Gravitational waves from first-order phase transitions: some developments in ultra-supercooled transitions

Ryusuke Jinno (DESY)

Based on

1707.03111 with Masahiro Takimoto (Weizmann)

1905.00899 with Hyeonseok Seong (IBS & KAIST), Masahiro Takimoto (Weizmann), Choong Min Um (KAIST)

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Introduction

GRAVITATIONAL WAVES: PROBE TO THE EARLY UNIVERSE

- Gravitational waves

  Transverse-traceless part of the metric

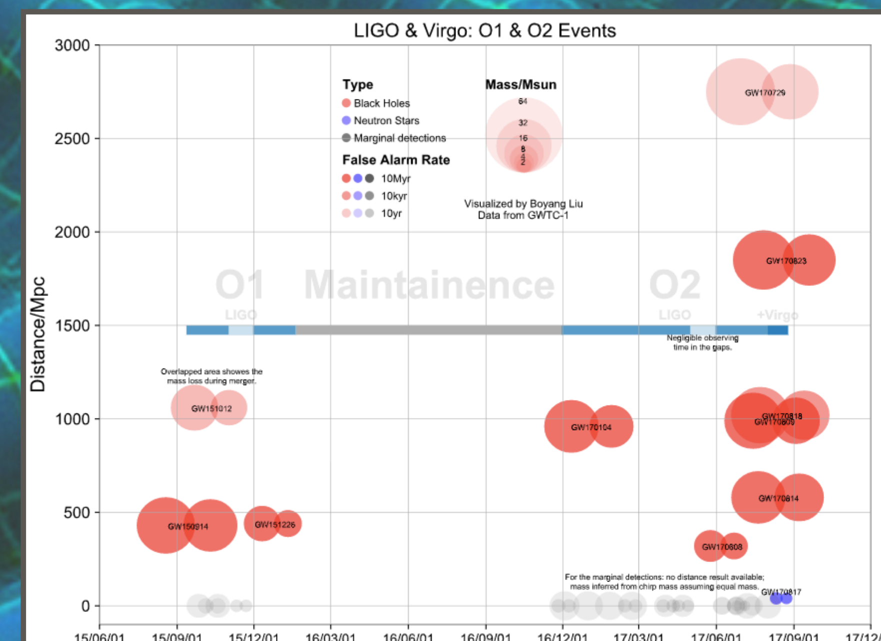
  $$ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij})dx^i dx^j$$

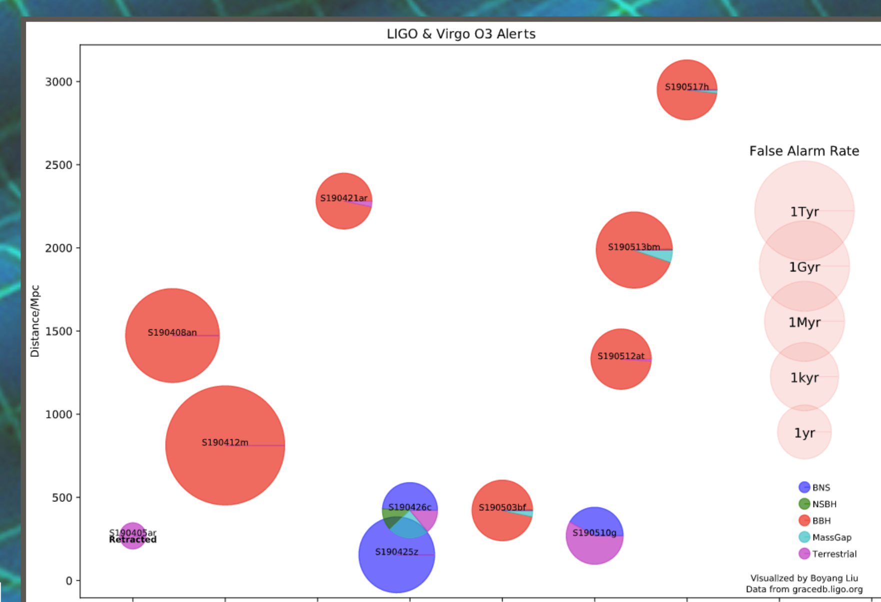
  sourced by the energy-momsntum tensor

  $$\Box h_{ij} \sim G T_{ij}$$

- GW detections by LIGO & Virgo

  have been exciting us

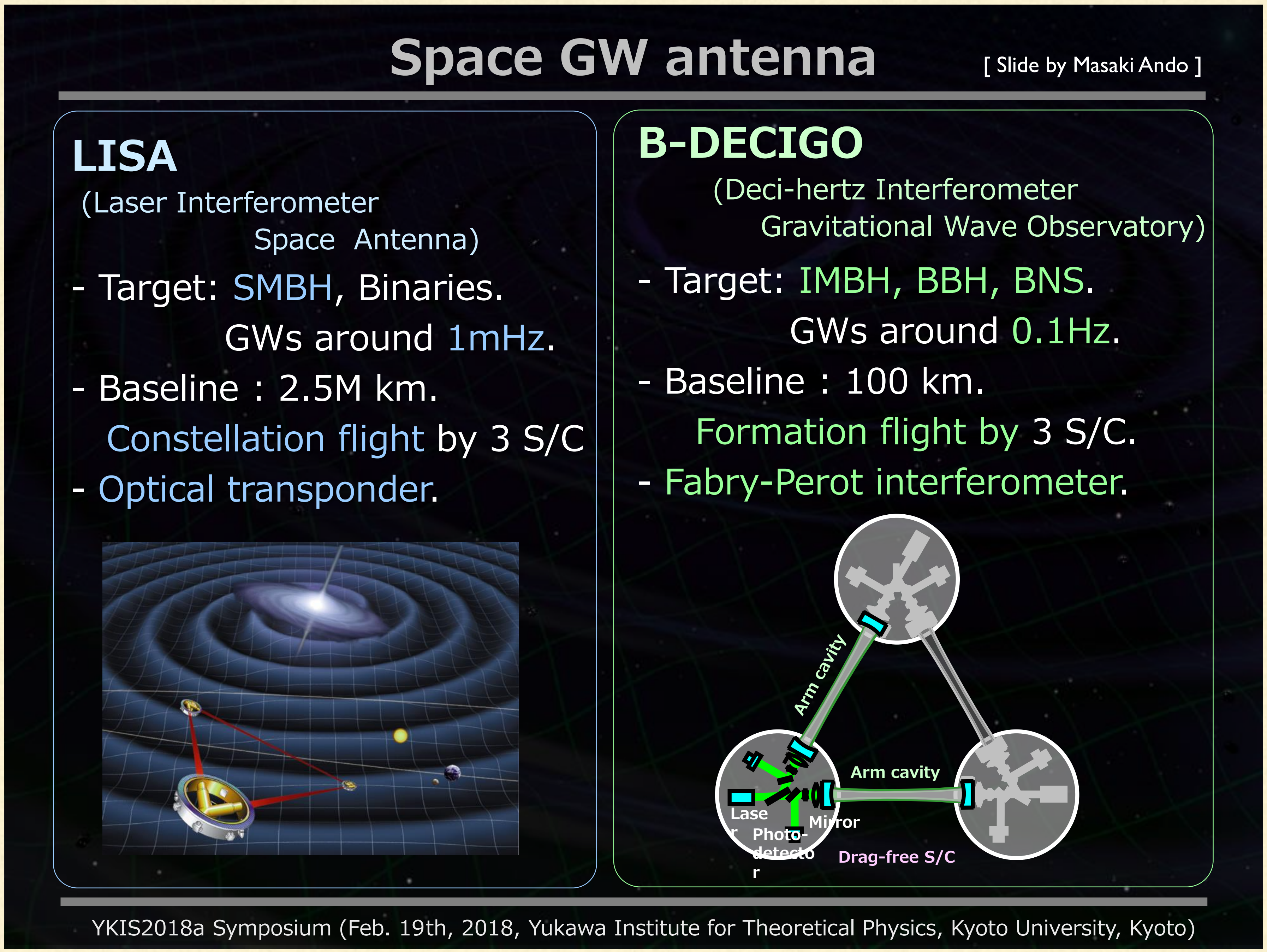




[Wikipedia "List of gravitational wave observations"]

[see also https://gracedb.ligo.org/superevents/public/O3/]

FROM ASTROPHYSICAL TO COSMOLOGICAL GWS

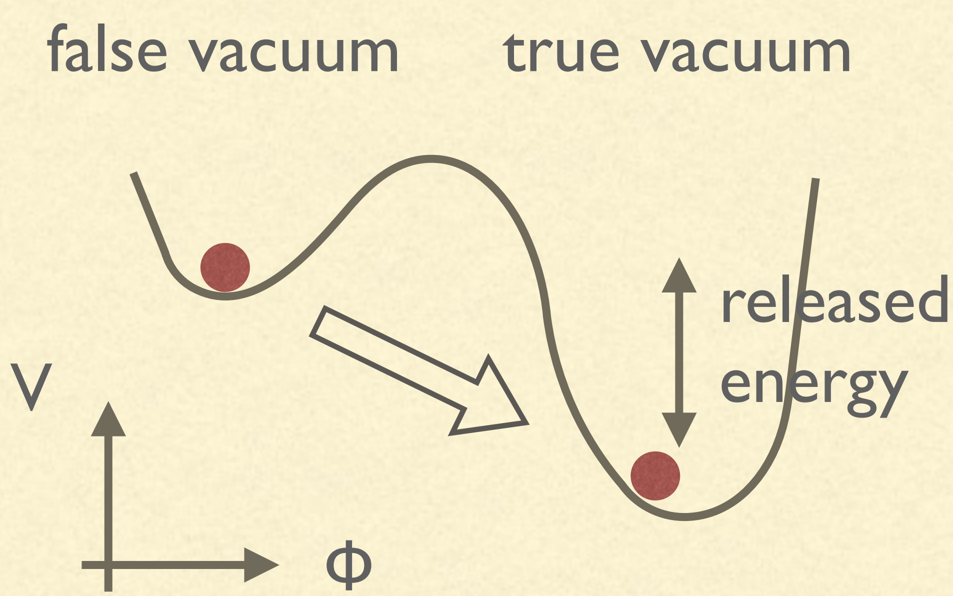


FIRST-ORDER PHASE TRANSITION & GWS

- Rough sketch of 1st-order phase transition & GW production

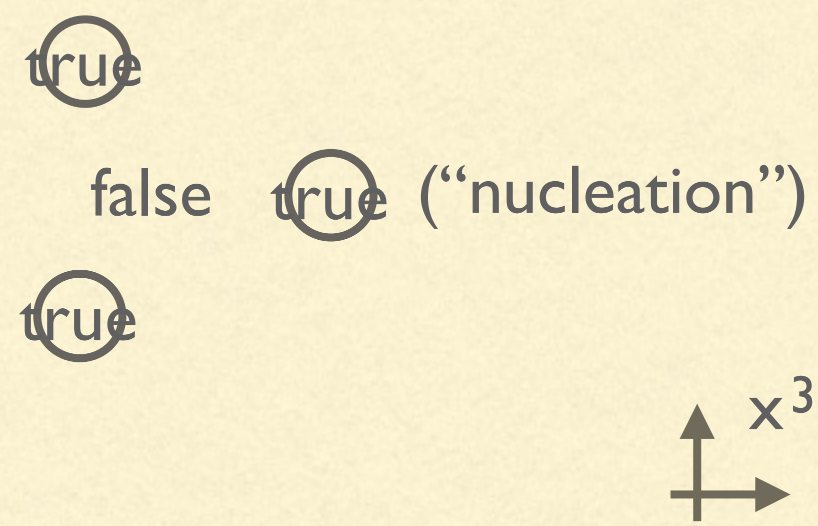
Bubbles nucleate, expand, collide and disappear, accompanying fluid dynamics

Field space



Quantum tunneling

Position space



Bubble formation & GW production

FIRST-ORDER PHASE TRANSITION & GWS

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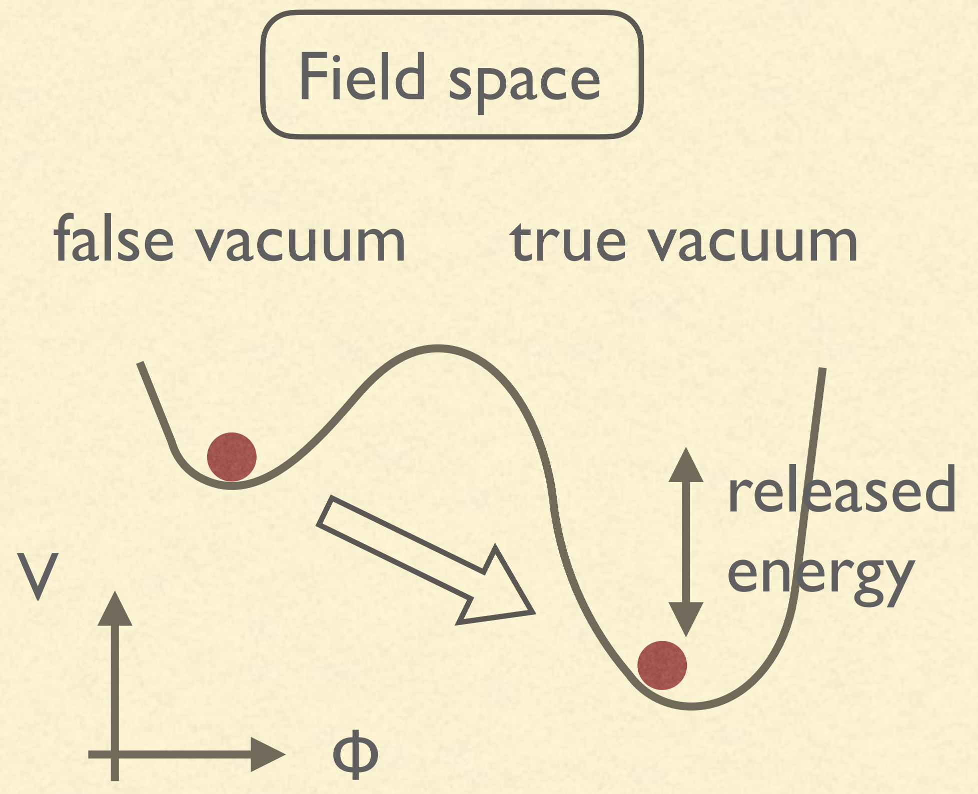
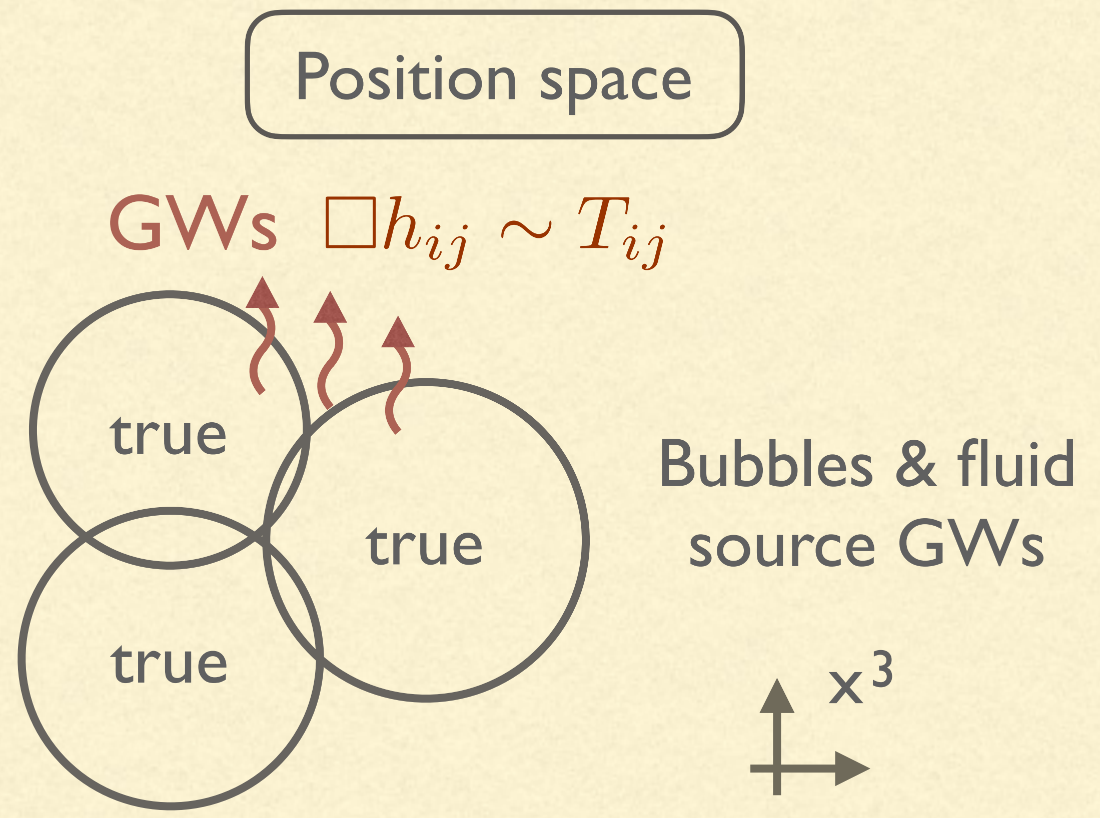


