Natural Mass Hierarchy in Potts-Yukawa Systems

& Its Implementations in Asymptotic Safety

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07.07.2020

Outline

- Motivation
- Statement of our Idea
- Introducing the Potts-Yukawa System (PYS)
- Inclusion of Gravitational Effects
- Discussion

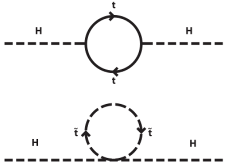
Motivation

Standard Model Limitations

Hierarchy problems

- Loop corrections to the higgs mass e.g,

$$\Delta M_{\text{H}} = -\frac{|\lambda_{\text{top}}|^2}{8\pi^2}\left[\Lambda_{\text{UV}}^2 + \cdots\right]$$



- Wide range of elementary particles' masses,

$$\frac{m_e}{m_\tau} \approx 10^{-6}, \quad \frac{m_{\text{H}}}{m_{\text{P}}} \approx 10^{-17}$$

SM :

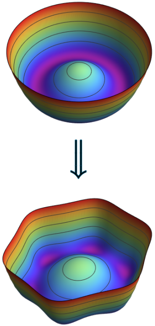
Effective field theory $\Rightarrow$ Limited validity at high UV (quantum triviality problem)

No widely accepted implementation of quantum gravity

Statement of our Idea

"Another" approach to the hierarchy problem

- Potential Expansion :
  Break $U(1)$-symmetry to a $\mathbb{Z}_n$-symmetry
- Goldstone boson $\rightarrow$ pseudo-Goldstone boson
  w/ non-vanishing mass $\left(\frac{m_T}{m_L} \ll 1 \text{ depending on } \mathbb{Z}_n\right)$
- Using FRG : flow from a UV-cutoff to IR. Why FRG ?
  - Inclusion of canonically irrelevant couplings
  - Dynamical generation of masses in the flow



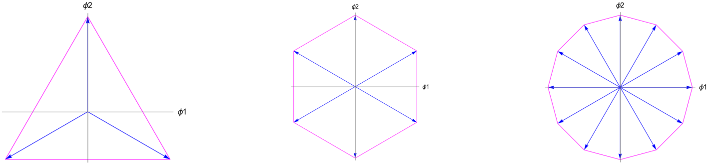
- Is it possible to Extend this mechanism to arbitrarily high scales ?
  $\Rightarrow$ Modify theory & search for an interactive, UV fixed point

The Potts-Yukawa System (PYS)

Scalar Invariants

We start w/ a complex field $\phi = \frac{1}{\sqrt{2}}(\phi_1 + \mathrm{i}\phi_2)$

- Projecting $\{\phi_1, \phi_2\}$ onto the unit vectors e^α : $\psi^\alpha \equiv e_i^\alpha \phi_i$



$\mathbb{Z}_3$ $\mathbb{Z}_6$ $\mathbb{Z}_{12}$

- Invariants constructed as power sum symmetric polynomials

$$P_k = \sum_\alpha (\psi^\alpha)^k$$

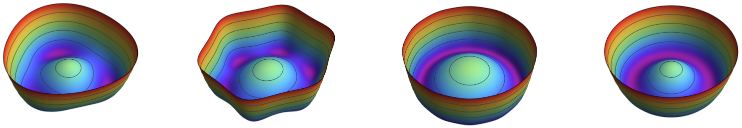
- We choose the linear combination,

$$\rho = \phi^* \phi \,, \quad \sigma_n = (\phi^{*\,n} + \phi^n) + (-1)^{n+1}\, 2\, \rho^{\frac{n}{2}}$$

$$\rho = \phi^*\phi\,, \quad \sigma_n = ({\phi^*}^n + \phi^n) + (-1)^{n+1}\, 2\, \rho^{\frac{n}{2}}$$

