Cold Quantum Coffee
30.06.2020

# QUANTUM SIMULATION OF LGT

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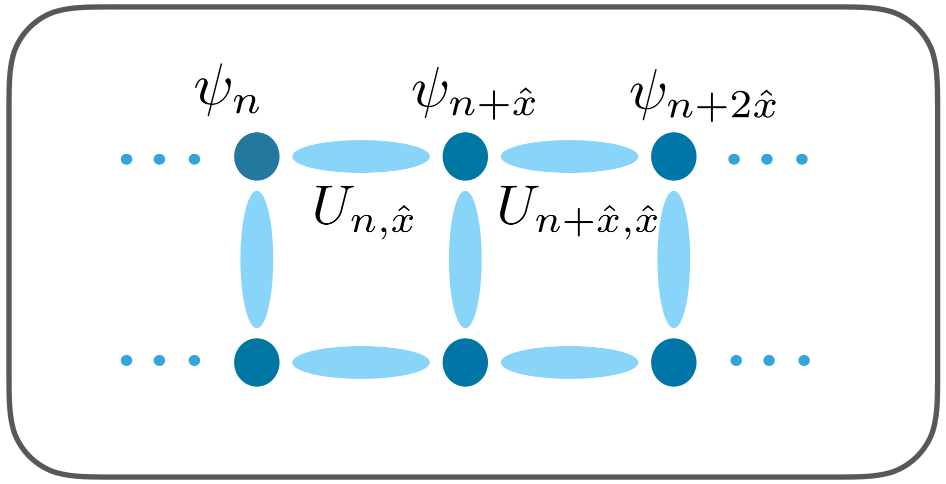
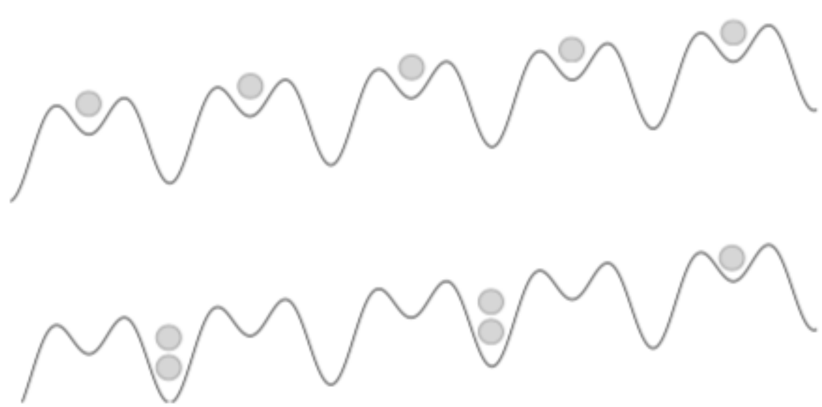
# OUTLINE

1. Lattice gauge theory & QS



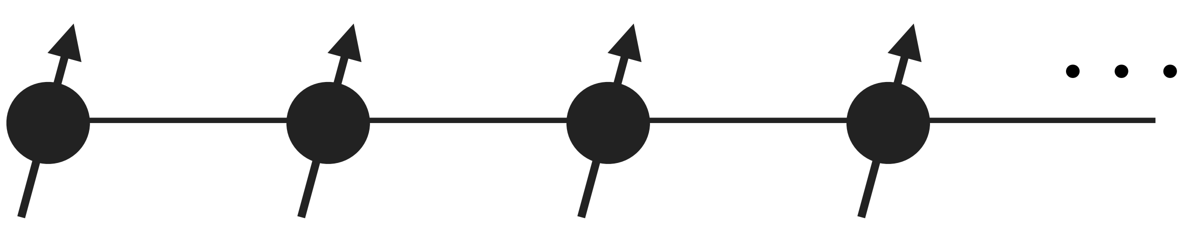

2. QS of U(1) quantum link model

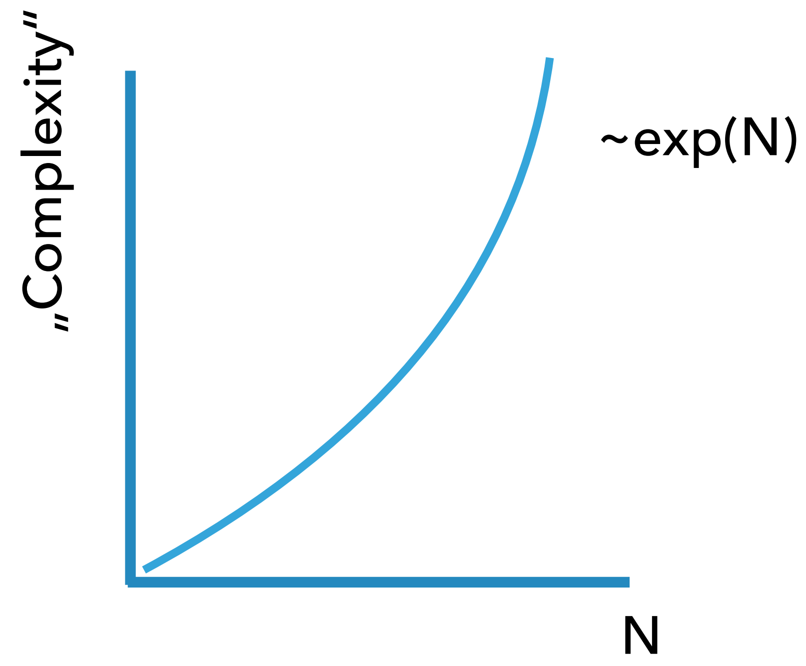


„And I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

*-Richard Feynman,* `Simulating Physics with Computers'

E.g.: N Spins-1/2 on a chain





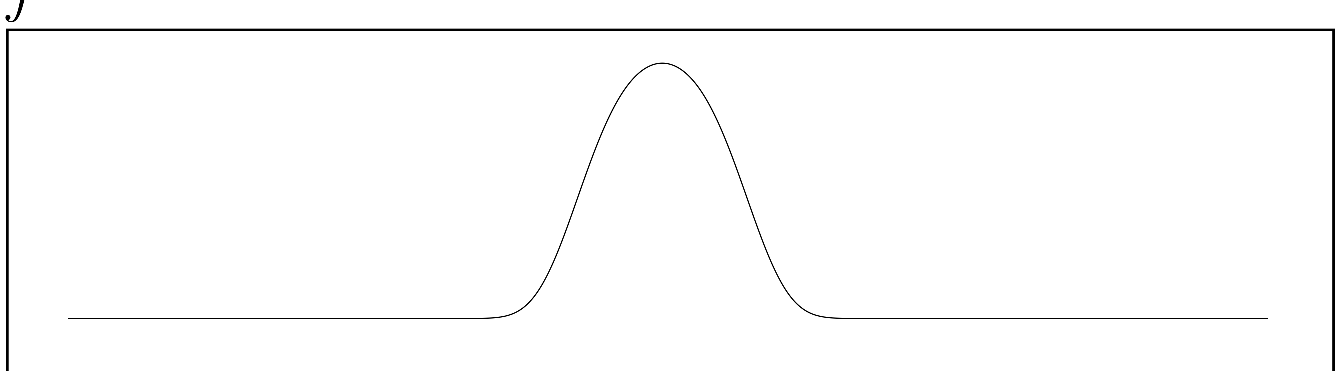
Path integral:

Monte Carlo integration of: $\int \mathcal{D}\phi \; e^{-S[\phi]}$

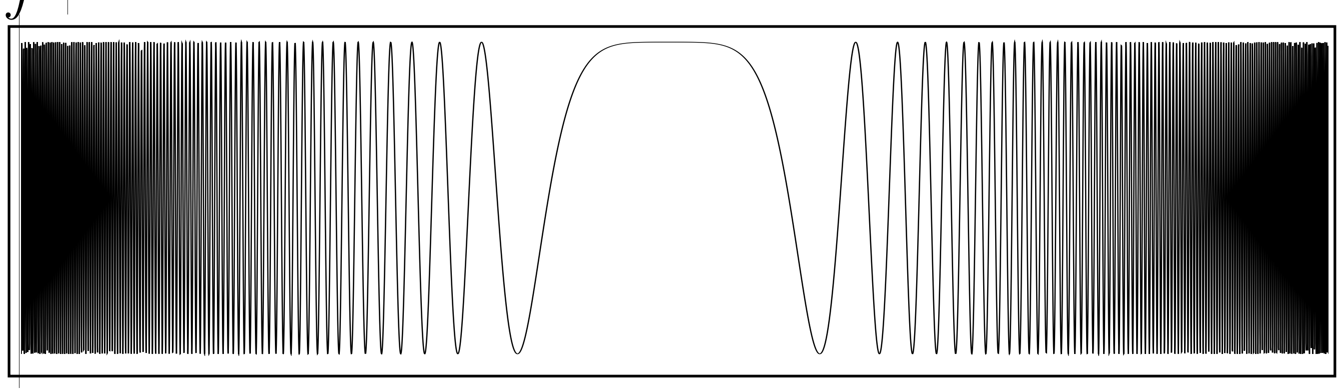
Monte Carlo integration of: $\int \mathcal{D}\phi \; e^{iS[\phi]}$