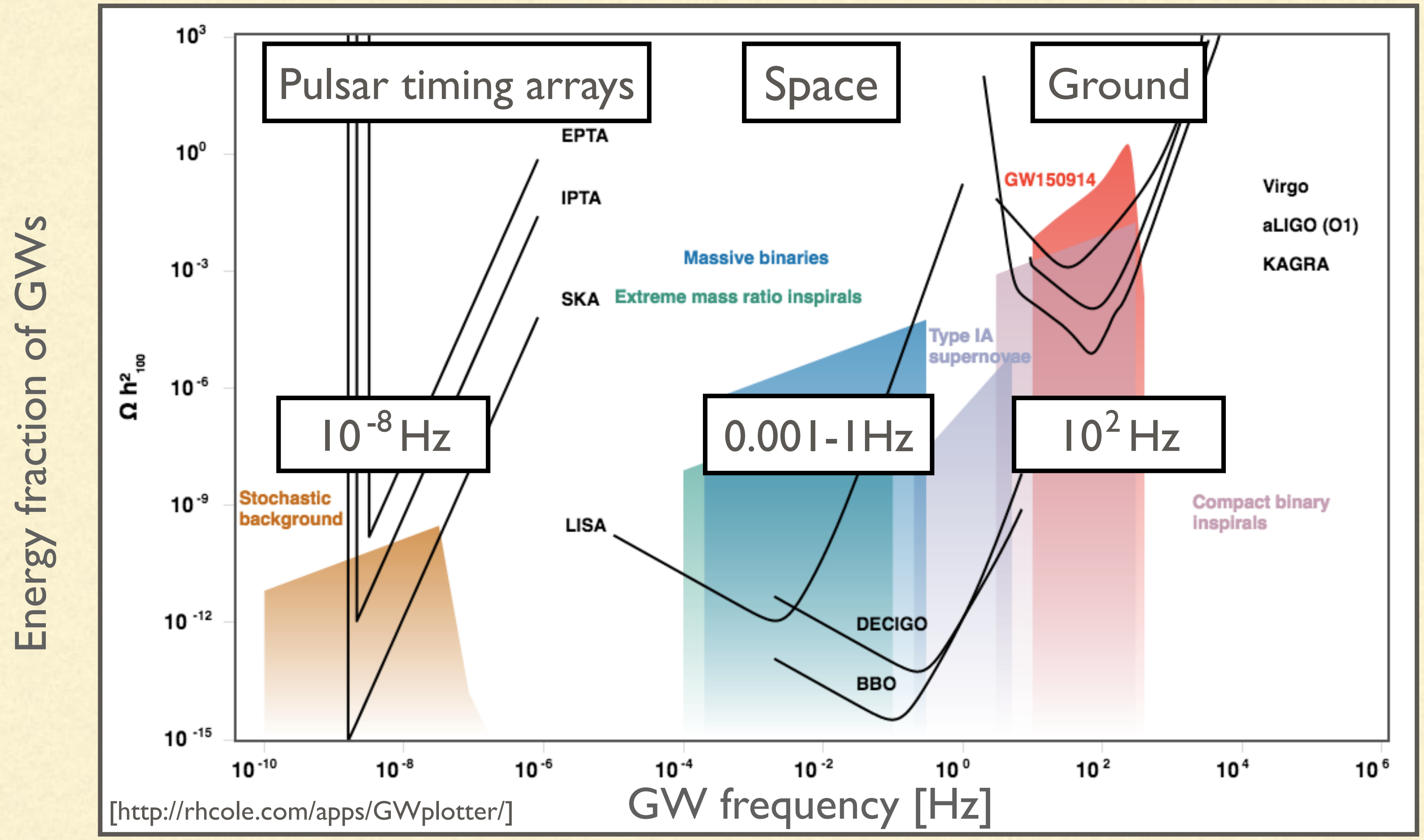
Quantum tunneling



Bubble formation & GW production

FIRST-ORDER PHASE TRANSITION & GWS

- $10^{-3} \sim 1$Hz GWs correpond to electroweak physics and beyond



Temperature of the Universe @ transition time ← MeV GeV TeV PeV

Note : $\beta/H_* \sim 10^3$

TALK PLAN

✔1. Introduction

2. Brief review of bubble dynamics and GW production

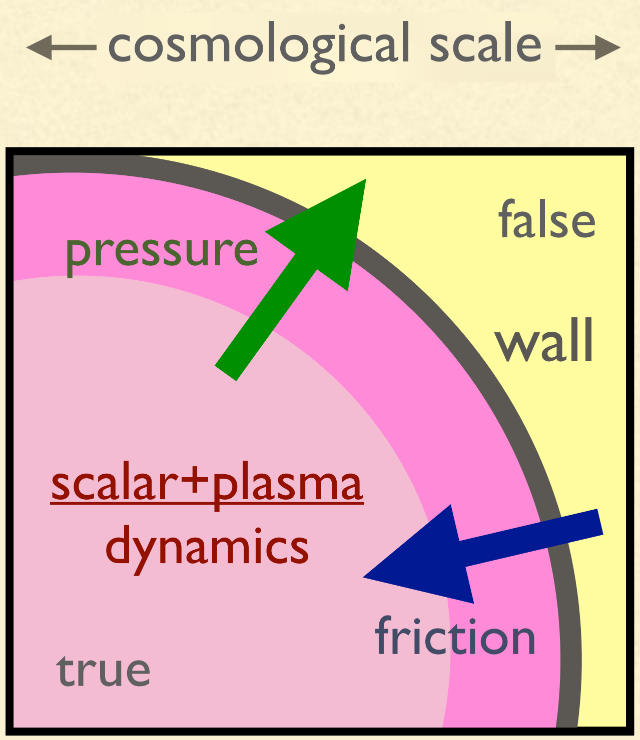
3. GW production in ultra-supercooled transitions:

   Effective description of fluid propagation & Implications to GW production

4. Summary

BUBBLE DYNAMICS BEFORE COLLISION

- "Pressure vs. friction" determines behavior of bubbles



- Two main players : **scalar field and plasma**

- Walls want to expand ("pressure")

  Parametrized by $\alpha \equiv \dfrac{\rho_{\rm vac}}{\rho_{\rm plasma}}$ [Several definitions exist... see e.g. Giese, Konstandin, van de Vis '20]

- Walls are pushed back by plasma ("friction")

  Parametrized by coupling η btwn. scalar and plasma

- Let's see how bubbles behave for different α

  (with fixed coupling η)

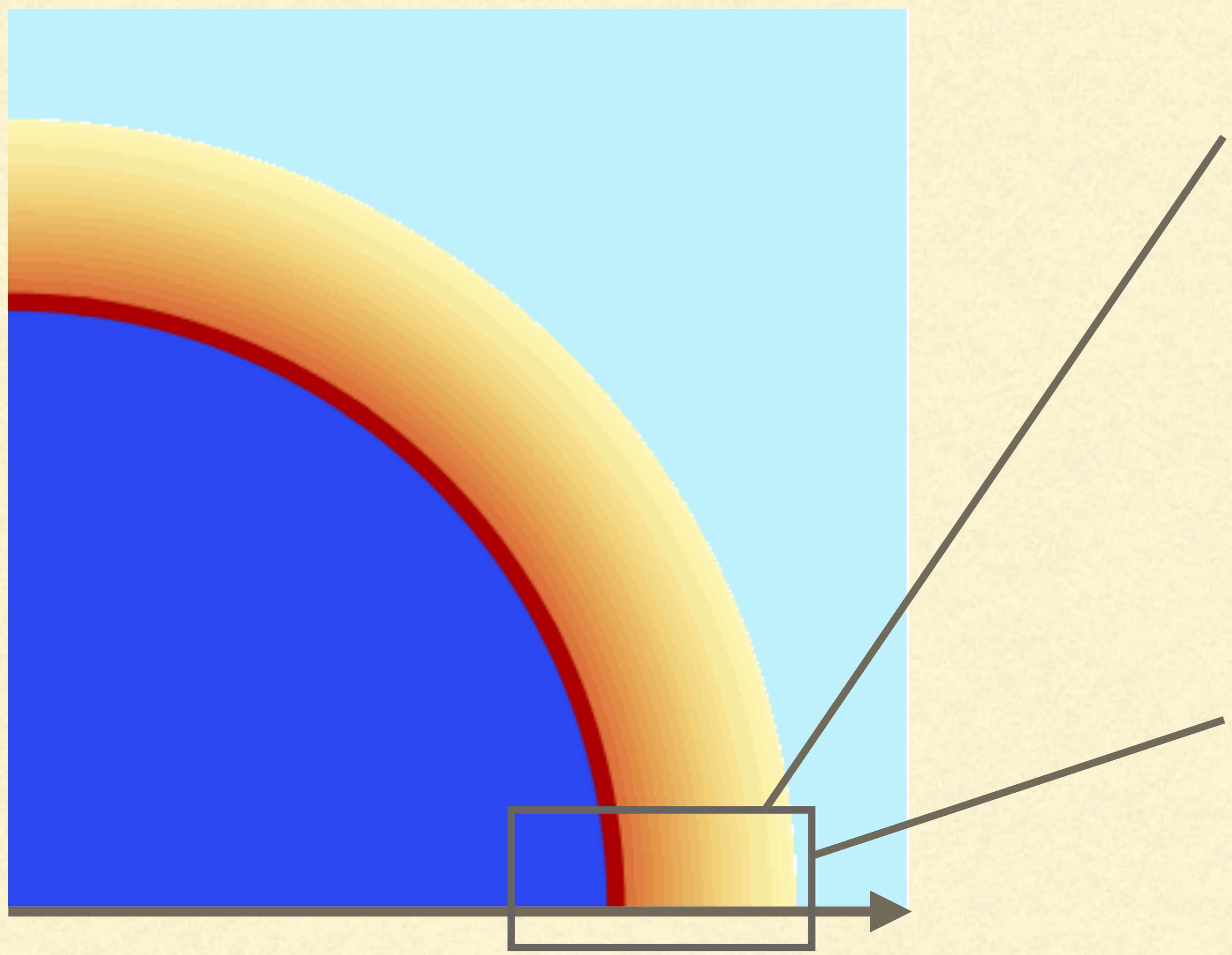
BUBBLE DYNAMICS BEFORE COLLISION

$$\alpha \equiv \frac{\rho_{\rm vac}}{\rho_{\rm plasma}}$$

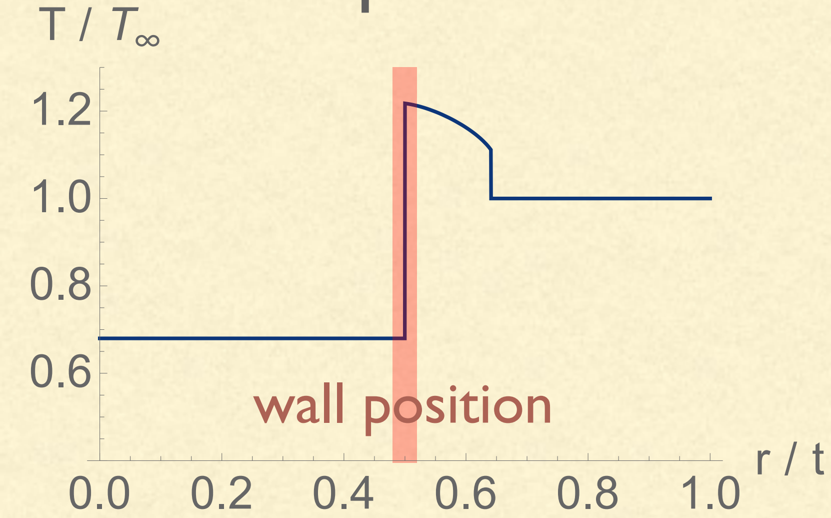
[Espinosa, Konstandin, No, Servant '10]

- Small α (say, $\alpha \lesssim \mathcal{O}(0.1)$)

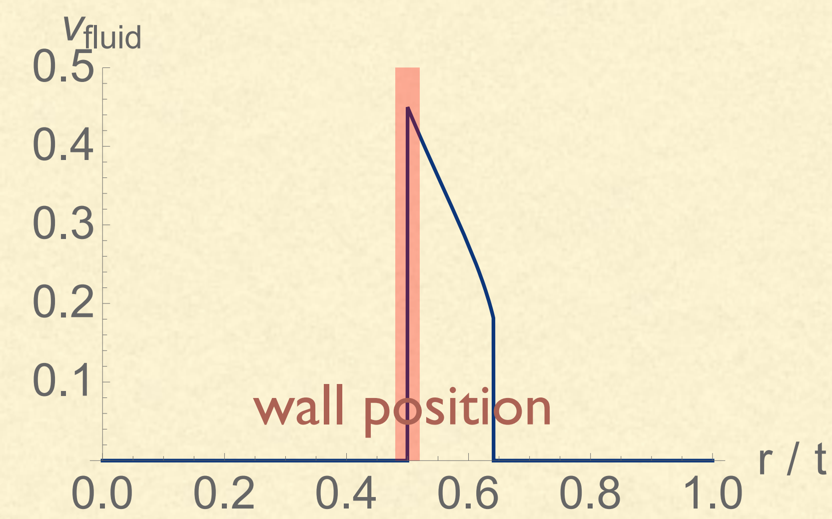
"deflagration"



Temperature



Fluid outward velocity



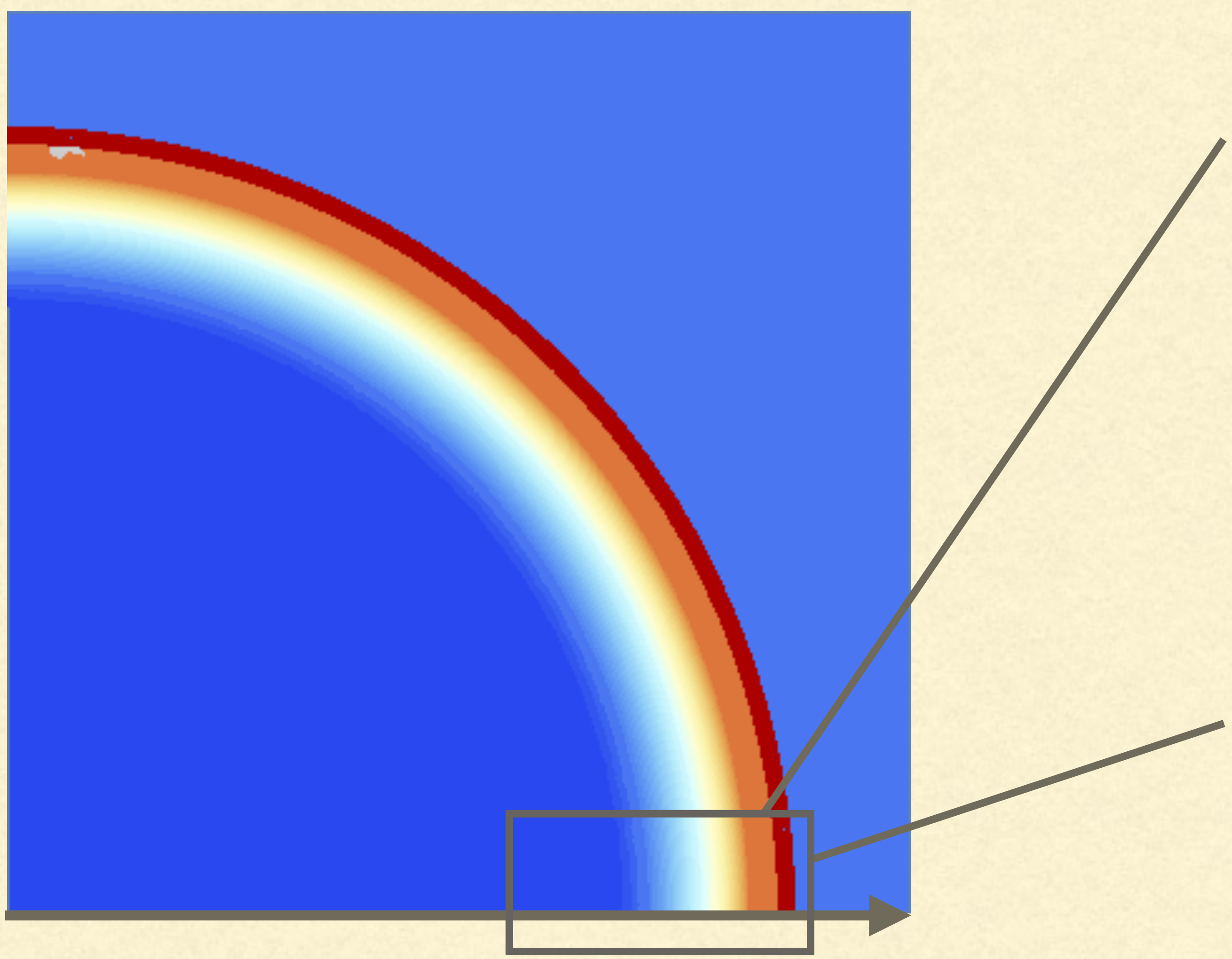
BUBBLE DYNAMICS BEFORE COLLISION

$$\alpha \equiv \frac{\rho_{\rm vac}}{\rho_{\rm plasma}}$$

[Espinosa, Konstandin, No, Servant '10]

- Small but slightly increased α

"detonation"



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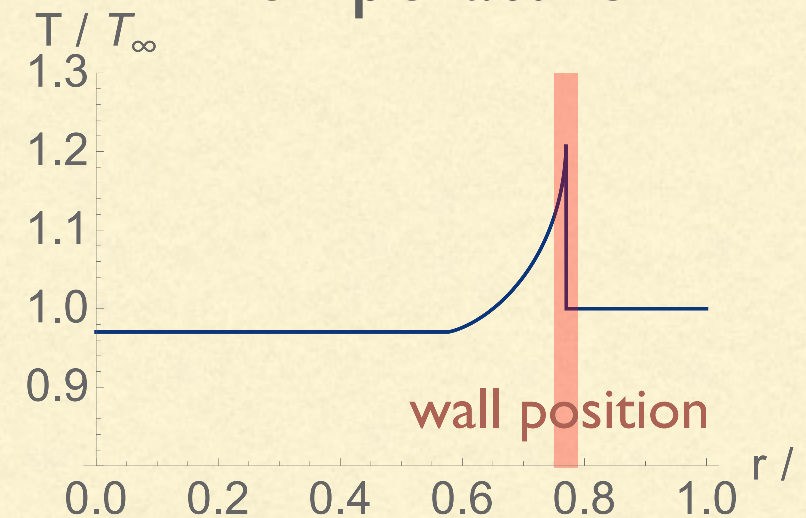

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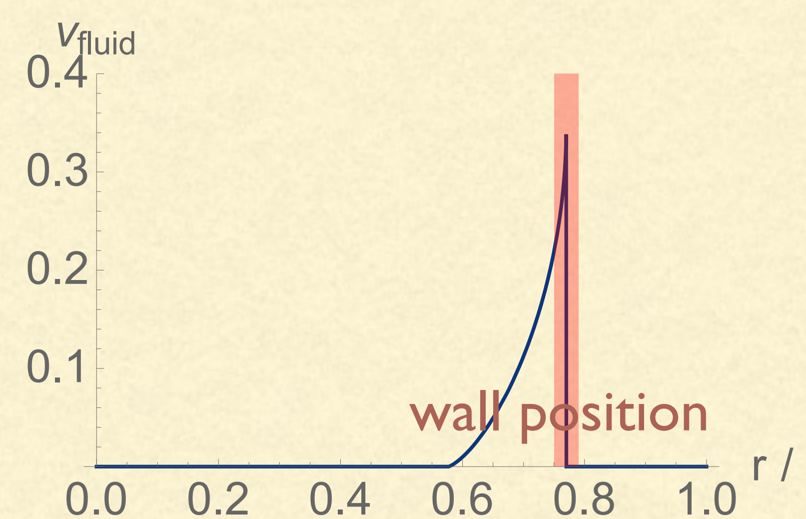
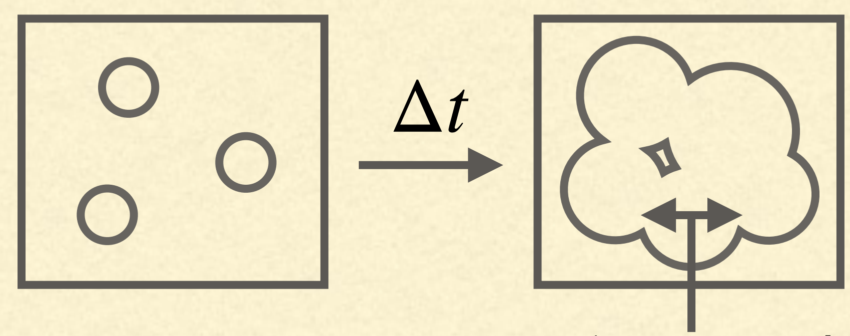
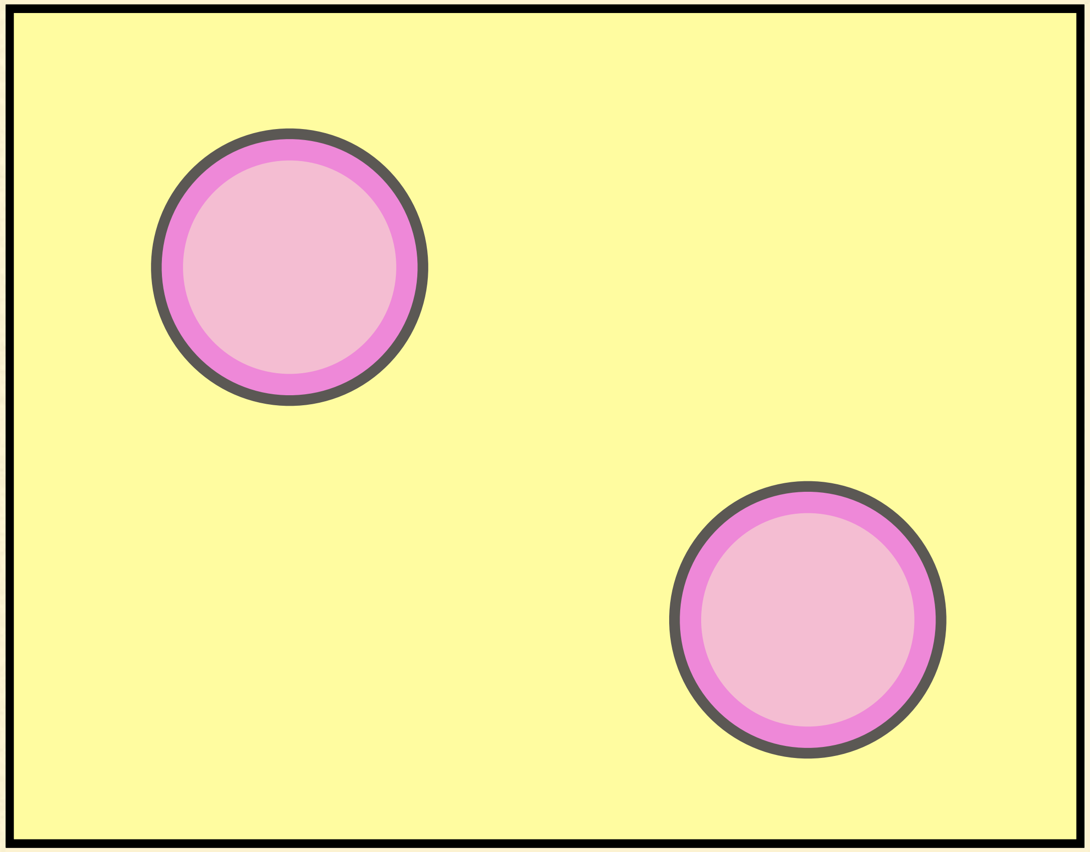