Expanding the potential,

$$U^{\mathbb{Z}_n}[\rho, \sigma_n; x] = \lambda_2(\rho - \kappa) + \frac{\lambda_4}{2}(\rho - \kappa)^2 + g_n\sigma_n + \cdots \quad \begin{cases} \lambda_2 = 0 & \text{SSB regime} \\ \kappa = 0 & \text{SYM regime} \end{cases}$$



$\mathbb{Z}_3$ $\mathbb{Z}_6$ $\mathbb{Z}_{12}$ $\mathbb{Z}_{n\to\infty} \sim \mathcal{U}(1)$

W/ Longitudinal and Transverse masses *{in SSB}*

$$\boldsymbol{m_L^2 = 2\lambda_4\kappa \quad , \quad m_T^2 = n^2\kappa^{\frac{n}{2}-1} g_n}$$

$\boldsymbol{\kappa}$ **:** needs fine-tuning, $\boldsymbol{\lambda_4}$ **:** runs logarithmically, $\boldsymbol{g_n}$ **:** $(n > 4)$ faster than logarithmically

Microscopic (UV) Theory $\longrightarrow$ Macroscopic (IR) Theory :

The Functional Renormalization Group

The Functional Renormalization Group

Wilson's Idea : Integrate out modes along infinitesimal momentum shells.

$$\underbrace{\textbf{Full Effective action } \Gamma[\phi]}_{\text{(Legendre transform of } \exp\{\mathcal{Z}[\mathcal{J}]\})} \Rightarrow \underbrace{\textbf{Effective Average Action } \Gamma_k[\phi]}_{\text{(Obtained by adding a regulator term),}}$$

Adding to the action,

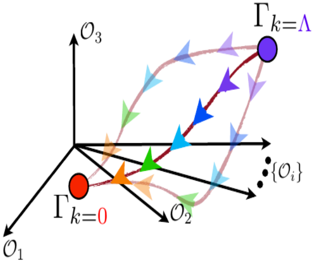
$$\Delta S_k[\phi] = \frac{1}{2}\int_q \phi(-q)R_k(q)\phi(q)$$

Where,

$$\lim_{q^2/k^2 \to 0} R_k(q) > 0$$

$$\lim_{k^2/q^2 \to 0} R_k(q) = 0$$

$$\lim_{k^2 \to \Lambda \to \infty} R_k(q) \to \infty$$



The Functional Renormalization Group

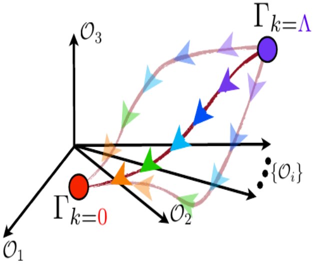
Given, $t = \ln\left(\frac{k}{\Lambda}\right)$, $\partial_t = -k\partial_k$

The Wetterich (flow) Eq :

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} (\partial_t R_k)\right]$$

- Solution : trajectory in Theory space.
- End-points are :
  The bare action *(UV-limit)*
  The full effective action *(IR-limit)*.
- Precise trajectory depends on R_k.
  We use the Litim regulator,

$$R_k \approx Z_{\phi,k}(k^2 - q^2)\Theta(k^2 - q^2)$$



PYS Flow

The Potts-Yukawa System Flow

Our microscopic action becomes,

$$S_{\text{PYS}} = S_{\phi}^{U(1)} + S_{\phi}^{\mathbb{Z}_n} + S_{\psi} + S_{\psi\phi},$$

Giving the effective average action,

$$\Gamma_k = \int_x \left\{ \frac{Z_{\phi,k}}{2} \left[(\partial_\mu \phi_1)^2 + (\partial_\mu \phi_2)^2 \right] + U_k^{\mathbb{Z}_n} + Z_{\psi,k} \overline{\psi}_j \partial\!\!\!/ \psi_j + h_k \overline{\psi}_j \left(\phi_1 + \mathrm{i}\gamma_5 \phi_2 \right) \psi_j \right\}$$

