Local quantum field theory in extreme environments

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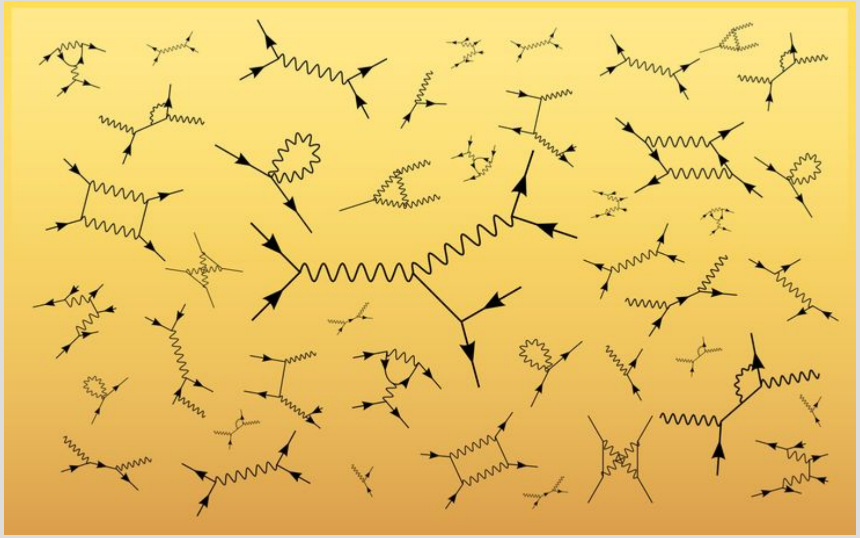
Motivation

- Perturbation theory has proven to be an extremely successful tool for investigating problems in particle physics



But by definition this procedure is only valid in a ***weakly interacting regime***

→ Non-convergence of perturbative series
→ Observables: form factors, parton distribution functions, hadronic properties, ...
→ Confinement in QCD

- This emphasises the need for a non-perturbative approach!

→ ***Local QFT*** is one such approach

Local QFT

- In the 1960s, A. Wightman and R. Haag pioneered an approach which set out to answer the fundamental question "*what is a QFT?*"
- The resulting approach, *Local QFT*, defines a QFT using a core set of physically motivated axioms

Axiom 1 (Hilbert space structure). *The states of the theory are rays in a Hilbert space $\mathcal{H}$ which possesses a continuous unitary representation $U(a, \alpha)$ of the Poincaré spinor group $\overline{\mathscr{P}_+^\uparrow}$.*

Axiom 2 (Spectral condition). *The spectrum of the energy-momentum operator P^μ is confined to the closed forward light cone $\overline{V}^+ = \{p^\mu \mid p^2 \geq 0,\, p^0 \geq 0\}$, where $U(a, 1) = e^{iP^\mu a_\mu}$.*

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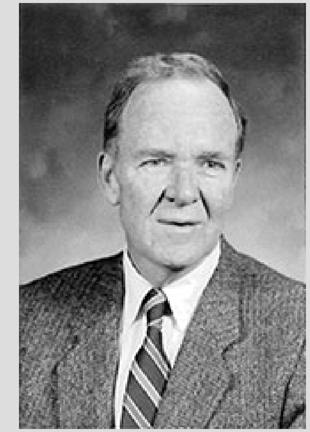
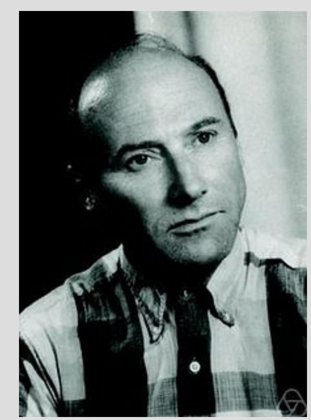


A. Wightman

[R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics, and all that* (1964).]



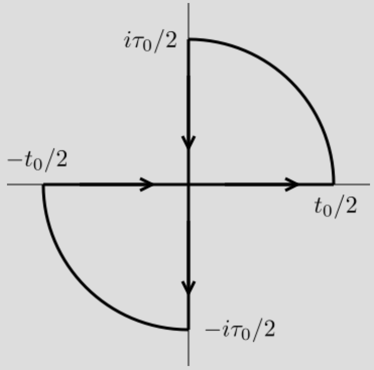
R. Haag

[R. Haag, *Local Quantum Physics*, Springer-Verlag (1992).]

Local QFT

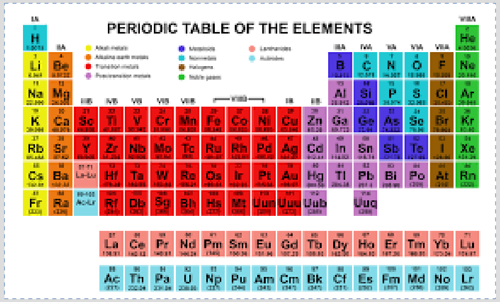
- Local QFT has led to many fundamental insights, including:

  → Relationship between Minkowski and Euclidean QFTs



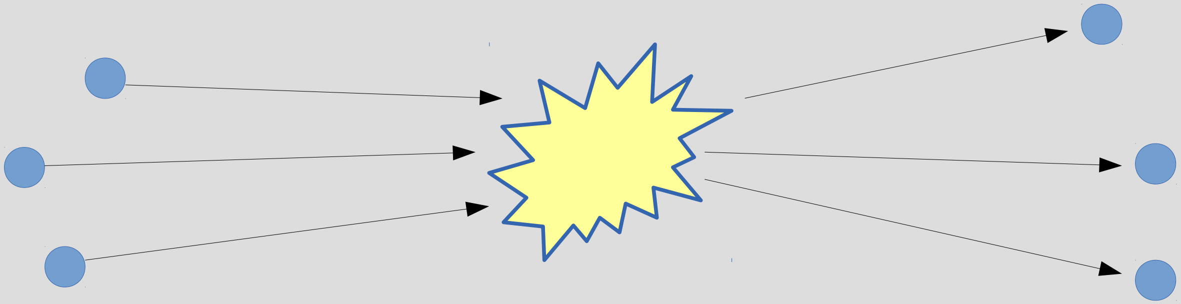
  → *CPT* is a symmetry of *any* QFT

  → Connection between spin & particle statistics



  → Existence of dispersion relations

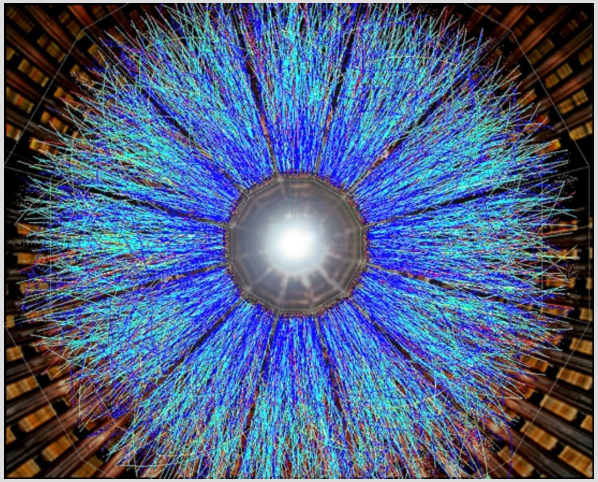
  → Scattering theory



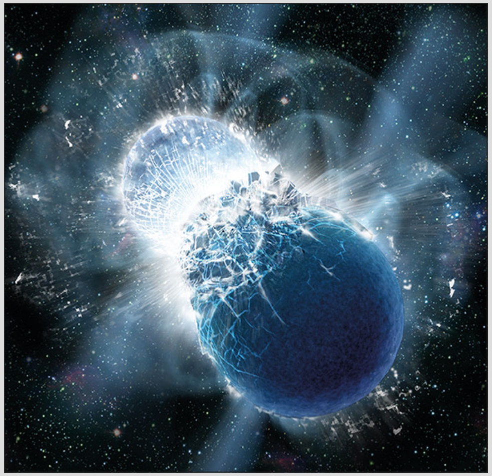
Local QFT beyond the vacuum

- But... local QFT only describes particle dynamics in the vacuum state

  → What about "extreme environments" where the system is either hot, dense, or both?



[Brookhaven National Lab]



[Skyworks Digital Inc.]

- Understanding local QFT in such environments is essential, and yet has received relatively little attention. Particularly important progress was made by J. Bros and D. Buchholz for non-vanishing temperature T

→ See: [Z. Phys. C **55** (1992) 509, hep-th/9606046, hep-th/9807099, hep-ph/0109136]

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- **Idea**: Look for a generalisation of the standard axioms that is compatible with $T>0$, and approaches the vacuum case for $T\to 0$

Axiom 1 (**Hilbert space structure**). *The states of the theory are rays in a Hilbert space $\mathcal{H}$ which possesses a continuous unitary representation $U(a,\alpha)$ of the Poincaré spinor group $\overline{\mathscr{P}_+^\uparrow}$.*

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→ ***H_β* is defined for fixed *$\beta = 1/T$***

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→ **Replaced by the KMS condition**

$$\langle \Omega_\beta | \phi(x_1) \cdots \phi(x_k) \phi(x_{k+1}) \cdots \phi(x_n) | \Omega_\beta \rangle$$
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✓ Locality is unaffected by the properties of the background state. *This is important!*

Non-perturbative implications

- By demanding fields to be local ($[\Phi(x),\Phi(y)]=0$ for $(x\text{-}y)^2<0$) this imposes significant constraints on the structure of correlation functions

  → For $T=1/\beta>0$, the scalar spectral function has the representation:

$$\rho(p_0,\vec{p}) := \mathcal{F}\left[\langle\Omega_\beta|\left[\phi(x),\phi(y)\right]|\Omega_\beta\rangle\right] = \int_0^\infty ds \int \frac{d^3\vec{u}}{(2\pi)^2}\,\epsilon(p_0)\,\delta\big(p_0^2-(\vec{p}-\vec{u})^2-s\big)\,\widetilde{D}_\beta(\vec{u},s)$$

"*Thermal spectral density*"

Note: this is a ***non-perturbative*** representation!

- In the limit of vanishing temperature one recovers the well-known *Källén-Lehmann* spectral representation:

$$\rho(p_0,\vec{p}) \xrightarrow{\beta\to\infty} 2\pi\,\epsilon(p_0)\int_0^\infty ds\;\delta\big(p^2-s\big)\,\rho(s)$$

e.g. $\rho(s)=\delta(s\text{-}m^2)$ for a massive free theory

Important question: *what does the thermal spectral density* $\widetilde{D}_\beta(\boldsymbol{u},s)$ *look like?*

Non-perturbative implications

- A natural decomposition [J. Bros, D. Buchholz, hep-ph/0109136] is:

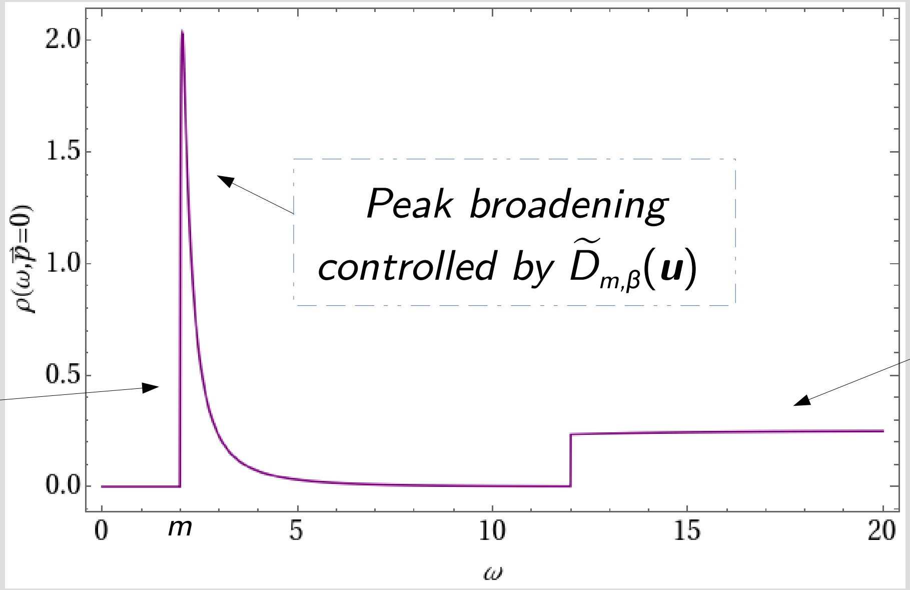
$$\widetilde{D}_{\beta}(\vec{u},s) = \widetilde{D}_{m,\beta}(\vec{u})\,\delta(s-m^2) + \widetilde{D}_{c,\beta}(\vec{u},s)$$