**Sign problem!**

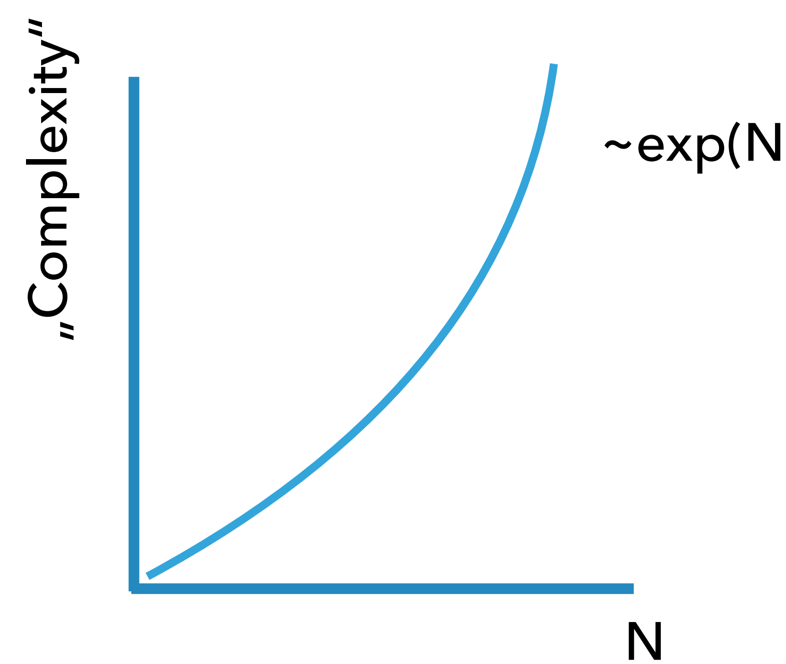
$\int \mathcal{D}\phi \; e^{-S[\phi]}$



$\int \mathcal{D}\phi \; e^{iS[\phi]}$



**Again:**



**Real-time/out-of-equilibrium phenomena:**

Schwinger Pair Production

Heavy Ion Collisions

Thermalization

. . .

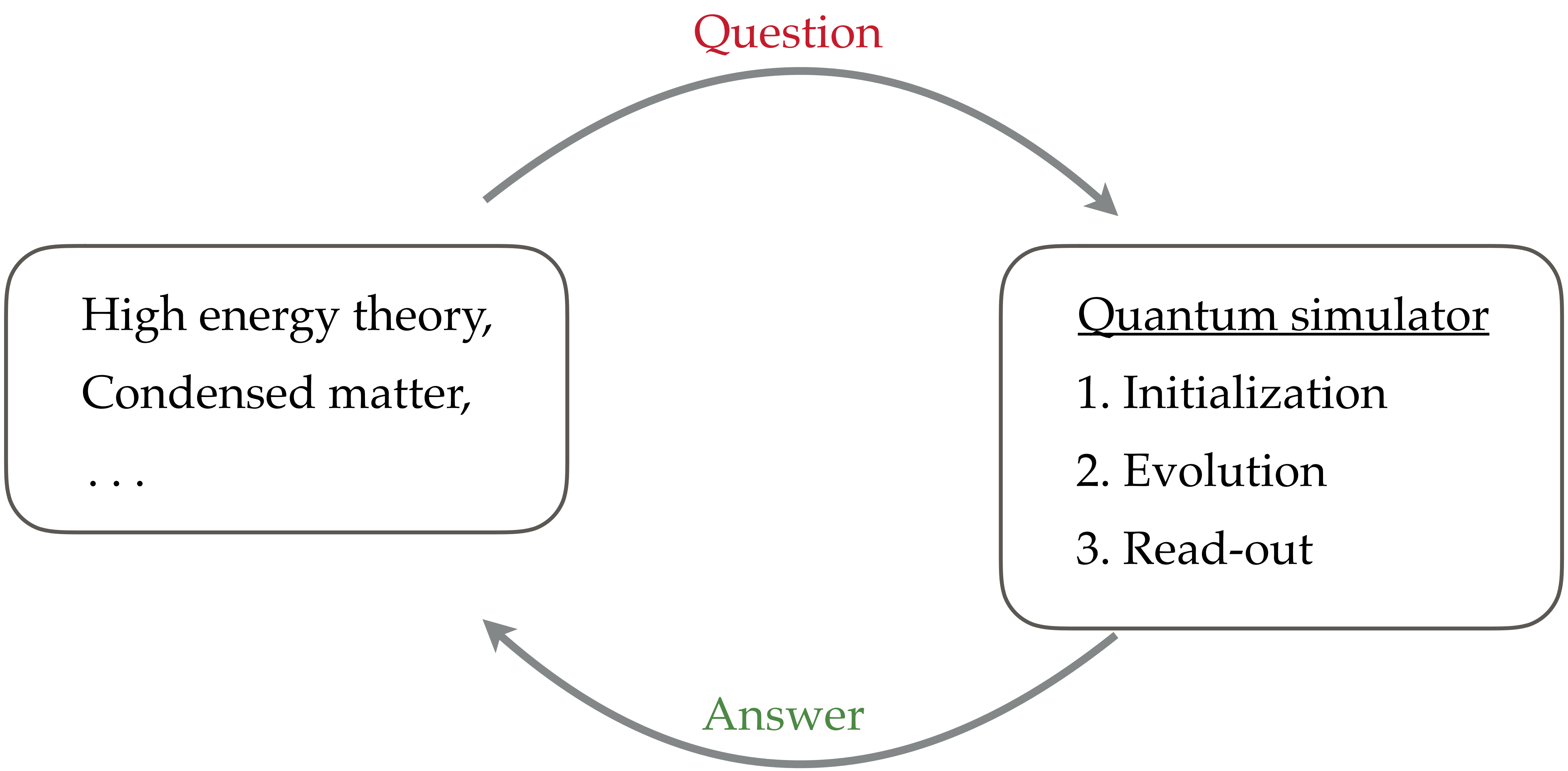
**Equilibrium phenomena:**

QCD phase diagram

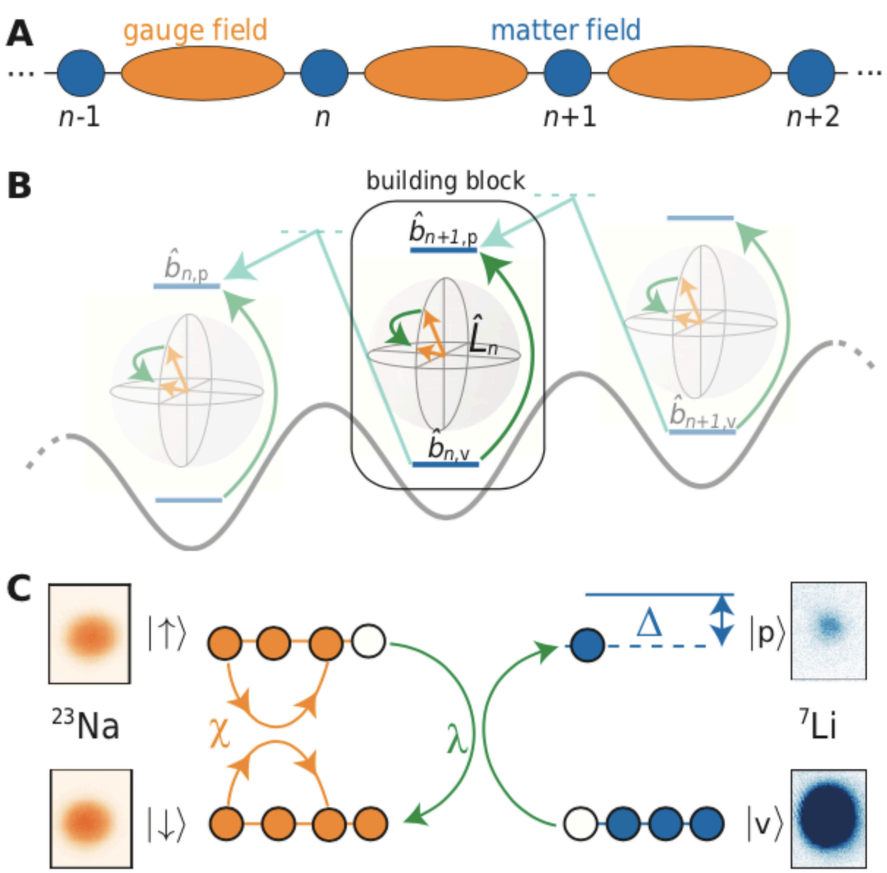
High $T_c$ superconductivity

. . .