Where, $j \in \{1, N_F\}$

With the potential expansion,

$$U_k^{\mathbb{Z}_n}[\rho, \sigma_n; x] = \lambda_2(\rho - \kappa) + \frac{\lambda_4}{2}(\rho - \kappa)^2 + \frac{\lambda_6}{3!}(\rho - \kappa)^3 + g_n \sigma_n$$

Generated Masses

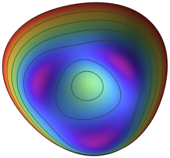
Theory's masses :

Eigen-values of Hessian of Γ_k,

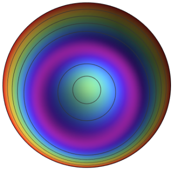
$$m_L^2 = 2\lambda_4\kappa$$

$$m_T^2 = n^2\kappa^{n/2-1}g_n$$

$$m_\psi^2 = h^2\kappa$$



$\mathbb{Z}_3$



$\mathbb{Z}_\infty$

For numerical analysis, the choices of $\{d = 4 \;\&\; n = 6\}$ are made !

Flow Equations

$$\partial_t u_k = -d u_k + \frac{1}{2}\left(d-2+\eta_\phi\right)\left(2\rho u_{k;\rho} + n\sigma_n u_{k;\sigma}\right)$$
$$+\frac{4\Omega_d}{d}\left[\left(1-\frac{\eta_\phi}{d+2}\right)\frac{1}{1+\boldsymbol{m_T^2}} + \left(1-\frac{\eta_\phi}{d+2}\right)\frac{1}{1+\boldsymbol{m_L^2}} - d_\gamma\left(1-\frac{\eta_\psi}{d+1}\right)\frac{1}{1+\boldsymbol{m_\psi^2}}\right]$$

- β-functions :

$$\partial_t\left(\partial_{\tilde\rho}^m u_k\Big|_{\substack{\tilde\rho=\kappa\\ \tilde\sigma_n=0}}\right) = \partial_t\lambda_{2m}, \quad \partial_t\left(\partial_{\tilde\sigma_n}^m u_k\Big|_{\substack{\tilde\rho=\kappa\\ \tilde\sigma_n=0}}\right) = \partial_t g_m$$

- The Yukawa coupling & Anomalous dimensions :

$$\eta_\phi\big|_{\mathrm{SYM}} = \frac{h^2}{8\pi^2} \quad , \quad \eta_\psi\big|_{\mathrm{SYM}} = \frac{h^2}{16\pi^2}\frac{1}{(1+\lambda_2)^2}$$
$$\partial_t h^2\big|_{\mathrm{SYM}} = \left(\eta_\phi + 2\eta_\psi\right)h^2 + \frac{h^2}{48\pi^2}\left(\frac{1}{(1+\lambda_2)^2} + \frac{1}{(1+\lambda_2)}\right)$$

The Running Couplings

- $d = 4, \quad d_\gamma = 4, \quad n = 6, \quad t_{\mathrm{UV}} = 15.$
- Ignoring η_φ, η_ψ in quantum corrections
- Inspiration from the SM values

$$\text{vev} \approx 246,\ m_t \approx 173,\ m_H \approx 125\ GeV$$

- Initial conditions estimates

$$\lambda_2 \approx 0.0055, \quad \lambda_4 \approx 0.087, \quad h^2 \approx 0.48$$

- $\lambda_2 \propto k^2, \quad \lambda_4 \propto k^0, \quad h^2 \propto k^0, \quad g_6 \propto k^{-2}$

