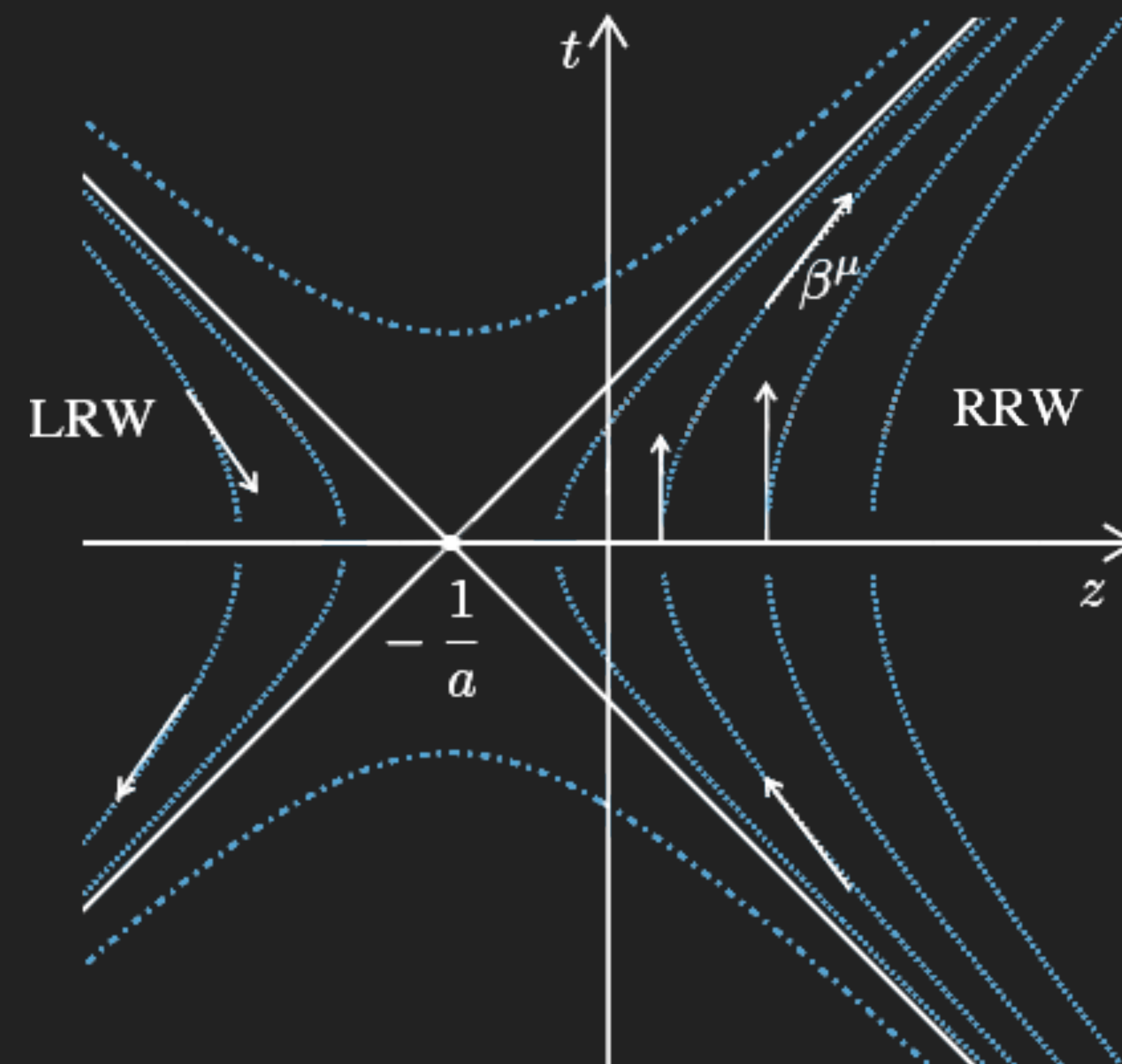
$$u^\mu = \frac{\beta^\mu}{\sqrt{\beta^2}} = \frac{1}{\sqrt{z'^2 - t^2}}(z', 0, 0, t), \qquad T = \frac{1}{\sqrt{\beta^2}} = \frac{T_0}{a\sqrt{z'^2 - t^2}}$$

- Proper four-acceleration

$$A^\mu = u^\nu \partial_\nu u^\mu = \frac{1}{z'^2 - t^2}(t, 0, 0, z')$$

  constant magnitude A^2 along flow lines, hence the name "*GTE with acceleration*".

- Decompose $\varpi_{\mu\nu} = \alpha_\mu u_\nu - \alpha_\nu u_\mu$ with $\alpha^\mu = \dfrac{\hbar}{ck_B}\dfrac{A^\mu}{T}$, hence $\alpha^2 = \dfrac{A^2}{T^2} = -\dfrac{a^2}{T_0^2}$ constant.