"Damping factor"

Causes $T=0$ mass pole m to be <u>screened</u> by thermal effects

Continuous component

Fixes T-dependence of continuous spectral contributions

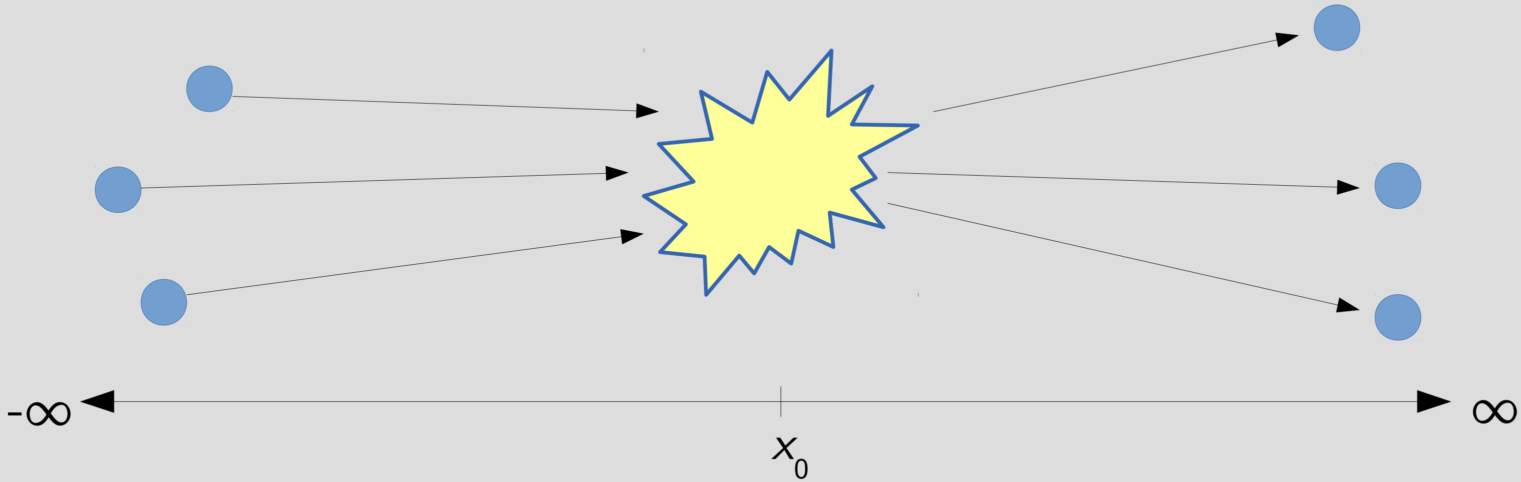


→ Damping factors hold the key to understanding in-medium effects!

Damping factors from asymptotic dynamics

- Since all observable quantities are computed using correlation functions, which are characterised by *damping factors*, one can use these to gain new insights into the properties of QFTs when $T>0$

- It has been proposed [Bros, Buchholz, hep-ph/0109136] that these quantities are controlled by the large-time x_0 dynamics of the theory



Important: Interactions with the thermal background persist, even for large x_0

→ ***Need to take this into account in definition of scattering states!***

Damping factors from asymptotic dynamics

- Idea: thermal scattering states are defined by imposing an asymptotic field condition (hep-ph/0109136):

Asymptotic fields Φ_0 are assumed to satisfy dynamical equations, but only at large x_0

In Φ^4 theory

"Asymptotic coupling"

$$(\partial^2 + m^2)\phi_0(x) + \frac{\lambda}{3!}\phi_0^3(x) \xrightarrow{|x_0|\to\infty} 0$$

"Asymptotic mass"

- Given that the thermal spectral density has the decomposition

$$\widetilde{D}_\beta(\vec{u}, s) = \widetilde{D}_{m,\beta}(\vec{u})\,\delta(s - m^2) + \widetilde{D}_{c,\beta}(\vec{u}, s)$$

it follows that:

1. The continuous contribution to $\langle\Omega_\beta|\phi(x)\phi(y)|\Omega_\beta\rangle$ is **suppressed** for large x_0

2. The particle damping factor $\widetilde{D}_{m,\beta}(\boldsymbol{u})$ is **uniquely fixed** by the asymptotic field equation

- This means that the non-perturbative thermal effects experienced by particle states are entirely controlled by the asymptotic dynamics!

