Prescaling in a far-from-equilibrium Bose gas

Cold Quantum Coffee

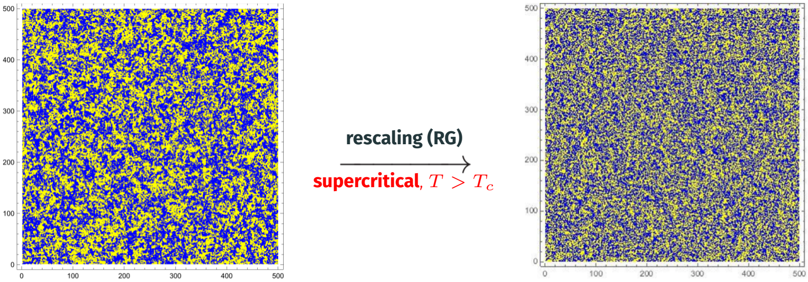
Aleksandr Mikheev

26 May, 2020

Based on C.-M. Schmied, AM, T. Gasenzer, *PRL* ***122***, *170404 (2019)*

Self-similarity

- Example from statistical field theory: **2D Ising model**



Figures adapted from D. Tong: Lectures on Statistical Field Theory

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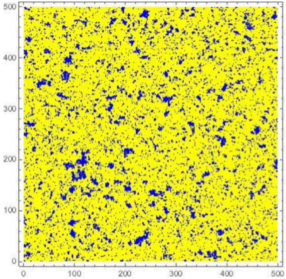
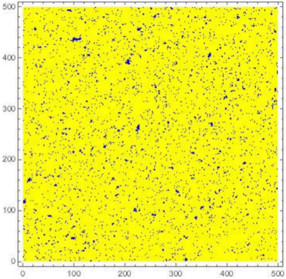


rescaling (RG)

$\longrightarrow$

subcritical, $T < T_c$



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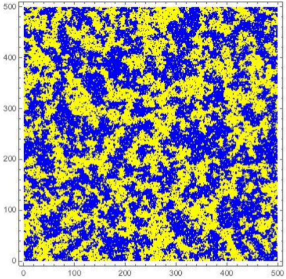
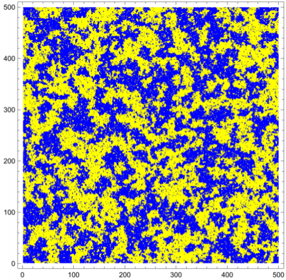


rescaling (RG)
$\longrightarrow$
critical, $T = T_c$



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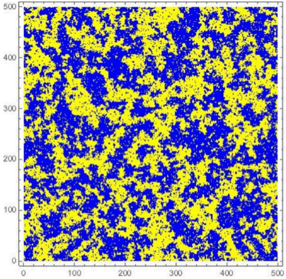

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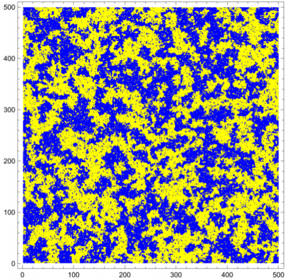

- Self-similarity/scale-invariance! Without going into details, for **correlators**, that roughly means

$$G^{(n)}(s\mathbf{p}_i) = s^{-\kappa_n} G^{(n)}(\mathbf{p}_i), \quad s \in \mathbb{R}_+$$

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Self-similarity

- Promote to **nonequilibrium** $\implies$ **nonthermal fixed point** (for simplicity, $n = 2$):

$$G\left(s^{-1/\beta}T, s^{z}\omega, s\mathbf{p}\right) = s^{-\alpha/\beta}G(T, \omega, \mathbf{p}), \quad s \in \mathbb{R}_{+},$$

or

$$G(T, \omega, \mathbf{p}) = T^{\alpha}G_{s}\left(T^{\beta z}\omega, T^{\beta}\mathbf{p}\right),$$

with $T = (t + t')/2$ – **central time** ($\sim$ evolution time), ω – frequency, i.e., Fourier transform with respect to **relative time** $t - t'$

