

# Exploring the IR sector of QCD out of equilibrium

*Based on ongoing work with J. Berges, K. Boguslavski, J. M. Pawłowski  
Related work ongoing by JB, KB, JMP and T. Butler, P. Radpay (resp.)*

Lillian de Bruin  
ITP, Universität Heidelberg

Cold Quantum Coffee seminar: 26 April 2022

# Outline

Motivation and objectives

Wilson Loops and Polyakov loop correlators

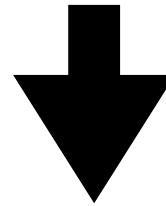
The scalar algebra field

Conclusions and ongoing/future work

# Initial overoccupation and thermalization

Thermalization  
dynamics from initial  
overoccupation

- heavy on collisions
- early universe re-heating
- ultracold quantum gases



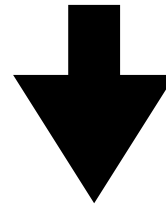
Bose condensation

- gauge invariant order param.  
field in QCD
- relativistic scalar inflation
- nonrelativistic Bose field

# Initial overoccupation and thermalization

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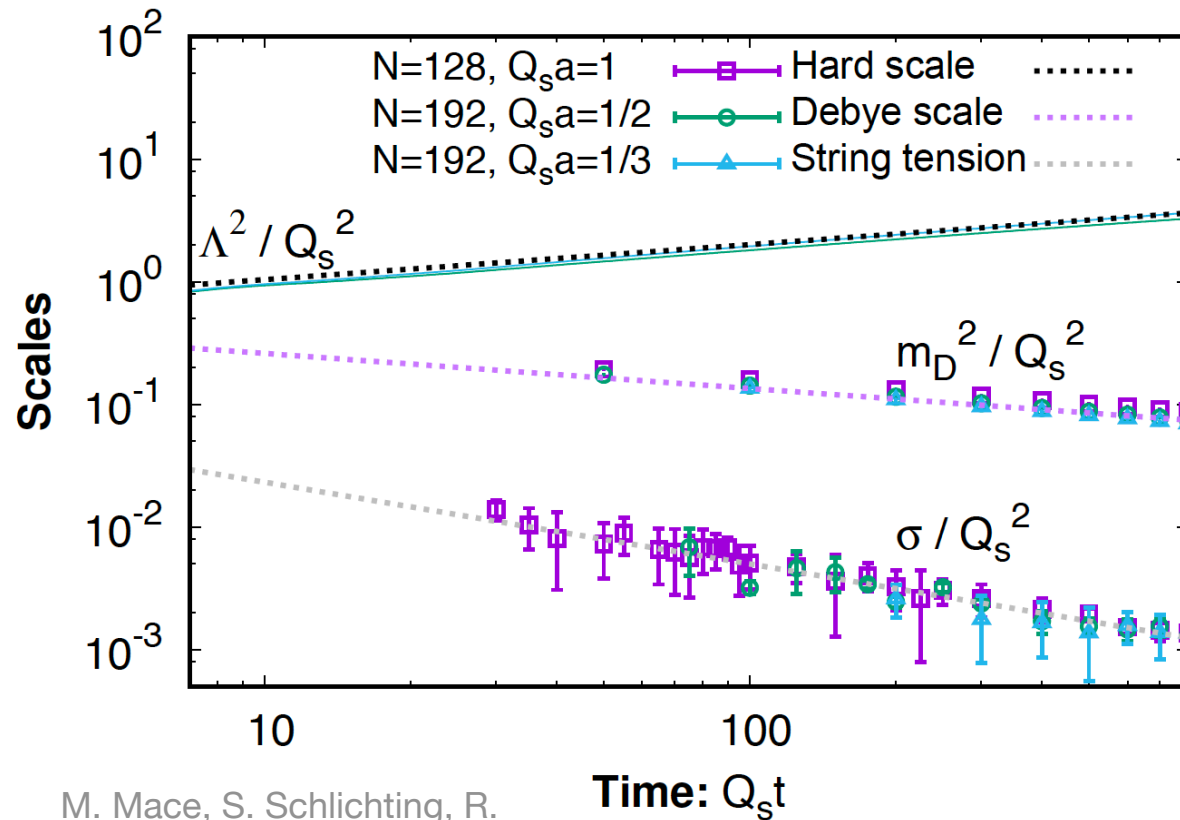
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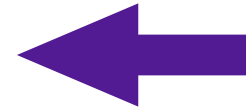
Bose condensation

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# Thermalization: separation of scales



**Ultrasoft** scale evolves faster than **soft** scale!



M. Mace, S. Schlichting, R. Venugopalan, PRD (2016)

# Objectives

One of the basic questions we need to answer is: to what extent do IR modes influence thermalization of the QGP out of equilibrium?