Φ^4 theory for $T > 0$

- Applying the asymptotic field condition for Φ^4 theory, the resulting damping factors have the form [hep-ph/0109136]:

→ **For $\boldsymbol{\lambda} < 0$:** $$D_{m,\beta}(\vec{x}) = \frac{\sin(\kappa|\vec{x}|)}{\kappa|\vec{x}|}$$ → **For $\boldsymbol{\lambda} > 0$:** $$D_{m,\beta}(\vec{x}) = \frac{e^{-\kappa|\vec{x}|}}{\kappa_0|\vec{x}|}$$

where κ is defined with $r = m/T$:

$$\kappa = T\sqrt{|\lambda|}K(r), \qquad K(r) = \sqrt{\int \frac{d^3\hat{\vec{q}}}{(2\pi)^3 2\sqrt{|\hat{\vec{q}}|^2 + r^2}} \frac{1}{e^{\sqrt{|\hat{\vec{q}}|^2+r^2}} - 1}}$$

→ *The parameter κ has the interpretation of a thermal width: $\kappa \to 0$ for $T \to 0$*, or equivalently *κ^{-1} is mean-free path*

- Now that one has the exact dependence of $D_{m,\beta}(\boldsymbol{x})$ on the external physical parameters, in this case T, m and λ, one can use this to calculate observables ***analytically***

Φ^4 theory for $T > 0$

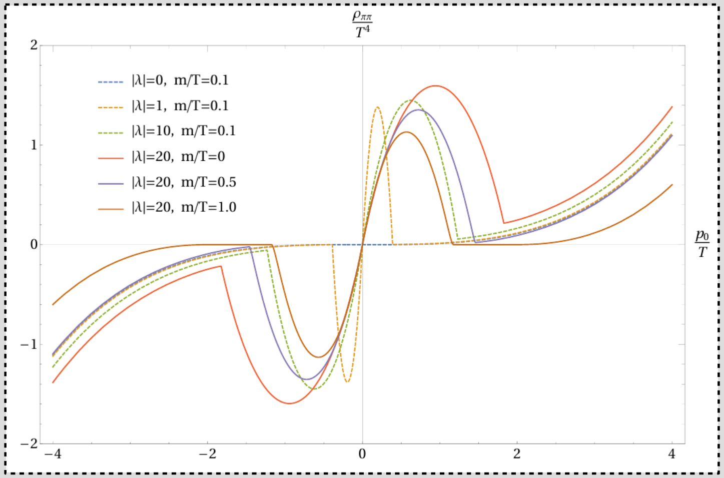
- Of particular interest is the *shear viscosity* η, which measures the resistance of a medium to sheared flow

  $\rightarrow$ This quantity can be determined from the spectral function of the spatial traceless energy-momentum tensor

$$\rho_{\pi\pi}(p_0) = \lim_{\vec{p}\to 0} \mathcal{F}\left[\langle\Omega_\beta|\left[\pi^{ij}(x), \pi_{ij}(y)\right]|\Omega_\beta\rangle\right](p)$$

  ... and η is recovered via the *Kubo relation* $\eta = \frac{1}{20}\lim_{p_0\to 0}\frac{d\rho_{\pi\pi}}{dp_0}$

- Using $D_{m,\beta}(\boldsymbol{x})$ for $\lambda < 0$, the EMT spectral function $\rho_{\pi\pi}$ has the form:

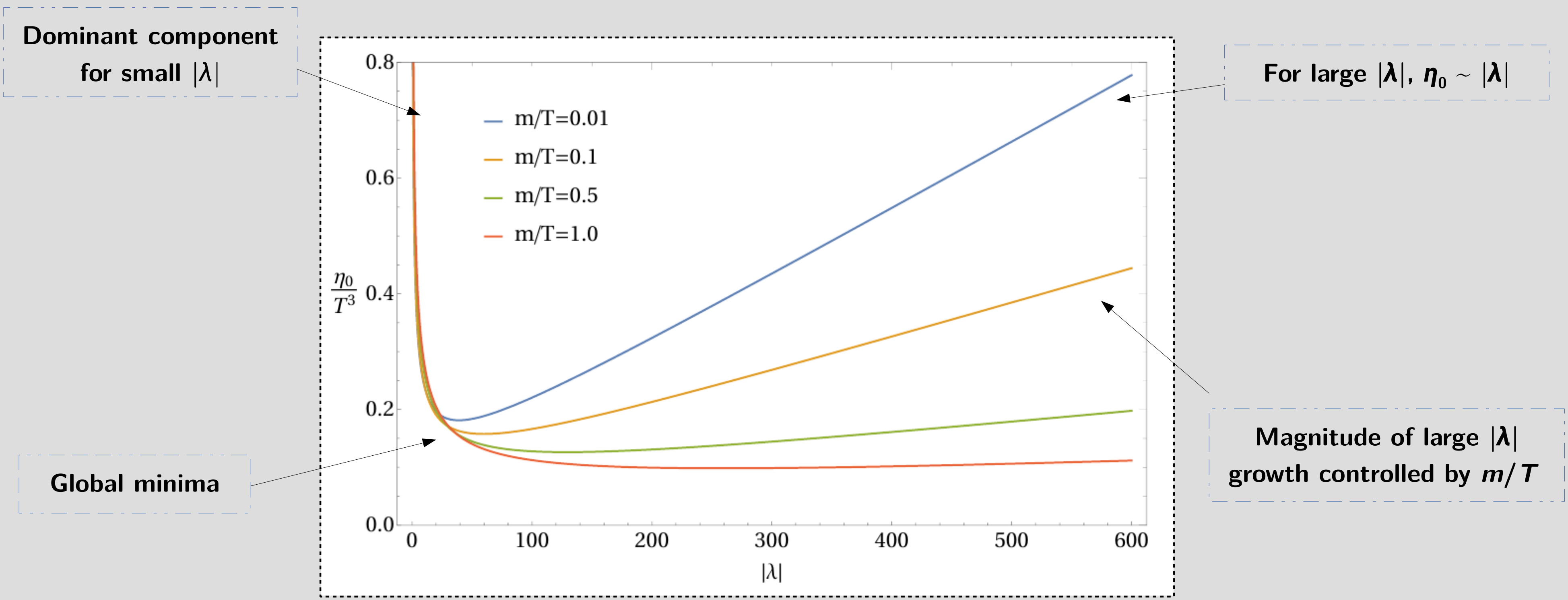


- The presence of interactions causes resonant peaks to appear $\rightarrow$ peaked when $p_0 \sim \kappa = 1/\ell$
- For $\lambda \rightarrow 0$ the free-field result is recovered, as expected
- The dimensionless ratio m/T controls the magnitude of the peaks