  $\Rightarrow$ Fine-tune λ_2 && choose $g_6 \approx 0.10$

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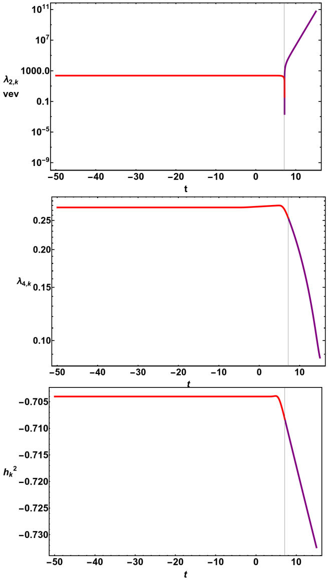
- $t_c \approx 7.1$ of SSB



The Running Couplings

λ_4 :

- Dashed : Global $U(1)$ $(g_6 = 0)$

  λ_4 runs logarithmically to IR

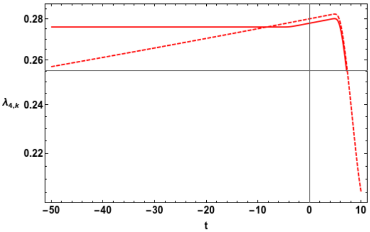
- Solid : $\mathbb{Z}_6$ symmetry (finite g_6)

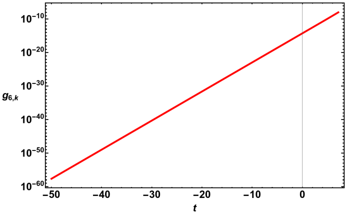
  λ_4 freeze out below scales $\sim \kappa^2 g_6$

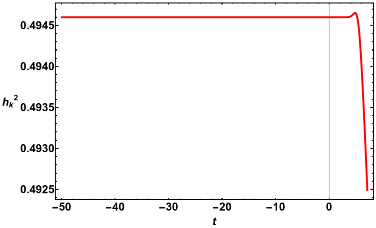
  Goldstone mode gains mass

g_6 :

- Dies off towards IR.







Generated Mass Hierarchy

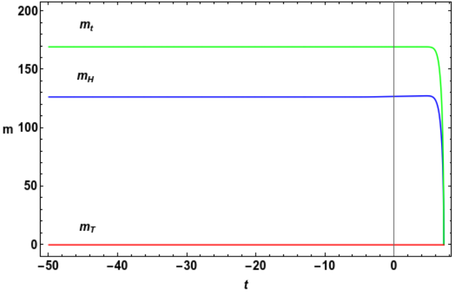
- At $t_{\mathrm{IR}} = -20$,

$$\text{vev} \approx 241 \text{ GeV}, \quad m_H \approx 127 \text{ GeV}, \quad m_T \approx 10^{-2} \text{ GeV}, \quad m_t \approx 170 \text{ GeV}$$

- With a hierarchy,

$$\frac{m_H}{m_t} \approx 0.75 \sim 1$$

$$\frac{m_T}{m_H} \approx 7.9 \cdot 10^{-5} \ll 1$$



- Hence we have,

$$\underset{0.5}{\mathcal{O}(h^2)} \sim \underset{0.1}{\mathcal{O}(\lambda_4)} \sim \underset{0.1}{\mathcal{O}(g_6)} \xrightarrow{\text{Flow to IR}} \mathcal{O}(m_t) \sim \mathcal{O}(m_H) \gg \mathcal{O}(m_T)$$

Asymptotic Safety Inclusion

Asymptotic Safety Inclusion

- Bypass the UV cutoff : Extend to arbitrarily high scales
- Combine our mechanism of mass-hierarchy w/ asymptotically-safe gravity

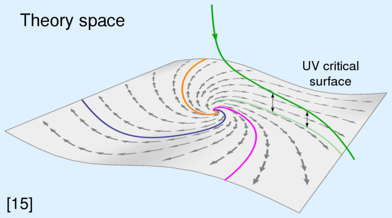
In asymptotic safety :

- We can thus have a non-trivial theory at arbitrarily high energies, w/ guaranteed non-divergence
- Provide a solid, non-perturbative approach to quantum gravity

Asymptotic Safety Inclusion

- Require our theory to have UV fixed point (FP)
- Observables in IR are governed by the FP
- If FP is interactive & UV-attractive,

  $\Rightarrow$ A predictive, asymptotically-safe theory is established.