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- In particular, for equal-time correlator

$$G(t,\mathbf{p}) = t^{\alpha}G_{s}\left(t^{\beta}\mathbf{p}\right)$$

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Non-thermal fixed points and initial conditions

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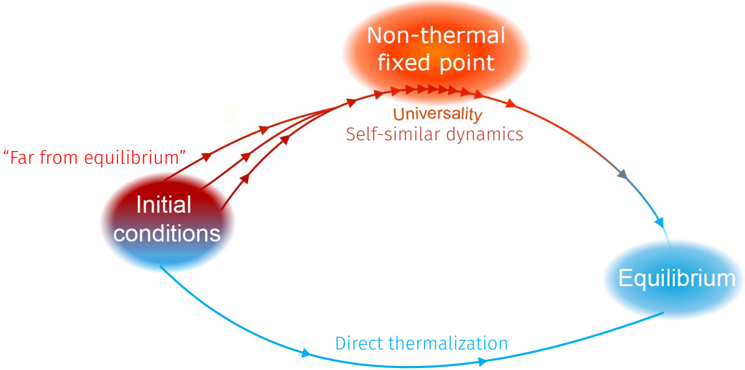


Figure adapted from M. Prüfer, P. Kunkel, H. Strobel, S. Lannig, D. Linnemann, C.-M. Schmied, J. Berges, T. Gasenzer, M. K. Oberthaler, *Nature* ***563***, *217 (2018)*

Universal self-similar dynamics

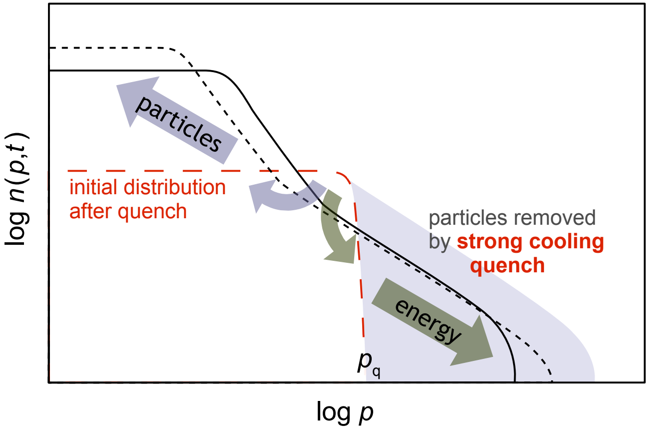
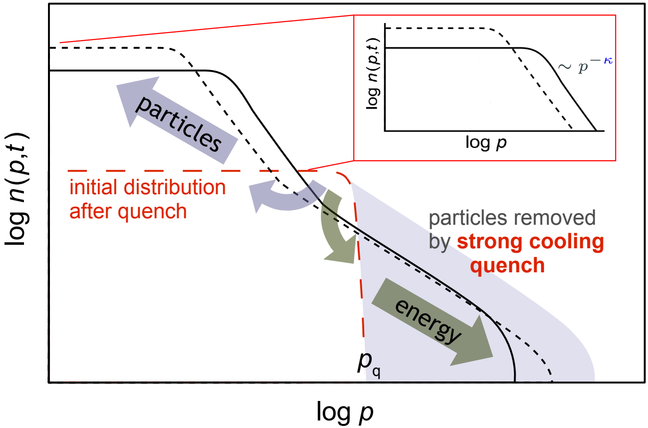


Figure taken from I. Chantesana, A. Piñeiro Orioli, T. Gasenzer, *PRA **99**, 043620 (2019)*

Universal self-similar dynamics

universal scaling dynamics: $n(t,p) = (t/t_{\text{ref}})^{\alpha} n_{\text{s}}([t/t_{\text{ref}}]^{\beta} p)$



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Universal self-similar dynamics

Numerical simulation of a $\mathrm{U}(3)$-symmetric Gross-Pitaevskii
Bose gas in $d = 3$ dimensions