Φ^4 theory for $T > 0$

- Applying Kubo's relation, the shear viscosity η_0 arising from the asymptotic states can be written [P.L., R.-A. Tripolt, J. M. Pawlowski, D. H. Rischke, 2104.13413]

$$\eta_0 = \frac{T^3}{15\pi}\left[\frac{\mathcal{K}_3\left(\frac{m}{T},0,\infty\right)}{\sqrt{|\lambda|}} + \sqrt{|\lambda|}\,\mathcal{K}_1\left(\frac{m}{T},0,\infty\right) + \frac{\mathcal{K}_4\left(\frac{m}{T},\sqrt{|\lambda|}K\left(\frac{m}{T}\right),\sqrt{|\lambda|}K\left(\frac{m}{T}\right)\right)}{4|\lambda|}\right]$$



$\rightarrow$ For fixed coupling, η_0/T^3 is entirely controlled by functions of m/T

Φ^4 theory for $T > 0$

- What about the case $\lambda > 0$? $\rightarrow$ η_0 *diverges!*

  Why? – *The particle damping factor $D_{m,\beta}(\boldsymbol{u})$ does not decay rapidly enough at large momenta*

- This characteristic is related to the "bad" UV behaviour of the quartic interaction, i.e. the triviality of Φ^4 appears to have an impact beyond $T=0$!

- In 2104.13413 it was shown more generally that the finiteness of η_0 is related to the existence of thermal equilibrium

> *If the KMS condition holds* $\Longrightarrow$ η_0 is finite

- This procedure demonstrates that asymptotic dynamics can be used to explore the non-perturbative properties of QFTs when $T>0$

  $\rightarrow$ *Can also calculate other observables, e.g. transport coefficients, entropy density, pressure, etc.*

Damping factors from Euclidean data

- The constraints imposed by locality offer new ways in which to understand, and compute, in-medium observables
- It turns out that these constraints also have significant implications in *Euclidean* spacetime

  → *Important to understand, since many non-perturbative techniques, e.g. lattice, functional methods (DSEs, FRG), are restricted to, or optimised for, calculations in imaginary time* τ

- In many instances $T>0$ Euclidean data is used to extract observables, e.g. spectral functions from $\mathcal{W}_E(\tau) = \int d^3x\, \mathcal{W}_E(\tau,\vec{x})$

$$\mathcal{W}_E(\tau) = \int_0^\infty \frac{d\omega}{2\pi} \frac{\cosh\left[\left(\frac{\beta}{2} - |\tau|\right)\omega\right]}{\sinh\left(\frac{\beta}{2}\omega\right)} \rho(\omega)$$

Determine $\rho(\omega)$ given $W_E(\tau)$

→ ***Inverse problem!***

- Problem is ill-conditioned, need additional information (see e.g. H. B. Meyer, 1104.3708 for review of different inversion approaches)

Damping factors from Euclidean data

- However, locality constraints imply that particle damping factors $D_{m,\beta}(\boldsymbol{x})$ can be directly calculated from Euclidean data, avoiding the inverse problem [P.L., 2201.12180]

$$D_{m,\beta}(\vec{x}) \sim e^{|\vec{x}|m} \int_0^\infty \frac{d|\vec{p}|}{2\pi}\, 4|\vec{p}| \sin(|\vec{p}||\vec{x}|)\, \widetilde{G}_\beta(0,|\vec{p}|).$$

p-space Euclidean propagator

Holds for large separation $|\boldsymbol{x}|$

- Like with the asymptotic calculations, $D_{m,\beta}(\boldsymbol{x})$ can then be used as input for phenomenological calculations

- In [P.L., R.-A. Tripolt, 2202.09142] pion propagator data from the quark-meson model (FRG calculation) was used to compute the damping factor at different values of T via the analytic relation above

- Fits to the resulting data were consistent with the form: $D_{\pi,\beta}(\vec{x}) = \alpha\, e^{-|\vec{x}|\gamma}$

$\rightarrow$ *Both parameters α and γ showed a significant T dependence*

Damping factors from Euclidean data

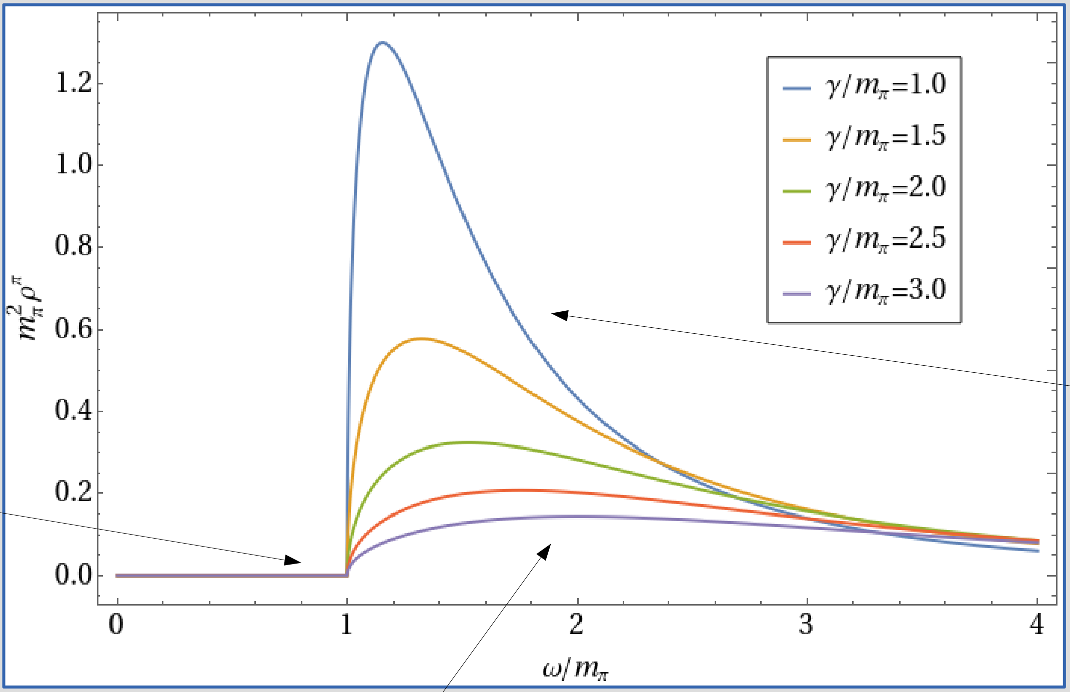
- Using the $T>0$ spectral representation one finds:

$$D_{\pi,\beta}(\vec{x}) = \alpha\, e^{-|\vec{x}|\gamma}$$

Implies

$$\rho^{\pi}(\omega) = \epsilon(\omega)\theta(\omega^2 - m_\pi^2)\frac{4\alpha\,\gamma\sqrt{\omega^2 - m_\pi^2}}{(\omega^2 - m_\pi^2 + \gamma^2)^2}$$

m is minimal energy needed to create particle with $\mathbf{p}=0$ $\rightarrow$ *cutoff at* $\omega = m_\pi$

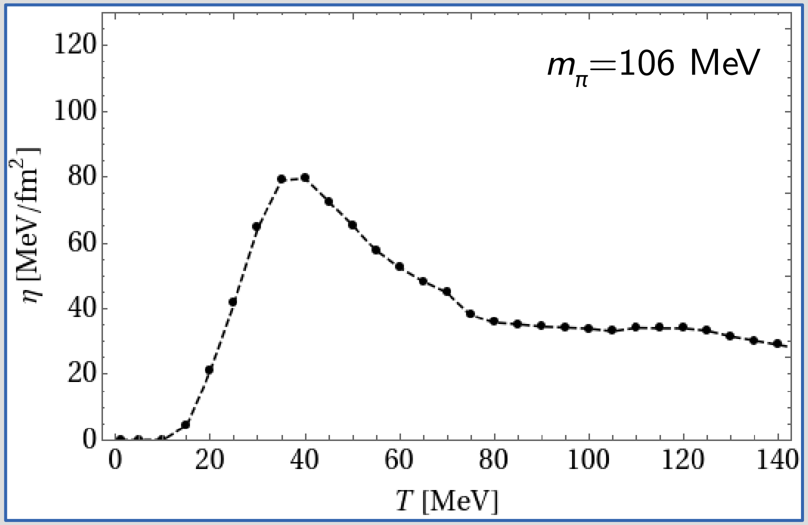


Width $\gamma\rightarrow 0$ *as* $T\rightarrow 0$, *causing increasingly peaked behaviour*

As T increases the width γ also increases, causing the thermal pion state to "melt"

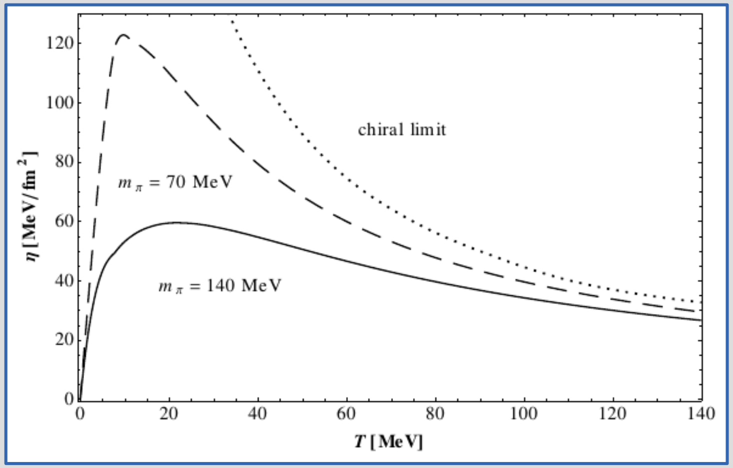
Damping factors from Euclidean data

- Using the analytic relations derived in [2104.13413] for the shear viscosity as a function of the damping factor, the numerically extracted values for $D_{\pi,\beta}(\boldsymbol{x})$ can be used to compute the shear viscosity



η vanishes for $T \to 0$, and appears to level out at large T

- Can compare these results with those obtained using chiral perturbation theory
  $\rightarrow$ *Very similar qualitative features!*



[R. Lang, N. Kaiser, W. Weise, 1205.6648]

Damping factors from Euclidean data

- In the FRG analysis we used *p*-space data to extract $D_{m,\beta}(\boldsymbol{x})$. Can we use *x*-space data intead? Yes!

  → A quantity of particular interest in lattice studies is the spatial correlator of particle-creating operators, defined:

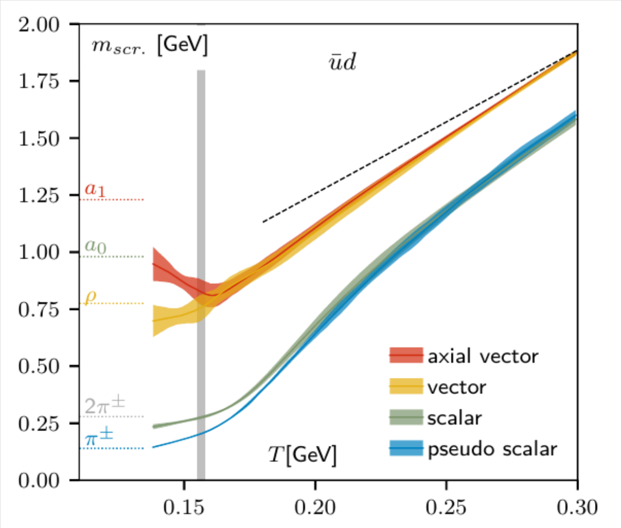
$$C(z) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} d\tau\, \mathcal{W}_E(\tau, \vec{x})$$

e.g. meson operators

$$\mathcal{W}_E(\tau, \vec{x}) = \langle \Omega_\beta | \overline{\psi}\Gamma\psi(x)\, \overline{\psi}\Gamma\psi(0) | \Omega_\beta \rangle$$