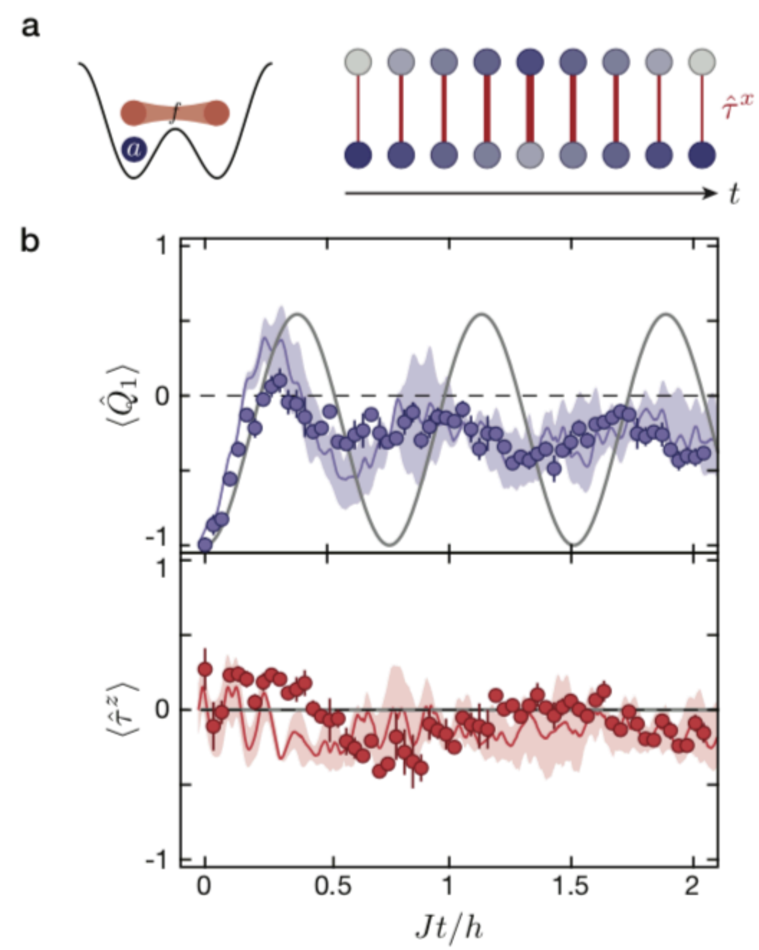
Quantum Simulation scheme:



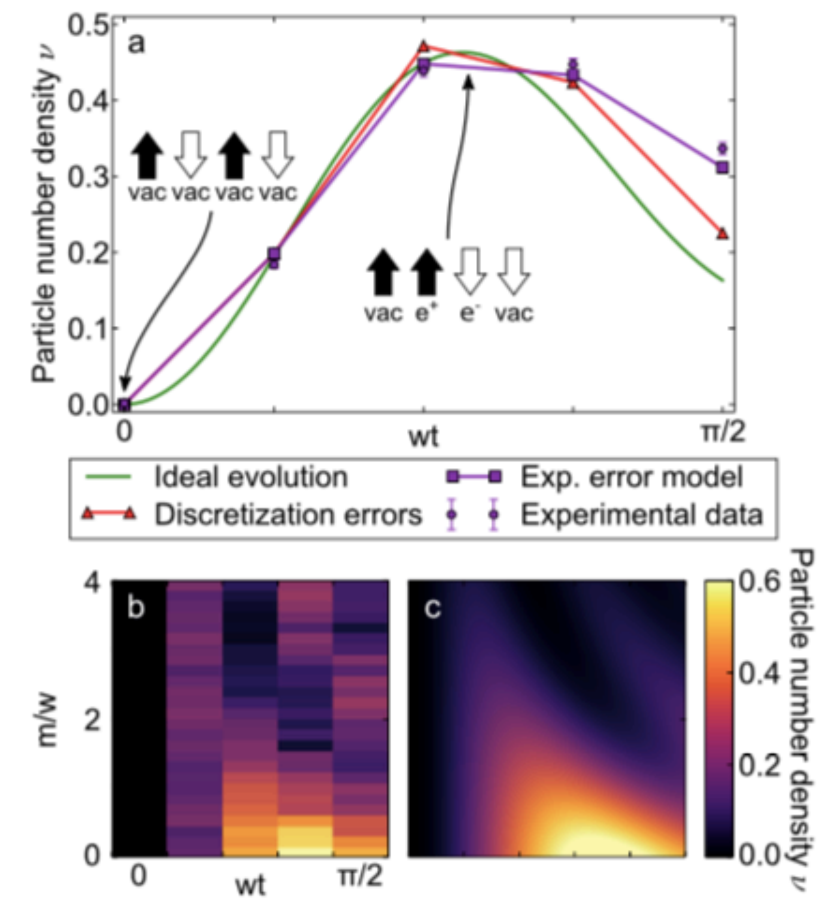
# Notable experiments:




Mil, Zache, Hedge, Xia, Bhatt, Oberthaler, Hauke, Berges and Jendrzejewski *Science* 367.6482 (2020): 1128-1130.




Schweizer, Grusdt, Berngruber, Barbiero, Demler, Goldman, Bloch and Aidelsburger, *Nature Physics* 15.11 (2019): 1168-1173.




Martinez, Muschik, Schindler, Nigg, Erhard, Heyl, Hauke, Dalmonte, Monz, Zoller and Blatt, *Nature* 534.7608 (2016): 516-519.

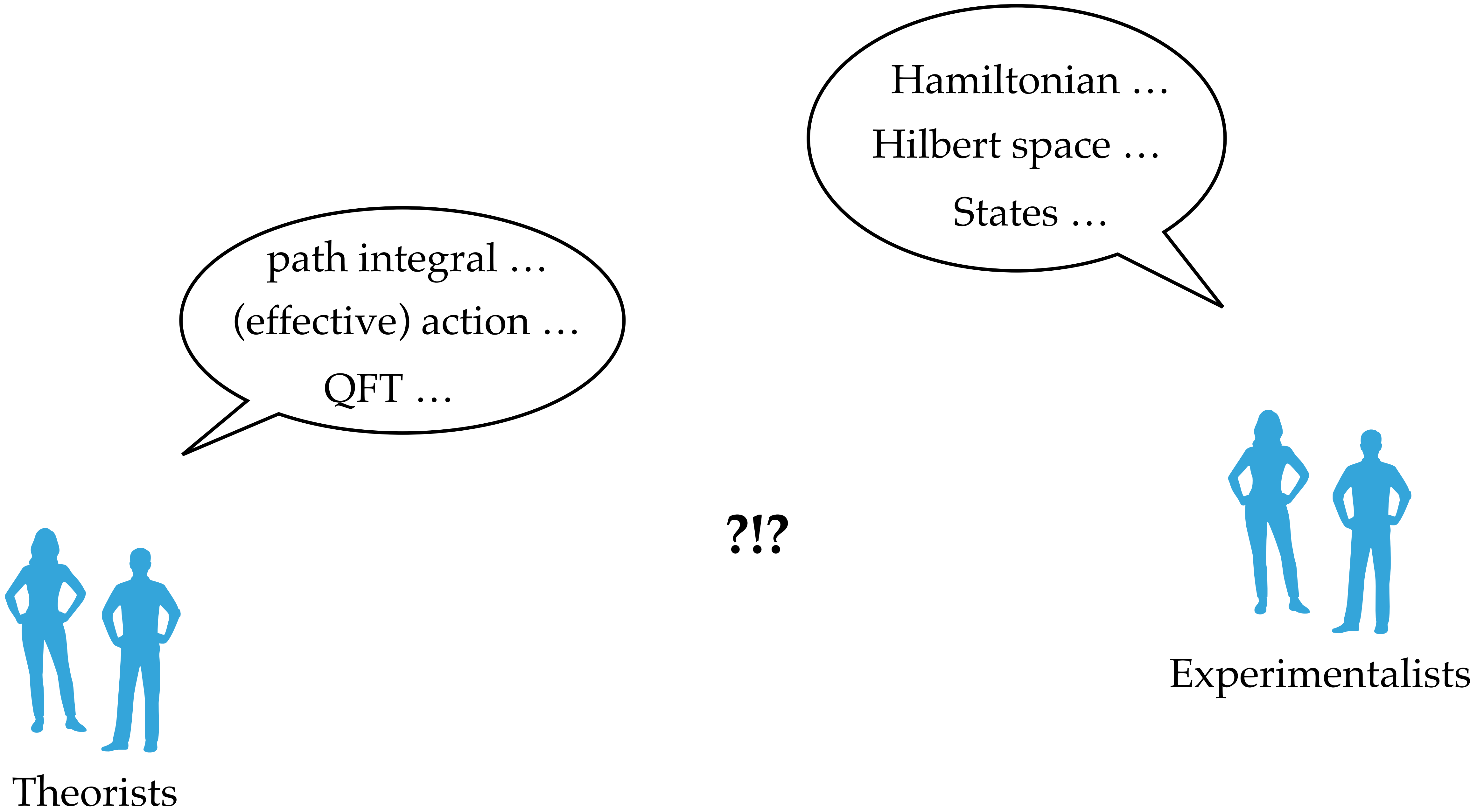
How to get QED into a quantum simulator? ⟶ **Identification of d.o.f.**