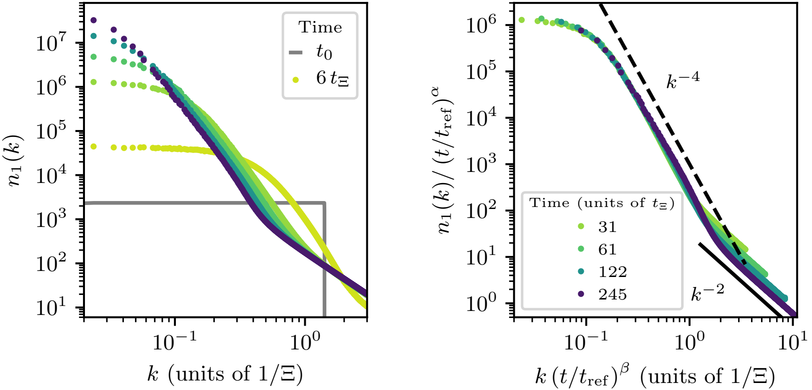
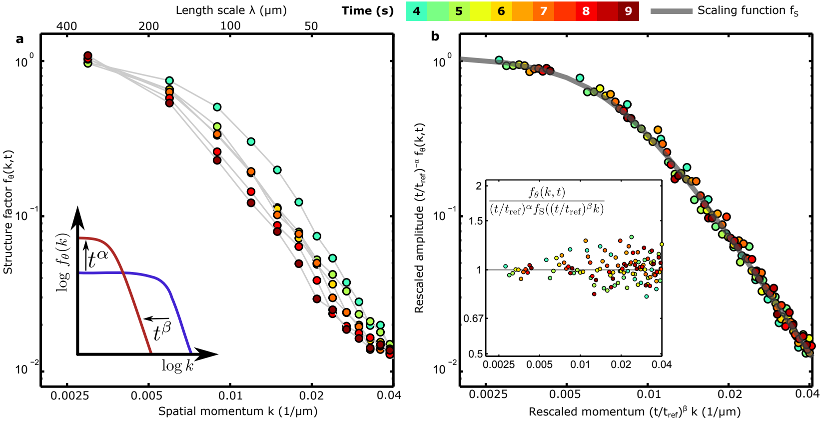


Figure taken from AM, C.-M. Schmied, T. Gasenzer, *PRA **99**, 063622 (2019)*

Universal self-similar dynamics

Experimental investigation of an $\mathrm{SO}(2) \times \mathrm{U}(1)$ spin-1 Bose gas in $d = 1$ dimension



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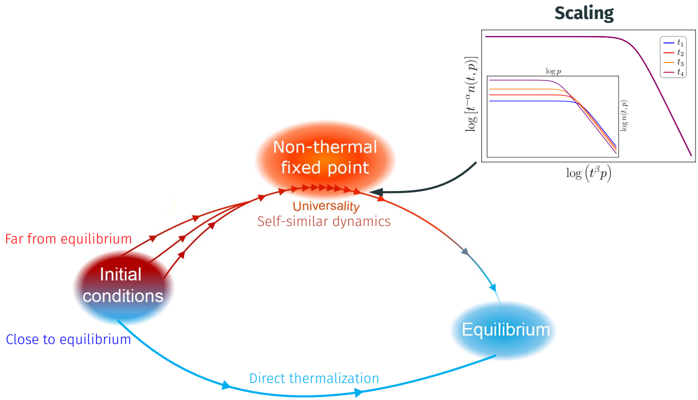
Prescaling



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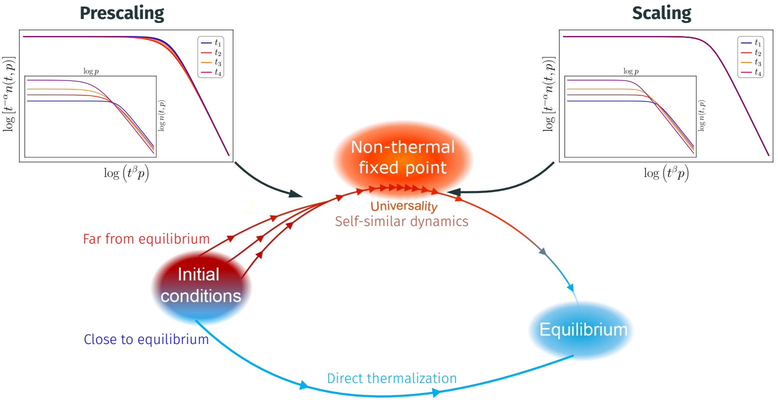