PARAMETERS CHARACTERIZING THE TRANSITION

	Definition	Properties
α	$\rho_{\rm vac}/\rho_{\rm plasma}$	Strength of the transition
β	Bubble nucleation rate Taylor-expanded around the transition time t_* $\Gamma(t) \propto e^{\beta(t-t_*)}$	Bubbles collide $\Delta t \sim 1/\beta$ after nucleation  Typical bubble size $\sim v_w \Delta t \sim v_w/\beta$
v_w	Wall velocity	Determined by the balance btwn. pressure & friction
T_*	Transition temperature	

DYNAMICS AFTER COLLISION

Bubbles nucleate & expand



- Nucleation rate (per unit time & vol)

$$\Gamma(t) \propto e^{\beta(t-t_*)}$$

- Typically the released energy is carried by fluid motion [Bodeker & Moore '17]

- Collide $\Delta t \sim 1/\beta$ after nucleation

DYNAMICS AFTER COLLISION

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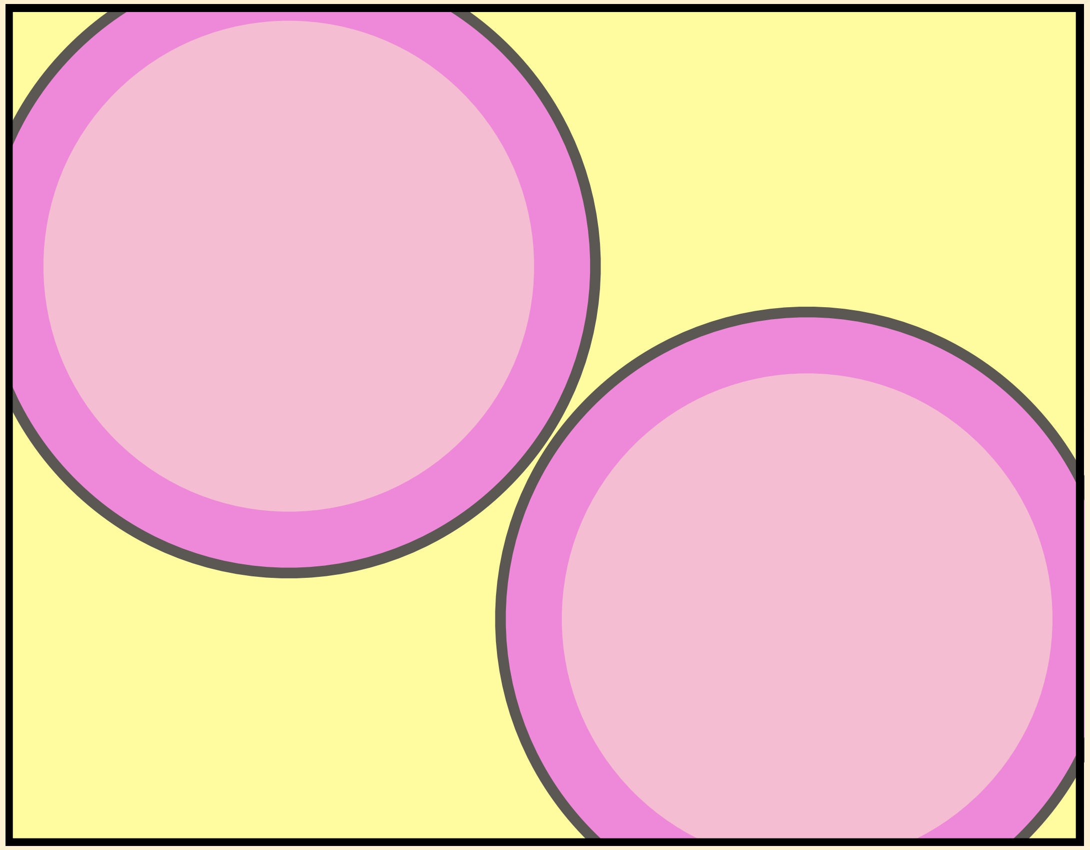

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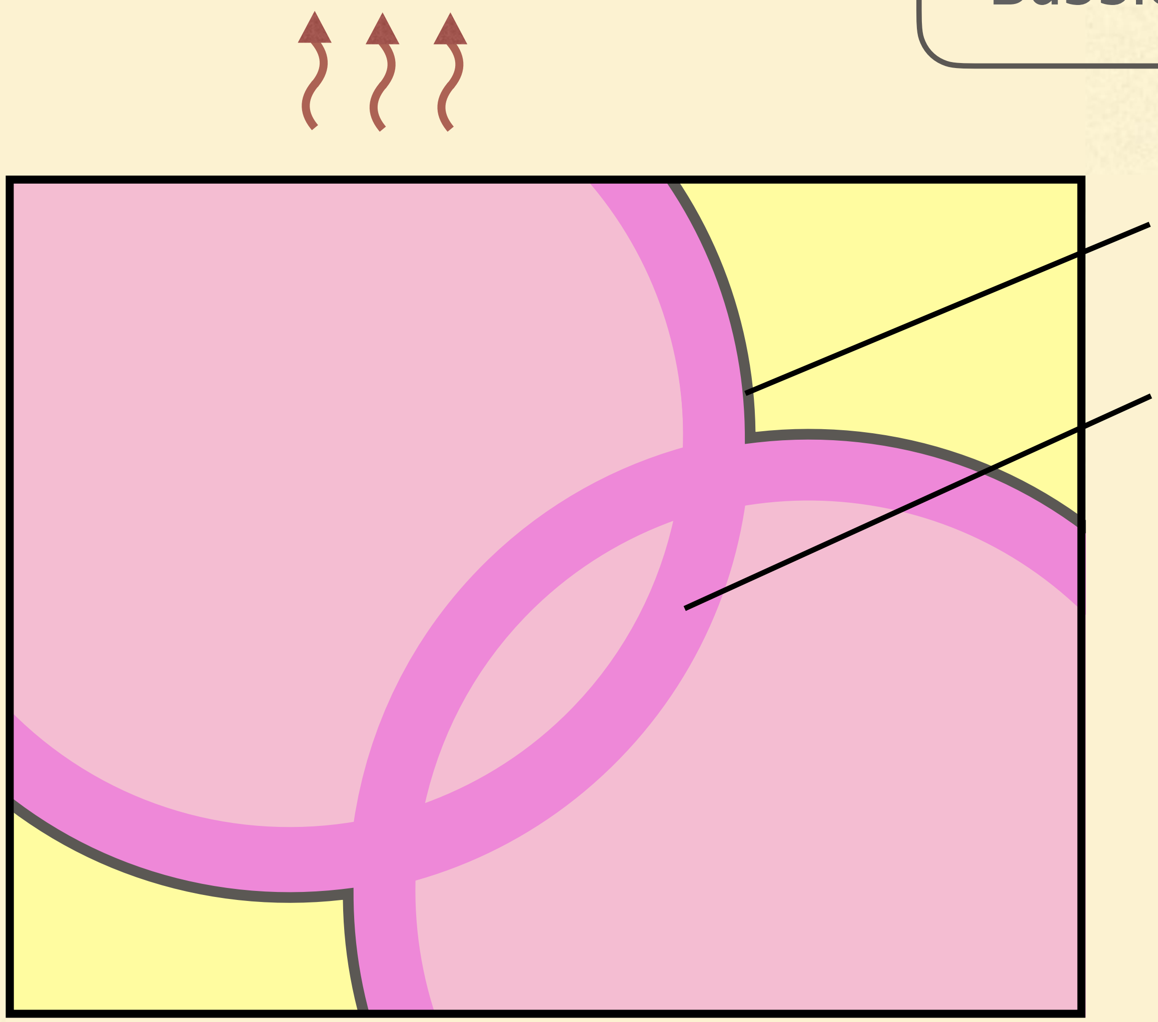
DYNAMICS AFTER COLLISION

GWs $\Box h_{ij} \sim T_{ij}$



Bubbles collide

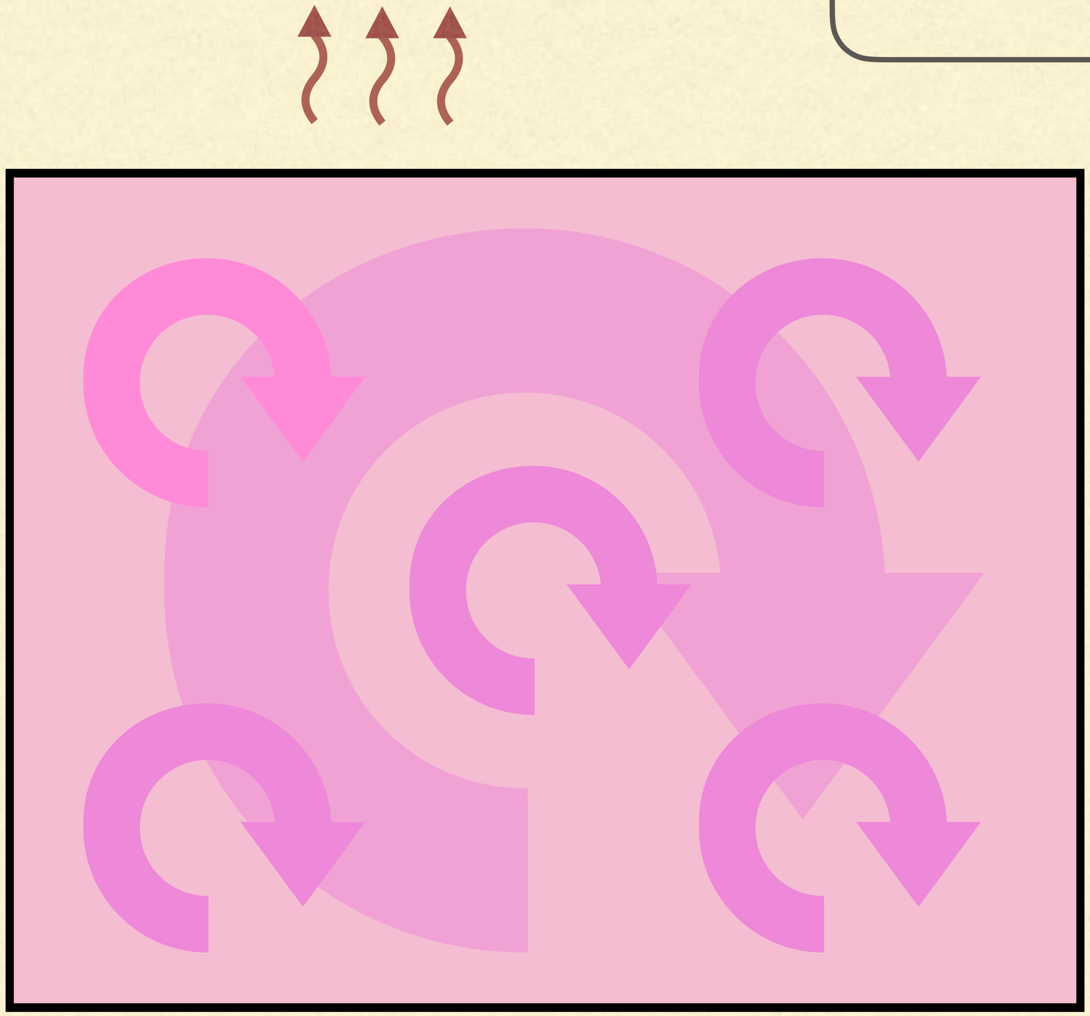
- Scalar field damps soon after collision
- For small α $(\lesssim \mathcal{O}(0.1))$, plasma motion is well described by linear approximation:

$$(\partial_t^2 - c_s^2 \nabla^2)\vec{v}_{\text{fluid}} \simeq 0 \quad \text{“sound waves”}$$

- In this case, fluid shell thickness is fixed at the time of collision

DYNAMICS AFTER COLLISION

GWs $\Box h_{ij} \sim T_{ij}$



Turbulence develops

- Nonlinear effects becomes important at late times

"turbulence"

SOURCES OF GWS IN FIRST-ORDER PHASE TRANSITION

- Time evolution of the system

  Bubble nucleation & expansion → Collision → Sound waves → Turbulence

- Resulting GW spectrum is classified accordingly: [Caprini et al. 1512.06239]

$$\Omega_{\rm GW} = \Omega_{\rm GW}^{\rm (coll)} + \Omega_{\rm GW}^{\rm (sw)} + \Omega_{\rm GW}^{\rm (turb)}$$

- Typically $\Omega_{\rm GW}^{\rm (sw)}$ is the largest, partly because of different parameter dependence:

[Hindmarsh, Huber, Rummukainen, Weir '13, '15, '17]

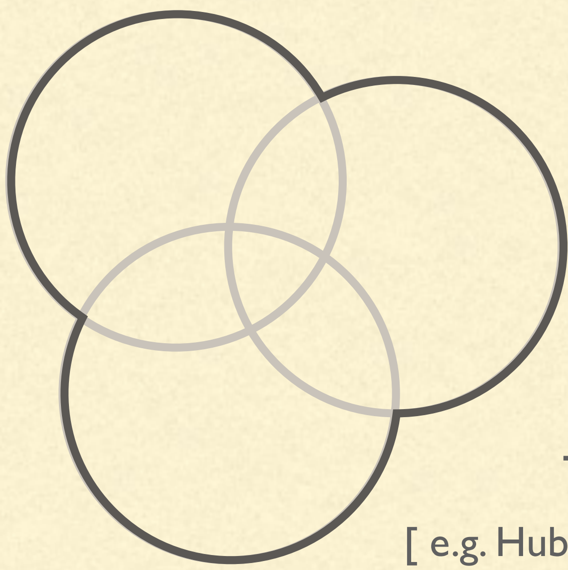
$\Omega_{\rm GW}^{\rm (coll)}$ (from scalar walls) $\propto \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{\beta}{H_*}\right)^{-2}$

$\Omega_{\rm GW}^{\rm (sw)}$ (from fluid shells) $\propto \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{\beta}{H_*}\right)^{-1}$

Note : $\frac{\beta}{H_*} \sim 10^{1-5} \gg 1$

GW ENHANCEMENT BY SOUND WAVES

Bubble collision



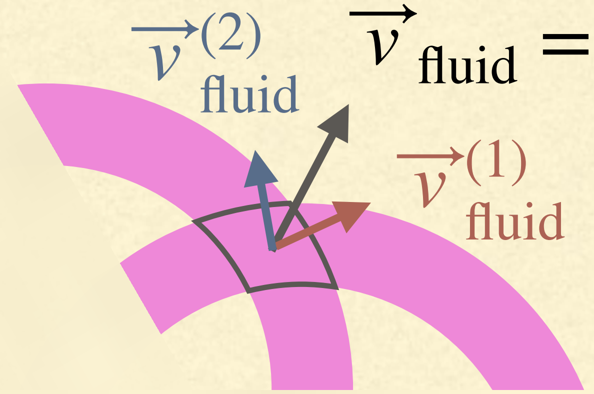
Thin source

[e.g. Huber & Konstandin '08

Jinno & Takimoto '16]

Sound waves

$$\vec{v}_{\rm fluid} = \vec{v}^{(1)}_{\rm fluid} + \vec{v}^{(2)}_{\rm fluid}$$



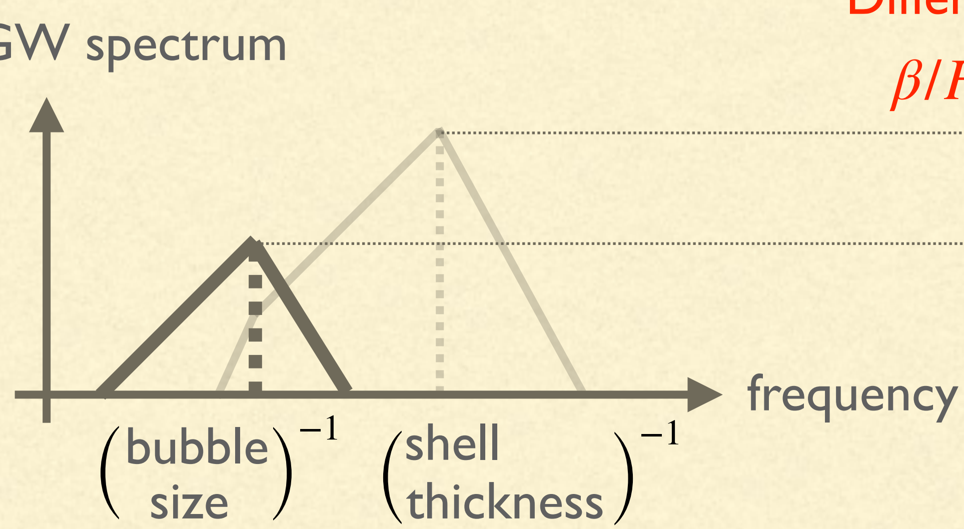
Thick source: sound shells continue to overlap everywhere during the whole Hubble time