- General form of β-functions : w/ g, Λ : gravitational couplings.

$$\beta_\lambda = b_0 + \left[b_1 + g f_\lambda(g,\Lambda)\right]\lambda + b_2\lambda^2 + b_3\lambda^3 + \cdots$$

- Sign of $\left[b_1 + g f_\lambda(g,\Lambda)\right]$ determine if FP is UV/IR attractive.
- If $b_0 = 0$. Non trivial solution : Must have a loop diagram $\propto \lambda^2$ or higher

Asymptotic Safety Inclusion

- Current Model
  - For $n > 4$: No one-loop diagram $\Rightarrow$ No non-trivial UV FP
  - For $n = 3$ or 4 :

    Possible Non-trivial FP

    g_n marginal, runs logarithmically to IR

    $\Rightarrow$ No mass hierarchy !
- Extension : a complex field χ transforming under an independent $U(1)$
- Include a $\mathbb{Z}_3$-symmetric χ-flavored term.

Microscopic action becomes

$$S^{g}_{\text{PYS}} = S^{U(1)}_{\varphi} + S^{U(1)}_{\chi} + S^{\mathbb{Z}_6}_{\varphi} + S^{\mathbb{Z}_3}_{\chi} + S_{\varphi\chi} + S_{\psi} + S_{\psi\varphi},$$

Ansatz for the effective action

$$\Gamma_k = \int_x \Big\{ Z_{\phi,k}\, \partial_\mu \phi^* \partial_\mu \phi + Z_{\chi,k}\, \partial_\mu \chi^* \partial_\mu \chi + \mathcal{U}^{\mathbb{Z}_{3,6}}_k [\phi, \phi^*, \chi, \chi^*; x] + iZ_{\psi,k} \overline{\psi} \partial\!\!\!/ \psi + h_k \overline{\psi} \left[(1-\gamma_5)\phi - (1+\gamma_5)\phi^* \right] \psi \Big\}$$

Implementation

Extend to 2 complex fields,

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \quad , \quad \chi = \frac{1}{\sqrt{2}}(\chi_1 + i\chi_2)$$

W/ charges s.t.

$$\phi \to e^{\frac{2\pi i}{6}}\phi \quad , \quad \chi \to e^{\frac{8\pi i}{6}}\chi$$

Obtaining the invariants :

$$\left.\begin{aligned}\rho_\phi &= \phi^*\phi \\ \rho_\chi &= \chi^*\chi\end{aligned}\right\} \qquad U(1) \text{ - symmetric}$$

$$\tau = \left(\chi^{*3} + \chi^3\right) \qquad \mathbb{Z}_3 \text{ - symmetric}$$

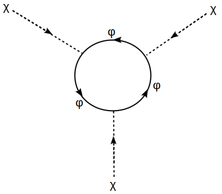
$$\sigma = \left(\phi^{*6} + \phi^6\right) \qquad \mathbb{Z}_6 \text{ - symmetric}$$

$$\left.\begin{aligned}\zeta_1 &= (\phi^{*2}\chi^* + \phi^2\chi) \\ \zeta_2 &= (\phi^{*4}\chi + \phi^4\chi^*) \\ \zeta_3 &= (\phi^{*2}\chi^{*4} + \phi^2\chi^4) \\ \zeta_4 &= (\phi^{*2}\chi^2 + \phi^2\chi^{*2})\end{aligned}\right\} \qquad \text{i.a terms}$$

Implementation

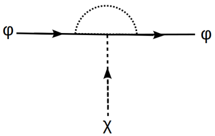
A closer look at the term $\left(\phi^2\chi + {\phi^*}^2\chi^*\right)$

Featuring : $\{\phi_1^2\chi_2\ ,\ \phi_1^2\chi_2\ ,\ \phi_1\phi_2\chi_1\}$



- Charges running through loop arising from the $\mathbb{Z}_{3,6}$ symmetries.

