Hydrodynamic approach to heavy-quark diffusion in the quark-gluon plasma

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Cold Quantum Coffee Seminar - 7.12.2021

Outline

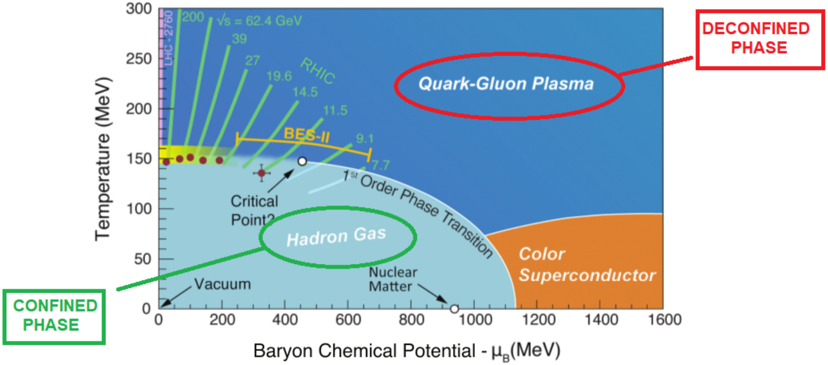
1. Motivations and introduction
2. Transport models
3. A new hydrodynamic approach
4. Results
5. Quantum corrections: do they matter?
6. Summary and outlook

Outline

Heavy-ion collisions

Purpose of heavy-ion collisions (HICs): studying the **QCD phase diagram**. Results from lattice-QCD and chiral Lagrangians show that there exist at least two phases.

- Goal: studying the **phase transition**!



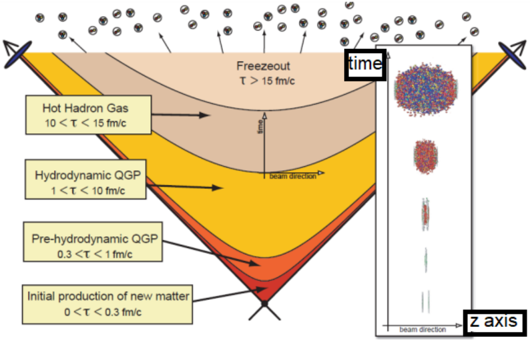
The quark-gluon plasma

Thus, it is necessary to provide a realistic description of the deconfined medium produced in HICs i.e. the **quark-gluon plasma** (QGP):

- plasma of thermalized quarks and gluons;
- expands because of its own thermal pressure (*fireball*);
- its presence affects the final observables measured with the experiments (e.g. particles momentum and angular distributions).

Different features of the QGP need to be studied with different tools:

- **soft probes**, e.g. low-momentum hadrons, investigate the collective behavior of the medium. Experimental results impressively reproduced by hydrodynamics
- **hard probes**, e.g heavy quarks, high-momentum hadrons/jets, quarkonia, are the results of *hard* processes happening at the beginning of the collisions; they go through all the stages of the expanding fireball!



In this work, we focus on heavy quarks.

How *heavy* are heavy quarks?

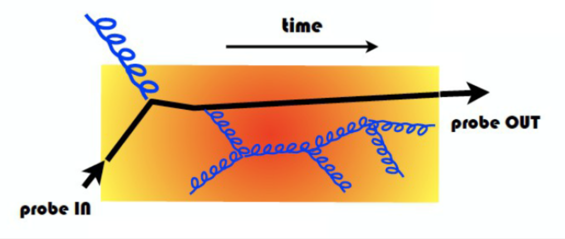
Charm (1.5 GeV) and bottom (4.8 GeV) are considered heavy because:

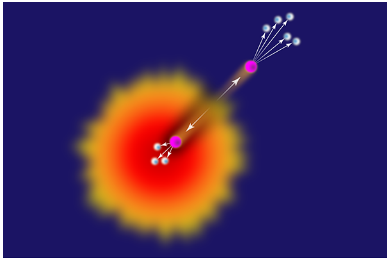
- $M \gg \Lambda_{\mathrm{QCD}}$: the initial production can be well described with pQCD;
- $M \gg T$: the heavy-quark thermal production in the QGP is negligible. Number of HQs is fixed by the initial production;
- $M \gg gT$: typical exchanged momentum is very small $\rightarrow$ several scatterings required to change the HQ momentum significantly.