In order to study dynamics at this scale, we need to identify an appropriate **order parameter**

# Wilson loops and Polyakov loop correlators

# Characteristics of over-occupied QGP

Gluons produced in heavy ion collisions are expected to have typical momenta on order of saturation scale  $Q \rightarrow$  Over-occupation of gluons at time  $t \sim 1/Q$

Running gauge coupling is small:  $\alpha_s(Q_s) \ll 1$

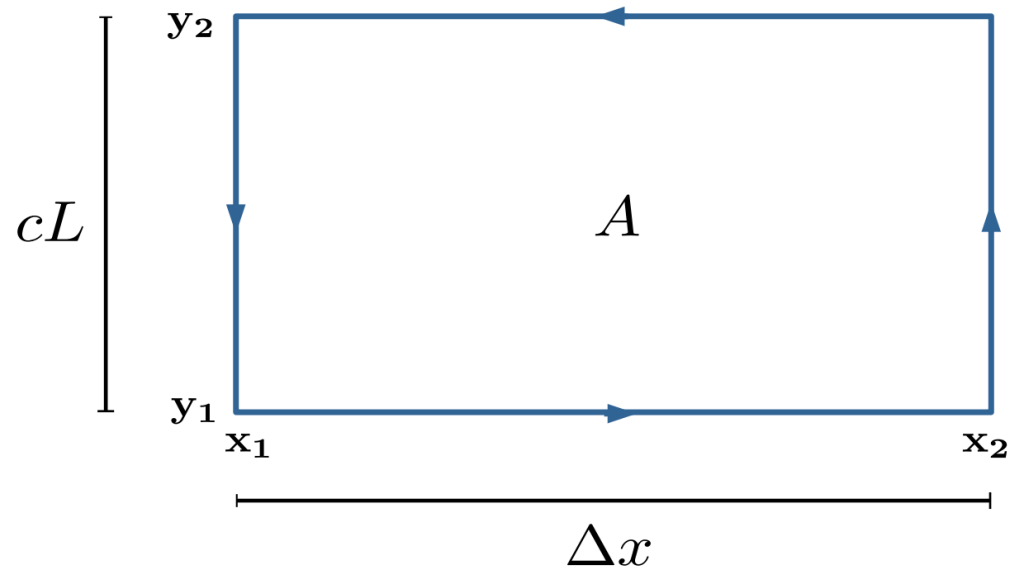
System is considered strongly correlated due to high gluon occupancy

Non-perturbative quantum problem can be mapped to classical-statistical lattice gauge theory