- SYM :
  - Charges $\overset{!}{=} 0 \quad \Rightarrow$ No λ^3 term under $\mathbb{Z}_{3,6}$
  - Including a scale-dependent $\eta_{\phi/\chi}$ can induce a λ^3 term via :



- SSB : Acquired vev in ϕ sector
  - $\phi^*/\phi\ ,\ \phi/\phi$ i.a. can arise $\Rightarrow$ Possibility to generate λ^3 term.

Selected β-Functions : SYM

$$\eta_\varphi = \frac{g_{2,1}^2}{4\pi^2\left(\lambda_{0,2}+1\right)^2\left(\lambda_{2,0}+1\right)^2} + \frac{h^2}{8\pi^2}$$

$$\eta_\chi = \frac{9g_{0,3}^2}{8\pi^2\left(\lambda_{0,2}+1\right)^4} + \frac{g_{2,1}^2}{8\pi^2\left(\lambda_{2,0}+1\right)^4}$$

$$\beta_{g_{2,1}} = g_{2,1}\left(f_\lambda + \eta_\varphi + \frac{\eta_\chi}{2} - 1\right) + \frac{g_{2,1}\lambda_{2,2}}{8\sqrt{2}\pi^2\left(\lambda_{0,2}+1\right)^2\left(\lambda_{2,0}+1\right)} + \mathcal{O}\left(g_{2,1}\right)$$

$$\beta_{g0,3} = g_{0,3}\left(f_\lambda + \frac{3}{2}\eta_\chi - 1\right) + \frac{1}{16\pi^2}\left(-\frac{g_{2,3a}}{\left(\lambda_{2,0}+1\right)^2} + \frac{4g_{2,1}g_{2,2}}{\left(\lambda_{2,0}+1\right)^3} + \frac{6g_{0,3}\lambda_{0,4}}{\left(\lambda_{0,2}+1\right)^3}\right)$$

Indeed we see both η_ϕ **&** η_χ supplying $\beta g_{2,1}$ with $g_{2,1}^3$ contributions

Fixed Point Solutions

FP 1 initial conditions :

$$\{g_{0,3} \to 0.15,\ g_{2,1} \to 0.30,\ h \to 0.42,\ f_\lambda \to 0.99,\ f_h \to -0.0025\}$$

FP values

Coupling	Value	Coupling	Value
$\lambda_{2,0}$	0.00657	$\lambda_{0,2}$	0.00774
$g_{0,3}$	0.147	$g_{2,1}$	0.331
$\lambda_{4,0}$	−0.00505	$\lambda_{0,4}$	−0.0108
$\lambda_{2,2}$	−0.0145	$g_{2,2}$	−0.00347
$g_{2,3}$	0.000358	$g_{2,3a}$	0.000237
$g_{4,1}$	0.0000404	$g_{4,1a}$	0.000169
$\lambda_{6,0}$	−0.00249	$\lambda_{0,6}$	−0.00812
$\lambda_{4,2}$	−0.0103	$\lambda_{2,4}$	−0.0147
$g_{6,0}$	−0.0000000707	$g_{0,6}$	−0.000000468
$g_{4,2}$	−0.00000157	$g_{4,2a}$	−0.00165
$g_{2,4}$	−0.00000225	$g_{2,4a}$	−0.00230
h	0.280		

Fixed Point Solutions

FP 2 initial conditions :

$$\{g_{0,3} \rightarrow 1.0, \quad g_{2,1} \rightarrow 1.3, \quad h \rightarrow 0.42, \quad f_\lambda \rightarrow 0.99, \quad f_h \rightarrow -0.0060\}$$