**Example** for cold atoms:

| Lattice Gauge Theory | mapping ⟶ | Quantum Simulator |
|---|---|---|
| space, time | ⟶ | optical Lattice,<br>Hamiltonian time evolution |
| gauge symmetry | ⟶ | angular momentum conservation,<br>effective theory |
| gauge fields | ⟶ | bosonic atoms |
| fermions | ⟶ | fermionic atoms |

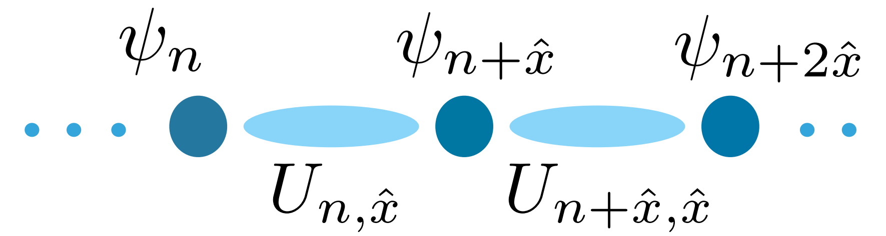
**In general**: Any kind of mapping, which (in a certain limit) reproduces the original theory.

Identification



Hamiltonian of QED in **one** (spatial) **dimension**?

Start from lattice field theory:




Discretize space: (for naive fermions here)

$$\psi_n^\dagger \partial_x \psi_n \xrightarrow{\text{lattice}} \frac{1}{2a}\psi_n^\dagger \left(\psi_{n+1} - \psi_{n-1}\right)$$

Gauge transformations:

$$\psi_n \to e^{-ie\alpha_n}\psi_n \qquad \text{i.e.} \quad \frac{1}{2a}\psi_n^\dagger \left(\psi_{n+1} - \psi_{n-1}\right) \quad \text{breaks gauge symmetry!}$$

Introduce gauge links:

$$U_{n,n+1} \to e^{ie\alpha_n} U_{n,n+1} e^{ie\alpha_{n+1}} \quad \text{as} \quad \frac{1}{2a}\psi_n^\dagger \left(U_{n,n+1}\psi_{n+1} - U_{n,n-1}\psi_{n-1}\right)$$

electric field: $[E_{n,n+1}, U_{m,m+1}] = e\delta_{nm}U_{m,m+1}$

$$\psi_n \to e^{-ie\alpha_n}\psi_n$$
$$U_{n,n+1} \to e^{ie\alpha_n} U_{n,n+1} e^{ie\alpha_{n+1}}$$

Gauge invariant Hamiltonian:

$$H = \frac{1}{2}\sum_n E^2_{n,n+1} + m\sum_n \psi_n^\dagger\psi_n + \frac{1}{2a}\sum_n(\psi_n^\dagger U_{n,n+1}\psi_{n+1} + h.c.)$$

General gauge transformation:

$$\mathcal{O} \to V^\dagger \mathcal{O} V \quad \text{with} \quad V = e^{i\sum_n \alpha_n G_n}$$

$$G_n = E_{n,n+1} - E_{n,n-1} - ea\psi_n^\dagger\psi_n$$

Gauss law:

$$G_n|\psi\rangle = 0$$

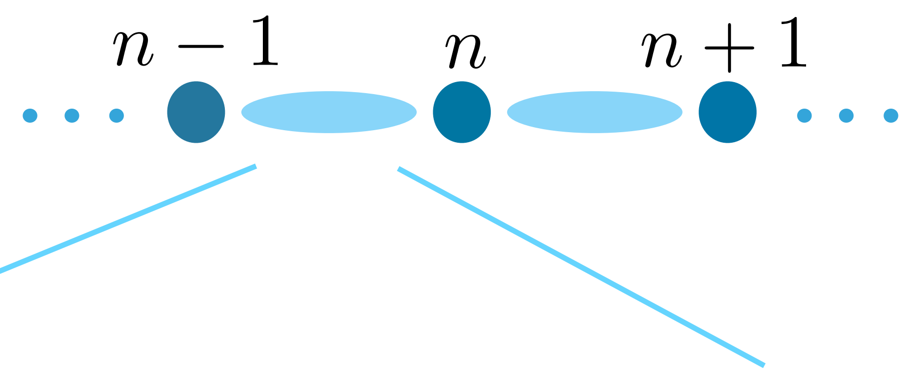
more generally: $G_n|\psi\rangle = q_n|\psi\rangle$

↖ external charges

Gauge invariant dynamics:

$$H \to H \quad \text{i.e.} \quad [H, G_n] = 0 \longrightarrow \boxed{G_n|\psi(t)\rangle = 0}$$

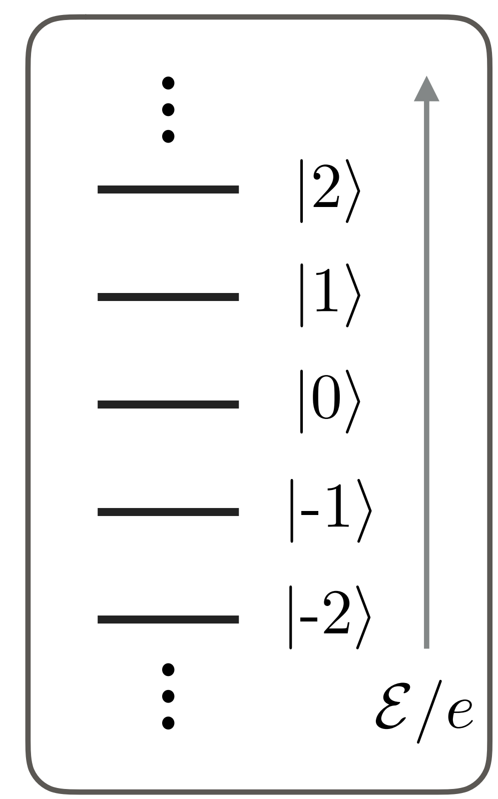
`Single link' Hilbert space:




Electric field basis: $E|\mathcal{E}\rangle = \mathcal{E}|\mathcal{E}\rangle$

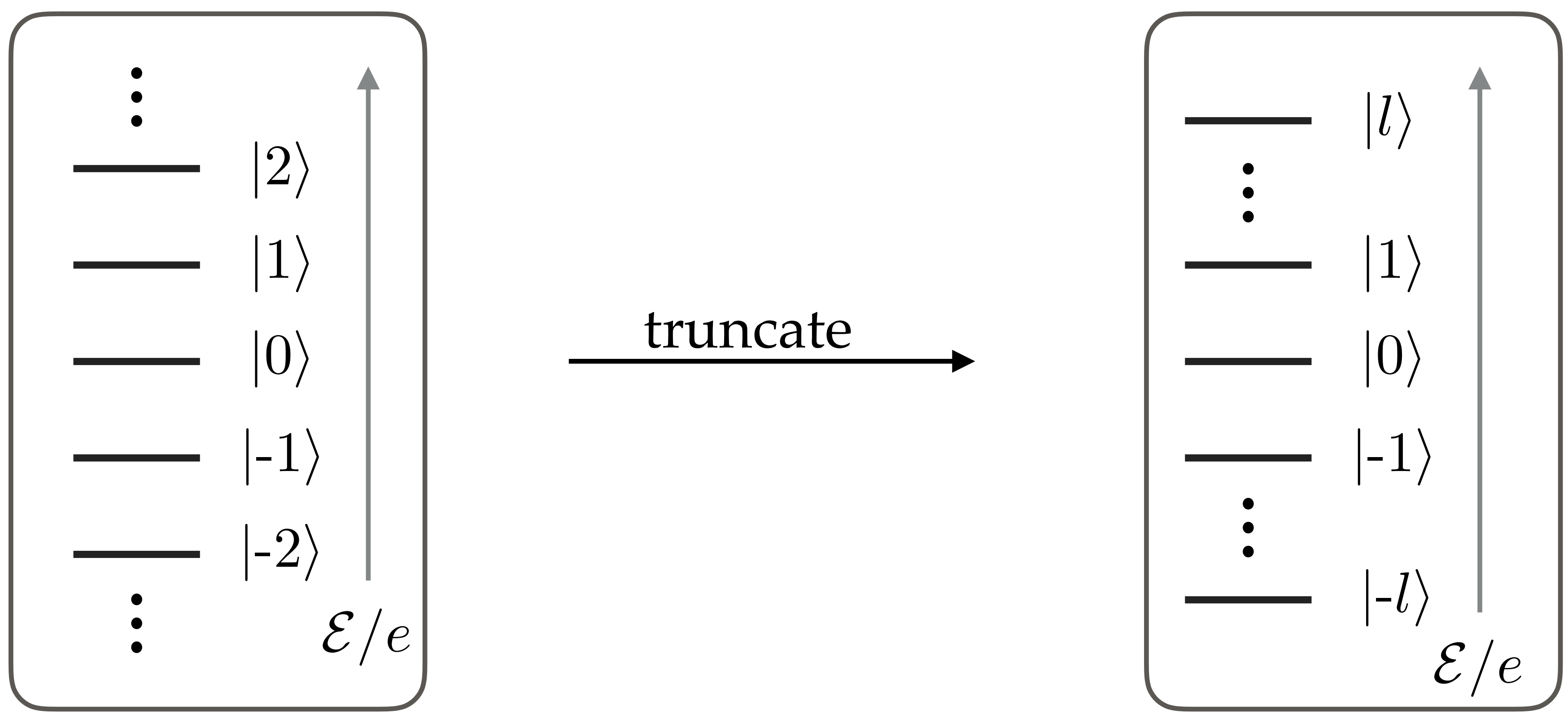
Using $[E, U] = U$: $\longrightarrow$ $U|\mathcal{E}\rangle = |\mathcal{E}+1\rangle$

Hilbert space: $\mathcal{H} = \mathrm{span}\{|\mathcal{E}\rangle \mid \in e\mathbf{Z}\}$



## Quantum link models (QLMs): new “class” of gauge theories

Chandrasekharan, S. & Wiese, U. J.
Nucl. Phys. B 492, 455–471 (1997)



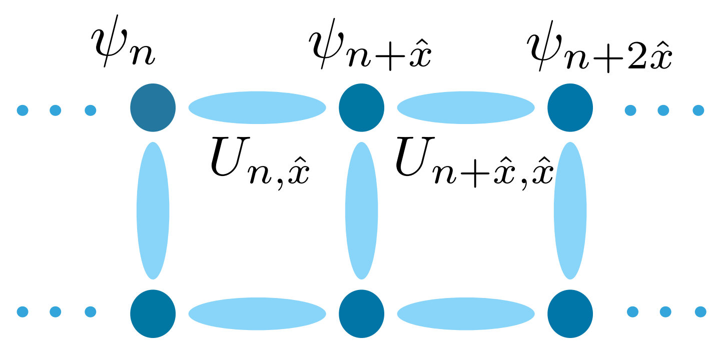

QED QLM

$$E \to eL_z \qquad U \to \frac{L^+}{\sqrt{l(l+1)}}$$

$$H = \frac{1}{2}\sum_n (L^z_{n,n+1})^2 + m\sum_n \psi_n^\dagger \psi_n + \frac{1}{2a\sqrt{l(l+1)}}\sum_n (\psi_n^\dagger L^+_{n,n+1}\psi_{n+1} + h.c.)$$

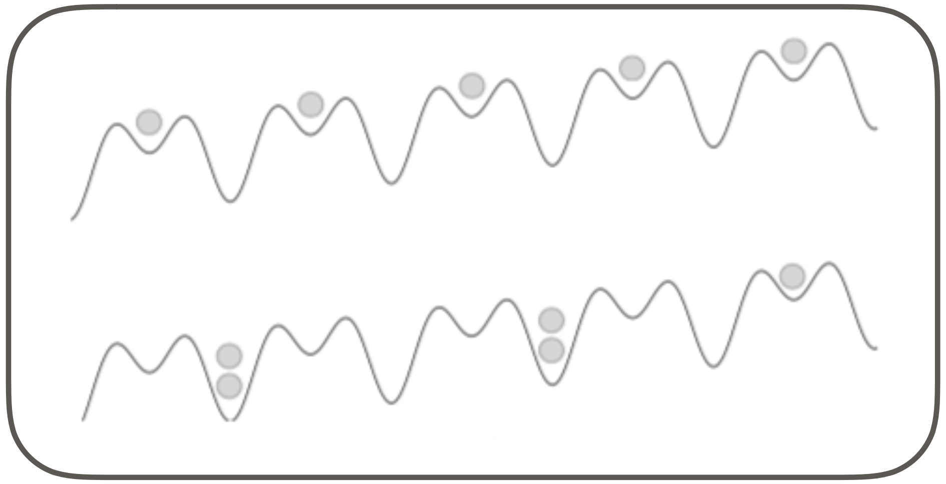
# OUTLINE

1 Lattice gauge theory & QS




2 QS of U(1) quantum link model

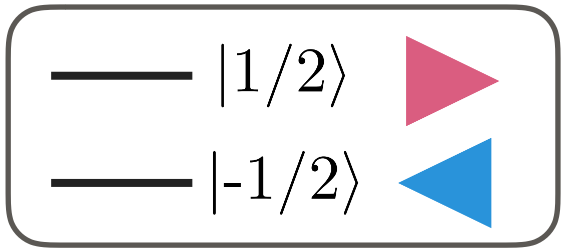
(based on arXiv:2003.08945v1)



# QS of U(1) QLM

$$H = \frac{1}{2}\sum_n (L^z_{n,n+1})^2 + m\sum_n \psi^\dagger_n \psi_n + \frac{1}{2a\sqrt{l(l+1)}}\sum_n (\psi^\dagger_n L^+_{n,n+1}\psi_{n+1} + h.c.)$$

Spin-1/2 link:

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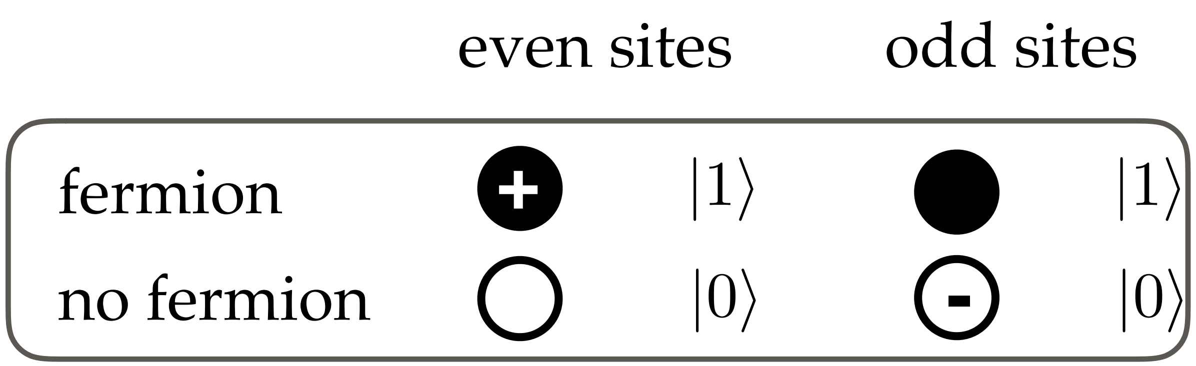
Staggered fermions:

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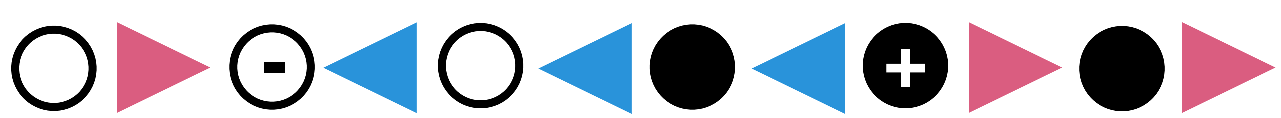
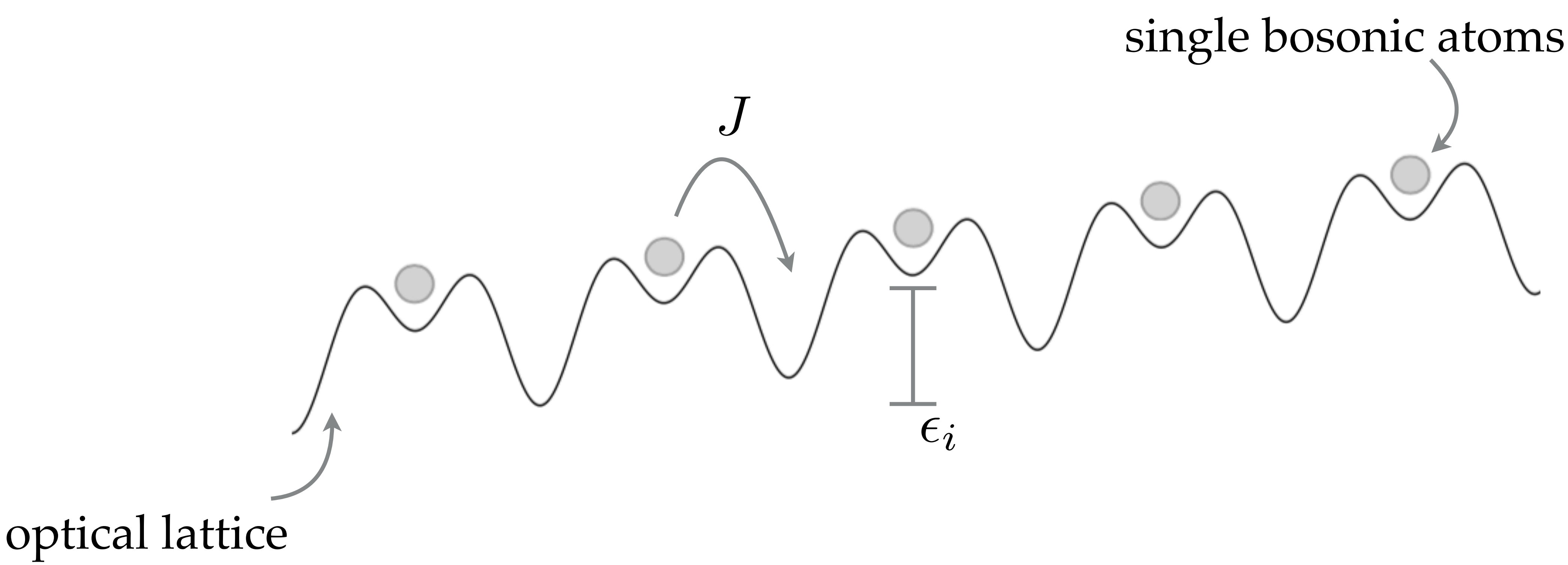
Spin-1/2 **QLM**:

$$H = m\sum_n \psi^\dagger_n \psi_n + \tilde{t}\sum_n (\psi^\dagger_n S^+_{n,n+1}\psi_{n+1} + h.c.)$$

E.g.:

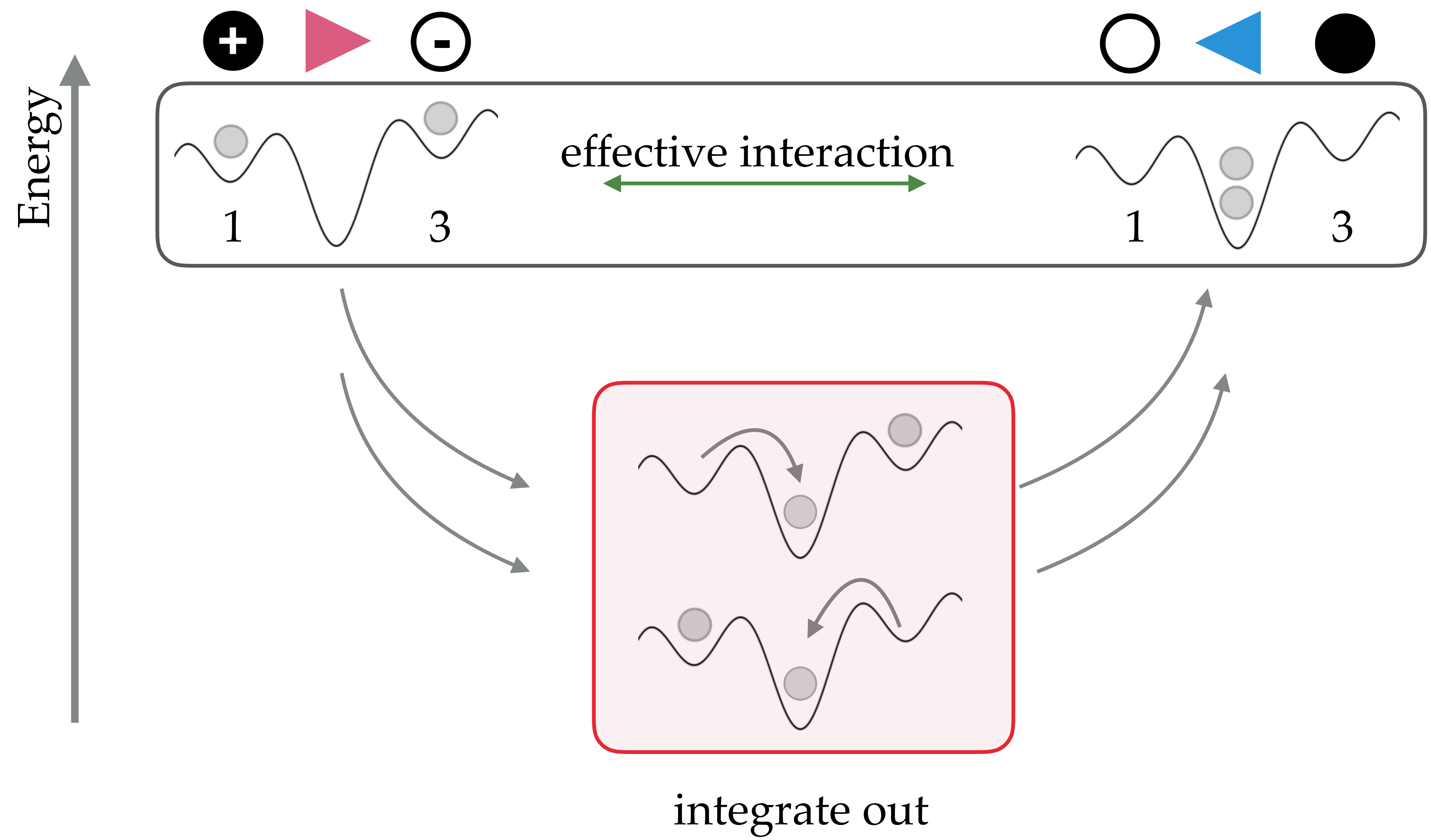


Bose Hubbard model:




$$H_{\rm BH} = J\sum_i (b_i^\dagger b_{i+1} + h.c.) + \sum_i \epsilon_i b_i^\dagger b_i + \frac{U}{2}\sum_i b_i^\dagger b_i^\dagger b_i b_i$$