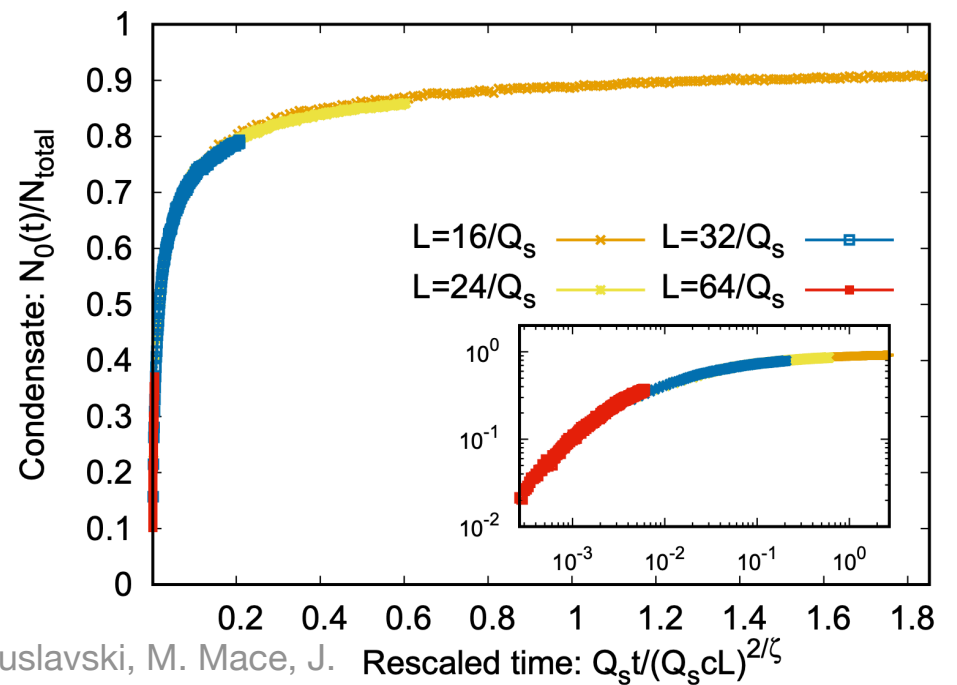
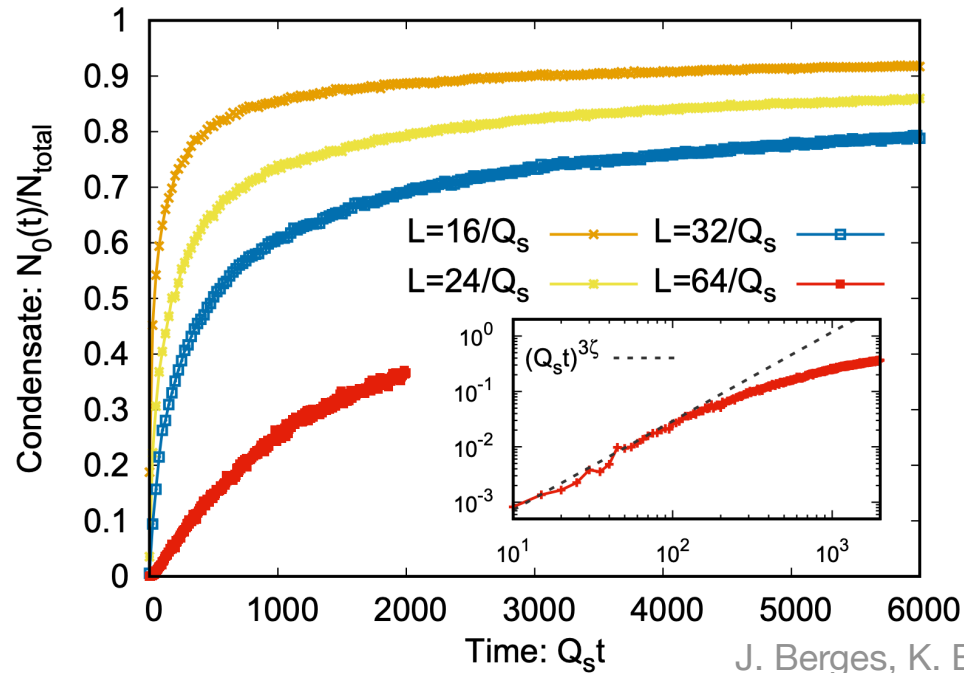
# Spatial Wilson loops

Gauge invariant quantity that captures long-distance behavior of gauge fields



$$\langle W(\Delta x, \Delta y; t) \rangle = \left\langle \frac{1}{N_c} \text{Tr} \mathcal{P} e^{-ig \int_{C[\Delta x, \Delta y]} \mathcal{A}_i(\mathbf{z}; t) dz_i} \right\rangle$$