FP values

Coupling	Value	Coupling	Value
$\lambda_{2,0}$	0.0184	$\lambda_{0,2}$	0.0414
$g_{0,3}$	0.362	$g_{2,1}$	0.544
$\lambda_{4,0}$	-0.0549	$\lambda_{0,4}$	-0.486
$\lambda_{2,2}$	-0.280	$g_{2,2}$	-0.0573
$g_{2,3}$	0.0793	$g_{2,3a}$	0.0538
$g_{4,1}$	0.00396	$g_{4,1a}$	0.0194
$\lambda_{6,0}$	-0.103	$\lambda_{0,6}$	-3.40
$\lambda_{4,2}$	-0.747	$\lambda_{2,4}$	-2.19
$g_{6,0}$	-0.0000479	$g_{0,6}$	-0.00405
$g_{4,2}$	-0.00232	$g_{4,2a}$	-0.106
$g_{2,4}$	-0.00706	$g_{2,4a}$	-0.284
h	0.429		

Fixed Point Solutions

As for the Stability Matrices

FP 1

$$\text{Sorted -ve Eigenvalues} = \{1.02,\ 1.00,\ -0.000759,\ -0.00444,\ -0.00783,\ -0.917,\ -0.948,\ -0.968,\ -1.00,\ -1.92,\ -1.95,\ -1.98,\ -2.00,\ -2.84,\ -2.88,\ -2.90,\ -2.93,\ -2.95,\ -3.00,\ -3.00,\ -3.00,\ -3.01,\ -3.01\}$$

FP 2

$$\text{Sorted -ve Eigenvalues} = \{1.11,\ 1.01,\ 0.0572,\ 0.0189,\ -0.00637,\ -0.592,\ -0.679,\ -0.906,\ -1.02,\ -1.65,\ -1.80,\ -1.95,\ -1.99,\ -2.34,\ -2.56,\ -2.67,\ -2.77,\ -2.83,\ -2.96,\ -2.99,\ -3.00,\ -3.01,\ -3.02\}$$

Discussion

- Broken $U(1)$ global symmetry to a $\mathbb{Z}_n$-symmetry
- Established a mass hierarchy in scalar sector in IR

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- FP values are very sensitive to f_h initial condition
- FP values signify an unstable potential
- In SYM : all $\propto \lambda^3$ vertices arise from anomalous dimensions contributions
- Next step : Do calculation in SSB to see if problem persists
- In SSB : Fermion sector would couple to χ field as well !
- Worth noting : Set discrete symmetry precludes gauge fields inclusion !

Thanks For Your Attention ☺

References

[1] Bertrand DELAMOTTE. "An Introduction to the Nonperturbative Renormalization Group". In : **Lecture Notes in Physics** (2012), p. 49-132. ISSN : 1616-6361. DOI : 10.1007/978-3-642-27320-9_2. URL : http://dx.doi.org/10.1007/978-3-642-27320-9_2.

[2] Astrid EICHHORN. **An asymptotically safe guide to quantum gravity and matter**. 2018. arXiv : 1810.07615 [hep-th].

[3] Astrid EICHHORN et Aaron HELD. "Viability of quantum-gravity induced ultraviolet completions for matter". In : **Physical Review D** 96.8 (oct. 2017). ISSN : 2470-0029. DOI : 10.1103/physrevd.96.086025. URL : http://dx.doi.org/10.1103/PhysRevD.96.086025.

[4] Astrid EICHHORN, Aaron HELD et Jan M. PAWLOWSKI. "Quantum-gravity effects on a Higgs-Yukawa model". In : **Phys. Rev. D** 94 (10 nov. 2016), p. 104027. DOI : 10.1103/PhysRevD.94.104027. URL : https://link.aps.org/doi/10.1103/PhysRevD.94.104027.