- $|z'| = t$ is bifurcated *Killing horizon*: β^μ timelike and future-oriented only in *Right Rindler Wedge* (*RRW*).

GTE WITH ACCELERATION IN MINKOWSKI SPACETIME

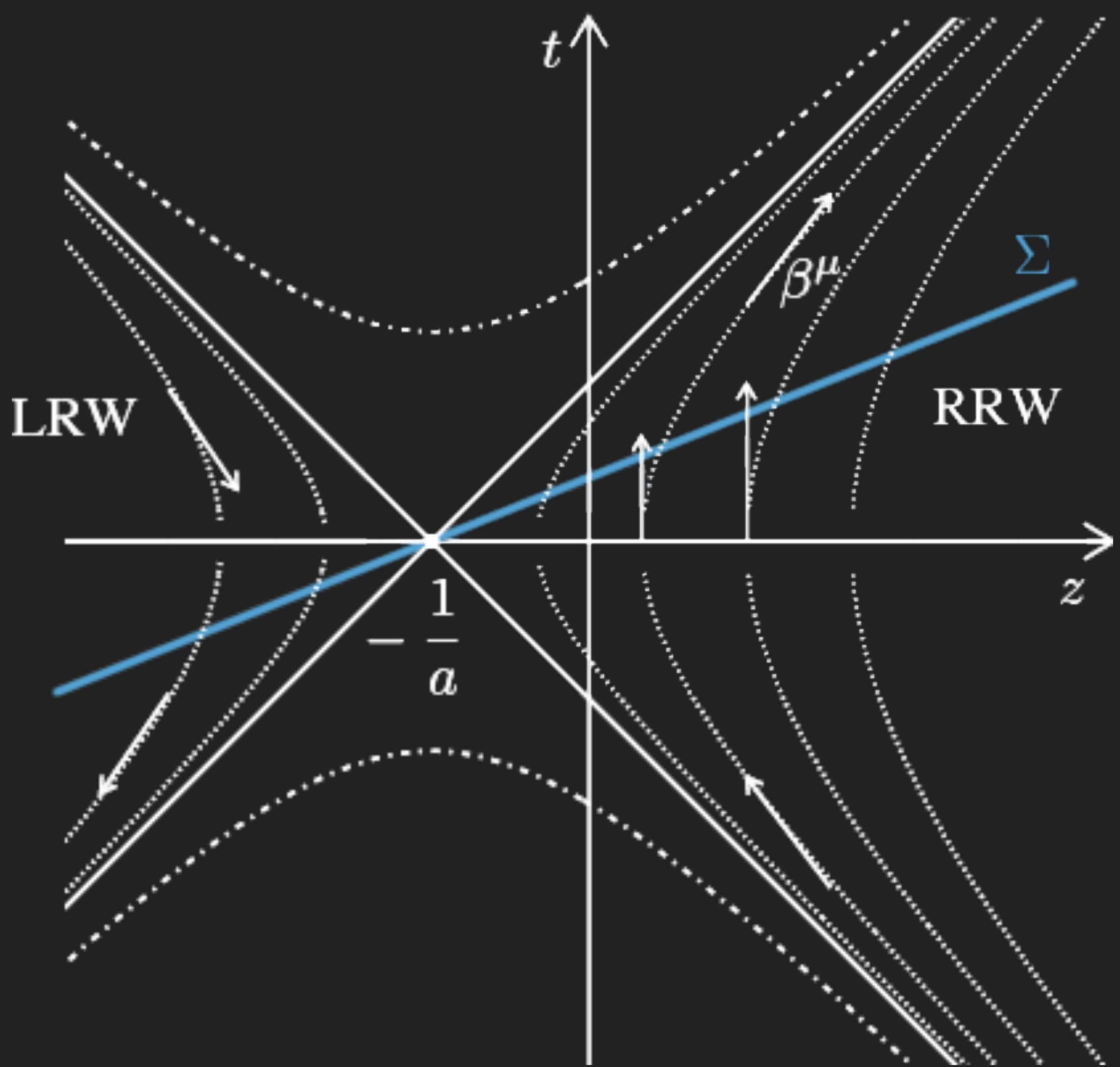
- *Recall*: $\hat{\rho} = \frac{1}{Z} \exp\left[-\int_\Sigma \mathrm{d}\Sigma\, n_\mu \hat{T}^{\mu\nu} \beta_\nu\right]$ is Σ-independent.
- *Note*: $\beta^\mu = 0$ at $z' = 0$.
- *Consequence*: For any Σ through $z' = 0$

$$\hat{\rho} = \hat{\rho}_{\mathrm{R}} \otimes \hat{\rho}_{\mathrm{L}}, \qquad [\hat{\rho}_{\mathrm{R}}, \hat{\rho}_{\mathrm{L}}] = 0$$

with $\hat{\rho}_{\mathrm{R/L}}$ involving DOFs only in RRW/LRW:

$$\hat{\rho}_{\mathrm{R}} = \frac{1}{Z_{\mathrm{R}}} \exp\left[-\int_{z'>0} \mathrm{d}\Sigma\, n_\mu \hat{T}^{\mu\nu} \beta_\nu\right],$$

$$\hat{\rho}_{\mathrm{L}} = \frac{1}{Z_{\mathrm{L}}} \exp\left[-\int_{z'<0} \mathrm{d}\Sigma\, n_\mu \hat{T}^{\mu\nu} \beta_\nu\right].$$