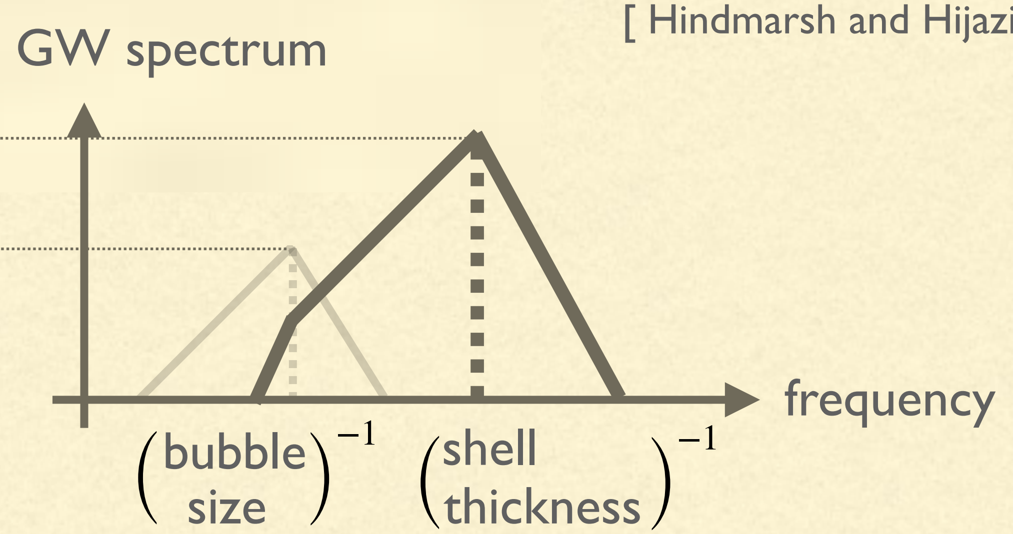
[Hindmarsh '18]

[Hindmarsh and Hijazi '19]

Difference is huge:

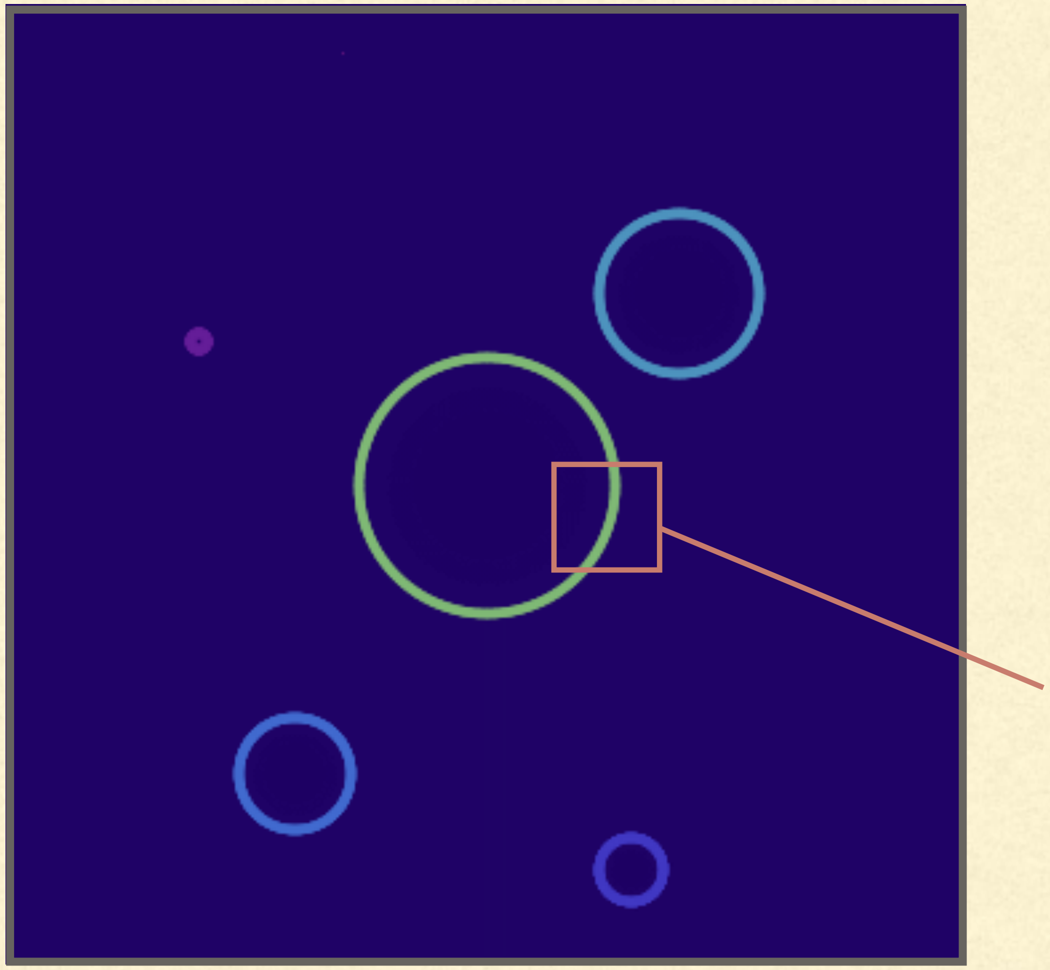
$\beta / H_* \sim 10^{1-5}$





GWS FROM THIN SOUCE (A BIT OF ADVERTISEMENT)

- GW production from thin source is strictly calculable [RJ & Takimoto 1707.03111]



- Cosmic expansion neglected

- Bubbles nucleate with rate Γ
  (Typically $\Gamma \propto e^{\beta t}$ in thermal transitions)

- Bubbles are approximated to be thin

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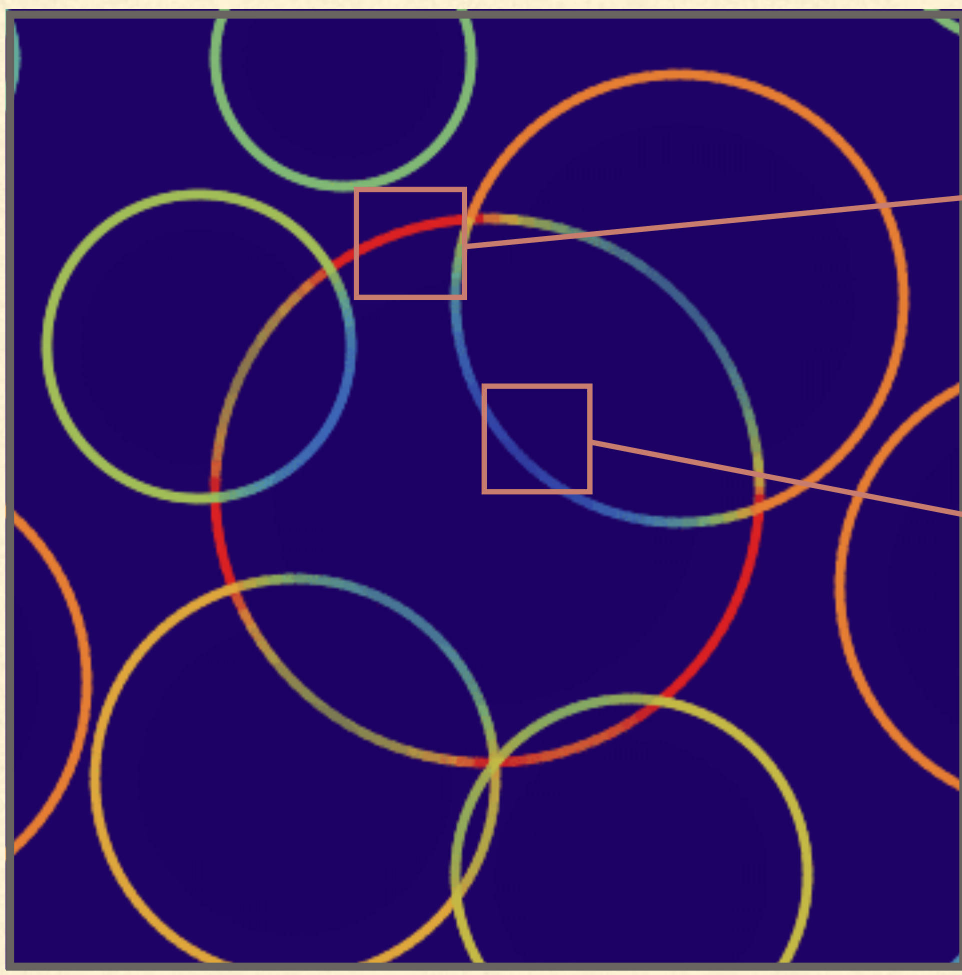

- Shells become more and more energetic

$$T_{ij} \propto \text{(bubble radius)}$$

- They lose energy & momentum after first collision

$$T_{ij} = T_{ij}\text{@ collision} \times \frac{\text{(bubble radius @ collision)}^2}{\text{(bubble radius)}^2}$$

$\times$ (arbitrary damping func. D)

GWS FROM THIN SOUCE (A BIT OF ADVERTISEMENT)

- GW production from thin source is strictly calculable [RJ & Takimoto 1707.03111]

$$\rho_{\mathrm{GW}}(k) \propto \Delta^{(s)} + \Delta^{(d)}$$

$$\Delta^{(s)} = \int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y \int_{v|t_{x,y}|}^{\infty} dr \int_{-\infty}^{t_{\max}} dt_n \int_{t_n}^{t_x} dt_{xi} \int_{t_n}^{t_y} dt_{yi}$$
$$\frac{k^3}{3}\left[\begin{array}{l} e^{-I(x_i,y_i)}\,\Gamma(t_n)\,\dfrac{r}{r_{xn}^{(s)} r_{yn}^{(s)}} \\ \times\left[j_0(kr)\mathcal{K}_0(n_{xn\times}, n_{yn\times}) + \dfrac{j_1(kr)}{kr}\mathcal{K}_1(n_{xn\times}, n_{yn\times}) + \dfrac{j_2(kr)}{(kr)^2}\mathcal{K}_2(n_{xn\times}, n_{yn\times})\right] \\ \times\,\partial_{txi}\left[r_B(t_{xi},t_n)^3 D(t_x,t_{xi})\right]\partial_{tyi}\left[r_B(t_{yi},t_n)^3 D(t_y,t_{yi})\right]\cos(kt_{x,y}) \end{array}\right]$$

GWS FROM THIN SOUCE (A BIT OF ADVERTISEMENT)

- GW production from thin source is strictly calculable [RJ & Takimoto 1707.03111]

$$\rho_{\rm GW}(k) \propto \Delta^{(s)} + \Delta^{(d)}$$

$$\begin{aligned}
\Delta^{(d)} = &\int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y \\
&\int_0^{\infty} dr \int_{-\infty}^{t_x} dt_{xn} \int_{-\infty}^{t_y} dt_{yn} \int_{t_{xn}}^{t_x} dt_{xi} \int_{t_{yn}}^{t_y} dt_{yi} \int_{-1}^{1} dc_{xn} \int_{-1}^{1} dc_{yn} \int_0^{2\pi} d\phi_{xn,yn} \\
&\frac{k^3}{3} \left[\begin{aligned} &\Theta_{\rm sp}(x_i, y_n)\Theta_{\rm sp}(x_n, y_i)e^{-I(x_i,y_i)}\Gamma(t_{xn})\Gamma(t_{yn}) \\
&\times r^2 \left[j_0(kr)\mathcal{K}_0(n_{xn}, n_{yn}) + \frac{j_1(kr)}{kr}\mathcal{K}_1(n_{xn}, n_{yn}) + \frac{j_2(kr)}{(kr)^2}\mathcal{K}_2(n_{xn}, n_{yn}) \right] \\
&\times \partial_{txi} \left[r_B(t_{xi}, t_{xn})^3 D(t_x, t_{xi}) \right] \partial_{tyi} \left[r_B(t_{yi}, t_{yn})^3 D(t_y, t_{yi}) \right] \cos(kt_{x,y}) \end{aligned} \right]
\end{aligned}$$

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✔1. Introduction

✔2. Brief review of bubble dynamics and GW production

3. GW production in ultra-supercooled transitions:

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4. Summary

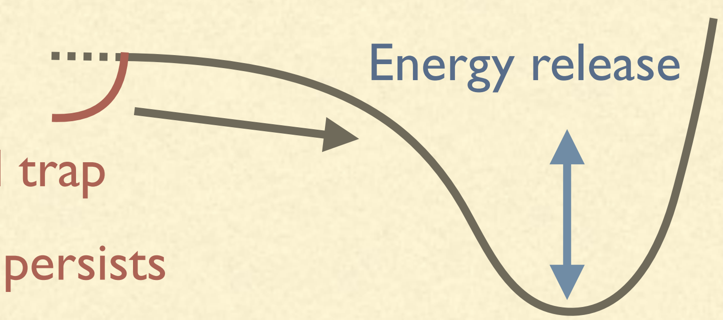
ULTRA-SUPERCOOLED TRANSITIONS

- $\alpha \gg 1$ occurs in a certain class of models [e.g. Randall & Servant '07, Konstandin & Servant '11, ...]

  - Thermal trap persists even at low temperatures → $\alpha \gg 1$

  - These models also give small β/H_* (i.e. large bubbles)

one example:



Zero-temerature:

Coleman-Weinberg potential

$\sim \lambda(\phi)\phi^4$

- So, at least naively, large amplitude of GWs is expected

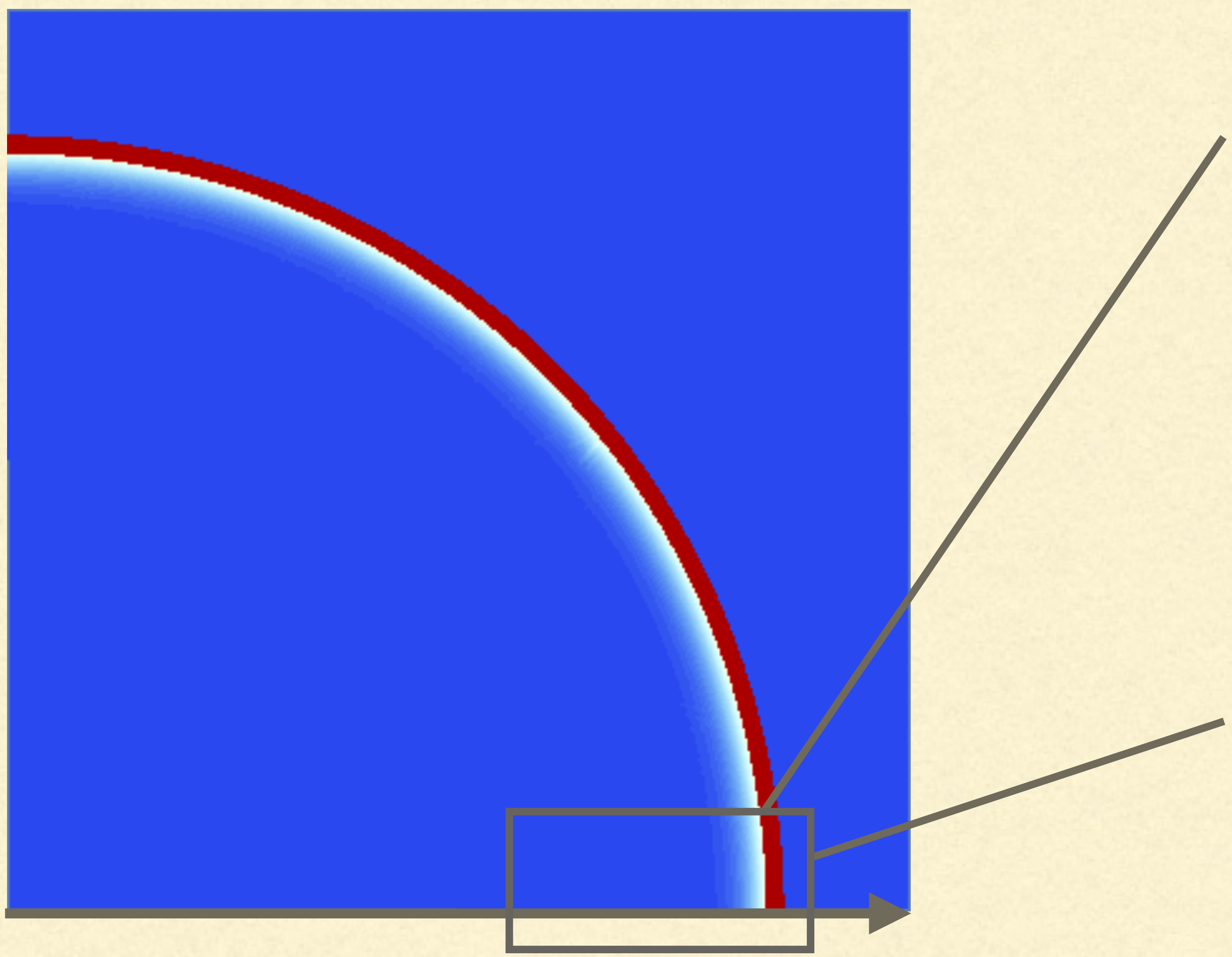
$$\Omega_{\rm GW}^{(\rm sw)} \propto \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{\beta}{H_*}\right)^{-1}$$

- However, the story is not so simple...

