# Non-Perturbative fRG Approaches in Dark Matter Structure Formation

Martina Zündel

Cold Quantum Coffee
Institut for Theoretical Physics - Heidelberg

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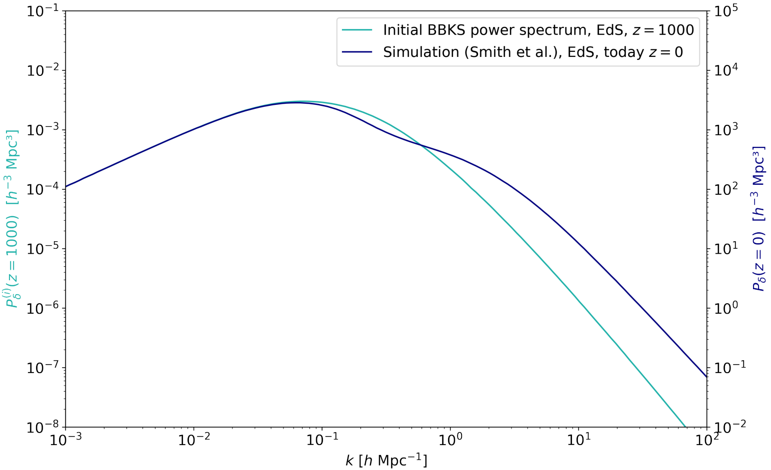
# Objective: Dark Matter Structure Growth

# Dark matter density contrast power spectrum

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# Dark matter density contrast power spectrum

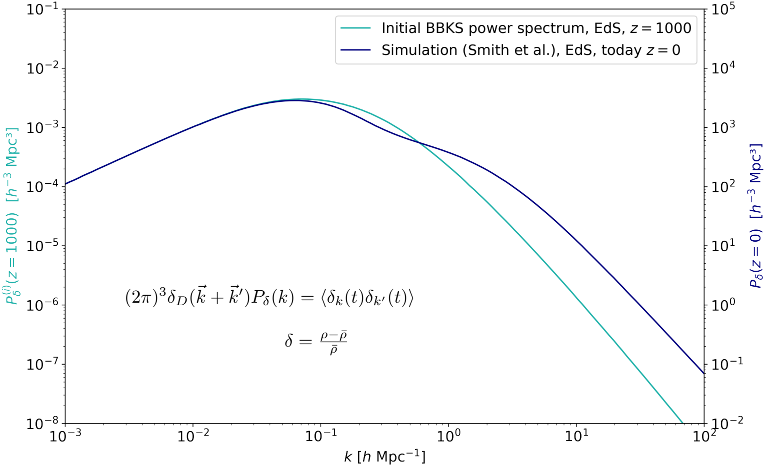


Figure: Initial BBKS and today's power spectrum; simulation by Smith et al. [8]; both in an EdS universe with DM only

# Linear structure growth

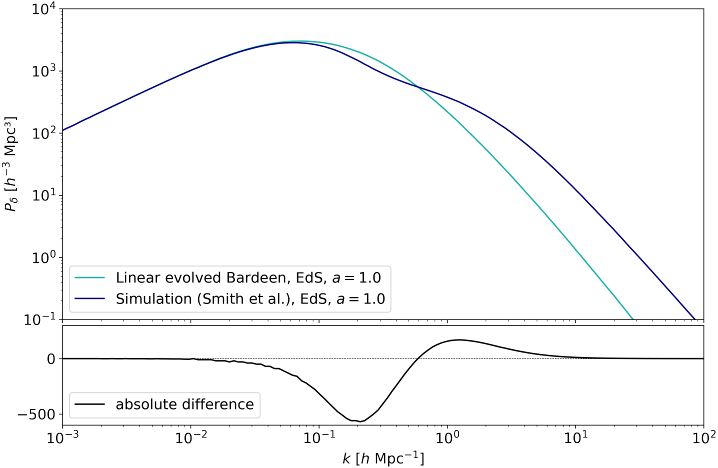


Figure: Initial BBKS spectrum linearly evolved until today, compared to simulation by Smith et al. [8]

# *Resummed* KFT

based on Lilow et al. [7, 6]

# Kinetic Field Theory

Non-relativistic description of $N$ particles moving in phase-space
$\rightarrow$ Phase-space trajectories $\vec{x}_j(t) = (\vec{q}_j(t), \vec{p}_j(t))$ for the particles $j \in \{1, ..., N\}$

$$\boldsymbol{x}(t) := \sum_{j=1}^{N} \vec{x}_j(t) \otimes \vec{e}_j$$

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1. Deterministic dynamics: E.o.m. $\boldsymbol{E}[\boldsymbol{x}(t)] = 0$
2. Interaction potential
3. Stochastic initial conditions $\boldsymbol{x}^{(i)} := \boldsymbol{x}(t^{(i)})$