- *Consequence*: If $x \in$ RRW, then

$$\langle \hat{\mathcal{O}}(x) \rangle = \mathrm{tr}(\hat{\rho}\hat{\mathcal{O}}(x)) = \mathrm{tr}_{\mathrm{R}}(\hat{\rho}_{\mathrm{R}}\hat{\mathcal{O}}(x)).$$

THERMAL EXPECTATION VALUES IN THE RRW AND UNRUH EFFECT

- Free scalar field theory in the RRW: Klein-Gordon equation

$$(\square + m^2)\hat{\phi} = 0.$$

- Introduce (hyperbolic) *Rindler coordinates*:

$$\tau = \frac{1}{2a}\log\left(\frac{z'+t}{z'-t}\right), \qquad \xi = \frac{1}{2a}\log\left[a^2\left(z'^2 - t^2\right)\right], \qquad \mathbf{x}_\mathrm{T} = (x, y).$$

- Solution:

$$\hat{\phi} = \int_0^{+\infty} \mathrm{d}\omega \int_{\mathbb{R}^2} \mathrm{d}^2\mathrm{k}_\mathrm{T} \left(u_{\omega,\mathbf{k}_\mathrm{T}} \hat{a}^\mathrm{R}_{\omega,\mathbf{k}_\mathrm{T}} + u^*_{\omega,\mathbf{k}_\mathrm{T}} \hat{a}^{\mathrm{R}\dagger}_{\omega,\mathbf{k}_\mathrm{T}} \right)$$

with modes

$$u_{\omega,\mathbf{k}_\mathrm{T}} = \sqrt{\frac{1}{4\pi^4 a}\sinh\left(\frac{\pi\omega}{a}\right)}\,\mathrm{K}_{i\frac{\omega}{a}}\left(\frac{m_\mathrm{T}\mathrm{e}^{a\xi}}{a}\right)\mathrm{e}^{-i(\omega\tau - \mathbf{k}_\mathrm{T}\cdot\mathbf{x}_\mathrm{T})}$$

orthonormalized with respect to Klein-Gordon inner product, $m_\mathrm{T}^2 = \mathbf{k}_\mathrm{T}^2 + m^2$.

- $\hat{a}^{\mathrm{R}\dagger}_{\omega,\mathbf{k}_\mathrm{T}}, \hat{a}^\mathrm{R}_{\omega,\mathbf{k}_\mathrm{T}}$ are creation and annihilation operators.

[Crispino et al. 2008]

THERMAL EXPECTATION VALUES IN THE RRW AND UNRUH EFFECT

- TEVs of physical interest can be calculated once the following are known

$$\langle \hat{a}^{\mathrm{R}\dagger}_{\omega,\mathbf{k}_{\mathrm{T}}} \hat{a}^{\mathrm{R}}_{\omega',\mathbf{k}'_{\mathrm{T}}} \rangle = \frac{1}{\mathrm{e}^{\omega/T_0}-1}\delta(\omega-\omega')\,\delta^2(\mathbf{k}_{\mathrm{T}}-\mathbf{k}'_{\mathrm{T}})$$

$$\langle \hat{a}^{\mathrm{R}}_{\omega',\mathbf{k}'_{\mathrm{T}}} \hat{a}^{\mathrm{R}\dagger}_{\omega,\mathbf{k}_{\mathrm{T}}} \rangle = \left(\frac{1}{\mathrm{e}^{\omega/T_0}-1}+1\right)\delta(\omega-\omega')\,\delta^2(\mathbf{k}_{\mathrm{T}}-\mathbf{k}'_{\mathrm{T}})$$

$$\langle \hat{a}^{\mathrm{R}}_{\omega,\mathbf{k}_{\mathrm{T}}} \hat{a}^{\mathrm{R}}_{\omega',\mathbf{k}'_{\mathrm{T}}} \rangle = \langle \hat{a}^{\mathrm{R}\dagger}_{\omega,\mathbf{k}_{\mathrm{T}}} \hat{a}^{\mathrm{R}\dagger}_{\omega',\mathbf{k}'_{\mathrm{T}}} \rangle = 0.$$