# Condensate fraction: Wilson loop



J. Berges, K. Boguslavski, M. Mace, J. M. Pawłowski, PRD (2020)

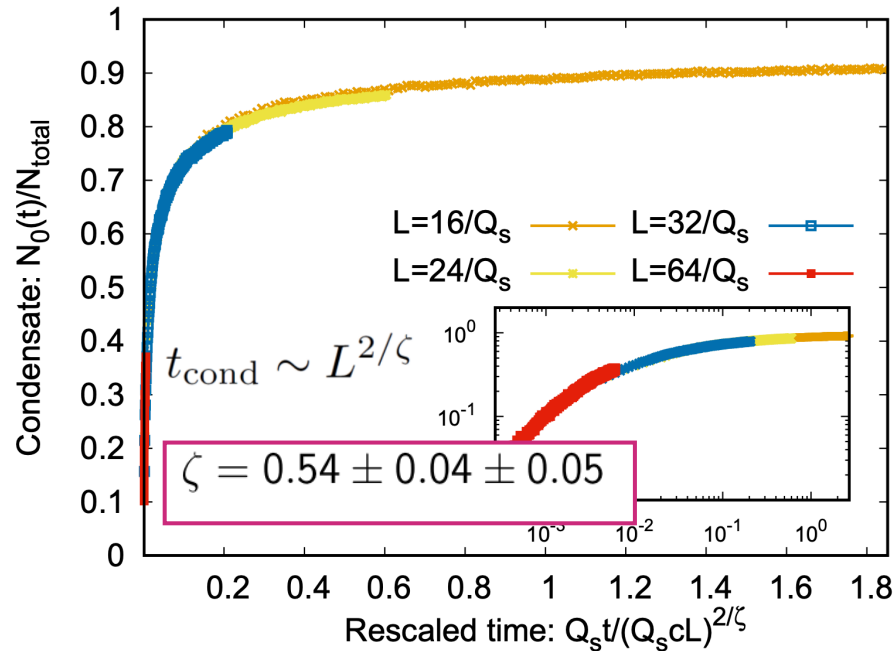
$$\begin{aligned} \frac{N_0(t, L)}{N_{\text{total}}} &= \frac{1}{V} \int_0^L d^d x \langle W(\Delta x, L, t) \rangle \\ &= \frac{1}{V} \int_0^L d^d \Delta x \omega_S(\Delta x L/t^\zeta) \end{aligned}$$