Heavy quarks as hard probes

Heavy quarks in heavy-ion collisions:

- lead to a deeper understanding of the parton-medium interaction;
- perform a *tomography* of the medium;
- they are produced out of equilibrium $\rightarrow$ can they reach thermalization in the expanding QGP?





Many transport models have been developed in the literature!

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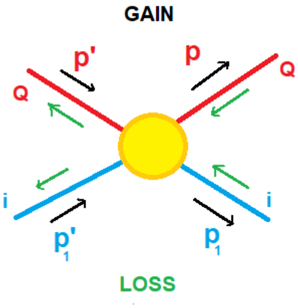
Transport models

Transport models are usually based on the Boltzmann equation, which studies how the distribution function associated to the heavy quark varies in time:

$$p^\mu \partial_\mu f_p = C[f_p] \quad (1)$$



Heavy quarks:

- brownian particles scattering with light partons from the medium;
- asymptotically reach thermal equilibrium

In this context, the community is mainly focused on the description of the **transport coefficients** which characterise the medium and parametrize the interaction between the heavy quarks and the light partons from the QGP.

The Boltzmann Equation

$$p^\mu \partial_\mu f_p = C[f_p] \tag{2}$$

$$C[f_Q] = \int d\boldsymbol{p}' d\boldsymbol{p}_1 d\boldsymbol{p}'_1 w(\boldsymbol{p}', \boldsymbol{p}'_1 | \boldsymbol{p}, \boldsymbol{p}_1)(f_Q(\boldsymbol{p}')f_i(\boldsymbol{p}'_1) - f_Q(\boldsymbol{p})f_i(\boldsymbol{p}_1))$$

The integral vanishes *if and only if* $f_Q(\boldsymbol{p}')f_i(\boldsymbol{p}'_1) = f_Q(\boldsymbol{p})f_i(\boldsymbol{p}_1)$, which entails:

$$f_Q(\boldsymbol{p}) = \exp\left(-\frac{E_p}{T}\right) \quad f_i(\boldsymbol{p}_1) = \exp\left(-\frac{E_{p_1}}{T}\right) \tag{3}$$

The Boltzmann equations makes the HQs relax to a **thermal distribution** at the same temperature of the medium!

The Fokker-Planck Equation

In the limit of *small momentum exchange* ([PRD 37 2484]) the Boltzmann equation reduces to the Fokker-Planck equation:

$$\frac{\partial f_Q(t,\boldsymbol{p})}{\partial t} = \frac{\partial}{\partial p^i}\{A^i(\boldsymbol{p})f_Q(t,\boldsymbol{p}) + \frac{\partial}{\partial p^j}[B^{ij}(\boldsymbol{p})f_Q(t,\boldsymbol{p})]\} \tag{4}$$

where

$$\begin{cases} A^i(\boldsymbol{p}) = A(p)p^i \quad \text{friction} \\ B^{ij}(\boldsymbol{p}) = (\delta^{ij} - p^i p^j)B_0(p) + p^i p^j B_1(p) \quad \text{momentum broadening} \end{cases}$$

The problem reduces to the evaluation of three transport coefficients!

The Fokker-Planck Equation

The FP equation admits the Boltzmann thermal distribution solution when the right-hand side of Eq. (4) vanishes. This entails that the transport coefficients satisfy Einstein's **fluctuation-dissipation** relation!