- The $+1$ gives rise to divergences $\Rightarrow$ needs renormalization.
- TEVs in Minkowski vacuum $|0_{\mathrm{M}}\rangle$: same TEVs as above with $T_0 = a/2\pi$. In particular

$$\langle 0_{\mathrm{M}} | \hat{a}^{\mathrm{R}\dagger}_{\omega,\mathbf{k}_{\mathrm{T}}} \hat{a}^{\mathrm{R}}_{\omega',\mathbf{k}'_{\mathrm{T}}} | 0_{\mathrm{M}} \rangle = \frac{1}{\mathrm{e}^{2\pi\omega/a}-1}\delta(\omega-\omega')\,\delta^2(\mathbf{k}_{\mathrm{T}}-\mathbf{k}'_{\mathrm{T}}).$$

This is the content of the *Unruh effect*, and $a/2\pi$ is the *Unruh temperature*.

THERMAL EXPECTATION VALUES IN THE RRW AND UNRUH EFFECT

- TEVs of operators quadratic in the field, once the $|0_{\rm M}\rangle$ contribution is subtracted, vanish at $T_0 = a/2\pi$ and become negative for $T_0 < a/2\pi$. For instance the energy density

$$\rho_{\rm Minkowski} = \left(\langle \hat{T}^{\mu\nu} \rangle - \langle 0_{\rm M} | \hat{T}^{\mu\nu} | 0_{\rm M} \rangle \right) u_\mu u_\nu$$