# Kinetic Field Theory – Assumptions for DM structure growth

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2. 1-particle interaction potential: Newtonian
3. Initial conditions $\boldsymbol{x}^{(\boldsymbol{i})} := \boldsymbol{x}(t^{(i)})$ drawn from a probability distribution $\mathcal{P}(\boldsymbol{x}^{(\boldsymbol{i})})$
   - Statistical properties of macroscopic density field are given
   - Poisson sample the positions $\vec{x}_j^{(i)}$ from these fields

# Kinetic Field Theory – Assumptions for DM structure growth

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$\Longrightarrow$ Partition function for $t \geq t_i$

$$Z = \int \mathrm{d}\boldsymbol{x}^{(\boldsymbol{i})} \mathcal{P}(\boldsymbol{x}^{(\boldsymbol{i})}) \int_{\boldsymbol{x}^{(\boldsymbol{i})}} \mathcal{D}\boldsymbol{x}\; \delta_D^{(2dN)} \left[\boldsymbol{x}(t) - \boldsymbol{x}^{(\boldsymbol{cl})}(\boldsymbol{x}^{(\boldsymbol{i})}, t)\right]$$
$$= \int \mathrm{d}\boldsymbol{x}^{(\boldsymbol{i})} \mathcal{P}(\boldsymbol{x}^{(\boldsymbol{i})}) \int_{\boldsymbol{x}^{(\boldsymbol{i})}} \mathcal{D}\boldsymbol{x}\mathcal{D}\boldsymbol{\chi}\; \mathrm{e}^{\mathrm{i}\boldsymbol{\chi}\cdot\boldsymbol{E}[\boldsymbol{x}]}$$

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$\Longrightarrow$ Partition function for $t \geq t_i$; with $\psi = (\boldsymbol{x}, \boldsymbol{\chi})$

$$Z = \int \mathrm{d}\boldsymbol{x}^{(i)} \mathcal{P}(\boldsymbol{x}^{(i)}) \int_{\boldsymbol{x}^{(i)}} \mathcal{D}\boldsymbol{x} \; \delta_D^{(2dN)} \left[\boldsymbol{x}(t) - \boldsymbol{x}^{(cl)}(\boldsymbol{x}^{(i)}, t)\right]$$

$$= \int \mathrm{d}\boldsymbol{x}^{(i)} \mathcal{P}(\boldsymbol{x}^{(i)}) \int_{\boldsymbol{x}^{(i)}} \mathcal{D}\psi \; \mathrm{e}^{\mathrm{i}\boldsymbol{\chi} \cdot \boldsymbol{E}[\boldsymbol{x}]}$$

# Collective field description

- Collective density field

$$\Phi_\rho(\vec{q}_r, t_r) := \sum_{j=1}^{N} \delta_D\left(\vec{q}_r - \vec{q}_j(t_r)\right)$$

- Collective response field

$$\Phi_B(\vec{q}_r, t_r) := \sum_{j=1}^{N} \vec{\chi}_{p_j}(t_r) \cdot \nabla_q \delta_D\left(\vec{q}_r - \vec{q}_j(t_r)\right)$$

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- Dressed collective response field

$$\Phi_F(r) := -\mathrm{i}\; v(k_r, t_r) \sum_{j=1}^{N} \vec{\chi}_{p_j} \cdot \vec{k}_r \mathrm{e}^{-\mathrm{i}\vec{k}_r \cdot \vec{q}_j(t_r)}$$

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Splitting $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I$, leads to an interaction term

$$S_{\psi,I}[\psi] = \Phi_\rho[\psi] \cdot \Phi_F[\psi]$$

# Overview - Derivation RKFT

- Microscopic partition function; field $\psi = (\boldsymbol{x}, \chi)$

$$Z = \int \mathrm{d}\boldsymbol{x}^{(i)} \mathcal{P}(\boldsymbol{x}^{(i)}) \int_{x^{(i)}} \mathcal{D}\psi \; \mathrm{e}^{\mathrm{i}S_{\psi,0}[\psi] + \mathrm{i}S_{\psi,I}[\psi]}$$

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- Collective field generating functional; field $\Phi = (\Phi_\rho[\psi], \Phi_F[\psi])$

$$Z_\Phi[H] = \int \mathrm{d}\mathbf{x}^{(i)} \mathcal{P}(\boldsymbol{x}^{(i)}) \int_{x^{(i)}} \mathcal{D}\psi \; \mathrm{e}^{\mathrm{i}S_{\psi,\mathbf{0}} + \mathrm{i}\Phi_\rho \cdot \Phi_F + \mathrm{i}H \cdot \Phi}$$