Let's see it in a simple case:

- ignoring the momentum dependence of the transport coefficients $\rightarrow \gamma = A(p) \quad D = B_0(p) = B_1(p)$
- starting form $f_Q(t=0, \boldsymbol{p}) = \delta(\boldsymbol{p} - \boldsymbol{p}_0)$

one gets

$$f_Q(t, \boldsymbol{p}) \propto \exp\left(-\frac{\gamma}{2D} \frac{[\boldsymbol{p} - \boldsymbol{p}_0 \exp(-\gamma t)]^2}{1 - \exp(-2\gamma t)}\right) \tag{5}$$

Asymptotically (for $t \to \infty$), the solution *forgets about the initial conditions* and tends to a thermal distribution:

$$f_Q(t, \boldsymbol{p}) \propto \exp(-\frac{\gamma M_Q}{D} \frac{\boldsymbol{p}^2}{2M_Q}) \tag{6}$$

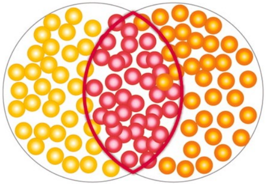
$\to D = \gamma M_Q T$ (*Einstein's FD relation*).

If we study the first momenta of the distribution we find:

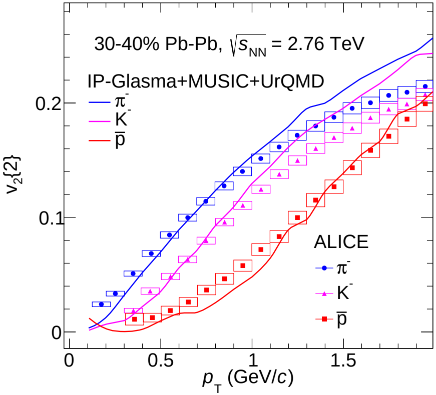
$$\begin{cases} \langle \boldsymbol{p}(t) \rangle = \exp\left(-\gamma t\right) & \text{friction} \\ \langle \boldsymbol{p}^2(t) \rangle - \langle \boldsymbol{p}(t) \rangle^2 \sim 6Dt \text{ for } t \to 0 & \text{momentum-diffusion coefficient} \end{cases}$$

Outline

Relativistic viscous hydrodynamics: powerful tool in order to capture the features of the collective motion of the medium. It successfully describes light-flavour observables such as elliptic flow of protons, pions and kaons [NPA 979 (2018)].

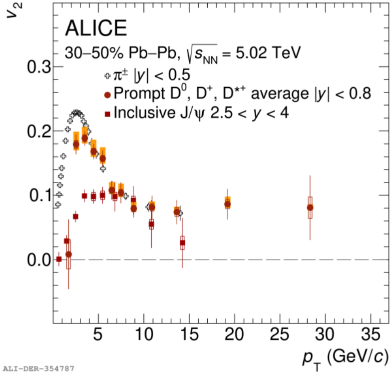


Elliptic flow is an important probe of collectivity in the system created in HICs: it is a response of the dense system to the initial conditions → sensitive to the early and strongly interacting phase of the evolution.



Signs of thermalization

Recent experimental results [PRB 813 (2021) 136054] show that charm quarks have positive elliptic flow $\rightarrow$ sign of thermalization!



$\rightarrow$ Idea: if heavy quarks had enough time to interact with light partons from the medium and thermalize, they could be treated as part of the medium itself!

A new approach

The question is:

how far indeed are heavy quarks from achieving thermal equilibrium?

We address this question from a new point of view: combining the microscopic description from the Boltzmann equation with a hydrodynamic framework.

→ In practice: we construct a heavy-quark current which propagates within the medium, following the conservation laws of hydrodynamics and study the connection between transport coefficients arising from hydrodynamics and standard transport theory.