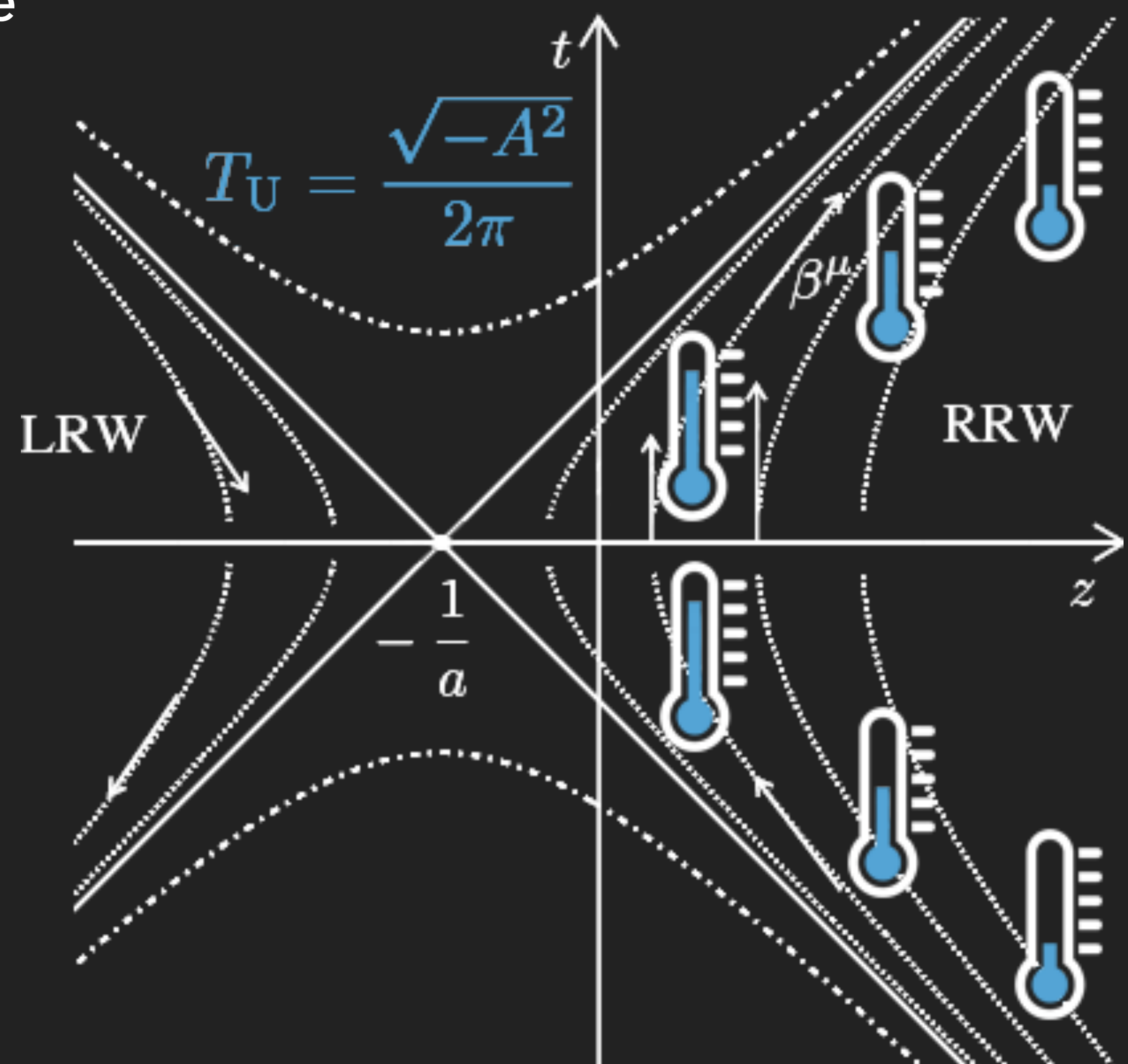
turns out to be

$$\rho_{\rm Minkowski} = \left(\frac{\pi^2}{30} - \frac{\alpha^2}{12} \right) T^4 \left[1 - \frac{\alpha^4}{(2\pi)^4} \right]$$

where at $T_0 = a/2\pi$ we have $\alpha^2 = -(2\pi)^2$.

- At $T_0 = a/2\pi$ the proper temperature is

$$\frac{A^2}{T^2} = -\frac{a^2}{T_0^2} \qquad \Rightarrow \qquad T = \frac{\sqrt{-A^2}}{2\pi} = T_{\rm U}.$$



In the Minkowski vacuum

ENTROPY CURRENT AND EXTENSIVITY

The *entropy current* is s^μ that makes the entropy S *extensive*:

$$S = -\operatorname{tr}(\hat\rho \log \hat\rho) = \int_\Sigma \mathrm{d}\Sigma\, n_\mu s^\mu.$$

At LTE

$$-\operatorname{tr}(\hat\rho_{\mathrm{LE}} \log \hat\rho_{\mathrm{LE}}) = \log Z_{\mathrm{LE}} + \int_{\Sigma(\tau)} \mathrm{d}\Sigma\, n_\mu \left(\langle \hat T^{\mu\nu} \rangle_{\mathrm{LE}} \beta_\nu - \zeta \langle \hat j^\mu \rangle_{\mathrm{LE}} \right).$$

where $\langle \hat{\mathcal{O}} \rangle_{\mathrm{LE}} = \operatorname{tr}(\hat\rho_{\mathrm{LE}} \hat{\mathcal{O}})$. If there is ϕ^μ such that (extensivity of $\log Z_{\mathrm{LE}}$)

$$\log Z_{\mathrm{LE}} = \int_{\Sigma(\tau)} \mathrm{d}\Sigma\, n_\mu \phi^\mu,$$

then

$$s^\mu = \phi^\mu + \langle \hat T^{\mu\nu} \rangle_{\mathrm{LE}} \beta_\nu - \zeta \langle \hat j^\mu \rangle_{\mathrm{LE}}.$$

ϕ^μ is the *thermodynamic potential current*.

ENTROPY CURRENT AND EXTENSIVITY

- Define $\hat{\Upsilon}$ as

$$\hat{\rho}_{\rm LE} = \frac{{\rm e}^{-\hat{\Upsilon}}}{Z_{\rm LE}}, \qquad \hat{\Upsilon} = \int_{\Sigma(\tau)} {\rm d}\Sigma\, n_\mu \left(\hat{T}^{\mu\nu}\beta_\nu - \zeta \hat{j}^\mu \right).$$

- Introduce λ as

$$\hat{\rho}_{\rm LE}(\lambda) = \frac{{\rm e}^{-\lambda\hat{\Upsilon}}}{Z_{\rm LE}(\lambda)}, \qquad \hat{\rho}_{\rm LE}(\lambda=1) = \hat{\rho}_{\rm LE}.$$

- Derive $\log Z_{\rm LE}(\lambda)$ with respect to λ

$$\frac{\partial \log Z_{\rm LE}(\lambda)}{\partial\lambda} = -\int_{\Sigma(\tau)} {\rm d}\Sigma\, n_\mu \left(\langle \hat{T}^{\mu\nu}\rangle_{\rm LE}(\lambda)\,\beta_\nu - \zeta \langle \hat{j}^\mu\rangle_{\rm LE}(\lambda) \right)$$

with $\langle \hat{\mathcal{O}}\rangle_{\rm LE}(\lambda) = {\rm tr}(\hat{\rho}_{\rm LE}(\lambda)\,\hat{\mathcal{O}})$.