C.-M. Schmied, AM, T. Gasenzer, *PRL* ***122****, 170404 (2019)*
A. Mazeliauskas, J. Berges, *PRL* ***122****, 122301 (2019)*

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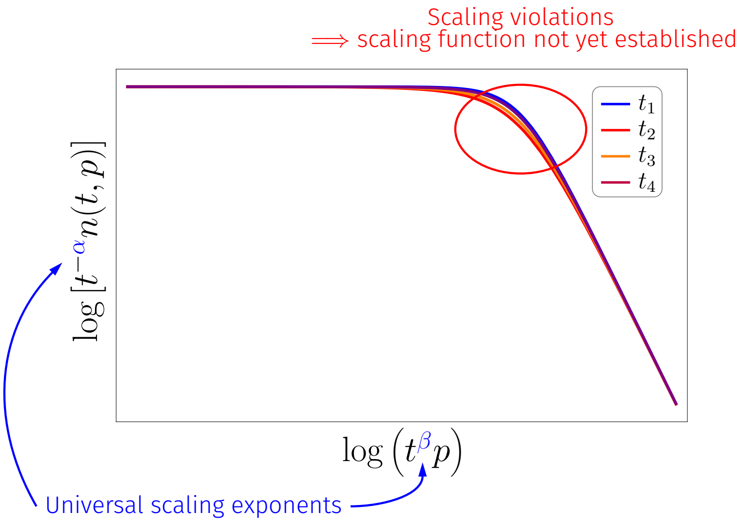

Scaling violations
⟹ scaling function not yet established
t_1
t_2
t_3
t_4
$\log\left[t^{-\alpha} n(t,p)\right]$
$\log\left(t^{\beta} p\right)$
Universal scaling exponents

- $\mathrm{U}(N)$-symmetric **Gross-Pitaevskii** model

$$\hat{H}_{\mathrm{U}(N)} = \int \mathrm{d}^d\mathbf{x} \left[\frac{1}{2m} \left(\nabla \hat{\Phi}_a^\dagger \right) \left(\nabla \hat{\Phi}_a \right) + \frac{g}{2} \hat{\Phi}_a^\dagger \hat{\Phi}_b^\dagger \hat{\Phi}_a \hat{\Phi}_b \right], \quad a, b \in \{1, \ldots, N\}$$

- Correlation functions

$$g_a^{(1)}(t, \mathbf{r}) \equiv \left\langle \hat{\Phi}_a^\dagger(t, \mathbf{x}+\mathbf{r}) \hat{\Phi}_a(t, \mathbf{x}) \right\rangle$$

$$n_a(t, \mathbf{p}) \equiv \mathcal{F}\left[g_a^{(1)} \right](t, \mathbf{p})$$

occupancy

and

$$g_{ab}^{(2)}(t, \mathbf{r}) \equiv \left\langle \hat{\Phi}_a^\dagger(t, \mathbf{x}+\mathbf{r}) \hat{\Phi}_b^\dagger(t, \mathbf{x}+\mathbf{r}) \hat{\Phi}_a(t, \mathbf{x}) \hat{\Phi}_b(t, \mathbf{x}) \right\rangle$$

$$C_{ab}(t, \mathbf{p}) \equiv \mathcal{F}\left[g_{ab}^{(2)} \right](t, \mathbf{p})$$

relative-phase correlations

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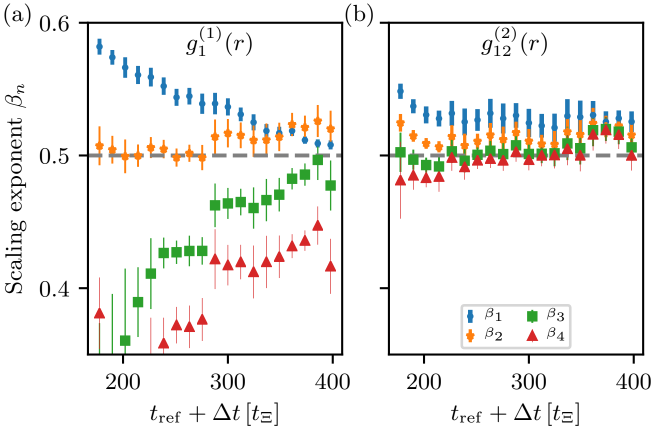
- Scaling and prescaling are **observable-dependent**!