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- Macroscopic partition function; field $\varphi = (\varphi_f, \varphi_\beta)$

$$Z = \int \mathcal{D}\varphi \; \mathrm{e}^{\mathrm{i}S[\varphi]}$$

Introduce a formally $\psi$-independent field $\varphi_f$ in $Z_\Phi$

$$Z_\Phi = \int \mathrm{d}\boldsymbol{x}^{(i)} \mathcal{P}(\boldsymbol{x}^{(i)}) \int_{x^{(i)}} \mathcal{D}\psi \int \mathcal{D}\varphi_f \; \mathrm{e}^{\mathrm{i}S_{\psi,\mathbf{0}} + \mathrm{i}\varphi_f \cdot \Phi_F} \, \delta_D(\varphi_f - \Phi_\rho[\psi])$$

# Macroscopic field description

Partition function for $\varphi = (\varphi_f, \varphi_\beta)$

$$Z = \int \mathcal{D}\varphi \; \mathrm{e}^{\mathrm{i}S[\varphi]}$$

with $n = n_f + n_\beta$ and

$$S[\varphi] = -\frac{1}{2}\varphi_{a_1}\Delta^{-1}_{a_1 a_2}\varphi_{a_2} + \sum_{n=0, n\neq 2}^{\infty} \frac{1}{n!}\left(\prod_{i=1}^{n}\varphi_{a_i}\right)\mathcal{V}^{(n)}_{a_1 \dots a_n}$$

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with

$$\mathcal{V}^{(n)}_{\beta\ldots\beta f\ldots f}(1, \ldots, n_\beta, n_\beta+1, \ldots, n) = i^{n-1} G^{(0)}_{\rho\ldots\rho F\ldots F}(1, \ldots, n_\beta, n_\beta+1, \ldots, n)$$

and the inverse tree-level propagator

$$\Delta^{-1}(1,2) = \begin{pmatrix} 0 & \mathcal{I} - iG^{(0)}_{F\rho} \\ \mathcal{I} - iG^{(0)}_{\rho F} & -iG^{(0)}_{\rho\rho} \end{pmatrix}(1,2)$$

where "free" collective-field cumulants

$$G^{(0)}_{\rho\ldots\rho F\ldots F}(1, \ldots, n_\rho, 1', \ldots, n'_F) = \prod_{u=1}^{n_\rho}\left(\frac{\delta}{i\delta H_\rho(u)}\right)\prod_{r=1'}^{n'_F}\left(\frac{\delta}{i\delta H_F(r)}\right) W_{\Phi,0}[H]\Big|_{H=0}$$

# Macroscopic field description - Tree-level propagator

$$\Delta(1,2) = \begin{pmatrix} 0 & \mathcal{I} - \mathrm{i}G^{(0)}_{F\rho} \\ \mathcal{I} - \mathrm{i}G^{(0)}_{\rho F} & -\mathrm{i}G^{(0)}_{\rho\rho} \end{pmatrix}^{-1} (1,2) \quad = \begin{pmatrix} \Delta_R \cdot \mathrm{i}G^{(0)}_{\rho\rho} \cdot \Delta_A & \Delta_R \\ \Delta_A & 0 \end{pmatrix} (1,2)$$

with

$$\begin{aligned} \Delta_R(1,2) = \Delta_A(2,1) &\coloneqq \left(\mathcal{I} - \mathrm{i}G^{(0)}_{\rho F}\right)^{-1}(1,2) \\ &= \sum_{n=0}^{\infty} \left(\mathrm{i}G^{(0)}_{\rho F}\right)^n (1,2) \end{aligned}$$

→ Infinite resummation of Newtonian interactions!

# Macroscopic field description - Tree-level propagator

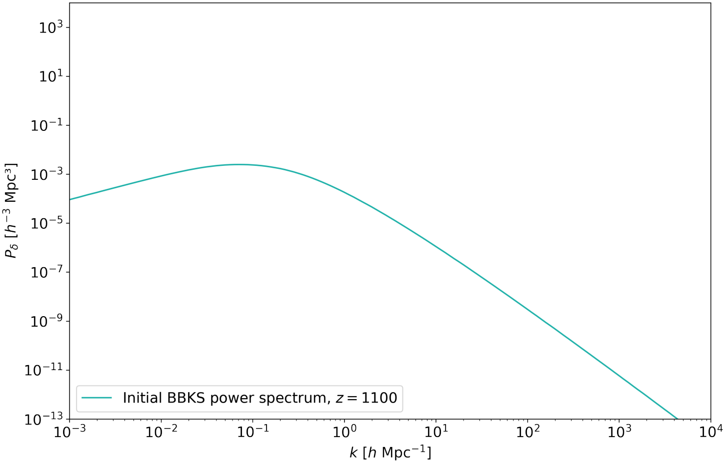


Figure: Comparison of the tree-level propagator (power spectrum)