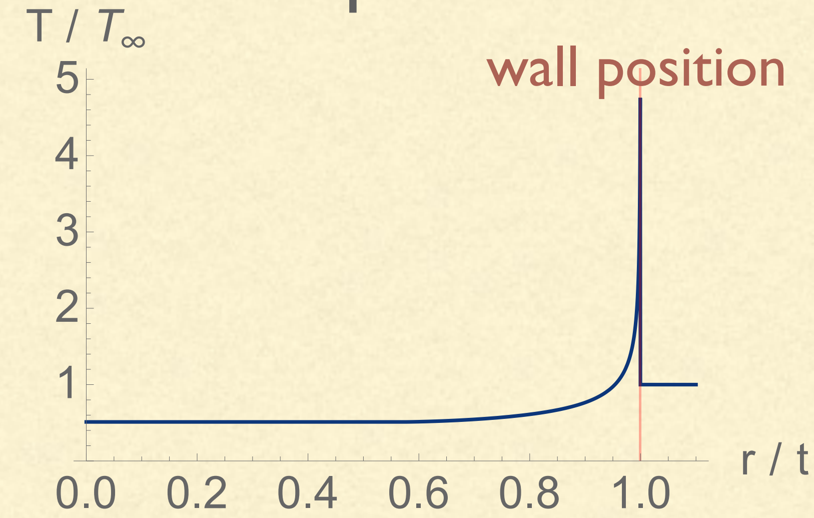
BUBBLE EXPANSION IN ULTRA-SUPERCOOLED TRANSITIONS

- Large α $(\gg 1)$

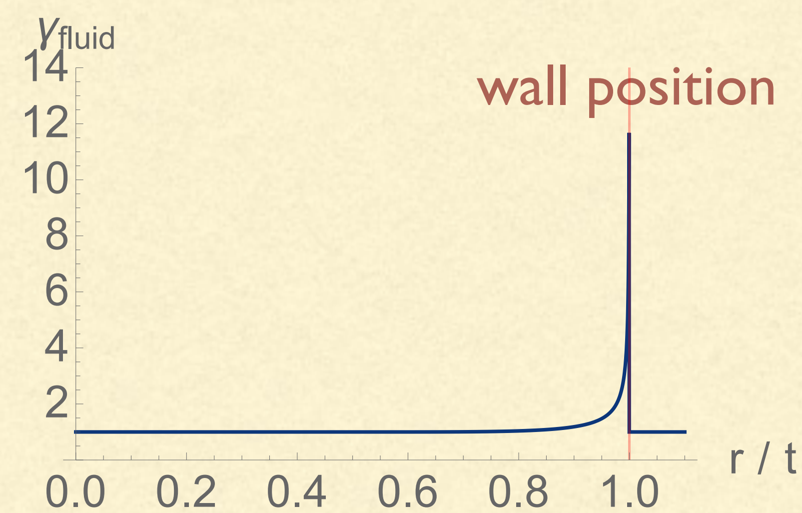
"strong detonation"



Temperature

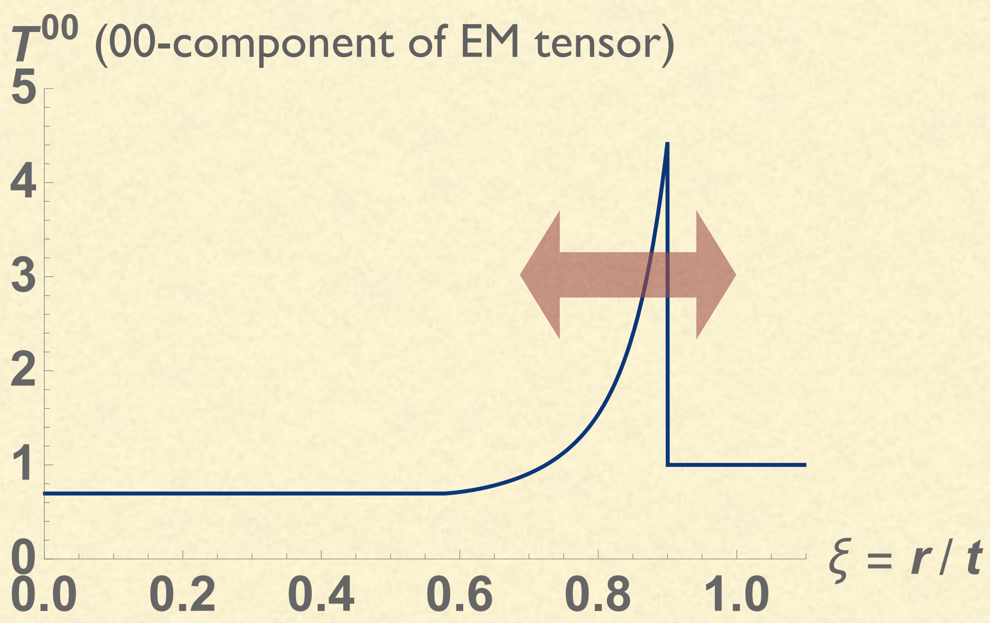


Fluid outward velocity

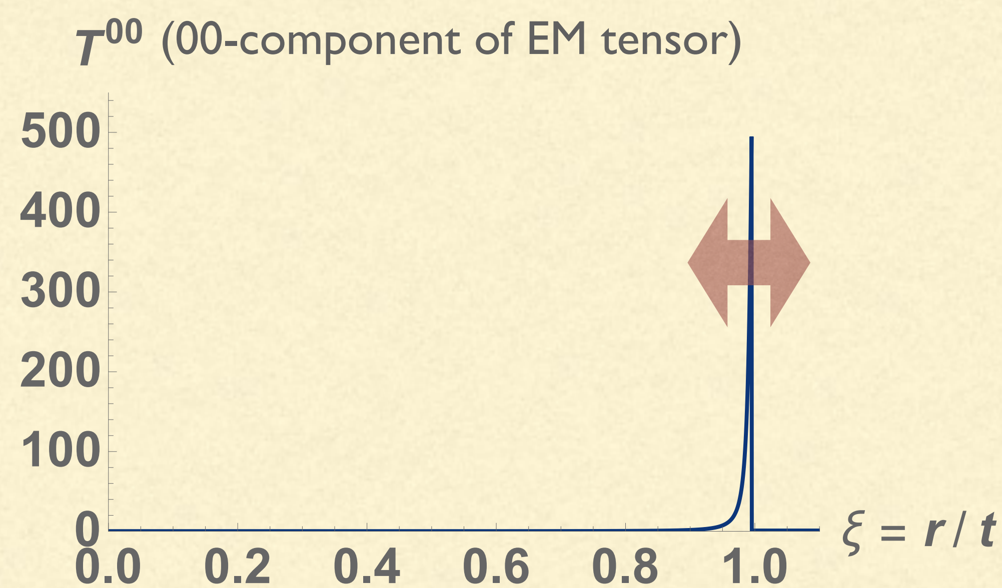


ENERGY LOCALIZATION IN ULTRA-SUPERCOOLED TRANSITIONS

- Fluid energy sharply localizes around bubble wall as α increases



$\alpha = 0.4, \quad v_w = 0.9$



$\alpha = 10, \quad v_w = 0.995$

- In realistic ultra-supercooled transitions, α is much larger, e.g. $\alpha \sim 10^{10}$

- As a result, huge hierarchy appears between bubble size and energy localization

→ Hard to simulate fluid dynamics after bubble collisions numerically

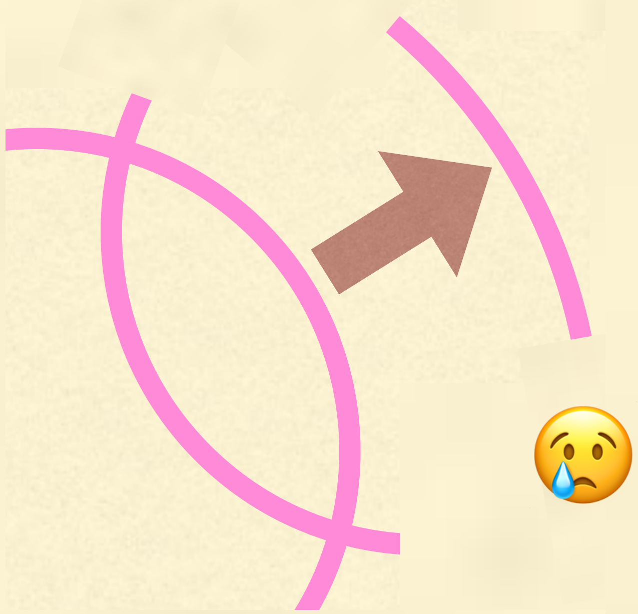
GW ENHANCEMENT CONDITION BY SOUND WAVES

- Necessary conditions to have GW enhancement by sound waves

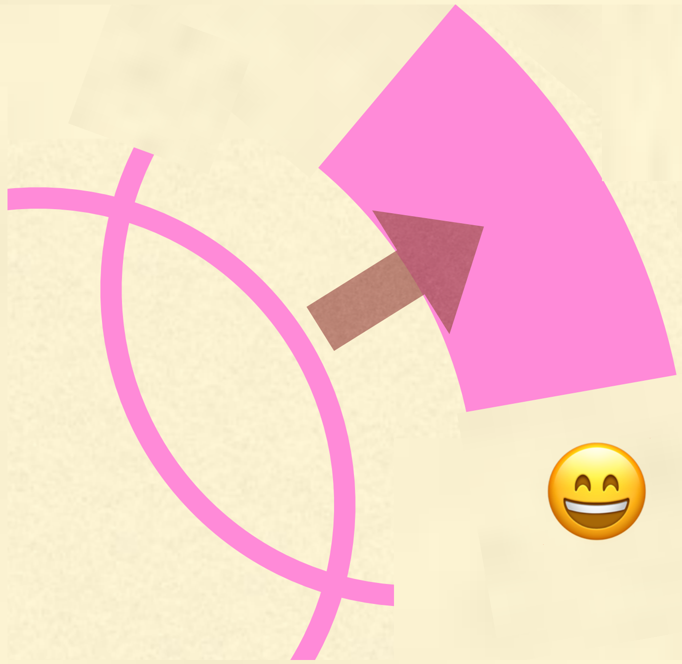
  - Delayed onset of turbulence

  - Sound shell overlap

- In order to have shell overlap, the energy localization has to break up:



or



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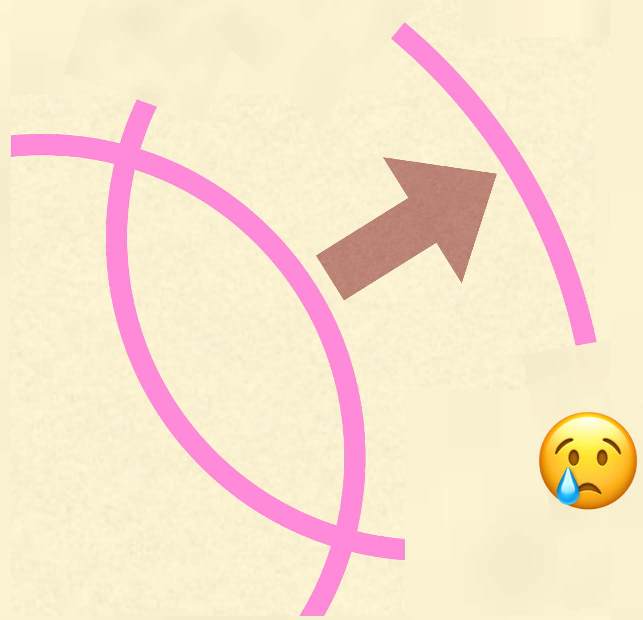

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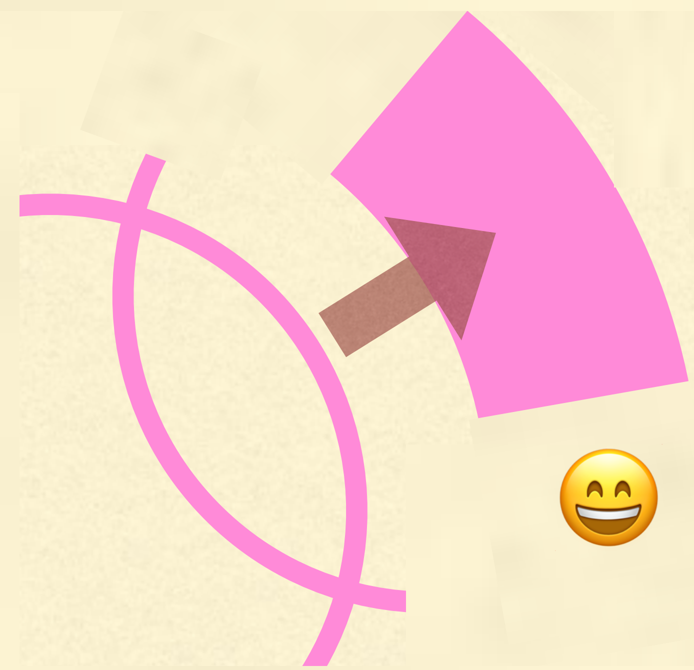


SUMMARY OF MOTIVATION

- Ultra-supercooled transitions ($\alpha \gg 1$) occur in some class of models,

  and they are theoretically and also observationally interesting

- Does GW enhancement by sound waves occur in these transitions?

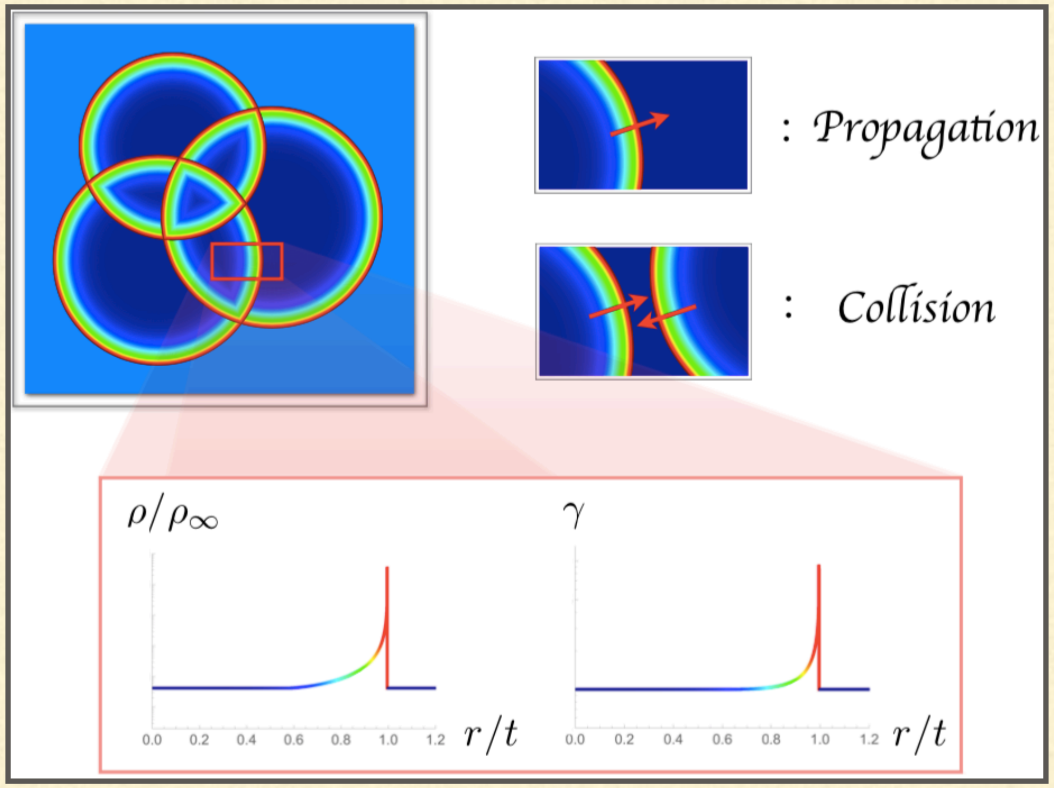
  More precisely: When does the energy localization break up and shell overlap start?

- Numerically difficult to study because of hierarchy in scales

What can we do?

REDUCING THE PROBLEM

- Let's devide the problem into small pieces:



(1) propagation of relativistic fluid

(2) collision of relativistic fluid

- Even propagation is nontrivial due to nonlinearity in fluid equation.

  We study propagation effects.

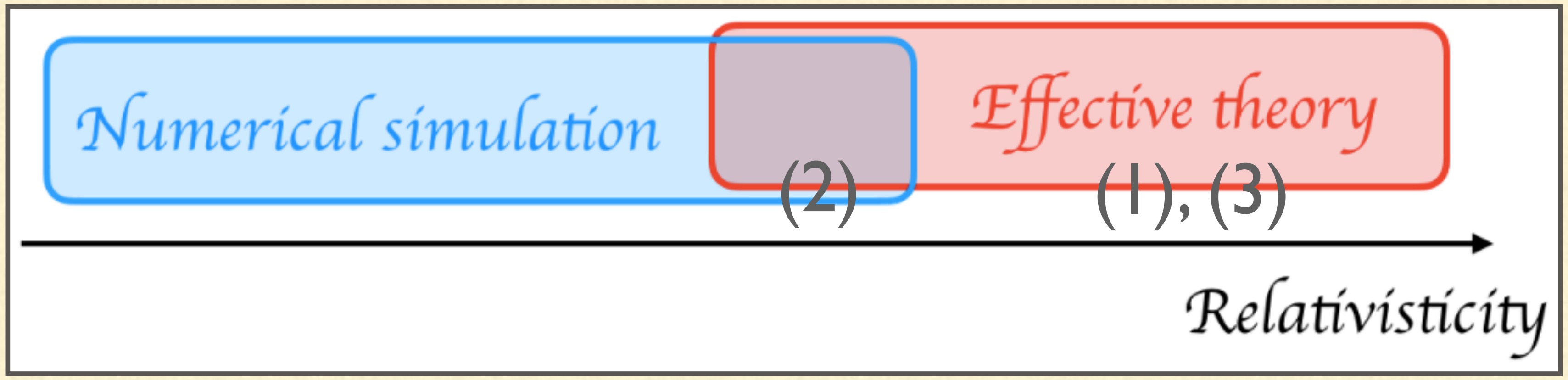
STRATEGY

- Our strategy:

(1) Develop an effective description of fluid propagation valid in highly relativistic regime

(2) Check the description against simulation in mildly-relativistic regime

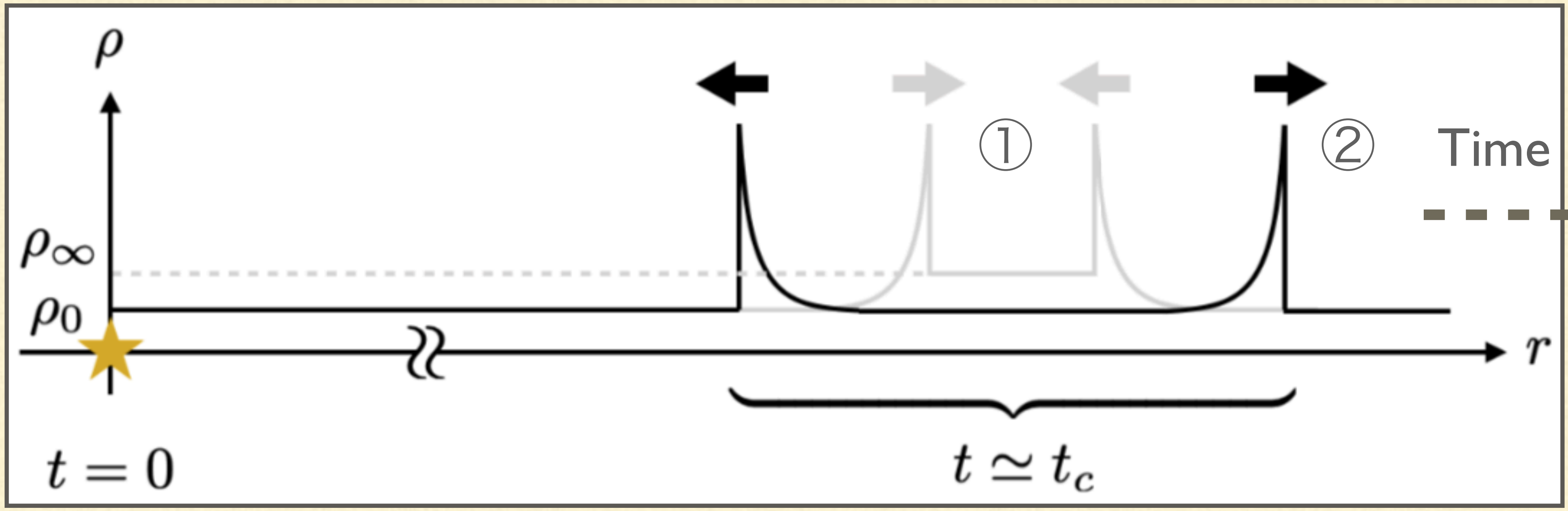
(3) Study implications of the effective description to GW production



(or simply the strength of transition α)

STRATEGY

- The setup we study



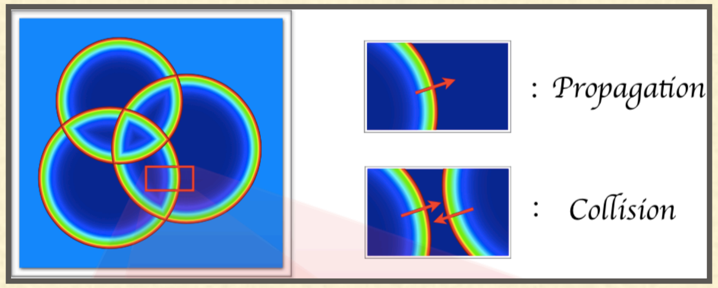
Time evolution

① Fluid profile just before collision: calculated from [Espinosa, Konstandin, No, Servant '10]