Numerical results



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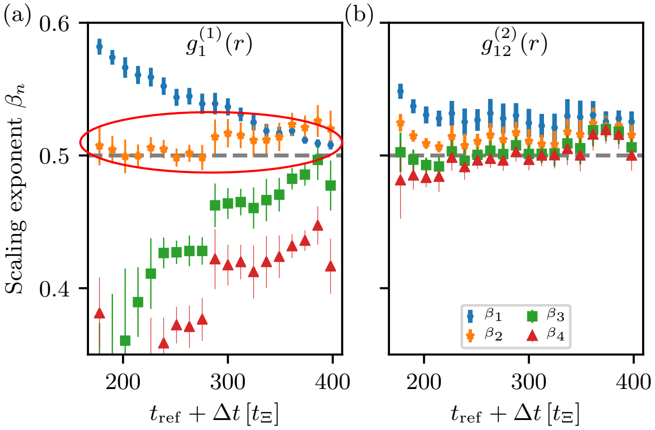

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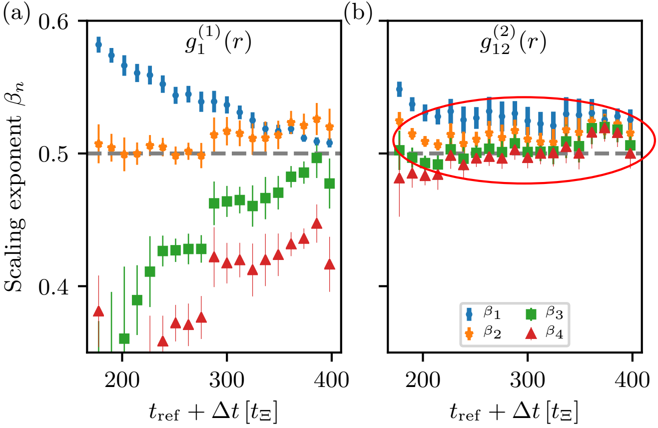

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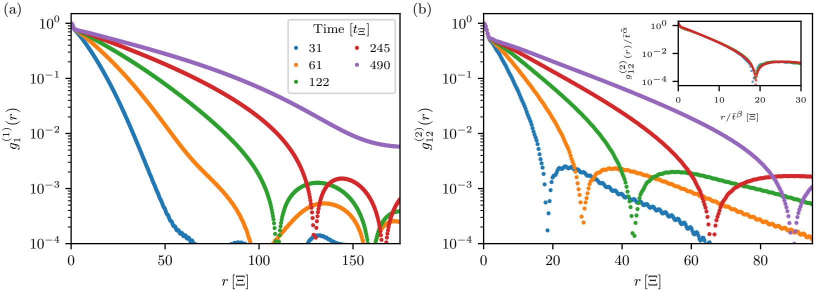
Summary and outlook

- System may reach asymptotic scaling dynamics at relatively early times at certain length/momentum scales $\Longrightarrow$ **partial fixed points**?
- Relevant degrees of freedom can be much **more sensitive to scaling** $\Longrightarrow$ **Goldstone excitations**?
- Check **symmetries** (both spacetime and internal) $\Longrightarrow$ **Ward identities**?

C.-M. Schmied, AM, T. Gasenzer, *PRL **122**, 170404 (2019)*
AM, C.-M. Schmied, T. Gasenzer, *PRA **99**, 063622 (2019)*
C.-M. Schmied, AM, T. Gasenzer, Int. J. Mod. Phys. A, **34 (29)**, 1941006 (2019)

Auxiliary slides

Correlation functions in real space



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"Flow" of scaling exponents