Heavy-quark current: new hydrodynamic approach to heavy quarks

We write the heavy-quark current associated to the conservation of $Q\bar{Q}$ pairs as:

$$N^\mu = nu^\mu + \nu^\mu \tag{7}$$

such that

$$\partial_\mu N^\mu = 0 \tag{8}$$

where n is the heavy-quark density and u^μ is the fluid four-velocity. ν^μ is the **particle-diffusion current** which obeys the equation of motion:

$$\tau_n \dot{\nu}^\mu + \nu^\mu = \kappa_n \nabla^\mu \left(\frac{\mu_{Q\bar{Q}}}{T} \right) \tag{9}$$

where $\mu_{Q\bar{Q}}$ is the $Q\bar{Q}$ chemical potential. The relaxation time τ_n tells us for how long the out-of-equilibrium component ν^μ contributes dynamically to the full particle current ("hydrodynamization"). The particle-diffusion coefficient κ_n tells us how strong the diffusion process is.

Link with transport theory

$$\tau_n \dot{\nu}^{\langle\mu\rangle} + \nu^\mu = \kappa_n \partial^\mu(\mu/T) \tag{10}$$

One can relate the relaxation time with the collision integral (e.g. method of moments [arxiv 1202.4551]):

$$\tau_n^{-1} = \int dK k^\mu \nu_\mu C[f_k] \tag{11}$$

with $dK = gd^3k/(2\pi)^3 k_0$ and g the internal degrees of freedom of the particle. I plug in the Fokker-Planck approximation in the collision integral.. and get relation between FP coefficients and hydro!

Method of moments: a brief overview

It's used to derive hydrodynamic equations from kinetic theory (Boltzmann): The Boltzmann equation reads:

$$k^{\mu}\partial_{\mu}f_k = C[f_k] = \frac{1}{\nu}\int dK' dP dP' W_{kk'\to pp'}(f_p f_{p'} - f_k f_{k'}) \tag{12}$$

The conserved particle current N^{μ} is expressed as

$$N^{\mu} = \langle k^{\mu} \rangle \tag{13}$$

where:

$$\langle ... \rangle = \int dK (...) f_k \tag{14}$$

We decompose the distribution function in a equilibrium part + a deviation: $f_k = f_{0k} + \delta f_k$ and introduce the average over the equilibrium distribution function f_{0k}:

$$\langle ... \rangle_0 = \int dK (...) f_{0k} \quad f_{0k} = \left[\exp\left(E_k/T - \mu/T\right)\right]^{-1} \tag{15}$$

and the average over the deviation from equilibrium

$$\langle ... \rangle_\delta = \langle ... \rangle - \langle ... \rangle_0 \tag{16}$$

We impose *Landau matching conditions*:

$$n = n_0 = \langle E_k \rangle_0 \tag{17}$$

The diffusion current is given by $\nu^\mu = \langle k^{\langle \mu \rangle} \rangle_\delta$.

Irreducible moments of the distribution

The distribution function can be expanded in terms of the moments

$$\rho_n^{\mu_1 \cdots \mu_l} = \langle E_k^n k^{\langle \mu_1 \cdots} k^{\mu_l \rangle} \rangle_\delta \tag{18}$$

such that

$$\delta f_k = f_{0k} \sum_{l=0}^{\infty} \sum_{n=0}^{N_l} H_{kn}^{(l)} \rho_n^{\mu_1 \cdots \mu_l} k_{\langle \mu_1 \ldots} k_{\mu_l \rangle} \tag{19}$$

Notice: $\rho_0^\mu = \langle k^{\langle \mu \rangle} \rangle_\delta = \nu^\mu$!
The Boltzmann equation can be rewritten as

$$E_k \dot{\delta} f_k = -E_k - k^i \partial_i f_{0k} - k^i \partial_i \delta f_k - \dot{f}_{0k} + C[f_k] \tag{20}$$

One can use this to calculate equations of motion for the moments, e.g.

$$\dot{\rho}_n^\mu = u^\mu \partial_\mu \langle E_k^n k^{\langle \mu \rangle} \rangle_\delta \tag{21}$$