↓ Assumption: the first fluid collision does not change the profile significantly

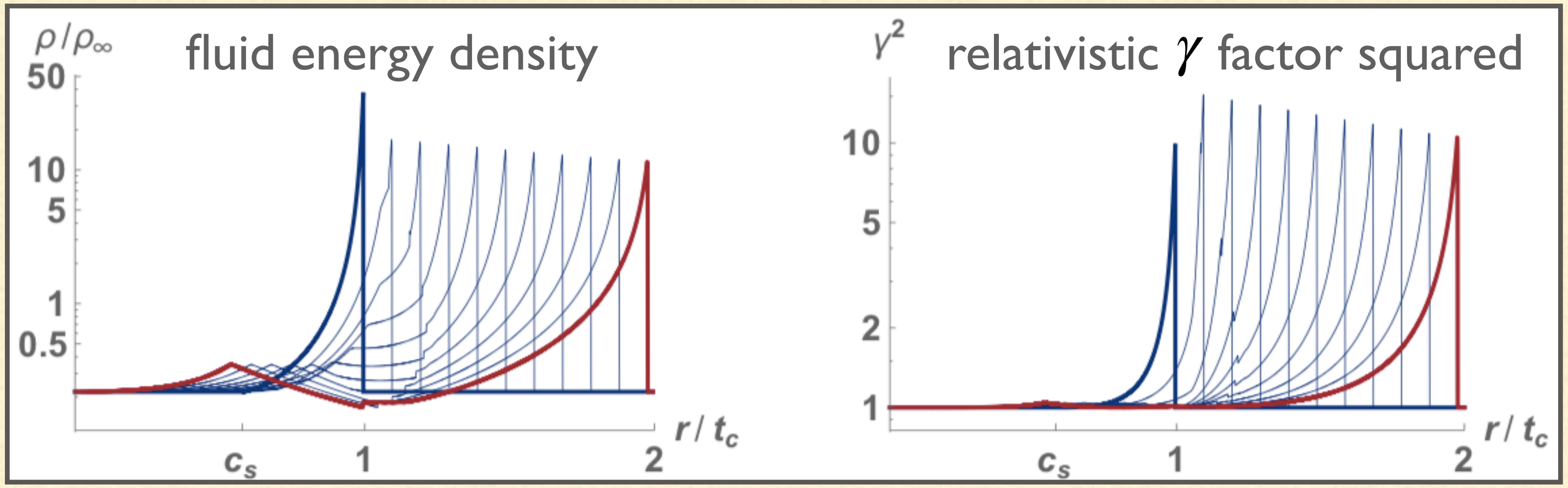
② Fluid profile just after collision: our interest is in the time evolution from here

EFFECTIVE THEORY OF FLUID PROPAGATION

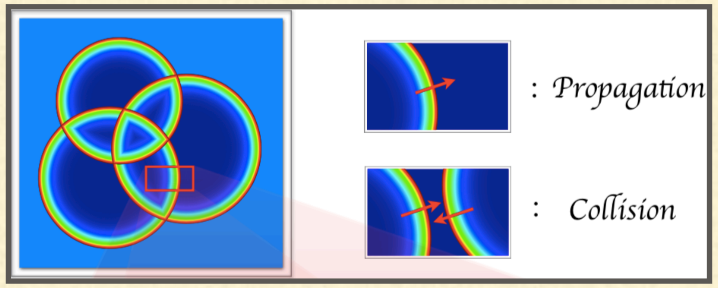


- Before constructing a theory, let's see the result of numerical simulation

(Perfect fluid $T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p\eta_{\mu\nu}$ & relativistic eos $\rho = 3p$)



EFFECTIVE THEORY OF FLUID PROPAGATION



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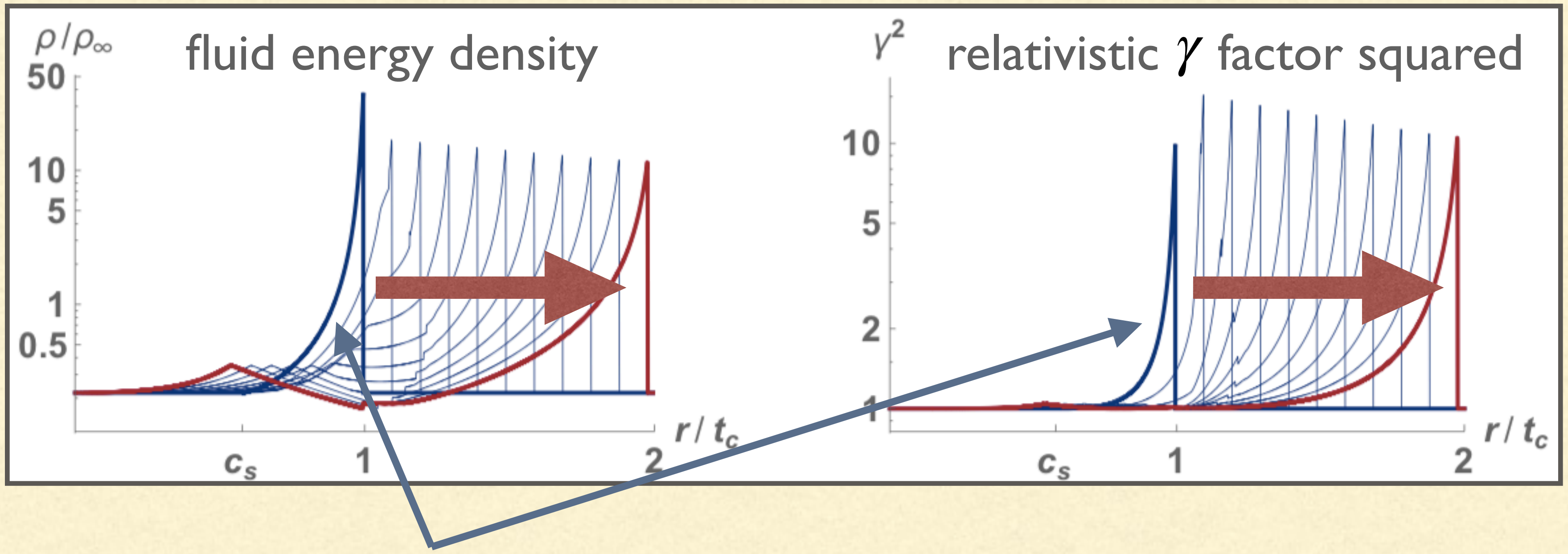
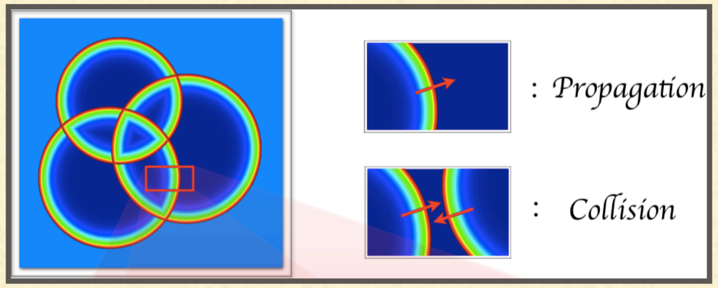
(Perfect fluid $T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p\eta_{\mu\nu}$ & relativistic eos $\rho = 3p$)



- Initial fluid profile (blue) propagates inside the other bubble (red)

EFFECTIVE THEORY OF FLUID PROPAGATION



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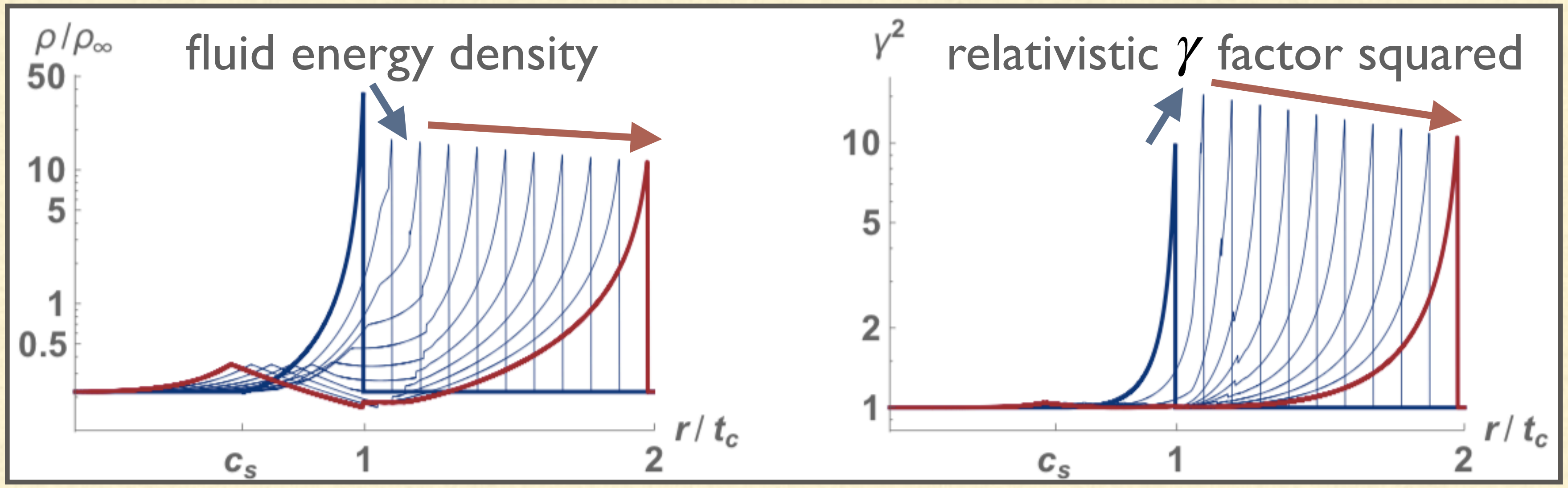

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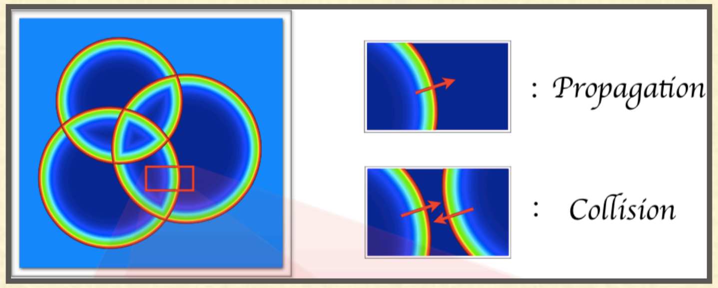
- Peaks rearrange to new initial values, and gradually become less energetic

EFFECTIVE THEORY OF FLUID PROPAGATION



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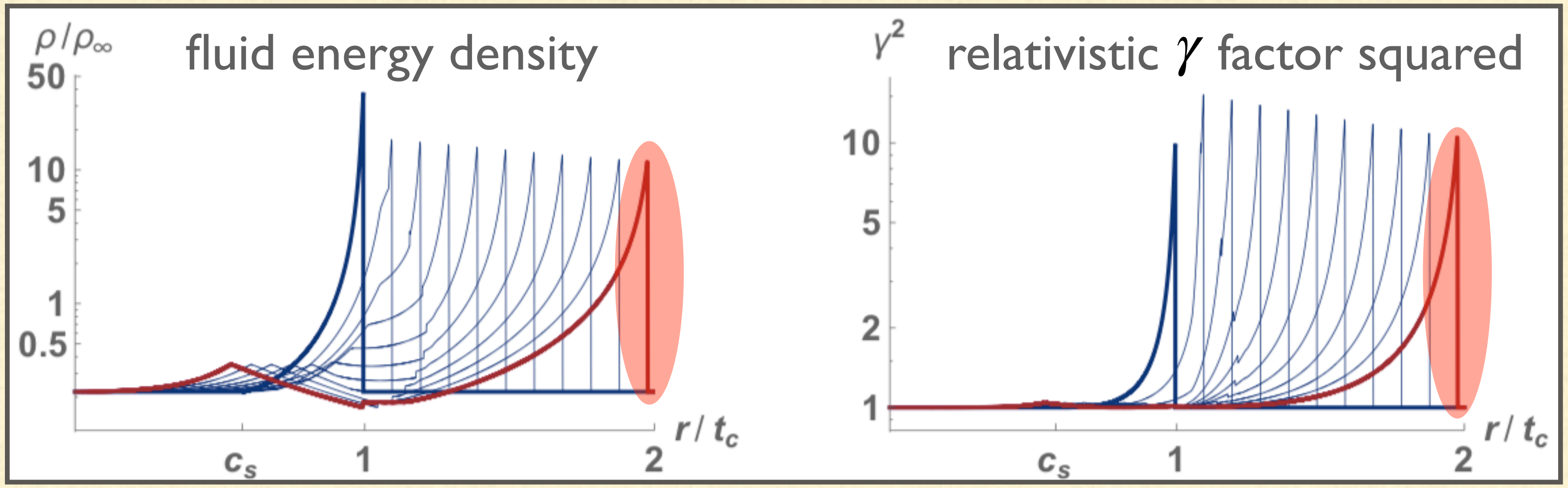
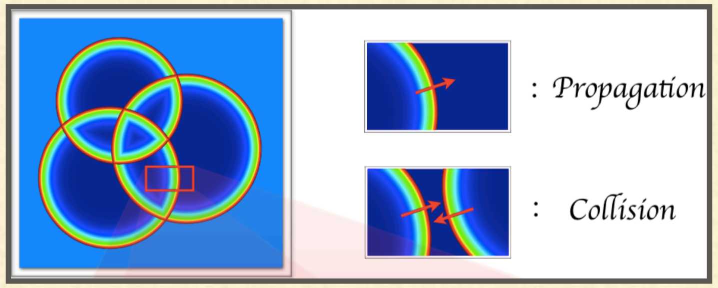


- Initial fluid profile (blue) propagates inside the other bubble (red)

- Peaks rearrange to new initial values, and gradually become less energetic

- Strong shocks (i.e. discontinuities) persist during propagation

EFFECTIVE THEORY OF FLUID PROPAGATION



- Can we construct an effective description?

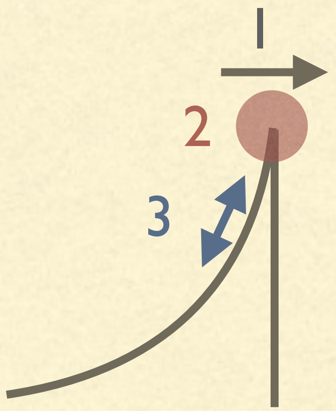
  - From the viewpoint of GW production, we are interested only in PEAKS, not TAILS

  - Can we describe the time evolution of peak-related quantities?

    1) Shock velocity: v_s

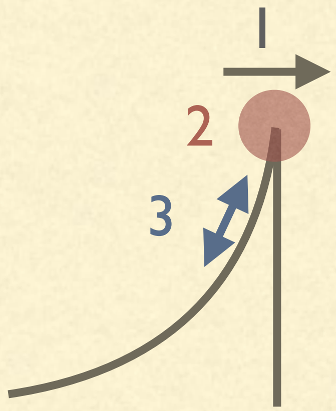
    2) Peak values: $\rho_{\rm peak}, v_{\rm peak}$ (equivalently $\rho_{\rm peak}, \gamma^2_{\rm peak}$)

    3) Derivatives at the peak: $\dfrac{d\rho_{\rm peak}}{dr}, \dfrac{dv_{\rm peak}}{dr}$ @ peak



  - We would like to construct a closed system for these quantities

HOW TO CONSTRUCT A CLOSED SYSTEM



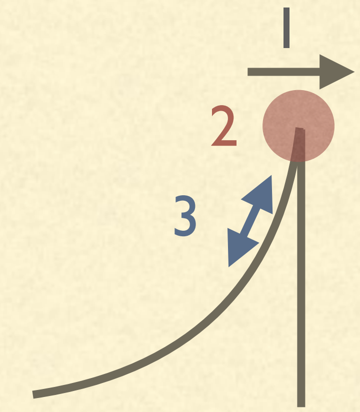
- Closed system for 5 quantities $\gamma_s^2,\ \rho_{\rm peak},\ \gamma_{\rm peak}^2,\ \dfrac{d\rho_{\rm peak}}{dr},\ \dfrac{d\gamma_{\rm peak}^2}{dr}$

  - Rankine-Hugoniot conditions across the shock : 2 constraints

    (corresponding to energy and momentum conservation across the shock)

$$p_{\rm peak} = \frac{p_0 + \rho_0 v_{\rm peak} v_s}{1 - v_{\rm peak} v_s}, \qquad v_s = \frac{(p_{\rm peak} + \rho_{\rm peak}) v_{\rm peak}}{p_{\rm peak} v_{\rm peak}^2 + \rho_{\rm peak} - \rho_0 (1 - v_{\rm peak}^2)}$$

HOW TO CONSTRUCT A CLOSED SYSTEM



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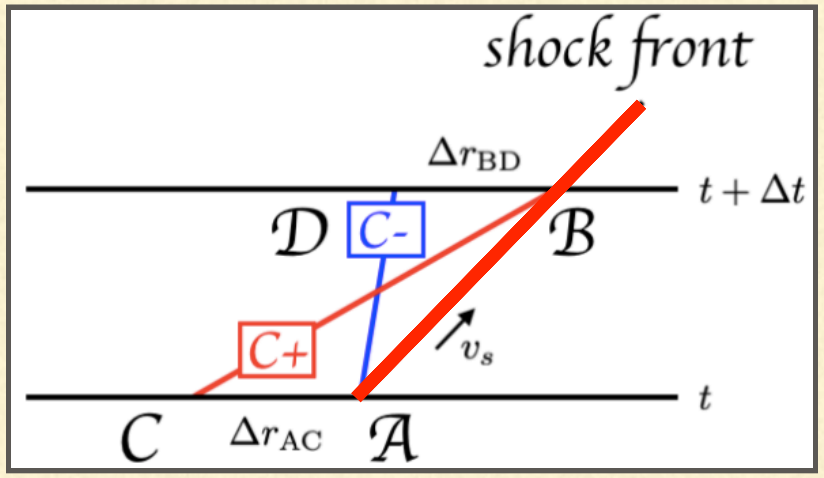
  - Time evolution equations : 2 evolution equations

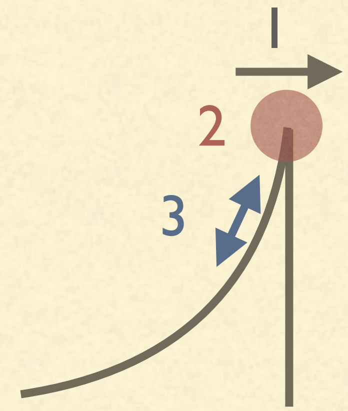
    (corresponding to temporal & spatial part of $\partial_\mu T^{\mu\nu}_{\rm fluid} = 0$ behind the shock)



Advanced note

Easily derived from the conservation of Riemann invariants along C_+ & C_-

HOW TO CONSTRUCT A CLOSED SYSTEM



- Closed system for 5 quantities $\gamma_s^2,\ \rho_{\rm peak},\ \gamma_{\rm peak}^2,\ \frac{d\rho_{\rm peak}}{dr},\ \frac{d\gamma^2_{\rm peak}}{dr}$

  - Rankine-Hugoniot conditions across the shock : 2 constraints

    (corresponding to energy and momentum conservation across the shock)

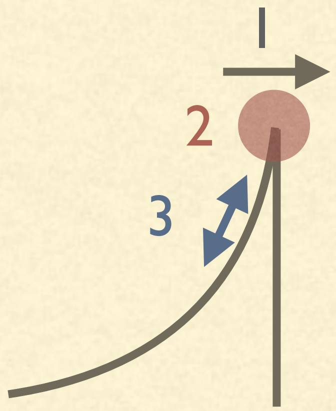
  - Time evolution equations : 2 evolution equations

    (corresponding to temporal & spatial part of $\partial_\mu T^{\mu\nu}_{\rm fluid} = 0$ behind the shock)

$$\frac{\sqrt{3}}{2}\partial_t \ln \rho_{\rm peak} + \partial_t \ln \gamma^2_{\rm peak} = -\frac{2\sqrt{3}-3}{4}\frac{1}{\gamma^2_{\rm peak}}\left[\frac{\sqrt{3}}{2}\ln\rho' + \ln\gamma^{2\prime}\right] - \frac{(\sqrt{3}-1)(d-1)}{t}$$

$$-\frac{\sqrt{3}}{2}\partial_t \ln \rho_{\rm peak} + \partial_t \ln \gamma^2_{\rm peak} = \frac{2\sqrt{3}+3}{4}\frac{1}{\gamma^2_{\rm peak}}\left[-\frac{\sqrt{3}}{2}\ln\rho' + \ln\gamma^{2\prime}\right] + \frac{(\sqrt{3}+1)(d-1)}{t}$$

HOW TO CONSTRUCT A CLOSED SYSTEM



- The last equation?