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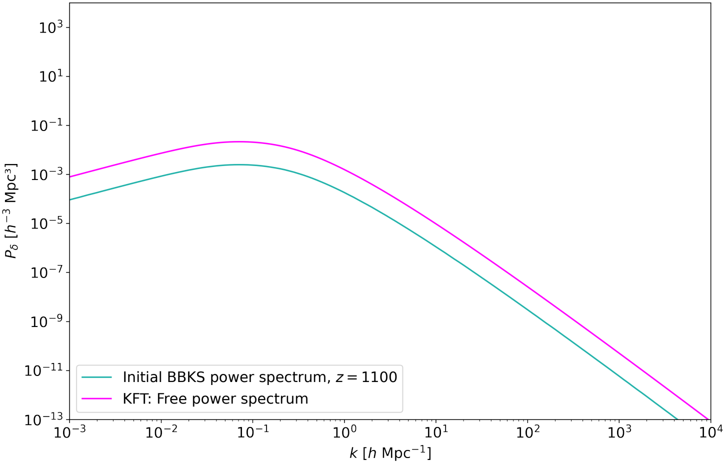


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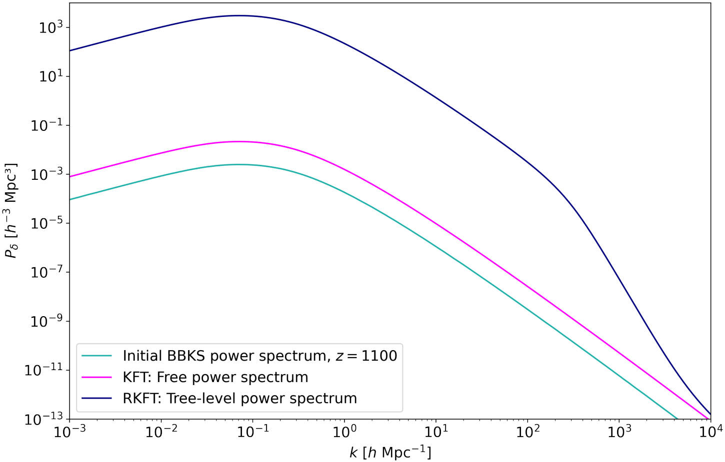


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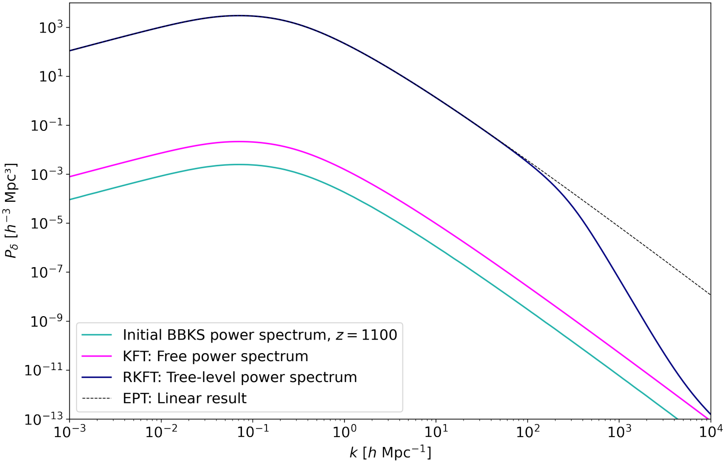


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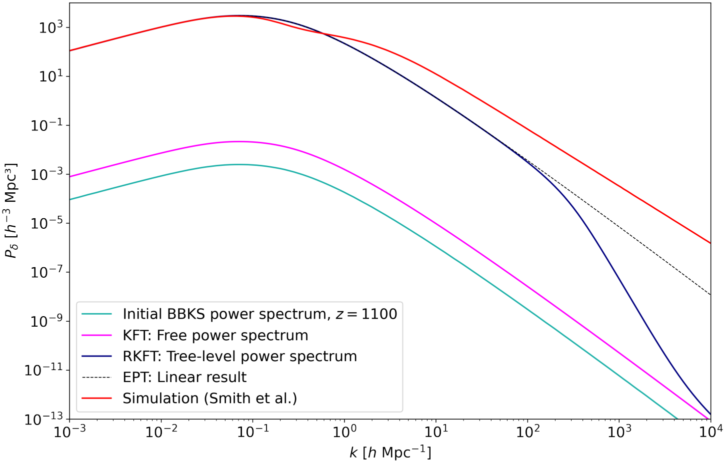


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# Non-perturbative ansatz

RKFT can describe the power spectrum on large scales perturbatively.
→ We flow to smaller scales, i.e., higher $k$.
→ We need a UV regulator.