$$t_{\text{cond}} \sim L^{2/\zeta}$$

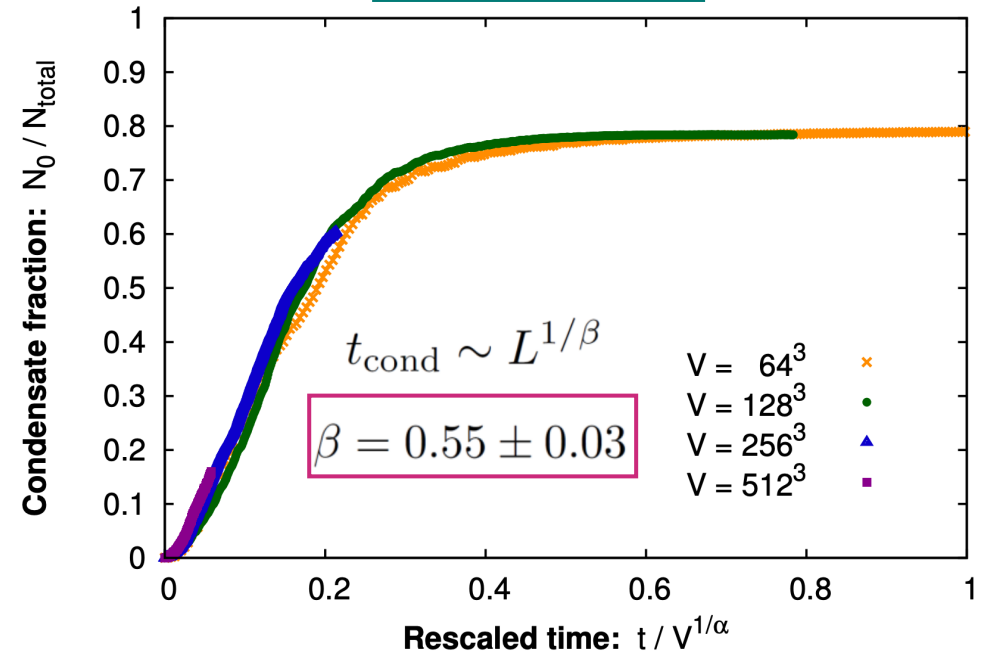
$$\zeta = 0.54 \pm 0.04 \pm 0.05$$

# Condensation from initial overoccupation

Gauge fields



Scalar fields

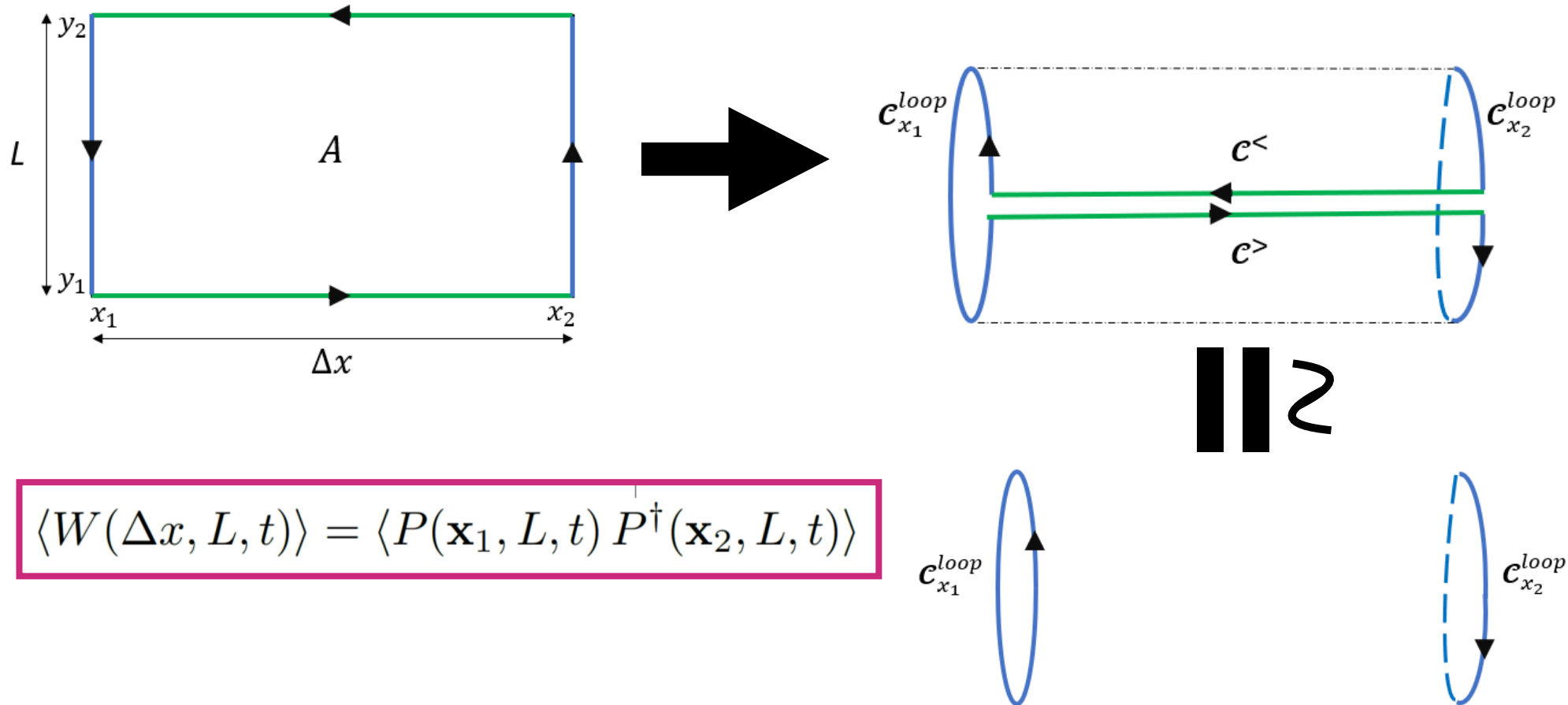


$$\begin{aligned} \frac{N_0(t, L)}{N_{\text{total}}} &= \frac{1}{V} \int_0^L d^d x \langle W(\Delta x, L, t) \rangle \\ &= \frac{1}{V} \int_0^L d^d \Delta x \omega_S(\Delta x L / t^\zeta) \end{aligned}$$

$$\begin{aligned} N_{\text{total}}^\phi &= \langle \phi(\mathbf{x}, t) \phi^\dagger(\mathbf{x}, t) \rangle \\ \frac{N_0^\phi(t)}{N_{\text{total}}^\phi} &= \frac{1}{N_{\text{total}}^\phi V} \int_0^L d^d x \langle \phi(x) \phi^\dagger(0) \rangle \\ &= \frac{1}{N_{\text{total}}^\phi V} \int_0^L d^d \Delta x f_S(\Delta x / t^\beta) \end{aligned}$$