- Usually, the large-*z* behaviour of $C(z) \sim exp(-m_{scr}|z|)$ is used to extract particle *screening masses* m_{scr}

- This quantity is important for understanding phenomena such as *quarkonium melting* and (effective) chiral restoration in QCD



[HotQCD collaboration, 1908.09552]

Damping factors from Euclidean data

- Using an equivalent result to that in *p*-space, one obtains the following general relation between the damping factor and spatial correlator

$$D_{m,\beta}(z) \sim -2e^{m|z|} \frac{dC(z)}{dz}$$

← *Holds for large z*

- The implication of this relation is that the dependence of screening masses m_{scr} on the external *physical* parameters; T, m, etc. is dictated by the damping factors $D_{m,\beta}$

→ *Each particle experiences different in-medium effects!*

- The advantage of using spatial correlator data is that one can obtain systematically improvable data, i.e. use larger lattice sizes!
- Using this approach one can proceed to analyse the properties of meson/baryon damping factors in QCD, and use this for phenomenology

→ ***Work in progress!***

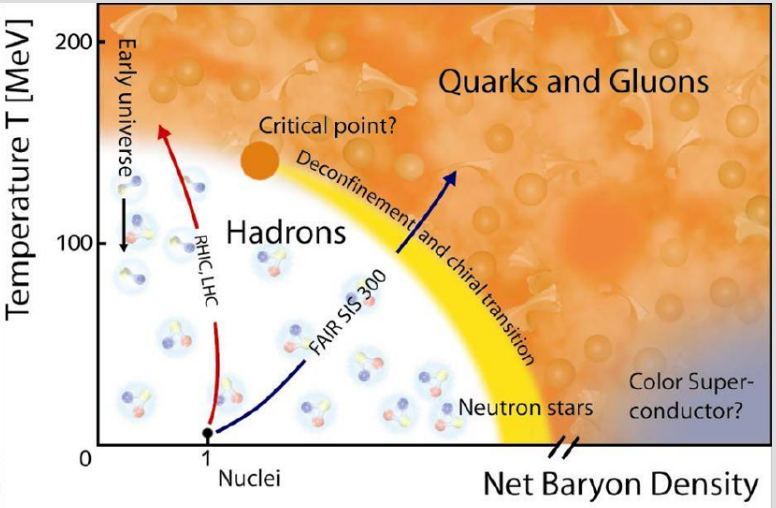
Framework generalisations

- So far we have only discussed the simplest situation: a real scalar field $\Phi(x)$ with T$>$0

  $\rightarrow$ *What about fields/states with higher spin?*

  $\rightarrow$ *What about regimes where the background environment is dense, characterised by a ground state with* $\mu \neq 0$*?*

- Answering these questions is essential for fully understanding the properties of particles in extreme environments, and in particular, unravelling the characteristics of the QCD phase diagram

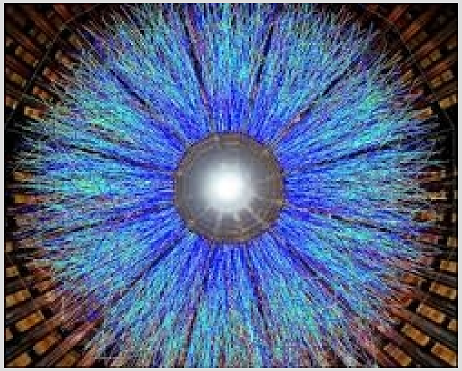


[GSI Homepage]

Summary & outlook

- Local QFT is an analytic framework that attempts to address the fundamental question "*what is a QFT?*"
- The framework can be extended to $T > 0$, and this has important implications, including:
  - → Connection to asymptotic dynamics
  - → Extraction of in-medium observables from Euclidean data
  - → Interpretation of screening masses
- So far only real scalar fields $\Phi(x)$ with $T > 0$ considered, but this approach can be extended (higher spin, $\mu \neq 0$). *Work in progress!*

→ This framework provides a way of obtaining **non-perturbative** insights into the phase structure of QFTs, and the resulting in-medium phenomena



[Brookhaven National Lab]

Backup

- For thermal asymptotic states, the spectral function $\rho_{\pi\pi}$ has the form

$$\rho_{\pi\pi}(p_0) = \sinh\left(\frac{\beta}{2}p_0\right) \int \frac{d^3\vec{q}}{(2\pi)^4}\frac{2}{3}|\vec{q}|^4 \int_{-\infty}^{\infty} dq_0\, \frac{\widetilde{C}_\beta(q_0,\vec{q})\,\widetilde{C}_\beta(p_0-q_0,\vec{q})}{\sinh\left(\frac{\beta}{2}q_0\right)\sinh\left(\frac{\beta}{2}(p_0-q_0)\right)}$$