  - So far, less equations (4 eqs.) than the number of quantities (5 quantities)

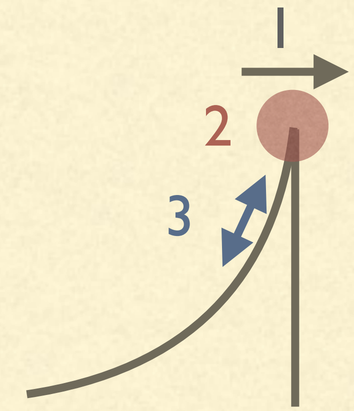
  - This is natural:

    the original system has infinite # of dof (i.e. # of spatial grids),

    so the system cannot be described strictly by finite number of dof

  - So, the last equality to close the system should be APPROXIMATE at best

HOW TO CONSTRUCT A CLOSED SYSTEM



- The last equation: energy domination by the peak

  - Any relation like "(peak T^{00}) × (thickness of the peak) = const." will work

  - In our parametrization, it will be like $\rho_{\rm peak}\gamma_{\rm peak}^2 \times \dfrac{1}{d\ln\rho_{\rm peak}/dr \quad \text{or} \quad d\ln\gamma_{\rm peak}^2/dr} = \text{const}\,.$

  - As an example, approximating $\rho_{\rm peak}$ and $\gamma_{\rm peak}$ to be exponential in r, we have

$$\sigma \simeq \begin{Bmatrix} 1 \\ t \\ t^2 \end{Bmatrix} \times \int dr\, \frac{4}{3}\rho\gamma^2 = \begin{Bmatrix} 1 \\ t \\ t^2 \end{Bmatrix} \times \frac{4}{3}\,\frac{\rho_{\rm peak}\gamma_{\rm peak}^2}{\ln\rho' + \ln\gamma^{2\prime}} \quad \text{for} \quad \begin{Bmatrix} d=1 \\ d=2 \\ d=3 \end{Bmatrix}$$

<u>Note</u> $d = 1,2,3$ corresponds to planar, cylindical, spherical

THEORY PREDICTION

- The resulting system can be solved analytically ($\delta = 10/13$)

1) Shock velocity:

$$\frac{1}{\gamma_s^2(t)} = \frac{8}{87}\left(\frac{\rho_0}{\sigma}\right)\left[t^3 - \left(\frac{t}{t_c}\right)^\delta t_c^3\right] + \frac{1}{\gamma_s^2(t_c)}\left(\frac{t}{t_c}\right)^\delta,$$

2) Peak values:

$$\frac{\rho_0}{\rho_{\rm peak}(t)} = \frac{1}{29}\left(\frac{\rho_0}{\sigma}\right)\left[t^3 - \left(\frac{t}{t_c}\right)^\delta t_c^3\right] + \frac{\rho_0}{\rho_{\rm peak}(t_c)}\left(\frac{t}{t_c}\right)^\delta,$$

$$\frac{1}{\gamma_{\rm peak}^2(t)} = \frac{16}{87}\left(\frac{\rho_0}{\sigma}\right)\left[t^3 - \left(\frac{t}{t_c}\right)^\delta t_c^3\right] + \frac{1}{\gamma_{\rm peak}^2(t_c)}\left(\frac{t}{t_c}\right)^\delta,$$

3) Derivatives at the peak:

$$\ln \rho'(t) = \frac{448}{117}\left(\frac{\rho_0}{\sigma}\right) t^2 \gamma_{\rm peak}^4(t) + \frac{24}{13}\frac{\gamma_{\rm peak}^2(t)}{t},$$

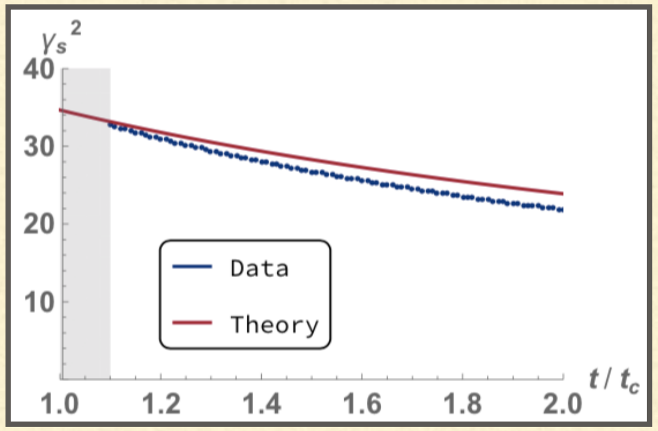
$$\ln \gamma^{2\prime}(t) = \frac{128}{39}\left(\frac{\rho_0}{\sigma}\right) t^2 \gamma_{\rm peak}^4(t) - \frac{24}{13}\frac{\gamma_{\rm peak}^2(t)}{t},$$

COMPARISON WITH NUMERICAL SIMULATION

- Analytic (red) vs. numerical (blue)

  with initial condition $\alpha = 10, \gamma_{\rm wall} = 10$

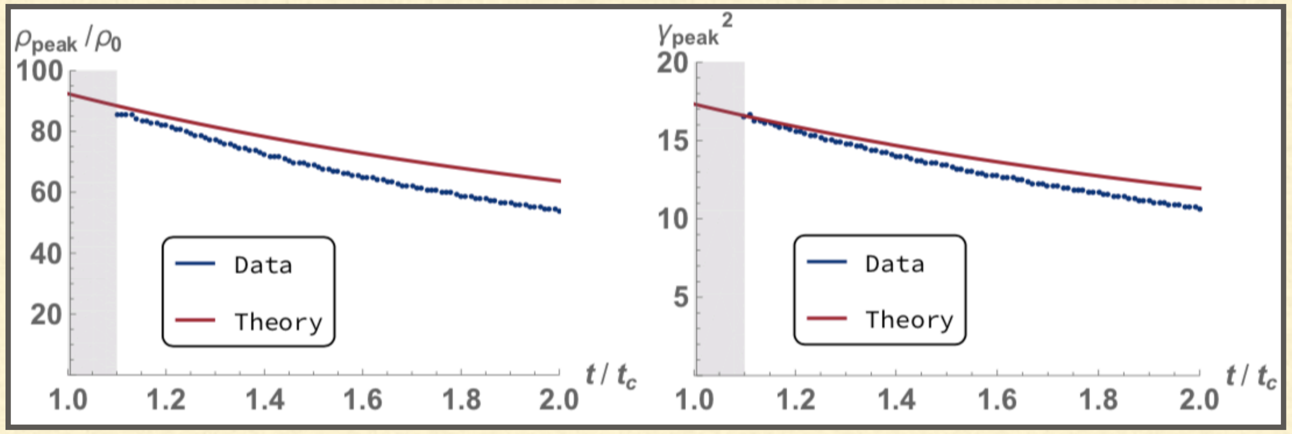
γ_s^2



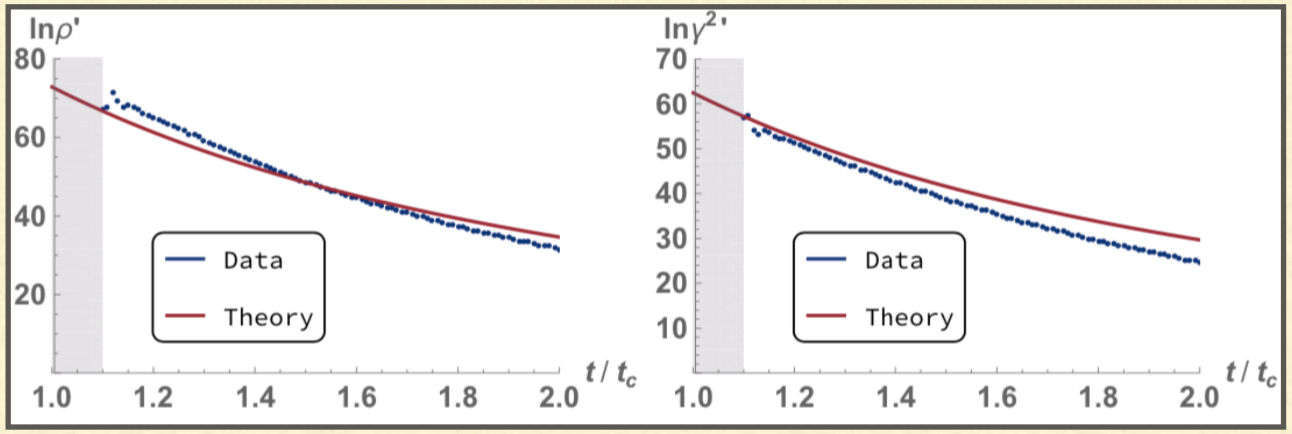
Qualitatively

OK!

$\rho_{\rm peak}, \gamma_{\rm peak}^2$



$$\frac{d\rho_{\rm peak}}{dr}, \frac{d\gamma_{\rm peak}^2}{dr}$$



IMPLICATIONS OF THE EFFECTIVE DESCRIPTION

- What can we learn?

  - All quantities have time dependence like

$$\frac{\rho_0}{\rho_{\rm peak}(t)} = \frac{1}{29}\left(\frac{\rho_0}{\sigma}\right)\left[t^3 - \left(\frac{t}{t_c}\right)^{\delta} t_c^3\right] + \frac{\rho_0}{\rho_{\rm peak}(t_c)}\left(\frac{t}{t_c}\right)^{\delta} \qquad \delta = 10/13$$

effect of increase in the surface area

effect of nonlinearity in fluid equation

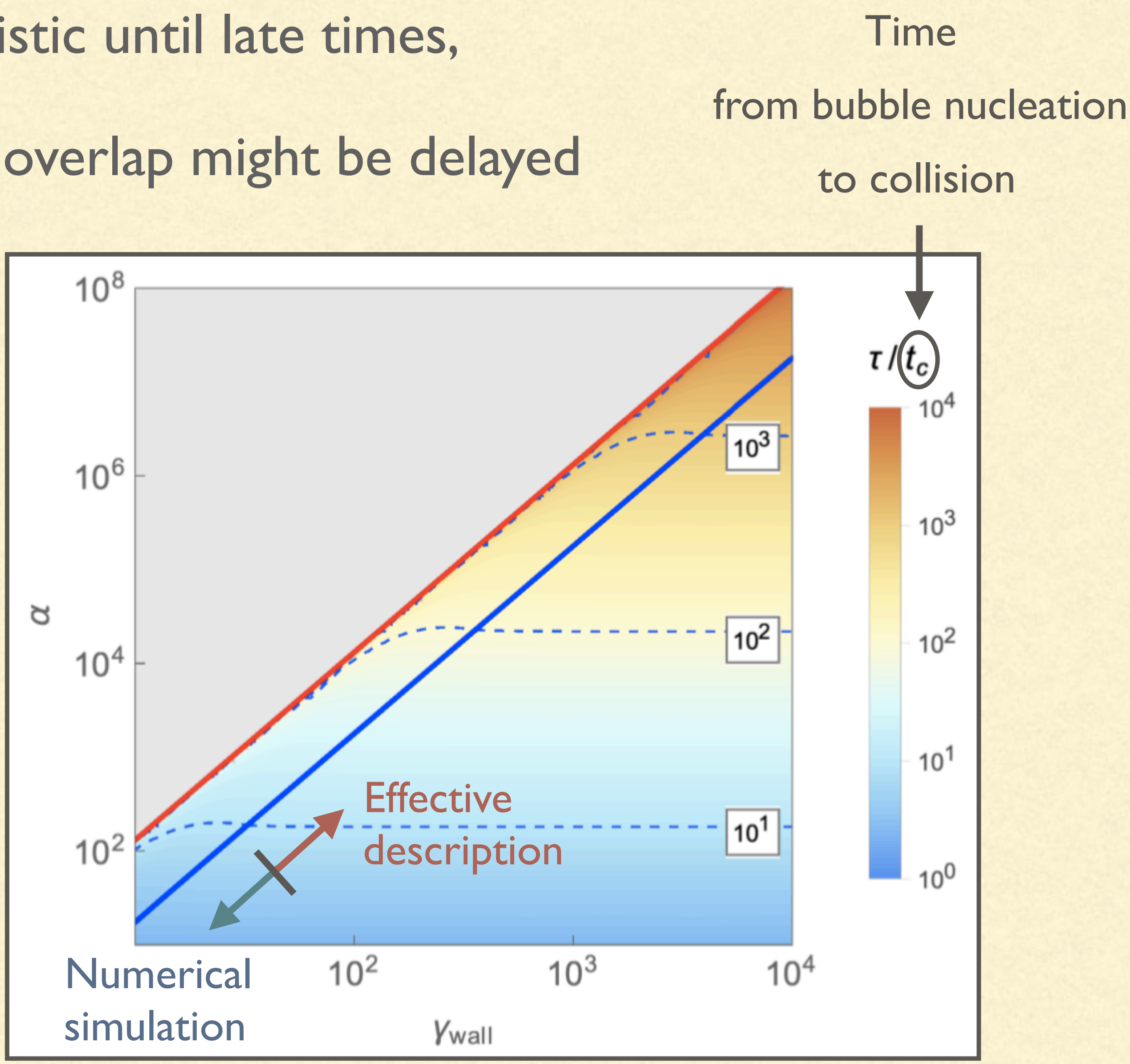
  - Surface area effect wins (3 > 10/13).

    In other words, nonlinearity is not effective in breaking up the energy localization.

  - Timescale for breaking up is governed by $\tau \equiv \left(\sigma/\rho_0\right)^{1/3}$

IMPLICATIONS TO GW PRODUCTION

- Fluid profile remains to thin and relativistic until late times, and therefore the onset of sound shell overlap might be delayed (as long as only fluid propagation is taken into account)
- Still we have to see the effect of fluid collisions in detail



TALK PLAN

✔1. Introduction

✔2. Brief review of bubble dynamics and GW production

✔3. GW production in ultra-supercooled transitions:

Effective description of fluid propagation & Implications to GW production

4. Summary

SUMMARY

- GW production in ultra-supercooled transitions $\alpha \gg 1$ is interesting

  but hard to simulate numerically

- We reduced the problem into (1) propagation and (2) collision, and tackled (1):

  - We constructed an effective description of relativistic fluid propagation

    and discussed implications to GW production

  - GW enhancement by sound waves might be delayed

- Questions to be addressed: Effect of fluid collision / Effect of turbulence

Back up

DEFINITION OF α

- Traditionally: bag eos [e.g. Espinosa, Konstandin, No, Servant '10]

$$p_s = a_+ T^4/3 - \epsilon \qquad p_b = a_- T^4/3$$

$$e_s = a_+ T^4 + \epsilon \qquad e_b = a_- T^4$$

$\rightarrow$

$$\alpha_\epsilon \equiv \epsilon \big/ a_+ T^4$$

(evaluated at nucleation temperature T_N)

- Other definitions?

$$\alpha_p \equiv -\frac{4(p_s(T) - p_b(T))}{3w_s(T)}$$

$$\alpha_\theta \equiv \frac{4(\theta_s(T) - \theta_b(T))}{3w_s(T)} \qquad \theta = e - 3p$$

[e.g. Hindmarsh, Huber, Rummukainen, Weir '15]

$$\alpha_e \equiv \frac{4(e_s(T) - e_b(T))}{3w_s(T)}$$

$$\alpha_{\bar\theta} \equiv \frac{4(\bar\theta_s(T) - \bar\theta_b(T))}{3w_s(T)} \qquad \bar\theta = e - p/c_s^2$$

[Giese, Konstandin, van de Vis '20]

TERMINAL VELOCITY VS. RUNAWAY

- It has been long thought that...

  Friction from the wall $\sim \Delta m^2 T^2 \sim \Phi^2 T^2$ (no γ dependence)

  → walls runaway once the pressure from the energy release wins over friction

- However, friction from transition radiation is found to be $\sim \gamma g^2 \Delta m T^3 \sim \gamma g^2 \Phi T^3$

  - Equating this with the released energy $\sim \lambda \Phi^4$ means that

    walls may reach terminal velocity with relativistic γ factor $\gamma \sim (\Phi/T)^3$

  - For $\Phi \sim \mathrm{EW\ scale}$ and $T \sim \mathrm{QCD}$, γ becomes $\gamma \sim 10^9$

AIM OF OUR PROJECT

- What we do when we predict GWs in particle physics models:

Particle physics model → Parameters relevant to phase transition → Prediction on GWs

$\mathcal{L}$

- Released energy (i.e. α)
- Nucleation rate (i.e. β)
- Transition temperature ... and so on

$\rho_{\rm GW}$

- To prepare for future observations, we have to understand [ ] well

- Currently, understanding on [ ] is mainly driven by numerical simulations

- However, numerical approach alone does not give good understanding of the system

→ We would like to develop an alternative approach e.g. CMB, Lattice QCD, ...