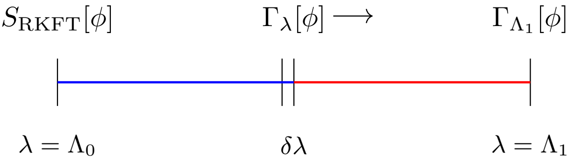


Figure: Starting from $S_{\mathrm{RKFT}}$ at $\lambda = \Lambda_0$ the scale-dependent effective action $\Gamma_\lambda$ includes modes (blue) of successively higher wave vectors $k$.

Full effective action by integrating the Wetterich equation

$$\Gamma[\phi] = \lim_{\Lambda_1 \to \infty} \int_{\Lambda_0}^{\Lambda_1} \mathrm{d}\lambda \, \partial_\lambda \Gamma_\lambda[\phi] + \Gamma_{\Lambda_0}[\phi]$$

# Closed time path formalism and the Keldysh-rotated basis

# Closed time path and rotated basis

Generating functional

$$Z[J,R] = \int \mathcal{D}'\varphi^{\pm} \exp \mathrm{i} \left\{ S[\varphi^+,\varphi^-] + \int_x (\varphi^+(x),\varphi^-(x)) \begin{pmatrix} J^+(x) \\ -J^-(x) \end{pmatrix} \right.$$
$$\left. + \frac{1}{2} \int_{xy} (\varphi^+(x),\varphi^-(x)) \begin{pmatrix} R^{++}(x,y) & -R^{+-}(x,y) \\ -R^{-+}(x,y) & R^{--}(x,y) \end{pmatrix} \begin{pmatrix} \varphi^+(x) \\ \varphi^-(x) \end{pmatrix} \right\}$$

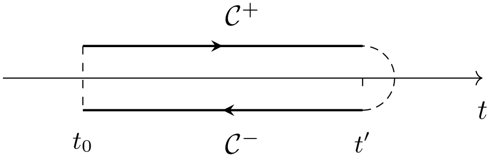


Figure: Illustration of the Schwinger-Keldysh contour $\mathcal{C}$ up to a time $t'$

# Closed time path and rotated basis

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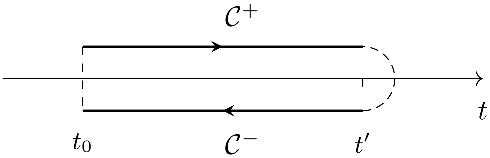
Rotation to a "physical basis" (Chou et al. [2])

$$\begin{pmatrix} \varphi \\ \tilde{\varphi} \end{pmatrix} := A \begin{pmatrix} \varphi^+ \\ \varphi^- \end{pmatrix} , \qquad A := \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & -1 \end{pmatrix}$$

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$$Z[J,R] = \int \mathcal{D}\varphi \mathcal{D}\tilde{\varphi} \exp \mathrm{i} \Big\{ S[\varphi,\tilde{\varphi}] + \int_x (\varphi(x),\tilde{\varphi}(x)) \begin{pmatrix} \tilde{J}(x) \\ J(x) \end{pmatrix}$$

$$+ \frac{1}{2} \int_{x,y} (\varphi(x),\tilde{\varphi}(x)) \begin{pmatrix} R^{\tilde{F}}(x,y) & R^{A}(x,y) \\ R^{R}(x,y) & R^{F}(x,y) \end{pmatrix} \begin{pmatrix} \varphi(y) \\ \tilde{\varphi}(y) \end{pmatrix} \Big\}$$

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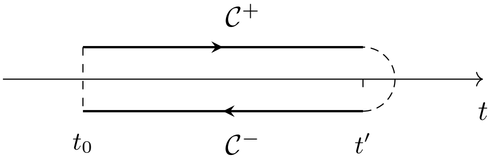


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# RKFT is a nonequilibrium theory

Identification

$$\begin{pmatrix} \varphi^f \\ \varphi^\beta \end{pmatrix}_{(RKFT)} \stackrel{?}{=} \begin{pmatrix} \varphi \\ \tilde{\varphi} \end{pmatrix}_{(CTP)}$$

Cumulants of RKFT have the same properties as a general nonequilibrium theory.
$\rightarrow$ Causality