ENTROPY CURRENT AND EXTENSIVITY

- Integrate in λ from some λ_0 to $\lambda = 1$ recalling $\log Z_{\rm LE}(\lambda = 1) = \log Z_{\rm LE}$ and exchange λ-integration with Σ-integration

$$\log Z_{\rm LE} - \log Z_{\rm LE}(\lambda_0) = -\int_{\Sigma(\tau)} {\rm d}\Sigma\, n_\mu \int_{\lambda_0}^{1} {\rm d}\lambda \left(\langle \hat{T}^{\mu\nu} \rangle_{\rm LE}(\lambda)\, \beta_\nu - \zeta \langle \hat{j}^\mu \rangle_{\rm LE}(\lambda) \right).$$

- If there exists λ_0 such that $\log Z_{\rm LE}(\lambda_0) = 0$, then

$$\phi^\mu = -\int_{\lambda_0}^{1} {\rm d}\lambda \left(\langle \hat{T}^{\mu\nu} \rangle_{\rm LE}(\lambda)\, \beta_\nu - \zeta \langle \hat{j}^\mu \rangle_{\rm LE}(\lambda) \right).$$

- *Assume*: $\hat{\Upsilon}$ bounded from below with Υ_0 non-degenerate lowest eigenvalue and $|0\rangle$ corresponding eigenvector.
- Shift $\hat{\Upsilon} \mapsto \hat{\Upsilon}' = \hat{\Upsilon} - \Upsilon_0 = \hat{\Upsilon} - \langle 0 | \hat{\Upsilon} | 0 \rangle$ and see that $\hat{\rho}_{\rm LE}$ (hence S) is invariant.
- *Consequence*: $Z'_{\rm LE} = Z_{\rm LE}[\hat{\Upsilon}']$ is such that $\log Z'_{\rm LE}(\lambda_0 = +\infty) = 0$.

ENTROPY CURRENT AND EXTENSIVITY

- *Conclusion*: If $\hat{\Upsilon}$ is bounded from below and the lowest eigenvalue Υ_0 is non-degenerate, then $\log Z_{\text{LE}}$ is extensive and ϕ^μ is given by

$$\phi^\mu = \int_1^{+\infty} \mathrm{d}\lambda \left[\left(\langle \hat{T}^{\mu\nu} \rangle_{\text{LE}} - \langle 0 | \hat{T}^{\mu\nu} | 0 \rangle \right)(\lambda)\, \beta_\nu - \zeta \left(\langle \hat{j}^\mu \rangle_{\text{LE}} - \langle 0 | \hat{j}^\mu | 0 \rangle \right)(\lambda) \right].$$

  In this case, s^μ exists and reads

$$s^\mu = \phi^\mu + \left(\langle \hat{T}^{\mu\nu} \rangle_{\text{LE}} - \langle 0 | \hat{T}^{\mu\nu} | 0 \rangle \right) \beta_\nu - \zeta \left(\langle \hat{j}^\mu \rangle_{\text{LE}} - \langle 0 | \hat{j}^\mu | 0 \rangle \right).$$

- *Result*: We showed that $\log Z_{\text{LE}}$ is extensive under general hypotheses and provided a method to calculate the entropy current at LTE.

- *Note*: $|0\rangle$ is just the lowest eigenvector of $\hat{\Upsilon}$, it does not necessarily correspond to the vacuum state of the theory.

ENTROPY CURRENT AT GTE WITH ACCELERATION IN THE RRW

At GTE with acceleration, $\langle \hat{T}^{\mu\nu} \rangle$ can depend on $b^\mu, \varpi^{\mu\nu}, x^\mu, g^{\mu\nu}$, however:

- dependence on x^μ only through $\beta_\mu = b_\mu + \varpi_{\mu\nu} x^\nu$
- $\varpi_{\mu\nu} = \alpha_\mu u_\nu - \alpha_\nu u_\mu$
- $\partial\beta \sim \varpi$ constant $\Rightarrow \partial^2\beta = 0$

hence

$$\langle \hat{T}^{\mu\nu} \rangle = F_1\, \beta^\mu \beta^\nu + F_2\, g^{\mu\nu} + F_3\, \alpha^\mu \alpha^\nu + F_4 \left(\beta^\mu \alpha^\nu + \beta^\nu \alpha^\mu \right), \qquad F_i = F_i(\beta^2, \alpha^2)$$

Note: $\beta^\mu \alpha^\nu + \beta^\nu \alpha^\mu$ breaks time-reversal symmetry, therefore $F_4 = 0$ for free scalar field

$$\langle \hat{T}^{\mu\nu} \rangle = \underbrace{F_1\, \beta^\mu \beta^\nu + F_2\, g^{\mu\nu}}_{\text{ideal terms}} + F_3\, \alpha^\mu \alpha^\nu .$$

Recall: $\alpha^\mu = \dfrac{\hbar}{c k_{\rm B}} \dfrac{A^\mu}{T}$, therefore $F_3\, \alpha^\mu \alpha^\nu$ is a quantum correction.