Equation of motion

$$\dot{\rho}_0^{\mu} - \int_K k^{\langle\mu\rangle} C[f_k] = \alpha_0^{(1)} \partial^{\mu} \frac{\mu}{T} + \ldots \tag{22}$$

The integral can be linearized in terms of the moments:

$$\dot{\rho}_0^{\mu} - A\rho^{\mu} = \alpha_0^{(1)} \partial^{\mu} \frac{\mu}{T} \tag{23}$$

which is a relaxation-type equation. Taking $\tau_n = A^{-1}$ one gets

$$\tau_n \dot{\rho}_0^{\mu} - \rho^{\mu} = \tau_n \alpha_0^{(1)} \partial^{\mu} \frac{\mu}{T} \tag{24}$$

where τ_n is the relaxation time and $\kappa_n \equiv \alpha_0^{(1)} \tau_n$ is defined as diffusion coefficient.

Outline

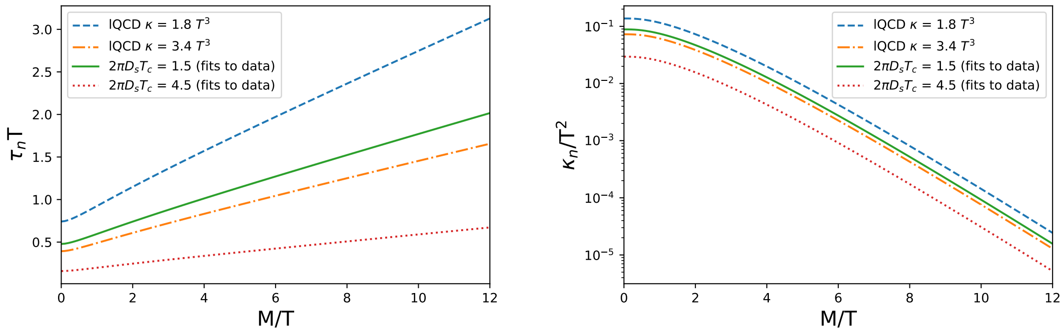


Figure 1: Left panel: heavy-quark relaxation time times temperature. Right panel: heavy-quark diffusion coefficient divided by squared temperature. Different lines correspond to different transport coefficients from transport theory.

Relaxation time and diffusion coefficient

Looking at a fixed temperature how the diffusion process changes with the HQ mass!

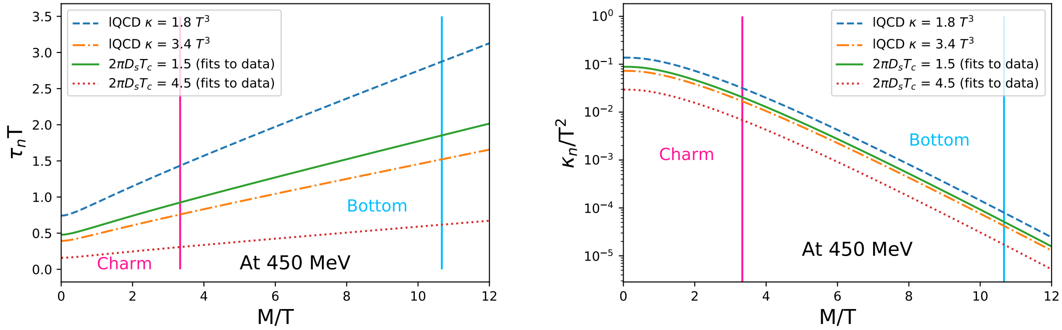


Figure 2: The diffusion process for charm quarks is faster and is mediated via a stronger diffusion coefficient compared to bottom quarks.