# Cumulants in the rotated basis

Schwinger functional

$$W[J,R] = -\mathrm{i}\ln Z[J,R]$$

Abbreviated notation

$$W^f(x) := \frac{\delta W}{\delta J^f(x)}$$

One-point function

$$W^f(x) =: \phi^f(x)$$
$$W^\beta(x) =: \phi^\beta(x)$$

Propagators

$$W^{ff}(x,y) =: \mathrm{i}F(x,y) \qquad \text{statistical propagator}$$
$$W^{f\beta}(x,y) =: G^R(x,y) = \rho(x,y)\theta(x^0 - y^0) \qquad \text{retarded propagator}$$
$$W^{\beta f}(x,y) =: G^A(x,y) = \rho(x,y)\theta(y^0 - x^0) \qquad \text{advanced propagator}$$
$$W^{\beta\beta}(x,y) =: \mathrm{i}\tilde{F}(x,y) \qquad \text{anomalous propagator}$$

Symmetries

$$G^R(x,y) = G^A(y,x)\,, \qquad F(x,y) = F(y,x)\,, \qquad \tilde{F}(x,y) = \tilde{F}(y,x)$$

# Ansatz 1: Nonequilibrium fRG

Paper: Berges and Mesterházy [1]
"Introduction to the nonequilibrium functional renormalization group"

# Flow equation

Wetterich equation in the Keldysh-rotated basis

$$\dot{\Gamma}_\lambda[\phi] = -\frac{\mathrm{i}}{2}\,\mathrm{Tr}\{G^R_\lambda \cdot \dot{R}^R_\lambda + G^A_\lambda \cdot \dot{R}^A_\lambda + iF_\lambda \cdot \dot{R}^{\tilde{F}}_\lambda + i\tilde{F}_\lambda \cdot \dot{R}^F_\lambda\}$$

Sufficient to choose the regulator as

$$R^{R,A}_\lambda \propto R_\lambda(k^2)\,, \qquad R^{F,\tilde{F}}_\lambda = 0$$

W.E. simplifies

$$\begin{aligned}\dot{\Gamma}_\lambda &= -\frac{\mathrm{i}}{2}\,\mathrm{Tr}\{G^R_\lambda \dot{R}^R_\lambda + G^A_\lambda \dot{R}^A_\lambda\} \\ &= -\mathrm{i}\ \mathrm{Tr}\ G^R_\lambda \dot{R}^R_\lambda\end{aligned}$$

Second derivative

$$\dot{\Gamma}^{\beta\beta}_{\lambda,ab} = -\mathrm{i}\frac{\delta^2 G^R_{\lambda,ch}}{\delta\phi^\beta_a\,\delta\phi^\beta_b}\dot{R}^R_{\lambda,hc}$$

$$\dot{\Gamma}^{f\beta}_{\lambda,ab} = -\mathrm{i}\frac{\delta^2 G^R_{\lambda,ch}}{\delta\phi^f_a\delta\phi^\beta_b}\dot{R}^R_{\lambda,hc}$$

# Propagators

$$G^R_\lambda = -\left[(\Gamma^{\beta f}_\lambda + R^R_\lambda) - \Gamma^{\beta\beta}_\lambda(\Gamma^{f\beta}_\lambda + R^A_\lambda)^{-1}\Gamma^{ff}_\lambda\right]^{-1}$$

$$G^A_\lambda = -\left[(\Gamma^{f\beta}_\lambda + R^A_\lambda) - \Gamma^{ff}_\lambda(\Gamma^{\beta f}_\lambda + R^R_\lambda)^{-1}\Gamma^{\beta\beta}_\lambda\right]^{-1}$$

$$\mathrm{i}F_\lambda = -\left[\Gamma^{ff}_\lambda - (\Gamma^{f\beta}_\lambda + R^A_\lambda)(\Gamma^{\beta\beta}_\lambda)^{-1}(\Gamma^{\beta f}_\lambda + R^R_\lambda)\right]^{-1}$$

$$\mathrm{i}\tilde{F}_\lambda = -\left[\Gamma^{\beta\beta}_\lambda - (\Gamma^{\beta f}_\lambda + R^R_\lambda)(\Gamma^{ff}_\lambda)^{-1}(\Gamma^{f\beta}_\lambda + R^A_\lambda)\right]^{-1}$$