ENTROPY CURRENT AT GTE WITH ACCELERATION IN THE RRW

Since in the RRW

$$\hat{\rho}_{\rm R} = \frac{1}{Z_{\rm R}} \exp\left[-\frac{1}{T_0}\int {\rm d}\omega\, {\rm d}^2{\rm k}_{\rm T}\, \omega\, \hat{a}^{{\rm R}\dagger}_{\omega,\mathbf{k}_{\rm T}} \hat{a}^{\rm R}_{\omega,\mathbf{k}_{\rm T}}\right],$$

$|0\rangle$ such that $\Upsilon_0 = \langle 0 |\, \hat{\Upsilon}\, | 0\rangle$ is minimum is the *Rindler vacuum* $|0\rangle = |0_{\rm R}\rangle$, different from Minkowski vacuum $|0_{\rm M}\rangle$. Moreover, $\Upsilon_0 = 0$.

Recall: $\phi^\mu = \displaystyle\int_1^{+\infty} {\rm d}\lambda \left(\langle \hat{T}^{\mu\nu}\rangle - \langle 0 |\, \hat{T}^{\mu\nu}\, | 0\rangle\right)(\lambda)\, \beta_\nu$. Thus

$$\left(\langle \hat{T}^{\mu\nu}\rangle - \langle 0 |\, \hat{T}^{\mu\nu}\, | 0\rangle\right)\beta_\nu = \left(F_1\beta^2 + F_2\right)\beta^\mu = \rho_{\rm Rindler}\, \beta^\mu$$

with $\rho_{\rm Rindler}$ the *energy density*. Hence

$$\phi^\mu = \int_1^{+\infty} {\rm d}\lambda\, \rho_{\rm Rindler}(\lambda)\, \beta^\mu.$$

ENTROPY CURRENT AT GTE WITH ACCELERATION IN THE RRW

For free real massless ($m = 0$) scalar field:

$$\rho_{\mathrm{Rindler}} = \left(\langle \hat{T}^{\mu\nu} \rangle - \langle 0 \,|\, \hat{T}^{\mu\nu} \,|\, 0 \rangle \right) u_\mu u_\nu = \frac{\pi^2}{30\beta^4} - \frac{\alpha^2}{12\beta^4}.$$

Result:

$$\phi^\mu = \left(\frac{\pi^2}{90\beta^4} - \frac{\alpha^2}{12\beta^4} \right) \beta^\mu, \qquad s^\mu = \left(\frac{2\pi^2}{45\beta^4} - \frac{\alpha^2}{6\beta^4} \right) \beta^\mu.$$

- *Note*: $\partial_\mu s^\mu = 0$, i.e. vanishing entropy production rate, as expected at GTE.
- The terms proportional to $\alpha^2 = \left(\frac{\hbar}{c k_{\mathrm{B}}} \right)^2 \frac{A^2}{T^2}$ are quantum corrections.

ENTANGLEMENT ENTROPY AND UNRUH EFFECT

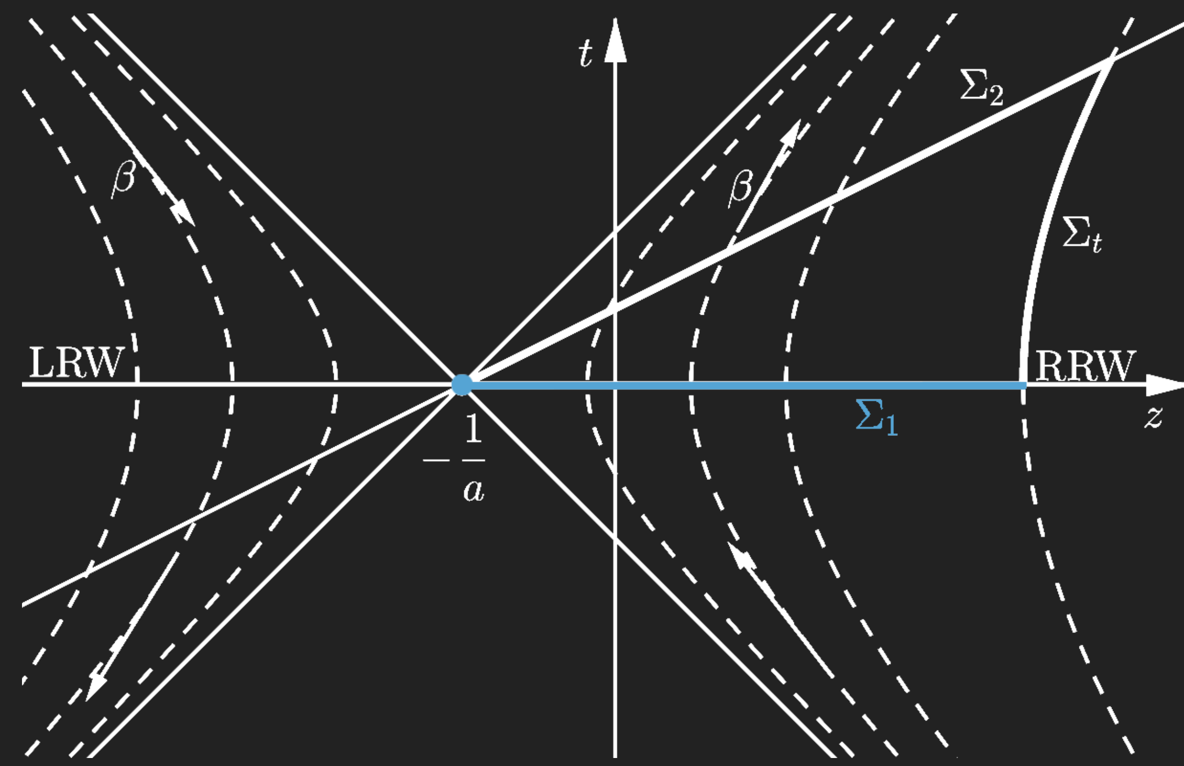
At GTE, S is independent of the choice of Σ.