[5] Astrid EICHHORN et al. "Quantum gravity fluctuations flatten the Planck-scale Higgs potential". In : **Physical Review D** 97.8 (avr. 2018). ISSN : 2470-0029. DOI : 10.1103/physrevd.97.086004. URL : http://dx.doi.org/10.1103/PhysRevD.97.086004.

[6] Lin FEI et al. **Yukawa CFTs and Emergent Supersymmetry**. 2016. arXiv : 1607.05316 [hep-th].

References

[7] Holger GIES. “Introduction to the Functional RG and Applications to Gauge Theories”. In : **Lecture Notes in Physics** (2012), p. 287-348. ISSN : 1616-6361. DOI : 10.1007/978-3-642-27320-9_6. URL : http://dx.doi.org/10.1007/978-3-642-27320-9_6.

[8] Holger GIES et Michael M. SCHERER. “Asymptotic safety of simple Yukawa systems”. In : **The European Physical Journal C** 66.3-4 (fév. 2010), p. 387-402. ISSN : 1434-6052. DOI : 10.1140/epjc/s10052-010-1256-z. URL : http://dx.doi.org/10.1140/epjc/s10052-010-1256-z.

[9] Clemens GNEITING. “Higgs Mass Bounds from Renormalization Flow”. Mém. de mast. Philosophenweg 16, 69120 Heidelberg, DE : Faculty of Physics et Astronomy, University of Heidelberg, 2005.

[10] Geoffrey R GOLNER. “Investigation of the potts model using renormalization-group techniques”. In : **Physical Review B** 8.7 (1973), p. 3419.

[11] WK74 Wilson KG et JB KOGUT. “The renormalization group and the\({{\epsilon}}\)-expansion”. In : **Phys. Rept** 12 (1974), p. 75-200.

[12] F. LÉONARD, B. DELAMOTTE et N. WSCHEBOR. **Naturally light scalar particles : a generic and simple mechanism**. 2018. arXiv : 1802.09418 [hep-ph].

References

[13] F Léonard, B Delamotte et Nicolás Wschebor. "Naturally light scalar particles : a generic and simple mechanism". In : **arXiv preprint arXiv :1802.09418** (2018).

[14] Daniel F. Litim. "Optimized renormalization group flows". In : **Physical Review D** 64.10 (oct. 2001). ISSN : 1089-4918. DOI : 10.1103/physrevd.64.105007. URL : http://dx.doi.org/10.1103/PhysRevD.64.105007.

[15] Andreas Nink. **UV Critical Surface**. Scholarpedia, 8(7) :31015. 2013. URL : http://www.scholarpedia.org/article/File:UVCriticalSurface.png.

[16] Stefan Rechenberger. "Asymptotic Safety of Yukawa Systems". Mém. de mast. Max-Wien-Platz 1, 07743 Jena, DE : Physikalisch-Astronomische Fakultät Friedrich Schiller Universiät Jena, 2010.

[17] Michael Scherer. "Introduction to Renormalization with Applications in Condensed-Matter and High-Energy Physics". In : (2018).

[18] Emilio Torres et al. "Fermion-induced quantum criticality with two length scales in Dirac systems". In : **Physical Review B** 97.12 (mar. 2018). ISSN : 2469-9969. DOI : 10.1103/physrevb.97.125137. URL : http://dx.doi.org/10.1103/PhysRevB.97.125137.

[19] Christof Wetterich. "Exact evolution equation for the effective potential". In : **Physics Letters B** 301.1 (1993), p. 90-94.

References

[20] Fa-Yueh Wu. "The potts model". In : **Reviews of modern physics** 54.1 (1982), p. 235.

[21] RKP Zia et DJ Wallace. "Critical behaviour of the continuous n-component Potts model". In : **Journal of Physics A : Mathematical and General** 8.9 (1975), p. 1495.

[22] Riccardo Ben Alì Zinati et Alessandro Codello. "Functional RG approach to the Potts model". In : **Journal of Statistical Mechanics : Theory and Experiment** 2018.1 (2018), p. 013206.