GW SPECTRUM AS EM TENSOR CORRELATOR

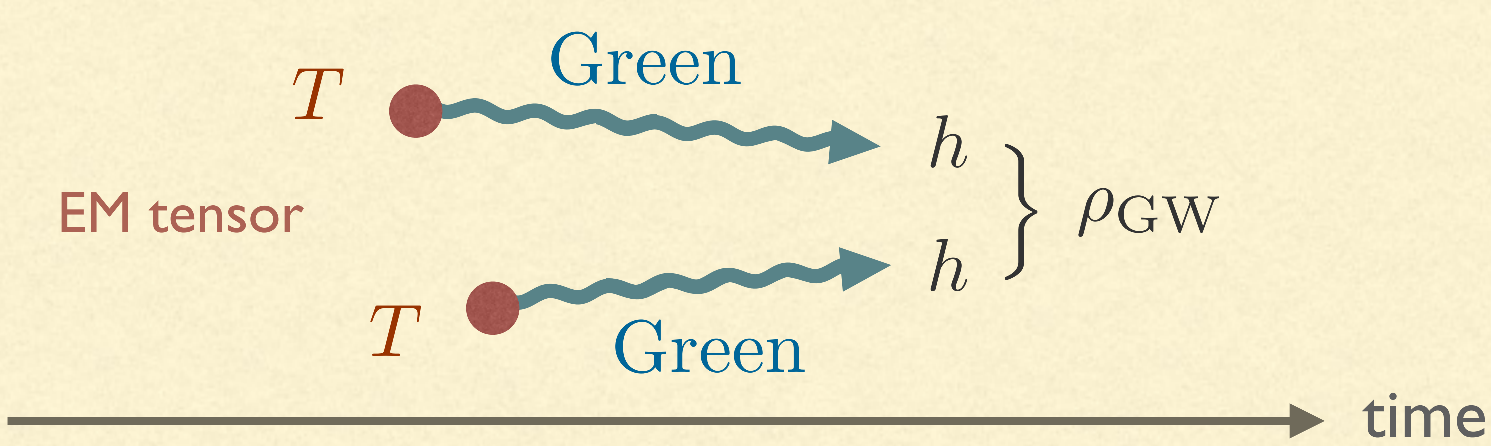
- Master formula:

[e.g. Caprini et al., PRD77 (2008)]

$$\rho_{\rm GW}(k) \sim \int dt_x \int dt_y \cos(k(t_x - t_y)) {\rm F.T.} \langle T_{ij}(t_x, \vec{x}) T_{ij}(t_y, \vec{y}) \rangle$$

GW energy density per each log wavenumber k — Green function — EM tensor

- Why? GW EOM : $\Box h \sim T$ → solution : $h \sim \int dt \; {\rm Green} \times T$



GW SPECTRUM AS EM TENSOR CORRELATOR

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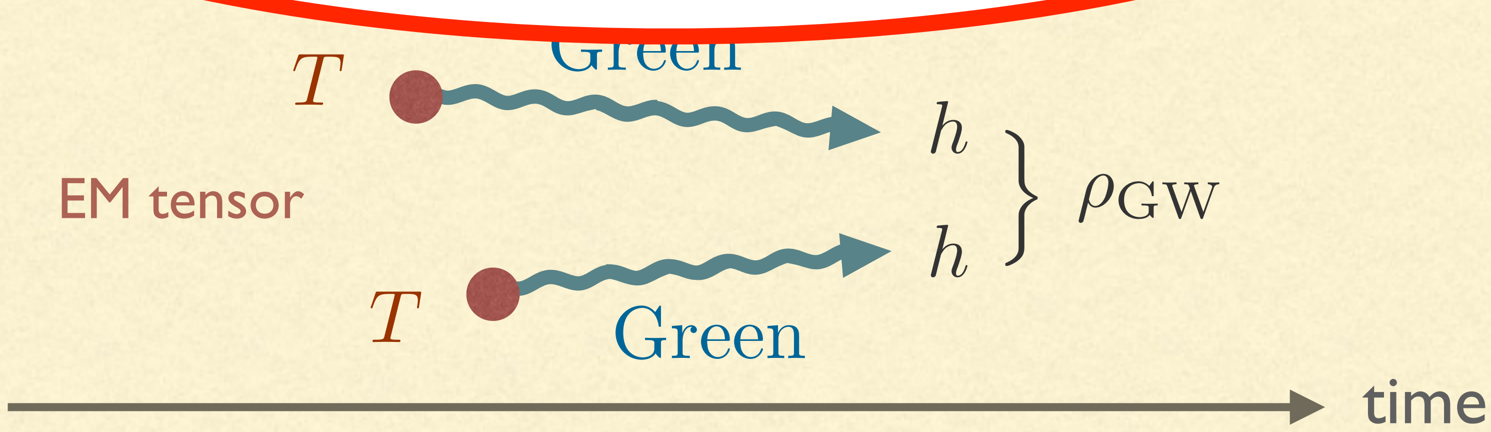
$$\rho_{\rm GW}(k) \sim \int dt_x \int dt \ldots \langle T_{ij}(t_x, \vec{x}) T_{ij}(t_y, \vec{y}) \rangle$$

GW

per e

- Why? G

GW spectrum is essentially
two-point ensemble average
< T T >



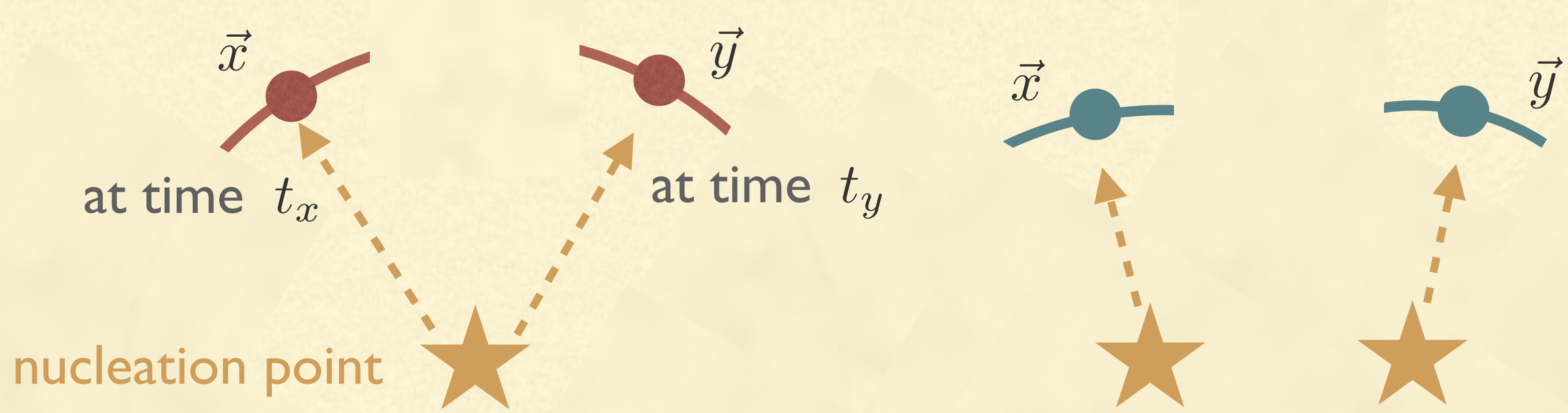
CALCULATION OF $\langle TT \rangle$

[Jinno & Takimoto '16 & '17]

- Calculating $\langle T(t_x,\vec{x})T(t_y,\vec{y})\rangle_{\rm ens}$ means ...

  - Fix spacetime points $x=(t_x,\vec{x})$ and $y=(t_y,\vec{y})$

  - Find bubble configurations s.t. EM tensor T is nonzero at x & y

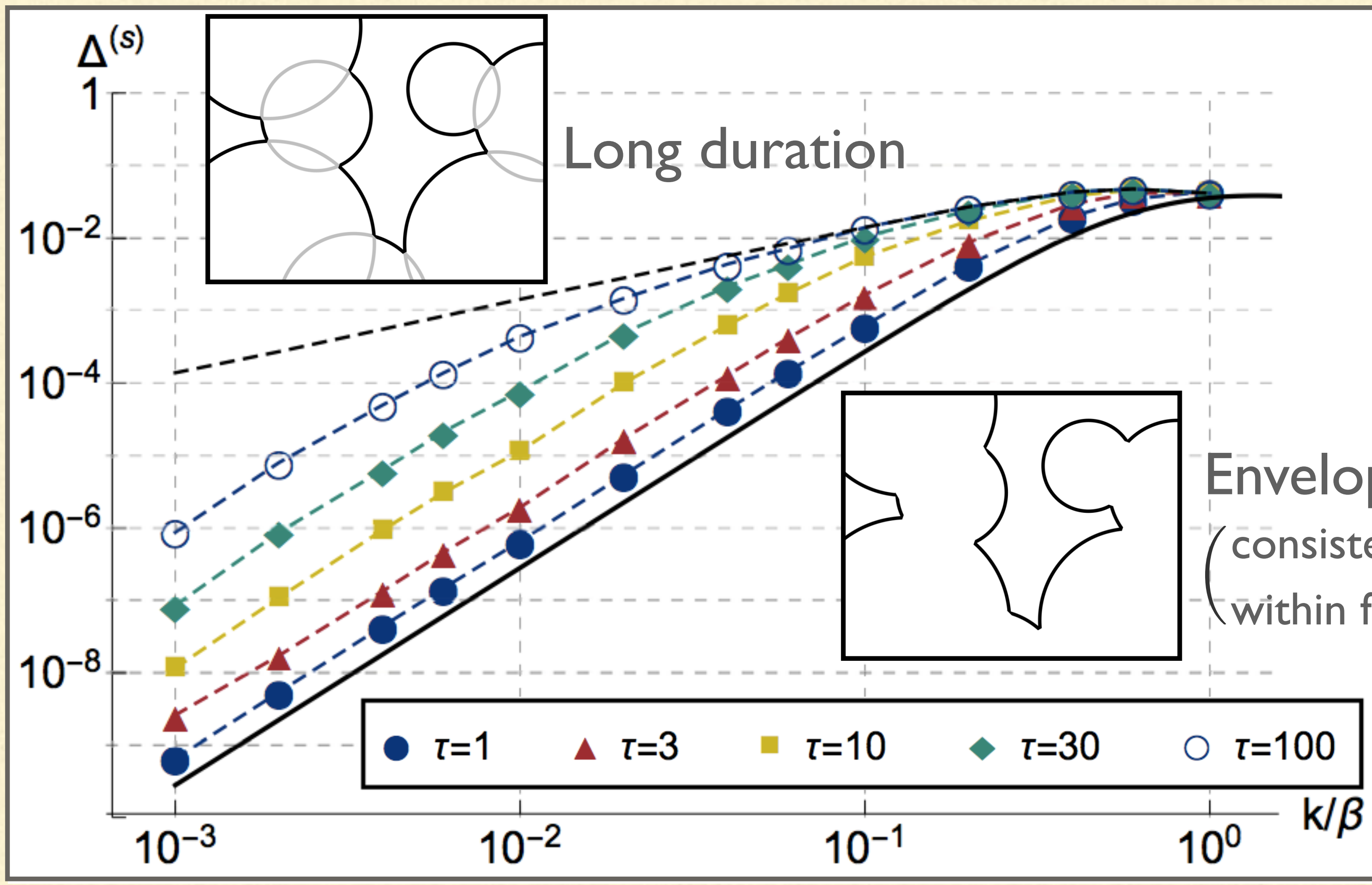


  - Calculate $\left\{ \begin{array}{c} \text{probability} \\ \text{value of } T(t_x,\vec{x})T(t_y,\vec{y}) \end{array} \right\}$ for such configurations and sum up

NUMERICAL PLOT

- Single-bubble spectrum $\Delta^{(s)}$ (Damping function $D = e^{-(t-t_i)/\tau}$, t_i : collision time)

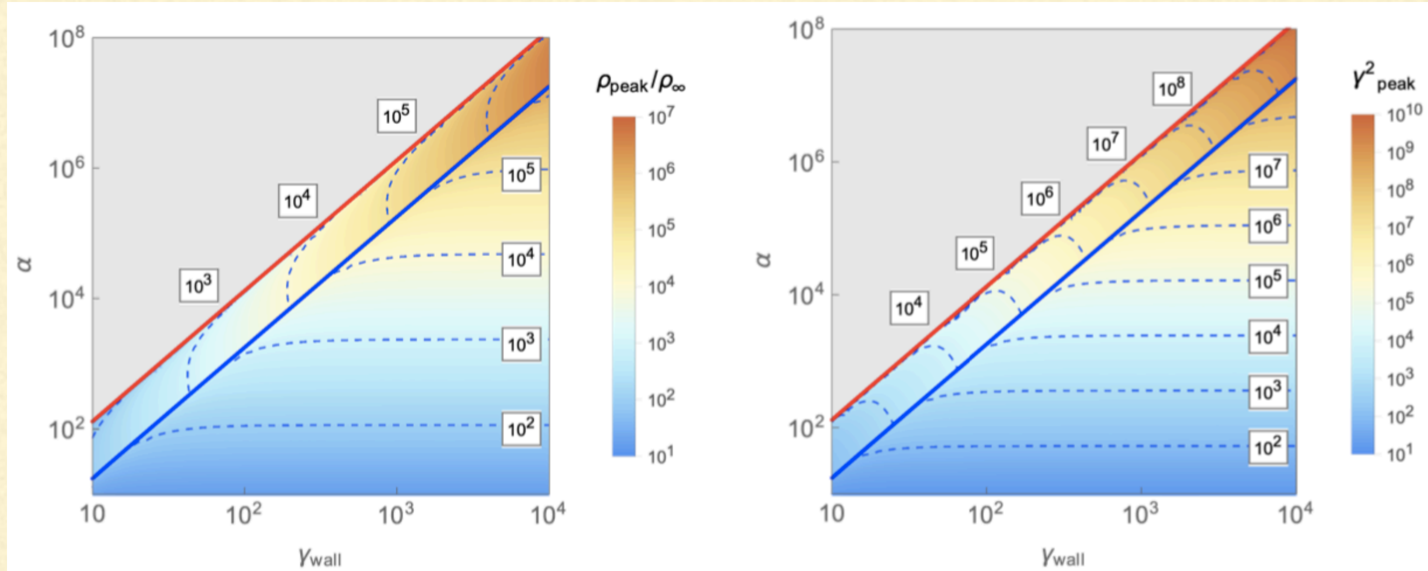
GW spectrum

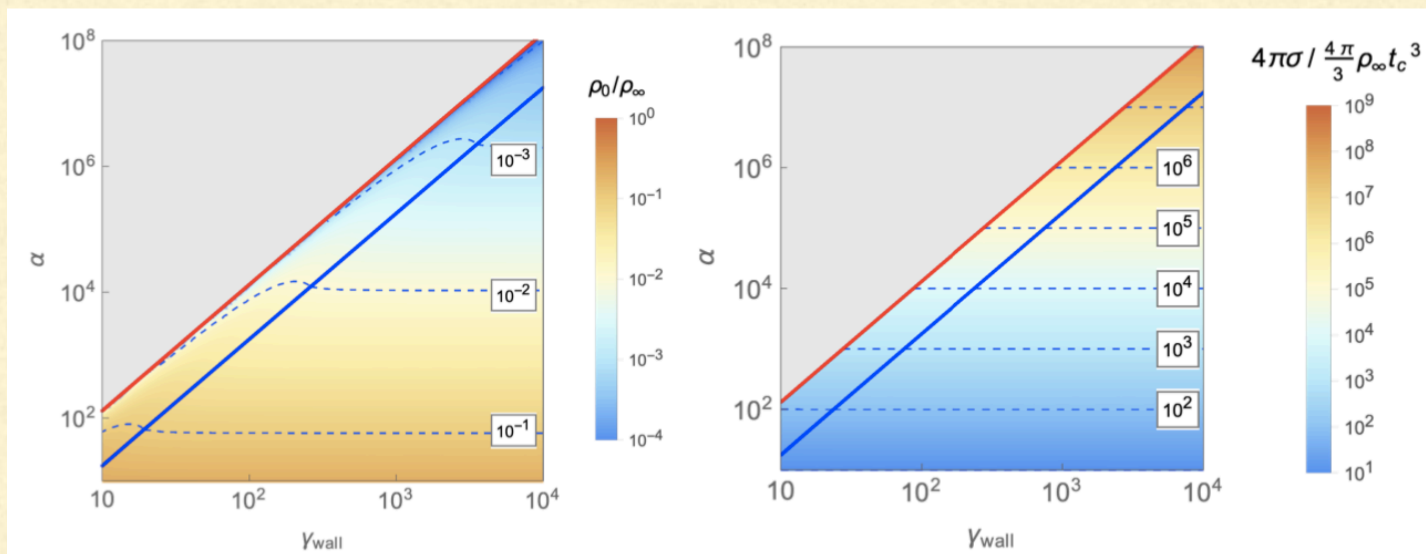


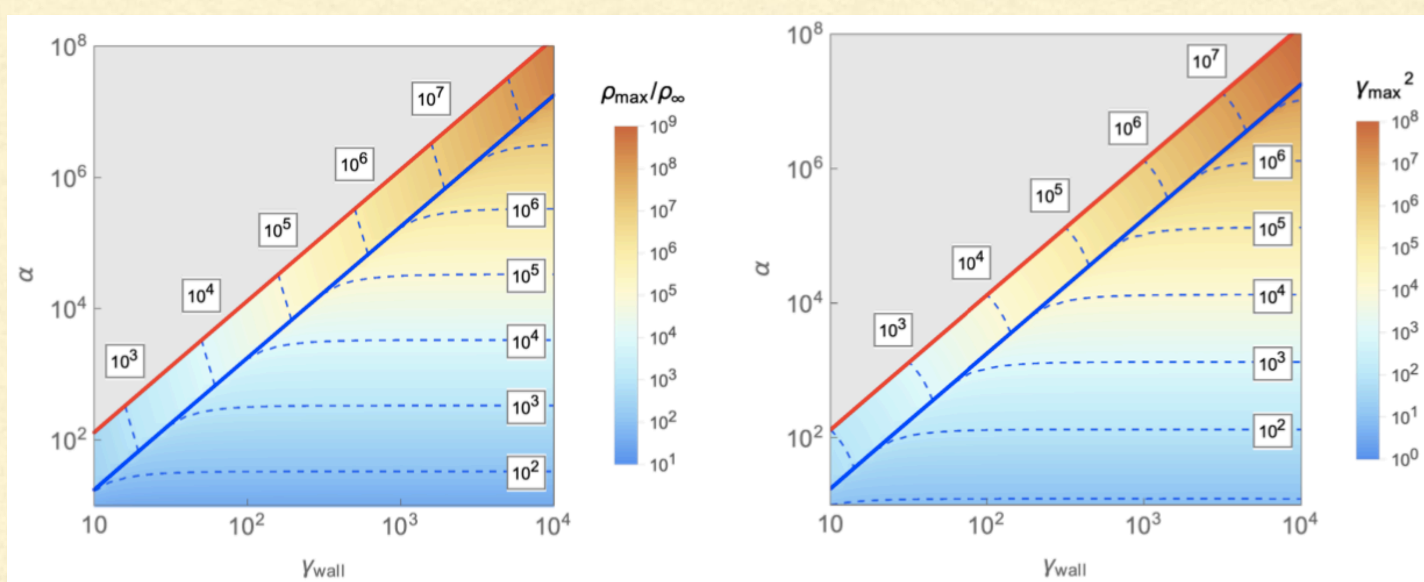
Envelope

(consistent with [Huber & Konstandin '08] within factor 2)

GW wavenumber


α
γwall
ρpeak/ρ∞
γ²peak
10³
10⁴
10⁵
10⁶
10⁷
10⁸
10²


α
γwall
ρ0/ρ∞
10⁻¹
10⁻²
10⁻³
4πσ / (4π/3) ρ∞ tc³
10²
10³
10⁴
10⁵
10⁶


α
γwall
ρmax/ρ∞
γmax²
10²
10³
10⁴
10⁵
10⁶
10⁷