Integrating s^μ in the RRW on $\Sigma_1 = \{t = 0, z' \geq 0\}$:

$$S_{\rm R} = \int_{\mathbb{R}^2} {\rm d}x\,{\rm d}y \left(\frac{2\pi^2}{45} - \frac{\alpha^2}{12}\right)\frac{T_0^3}{a^3}\lim_{z'\to 0}\frac{1}{2z'^2}.$$

- *Area law,*
- Divergence as $z' \to 0$.

[Bombelli et al. 1986]



- $\nabla_\mu s^\mu = 0$ at GTE $\Rightarrow$ there is a *potential* $\varsigma^{\mu\nu} = -\varsigma^{\nu\mu}$ such that $s^\mu = \nabla_\nu \varsigma^{\mu\nu}$, therefore $S_{\rm R}$ is

$$S_{\rm R} = -\frac{1}{4}\int_{\partial\Sigma} {\rm d}S^{\rho\sigma}\sqrt{|g|}\,\epsilon_{\mu\nu\rho\sigma}\,\varsigma^{\mu\nu}.$$

i.e. a surface integral. Solution:

$$\varsigma^{\mu\nu} = \frac{s}{2\alpha^2}(\beta^\mu\alpha^\nu - \beta^\nu\alpha^\mu)$$

with $s = s^\mu u_\mu$ the *entropy density*.

[Wald 1993]

ENTANGLEMENT ENTROPY AND UNRUH EFFECT

- *Recall*: At GTE with acceleration, $\hat{\rho} = \hat{\rho}_{\rm R} \otimes \hat{\rho}_{\rm L}$ with $\hat{\rho}_{\rm R} = {\rm tr}_{\rm L}(\hat{\rho})$ and $\hat{\rho}_{\rm L} = {\rm tr}_{\rm R}(\hat{\rho})$.
- *Consequence*: $S_{\rm R} = -\,{\rm tr}_{\rm R}(\hat{\rho}_{\rm R} \log \hat{\rho}_{\rm R})$ is the *entanglement entropy* of the RRW with the LRW.

- *Recall*: At GTE with acceleration, $T_{\rm U}$ is an absolute lower bound for T.
- *Consequence*: Non-vanishing entropy current in the Minkowski vacuum

$$s^\mu(T_{\rm U}) = \frac{32\pi^2}{45} T_{\rm U}^3 u^\mu.$$

- *Note*: s^μ depends on the choice of $\hat{T}^{\mu\nu}$. Usually there are two choices

$$\hat{T}^{\mu\nu}_{\rm can} = \frac{1}{2}\left(\partial^\mu\hat{\phi}\partial^\nu\hat{\phi} - \partial^\nu\hat{\phi}\partial^\mu\hat{\phi}\right) - \mathscr{L} g^{\mu\nu}, \qquad \hat{T}^{\mu\nu}_{\rm imp} = \hat{T}^{\mu\nu}_{\rm can} - \frac{1}{6}\left(\partial^\mu\partial^\nu - g^{\mu\nu}\Box\right)\hat{\phi}^2$$

with $\mathscr{L} = \frac{1}{2} g^{\mu\nu}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi} - \frac{1}{2}m^2\hat{\phi}^2$, hence two different entropy currents (for $m=0$)

$$s^\mu_{\rm can} = \left(\frac{2\pi^2}{45\beta^4} - \frac{\alpha^2}{6\beta^4}\right)\beta^\mu, \qquad s^\mu_{\rm imp} = \frac{2\pi^2}{45\beta^4}\beta^\mu.$$

SUMMARY

- We studied thermal QFT at GTE with acceleration.
- Accelerated observers in Minkowski vacuum see thermal radiation (Unruh effect).
- The Unruh temperature is an absolute lower bound for the proper temperature.
- We put forward a method to derive the entropy current at LTE.
- We calculated the entropy current at GTE with acceleration in the RRW.
- We found a relation with the Unruh effect and the entanglement entropy.

THANK YOU FOR YOUR ATTENTION