Self-consistency check: comparison with Bjorken flow

Bjorken expansion: $v_x = v_y = 0 \qquad v_z = z/t$
Longitudinal proper time: $\tau = \sqrt{t^2 - z^2}$

Typical expansion rate $\theta = \partial_\mu u^\mu = \tau^{-1} \rightarrow$ typical expansion time $\tau_{exp} = \theta^{-1} = \tau$

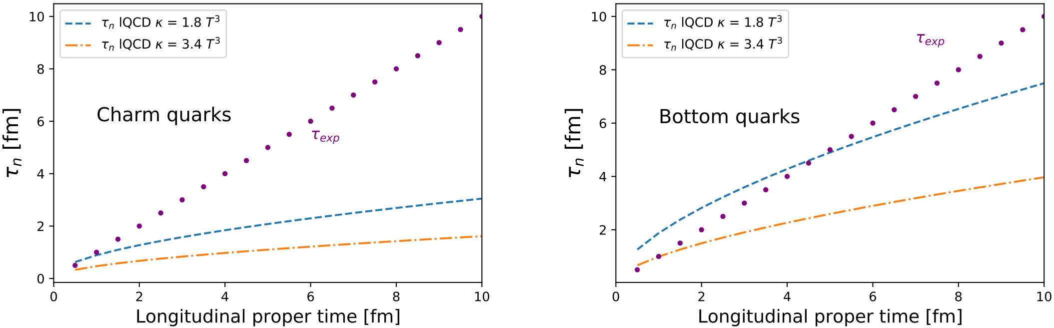


Figure 3: Relaxation time for charm (left) and bottom (right) compared to typical Bjorken expansion time: hydrodynamic conditions for charm are realized very fast!

Outline

If one considers the case of a single momentum-independent diffusion coefficient – namely $B_0 = B_1 \equiv D$ – the correspondent Fokker-Planck equation reads

$$C[f_k^Q] = k_0 \frac{\partial}{\partial k^i} \left\{ A(k) k^i f_k^Q \tilde{f}^Q - D g^{ij} \frac{\partial}{\partial k^j} f_k^Q] \right\}, \tag{25}$$

where $\tilde{f}_k = 1 - f_k$ accounts for Pauli blocking. This equation admits an analytical stationary solution in terms of a Fermi-Dirac distribution,

$$f_{0k}^Q = \left[\exp(-\frac{\mu_Q}{T}) \exp(\frac{E_k}{T}) + 1 \right]^{-1} . \tag{26}$$

Results

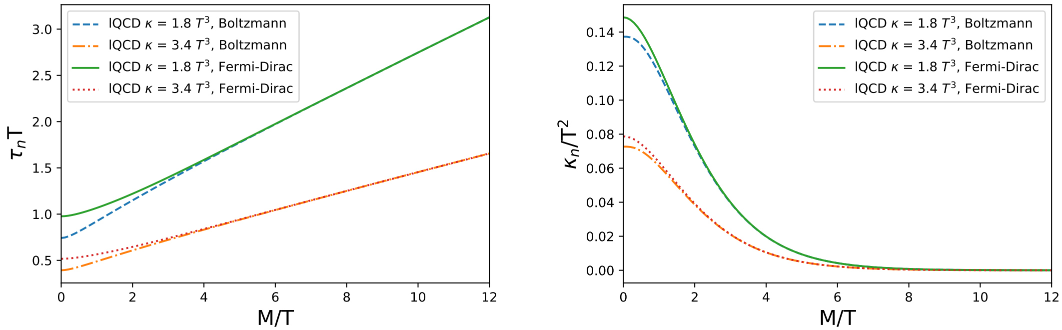


Figure 4: Corrections are visible only for $M \ll T$.

Outline

Summary and outlook

Summary:

- We constructed a heavy-quark current and computed the correspondent hydrodynamic transport coefficients connecting them to the FP equation
- By checking against Bjorken flow, we proved that a hydrodynamic description for heavy quarks is meaningful!

Exciting steps are ahead:

- Implement the heavy-quark current in the code simulating the hydrodynamic evolution of the QGP (FluiduM [PRC 100 014905]) and compare to data;
- study how the heavy-quark current can interact with other conserved currents (associated e.g. to baryon number, electric charge and strangeness conservation)