# Propagators

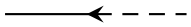
$$G_\lambda^R = -\left[(\Gamma_\lambda^{\beta f} + R_\lambda^R) - \Gamma_\lambda^{\beta\beta}(\Gamma_\lambda^{f\beta} + R_\lambda^A)^{-1}\Gamma_\lambda^{ff}\right]^{-1}$$

$$G_\lambda^A = -\left[(\Gamma_\lambda^{f\beta} + R_\lambda^A) - \Gamma_\lambda^{ff}(\Gamma_\lambda^{\beta f} + R_\lambda^R)^{-1}\Gamma_\lambda^{\beta\beta}\right]^{-1}$$

$$\mathrm{i}F_\lambda = -\left[\Gamma_\lambda^{ff} - (\Gamma_\lambda^{f\beta} + R_\lambda^A)(\Gamma_\lambda^{\beta\beta})^{-1}(\Gamma_\lambda^{\beta f} + R_\lambda^R)\right]^{-1}$$

$$\mathrm{i}\tilde{F}_\lambda = -\left[\Gamma_\lambda^{\beta\beta} - (\Gamma_\lambda^{\beta f} + R_\lambda^R)(\Gamma_\lambda^{ff})^{-1}(\Gamma_\lambda^{f\beta} + R_\lambda^A)\right]^{-1}$$

Sources $J$ set to zero:

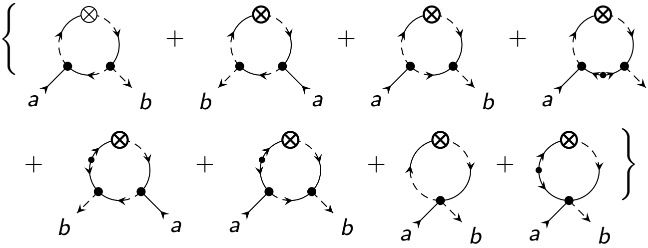
$$G_{\lambda,ab}^R = -(\Gamma_\lambda^{\beta f} + R_\lambda^R)_{ab}^{-1}$$ $= a$  $b$

$$G_{\lambda,ab}^A = -(\Gamma_\lambda^{f\beta} + R_\lambda^A)_{ab}^{-1}$$ $= a$  $b$

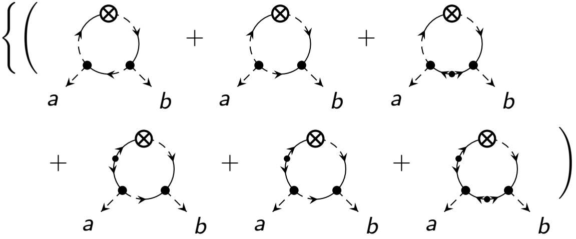
$$\mathrm{i}F_{\lambda,ab} = (G_\lambda^R \Gamma_\lambda^{\beta\beta} G_\lambda^A)_{ab}$$ $= a$  $b$

$$\mathrm{i}\tilde{F}_\lambda = 0$$

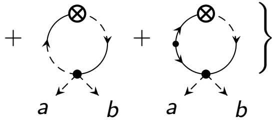
$$\dot{\Gamma}^{f\beta}_{\lambda,ab} = -\mathrm{i}$$




$$\dot{\Gamma}^{\beta\beta}_{\lambda,ab} = -\mathrm{i}$$




$+ \left(a \leftrightarrow b\right)$



# Simplifications

- Vertex expansion up to $N_V = 4$
- Static vertices $\Gamma^{(3)} = S^{(3)}$ and $\Gamma^{(4)} = S^{(4)} \implies$ only $\Gamma^{(2)}_\lambda$ flows
- Statistical spatial homogeneity: $n$-point functions depend only on $n-1$ wave vectors
- Statistical isotropy decreases the number of arguments further

Example: 1st term

$$\begin{aligned}\dot{\Gamma}_\lambda^{\beta\beta,(1\text{st})}(k; t_a, t_b) = &-\mathrm{i} G_\lambda^R(l; t_g, t_c) \cdot \dot{R}_\lambda^R(l) \cdot G_\lambda^R(l; t_c, t_d)\\ &\cdot \Gamma_\lambda^{\beta\beta f}(l, k, \cos\theta; t_d, t_a, t_e) \cdot G_\lambda^R(k; t_e, t_f)\\ &\cdot \Gamma_\lambda^{\beta\beta f}(|\vec{k} - \vec{l}|, k, \cos\vartheta; t_f, t_b, t_g)\end{aligned}$$

with $\theta = \measuredangle(\vec{k}, \vec{l})$ and $\vartheta = \measuredangle(\vec{k} - \vec{l}, \vec{k})$
This yields:

- 7 dimensional integration: Five time integrals and one integral over $l$ and the angle $\theta$
- For each grid point: $(\lambda, k, t_a, t_b)$