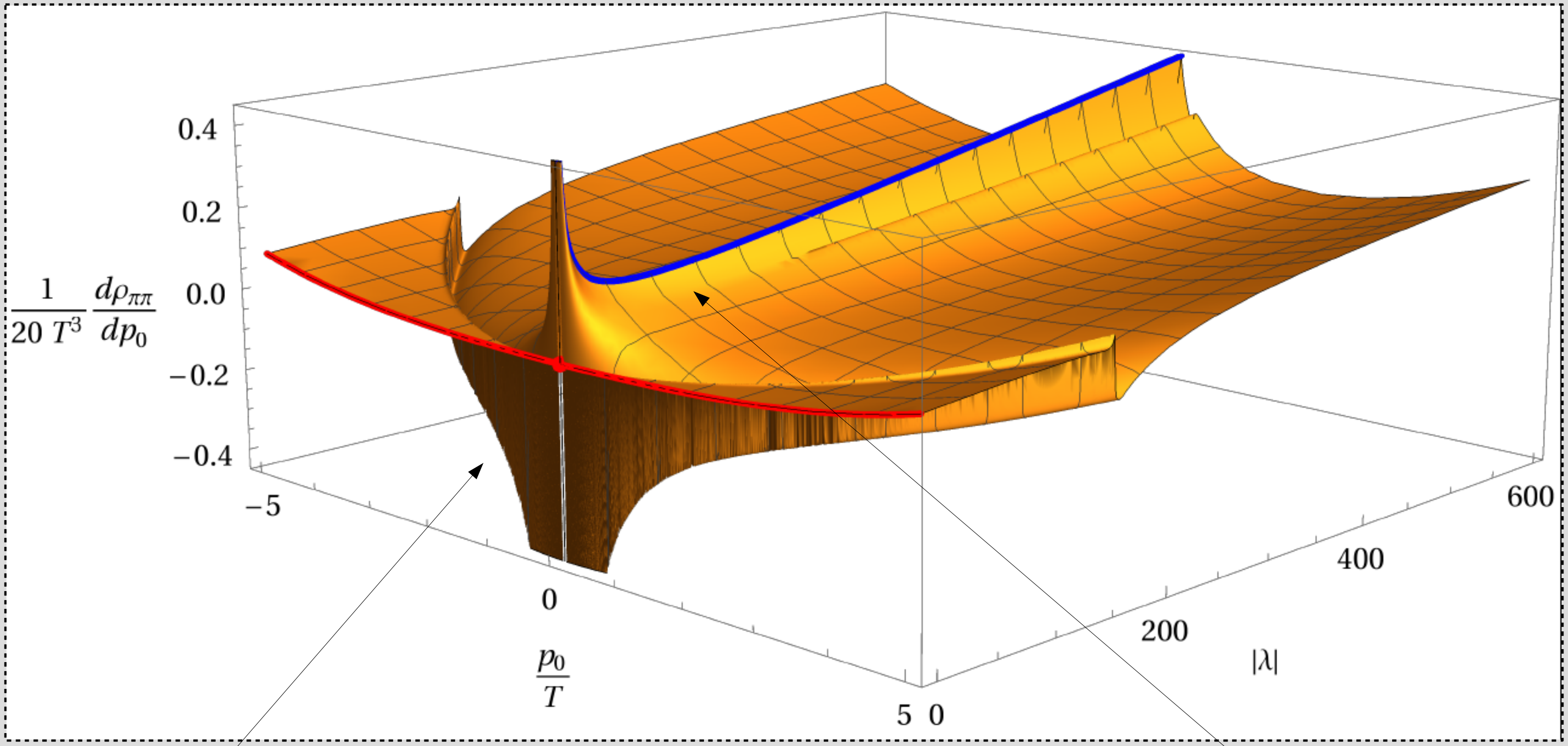
... which after applying the generalised KL representation, together with the Kubo relation, implies

$$\begin{aligned}\eta_0 = \frac{T^5}{240\pi^5}\int_0^{\infty} ds \int_0^{\infty} dt \int_0^{\infty} d|\vec{u}| \int_0^{\infty} d|\vec{v}|\; |\vec{u}||\vec{v}|\, \widetilde{D}_\beta(\vec{u},s)\, \widetilde{D}_\beta(\vec{v},t) \\ \times \left[4\left[1+\epsilon(|\vec{u}|-|\vec{v}|)\right]\left\{\frac{|\vec{v}|}{T}\mathcal{I}_3\left(\frac{\sqrt{t}}{T},0,\infty\right) + \frac{|\vec{v}|^3}{T^3}\mathcal{I}_1\left(\frac{\sqrt{t}}{T},0,\infty\right)\right\}\right. \\ \left. + \left\{\mathcal{I}_4\left(\frac{\sqrt{t}}{T},\frac{|\vec{v}|}{T},\frac{s-t+(|\vec{u}|+|\vec{v}|)^2}{2(|\vec{u}|+|\vec{v}|)T}\right) + \epsilon(|\vec{u}|-|\vec{v}|)\,\mathcal{I}_4\left(\frac{\sqrt{t}}{T},\frac{|\vec{v}|}{T},\frac{s-t+(|\vec{v}|-|\vec{u}|)^2}{2(|\vec{v}|-|\vec{u}|)T}\right)\right\}\right]\end{aligned}$$

- The model dependence of η_0 factorises, and is controlled by the thermal spectral density $D_\beta(\mathbf{u},s)$

Backup

- For $\lambda < 0$, $\rho_{\pi\pi}(p_0)$ and its derivative are ***non-analytic*** at $(p_0/T, |\lambda|)=(0,0)$



Setting λ=0 first, and then $p_0/T\to 0$, leads to a *vanishing* result

But, setting p_0/T=0 first, and then $\lambda\to 0$, leads to a *divergent* result

→ η_0 in the interacting theory is not a continuous perturbation of the free field result ($\eta_0 = 0$)

Backup

- One can use the assumptions of local QFT at finite-T to put constraints on the the structure of Euclidean correlation functions

  → From the KMS condition and locality:
$$\mathcal{W}_E(\tau,\vec{x}) = \frac{1}{\beta}\sum_{N=-\infty}^{\infty} w_N(\vec{x}) e^{\frac{2\pi i N}{\beta}\tau}$$

- The Fourier coefficients of the Euclidean two-point function are then related to the thermal damping factors as follows [P.L., 2201.12180]:

$$w_N(\vec{x}) = \frac{1}{4\pi|\vec{x}|}\left[D_m(\vec{x})e^{-|\vec{x}|\sqrt{m^2+\omega_N^2}} + \int_0^\infty ds\, e^{-|\vec{x}|\sqrt{s+\omega_N^2}} D_c(\vec{x},s)\right]$$

  → *The continuous component $D_c(\boldsymbol{x},s)$ is exponentially suppressed!*

- $\omega_N = 2\pi N T$ are the Matsubara frequencies. For $N=0$ this leads to:

$$\int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} d\tau\, \mathcal{W}_E(\tau,\vec{x}) \sim \frac{1}{4\pi|\vec{x}|} D_{m,\beta}(\vec{x})\, e^{-|\vec{x}|m}$$