A. Piñeiro Orioli,  
 J. Berges, K.  
 Boguslavski  
 PRD (2015)

# Spatial “Polyakov” loop

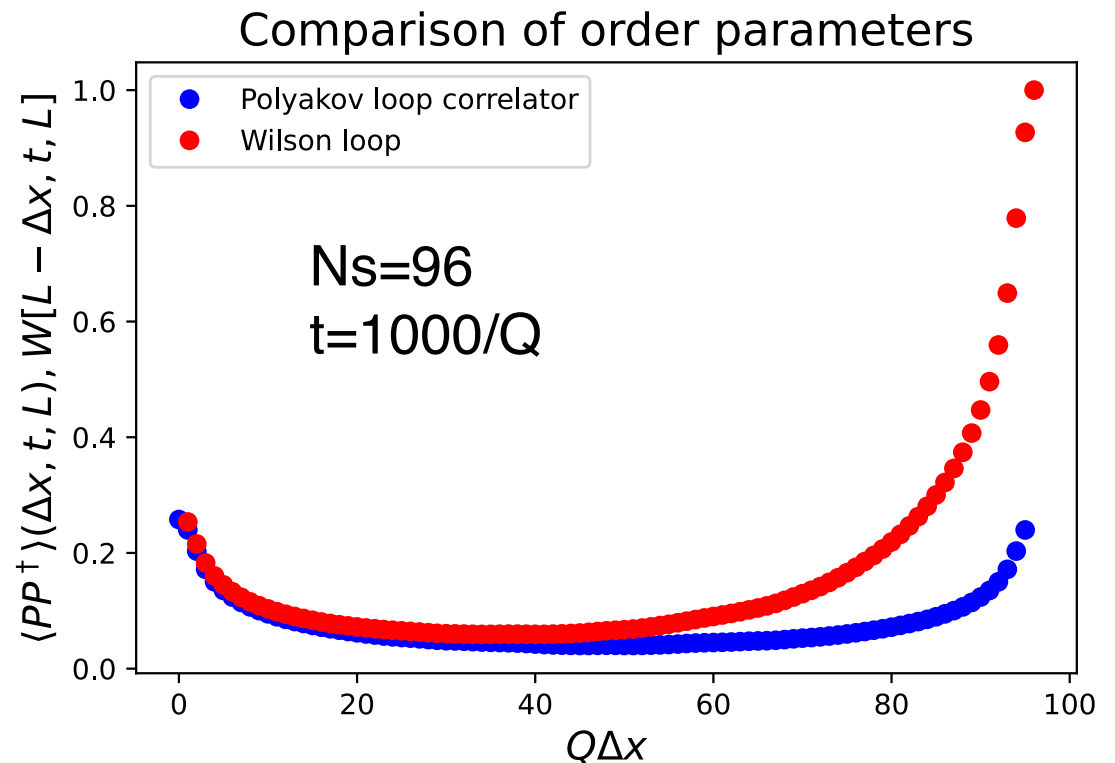


$$\langle W(\Delta x, L, t) \rangle = \langle P(\mathbf{x}_1, L, t) P^\dagger(\mathbf{x}_2, L, t) \rangle$$

T. Butler

# Spatial Polyakov loop cont'd.

$$P(\mathbf{x}) = \frac{1}{N_c} \text{Tr} \mathcal{P} e^{-ig \int_0^L dy A_y(x,y)}$$