# Numerical implementation

Solve integro-differential equation

$$\dot{\Gamma}^{\beta\beta}_{\lambda,ab} = -\mathrm{i}\frac{\delta^2 G^R_{\lambda,ch}}{\delta\phi_{\beta,a}\delta\phi_{\beta,b}}\dot{R}^R_{\lambda,hc}$$
$$\dot{\Gamma}^{f\beta}_{\lambda,ab} = -\mathrm{i}\frac{\delta^2 G^R_{\lambda,ch}}{\delta\phi_{f,a}\delta\phi_{\beta,b}}\dot{R}^R_{\lambda,hc}$$

with initial values at scale $\lambda = \Lambda_0$

$$\Gamma^{f\beta}_{\Lambda_0} = -(\Delta^{-1})_{f\beta} = -(\mathcal{I} - \mathrm{i}G^{(0)}_{F\rho})$$
$$\Gamma^{\beta\beta}_{\Lambda_0} = -(\Delta^{-1})_{\beta\beta} = \mathrm{i}G^{(0)}_{\rho\rho}$$

# Numerical implementation

Solve integro-differential equation

$$\dot{\Gamma}^{\beta\beta}_{\lambda,ab} = -\mathrm{i}\frac{\delta^2 G^R_{\lambda,ch}}{\delta\phi_{\beta,a}\delta\phi_{\beta,b}}\dot{R}^R_{\lambda,hc}$$

$$\dot{\Gamma}^{f\beta}_{\lambda,ab} = -\mathrm{i}\frac{\delta^2 G^R_{\lambda,ch}}{\delta\phi_{f,a}\delta\phi_{\beta,b}}\dot{R}^R_{\lambda,hc}$$

- MC Vegas integration on RHS
- Explicit Euler to integrate over the scale evolution: I.e.,

$$\Gamma^{(2)}_{\lambda_{n+1}} = \Gamma^{(2)}_{\lambda_n} + (\lambda_{n+1} - \lambda_n)\,\partial_\lambda \Gamma^{(2)}_\lambda\left[\Gamma^{(2)}_{\lambda_n}, R_{\lambda_n}, \lambda_n\right]$$

- Then, inversion of propagator on a time grid

$$G^R_\lambda = -(\Gamma^{\beta f}_\lambda + R^R_\lambda)^{-1}$$

and statistical propagator given by

$$\mathrm{i}F_\lambda = G^R_\lambda \Gamma^{\beta\beta}_\lambda G^A_\lambda$$

# Results

So far: Only tree-level, as the contributions from the flow are too small.

Chosen parameters:

- $t_1, t_2 \in [0, 7.0] \hat{=} z \in [1100, 0]$ on a linear axis with 70 points resolution,
- $k \in [10^{-3}h\text{Mpc}^{-1}, 10^{3}h\text{Mpc}^{-1}]$ on logarithmic axis with 140 points resolution, together with $k \in [0, 10^{-3}h\text{Mpc}^{-1}]$ on linear axis with 5 points resolution,
- $\lambda \in [0.01h\text{Mpc}^{-1}, 10.0h\text{Mpc}^{-1}]$ on a logarithmic axis with 50 points resolution.

# Chosen UV-Regulator

- Shape: Exponential regulator (with $c = 1$, $b = 2$), $y_{\mathrm{UV}} = \lambda^2/k^2$

$$r(y) = \frac{c \cdot y^{(b-1)}}{\exp\{y^b\} - 1}$$

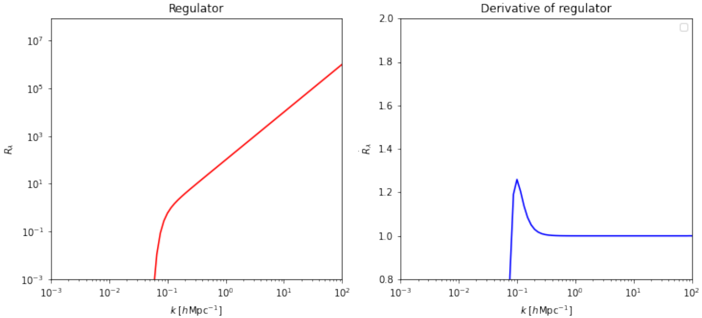



Figure: Exponential UV-regulator $r(y_{\mathrm{UV}})$

# Chosen UV-Regulator

- Shape: Exponential regulator (with $c = 1$, $b = 2$), $y_{\mathrm{UV}} = \lambda^2/k^2$

$$r(y) = \frac{c \cdot y^{(b-1)}}{\exp\{y^b\} - 1}$$

- Amplitude

$$R_\lambda(k) = \Gamma_\lambda^{(2)}(k)\; r(y)$$

- Time-dependence: local

$$R_\lambda(k; t_a, t_b) \propto \delta(t_a - t_b)$$

Since $\Gamma_\lambda^{\beta f}(k; t_a, t_a) = -1,\ \forall\, t_a, k, \lambda$, the regulator is

$$R_\lambda(k; t_a, t_b) = -\frac{y}{e^{y^2} - 1}\; \delta_D(t_a - t_b)$$

# Discussion

Thanks for your attention! I am happy about questions and ideas.

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