Unit cell:




effectively →

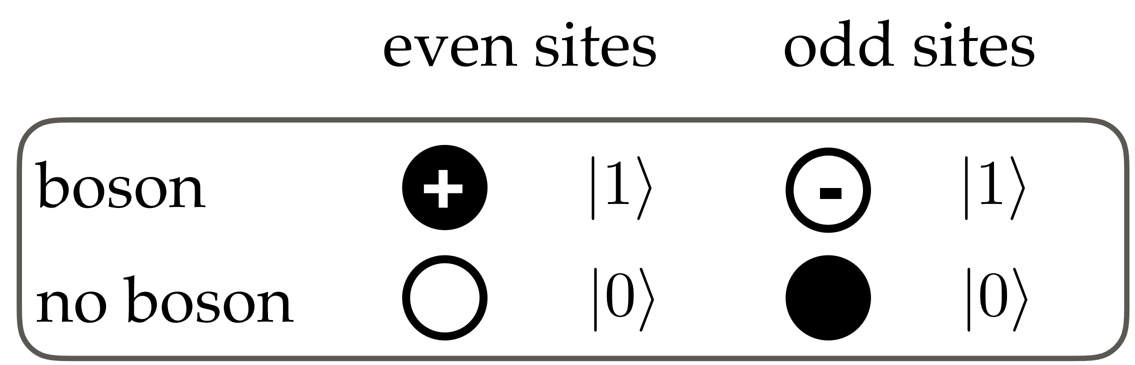
$$H_{\text{eff}} \sim \frac{J^2}{U}(b_1 b_2^\dagger b_2^\dagger b_3 + h.c.) + m(b_1^\dagger b_1 + b_3^\dagger b_3)$$

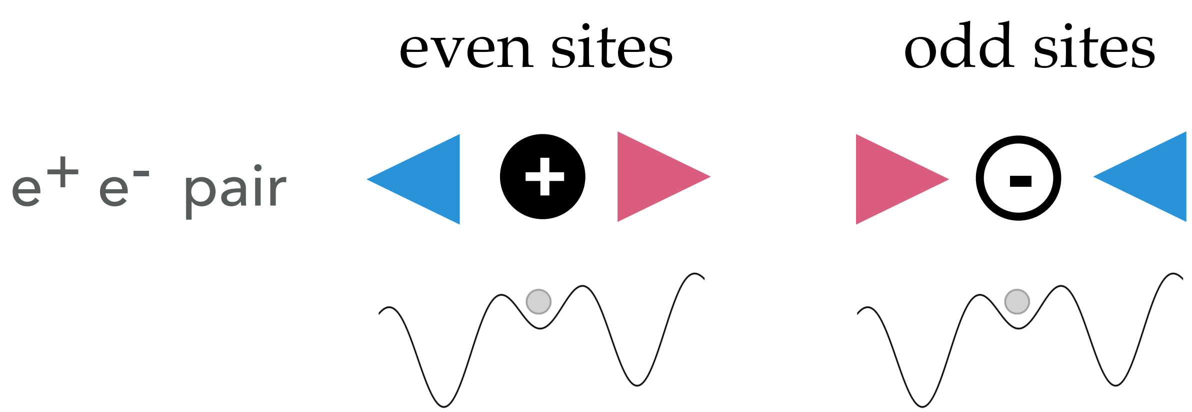
Jordan Wigner transformation

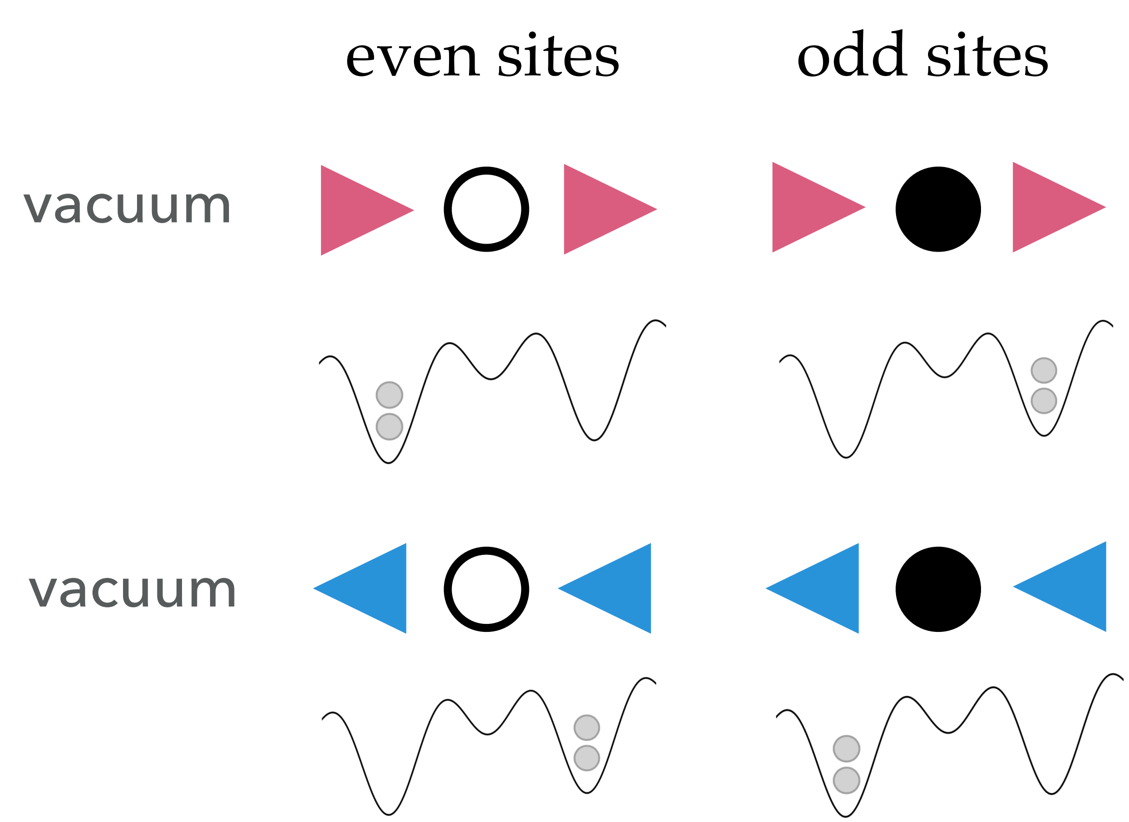
$$\tilde{t}(\psi_1^\dagger S_2^+ \psi_3 + h.c.)$$

energy imbalance $\delta - \frac{U}{2}$

Mapping to Bose-Hubbard model via Jordan Wigner transformation:



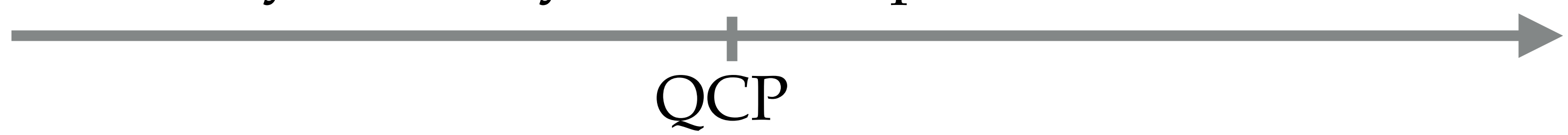




**Gauss law**

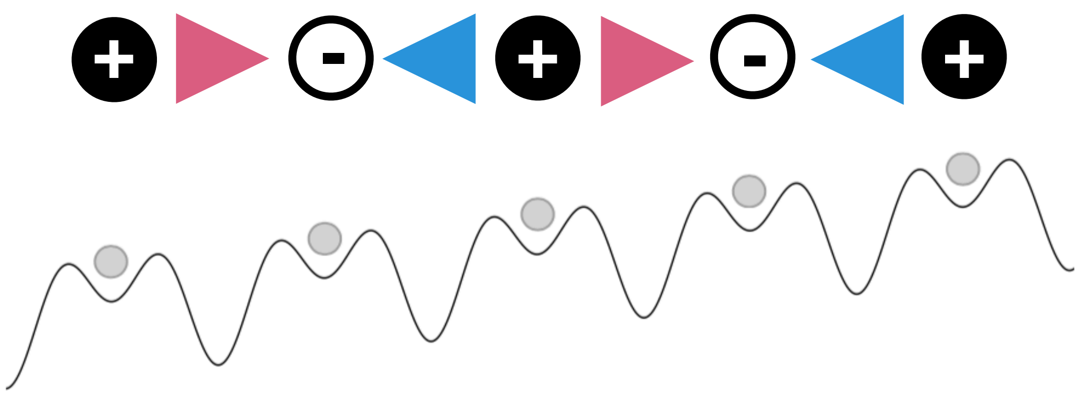
$$G = \nabla E - \rho = 0$$

Many-body system:

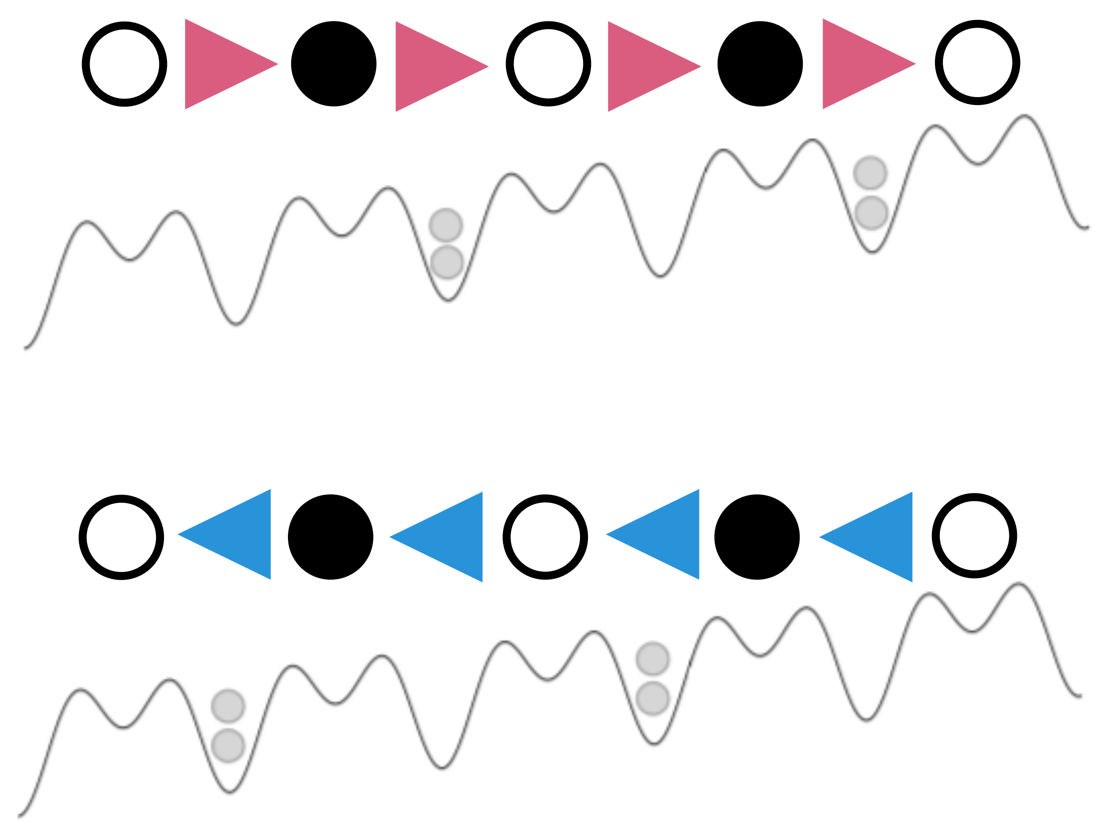
dynamically tune mass parameter




ground state at $m \to -\infty$

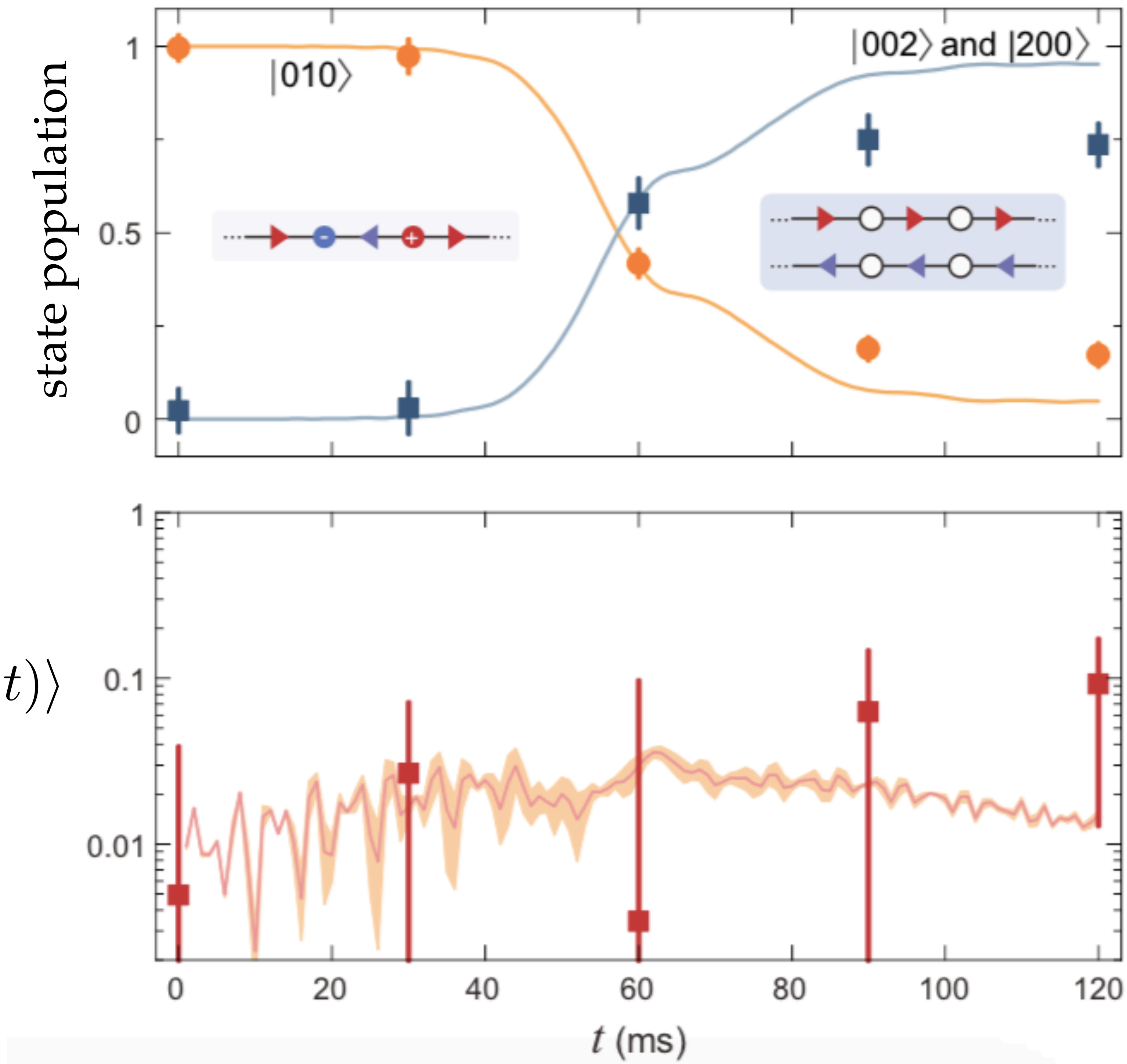


2 degenerate ground states at $m \to \infty$



Experimental and numerical results:

(arXiv:2003.08945v1)




$$\epsilon(t) = 1 - \frac{1}{N}\sum_i \langle\psi(t)|P_i|\psi(t)\rangle$$

projects onto states with $G_i|\psi\rangle = 0$

Summary:

Non-equilibrium dynamics is hard — Quantum Simulation?

We performed a quantum simulation of an extended U(1) gauge theory

Extension to higher dimensions, more complicated symmetry groups, etc.

Certification? How can we be sure the symmetry is there?

What other platforms can be used?

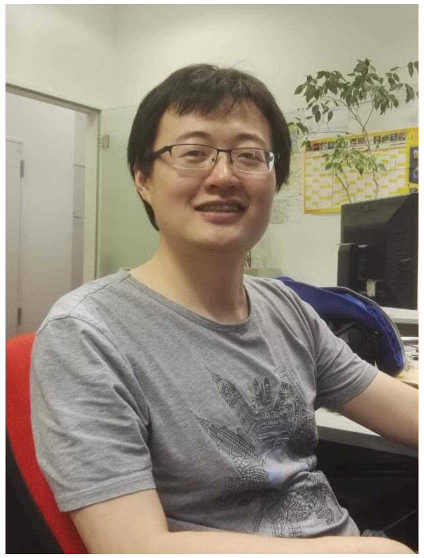
# THANK YOU FOR YOUR ATTENTION!


Torsten V. Zache (ITP)


Philipp Hauke (UniTN)


Jürgen Berges (ITP)


Bing Yang (PI HD)


Hui Sun (PI HD)


Jad Halimeh (UniTN)