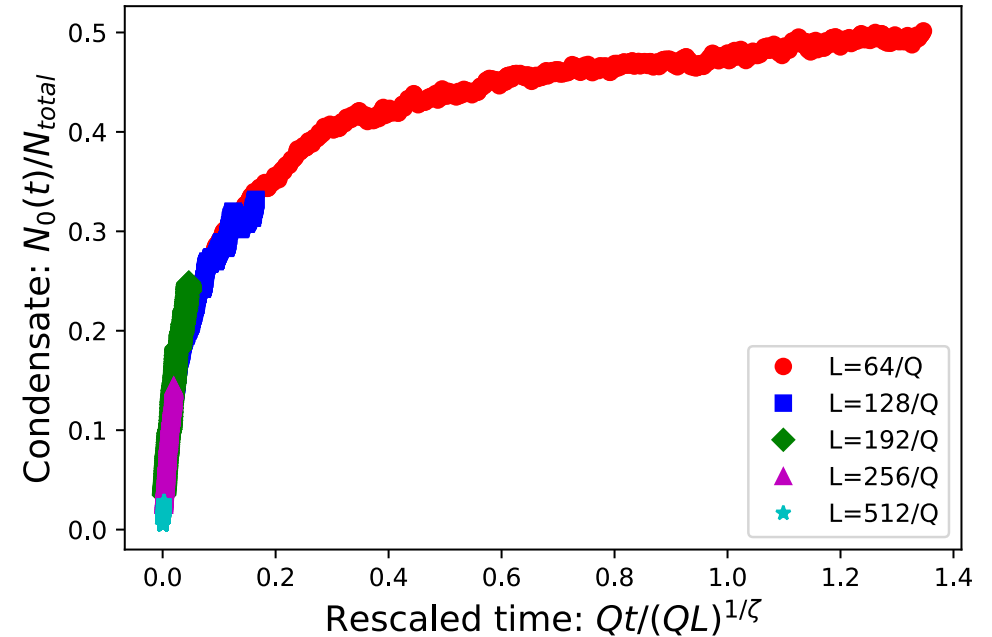
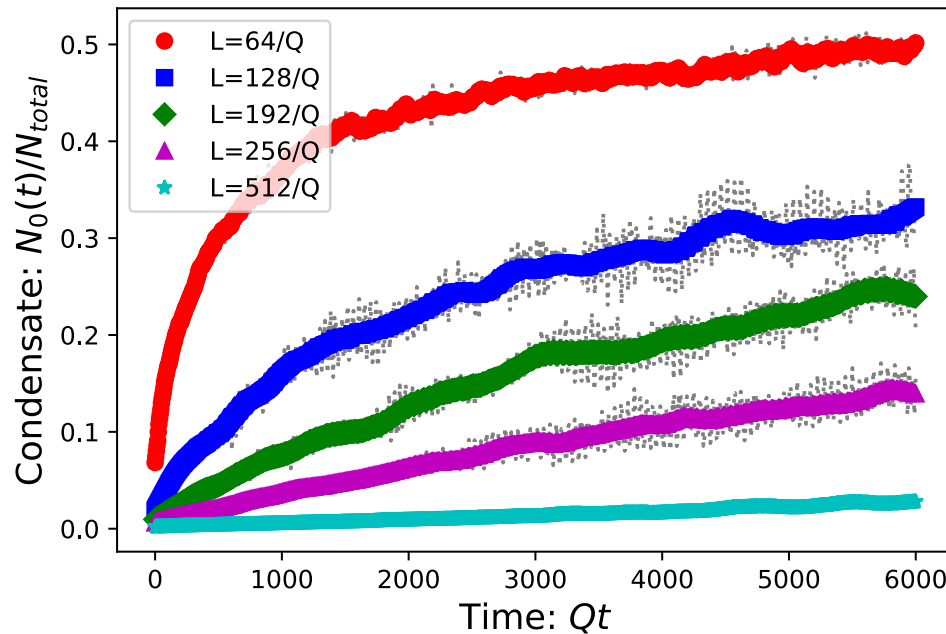


Same dynamics as the Wilson loop

T. Butler

Correlator is local, symmetric in  $x$  — cleaner order parameter!

# Spatial Polyakov loop cont'd.



$$\frac{N_0(t, L)}{N_{\text{total}}} \equiv \frac{1}{L} \int_0^L d\Delta x \frac{\langle PP^\dagger(t, \Delta x, L) \rangle}{\langle PP^\dagger(t, 0, L) \rangle}$$

J. Berges, K. Boguslavski, T. Butler, J. M. Pawłowski, (in preparation)

$$t_{\text{cond}} \sim (L)^{1/\zeta}$$

$$\zeta = 0.33 \pm 0.05 \text{ (stat.)}$$

# The scalar algebra field

# The scalar algebra field

We can rewrite the Polyakov loop

$$P(\mathbf{x}) = \frac{1}{N_c} \text{tr} \mathcal{P} \exp \left\{ -ig \int_0^L dy A_y(x, y) \right\},$$

as

$$P(\mathbf{x}) = \exp \{ i\phi \},$$

where  $\phi$  is the algebra-valued scalar field.



# Diagonalization of $P(\mathbf{x})$

$P(\mathbf{x})$  can be diagonalized by gauge transformations  $U \in SU(N)$ ,

$$P(\mathbf{x}) \rightarrow U^{-1}(y=0)P(\mathbf{x})U(y=cL)$$

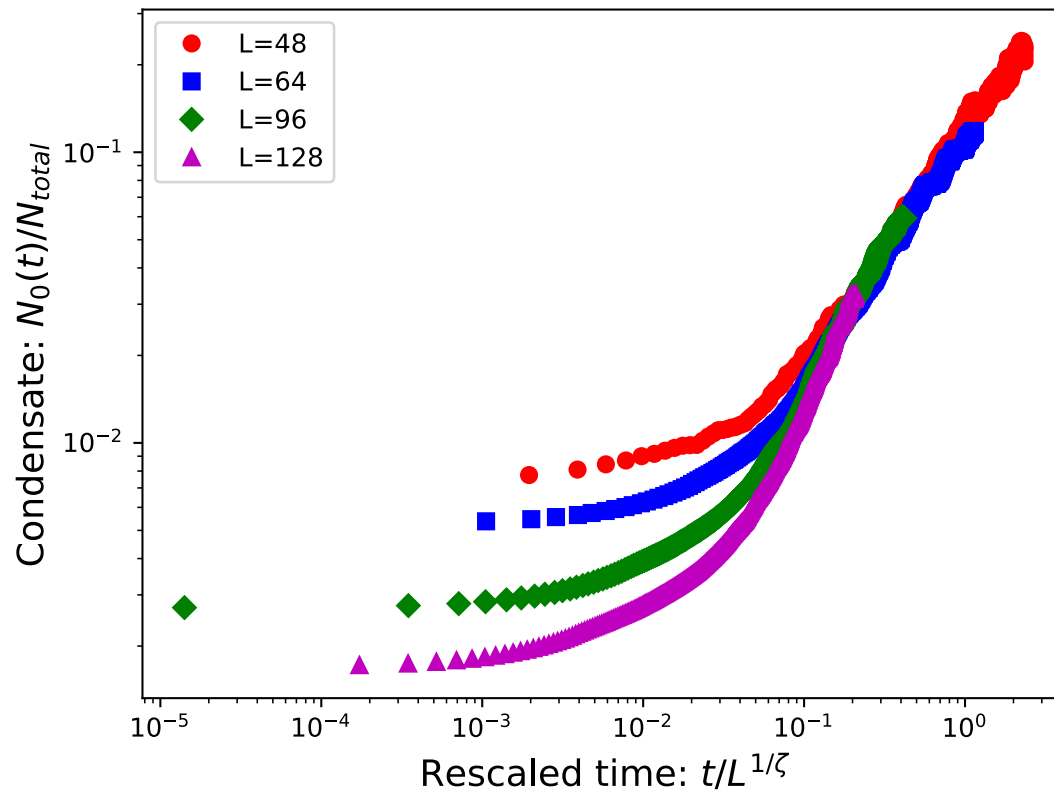
The eigenvalues  $\varphi_n$  of  $\phi$  are directly related to the eigenvalues of the Polyakov loop,  $\exp\{i\varphi_n\}$ , and are gauge invariant in the fundamental representation.

We want to explore this scalar field as an order parameter.

# Condensate fraction: algebra fields

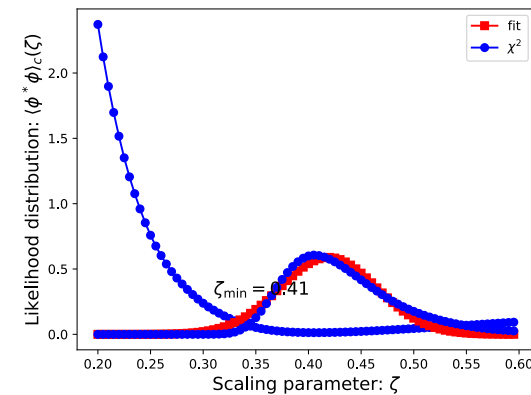
Connected correlator of algebra fields:

$$\langle \phi^* \phi \rangle_c (\Delta x, t) = \langle \phi^* \phi \rangle (\Delta x, t) - \langle \phi^*(x_1, t) \rangle \langle \phi(x_2, t) \rangle$$



$$t_{\text{cond}} \sim (L)^{1/\zeta}$$

$$\zeta = 0.41$$



# Related questions

Rigorously establishing the non-triviality of the Polyakov loop correlator, connected algebra field correlator —> Study higher order correlation functions of these order parameters

P. Radpay

# Conclusion

- The algebra-valued scalar field is a promising order parameter for gauge invariant condensation
- There's much to be done in this direction
  - e.g. study longtime limits