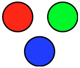
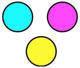
Meson spectrum from functional methods beyond Rainbow-Ladder

Cold-Quantum-Coffee Seminar, Heidelberg

Overview

Electromagnetic charge vs. colour-charge

	Charge	Anti-charge
Electromagnetism	⊖	⊕
Strong force		

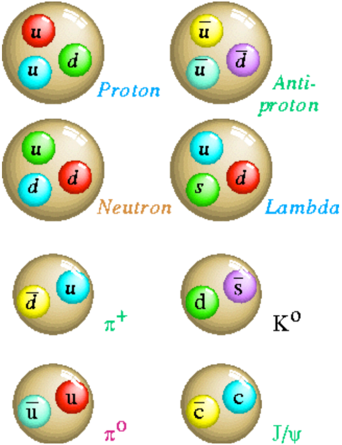
Confinement: All observable states are colourless!

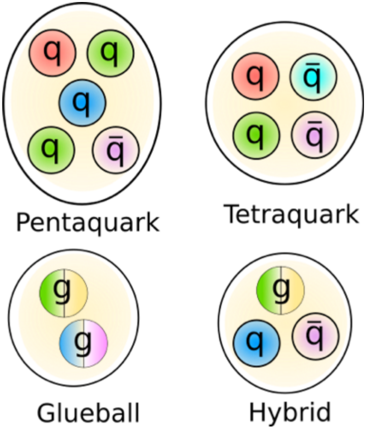
$\varepsilon^{ijk} q_i q_j q_k \qquad \varepsilon_{ijk} \overline{q}^i \overline{q}^j \overline{q}^k \qquad \overline{q}^i q_i$

Hadrons: Bound states of QCD

Baryons

Mesons





Quantum numbers (quark model)

Coupling a quark and an antiquark:

$$S : 1/2 \otimes 1/2 \to 0 \oplus 1$$

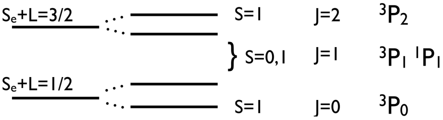
$$P : (-1)^{L+1}$$

$$C : (-1)^{L+S}$$

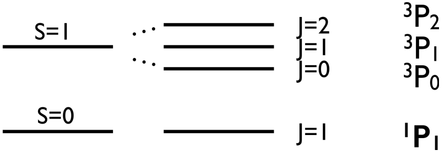
S	L	J^{PC}	$^{2S+1}L_J$
0	0	0^{-+}	$^1S_0$
1	0	1^{--}	$^3S_1$
0	1	1^{+-}	$^1P_1$
1	1	0^{++}	$^3P_0$
		1^{++}	$^3P_1$
		2^{++}	$^3P_2$

Spectrum for $L = 1$ states

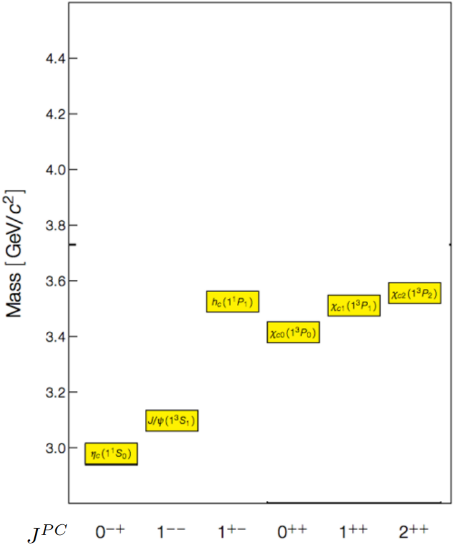
Dominant LS-coupling:



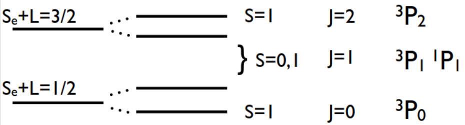
Dominant SS-coupling:



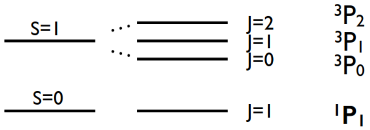
Spectrum of states in the charmonia region



dominant LS-coupling:

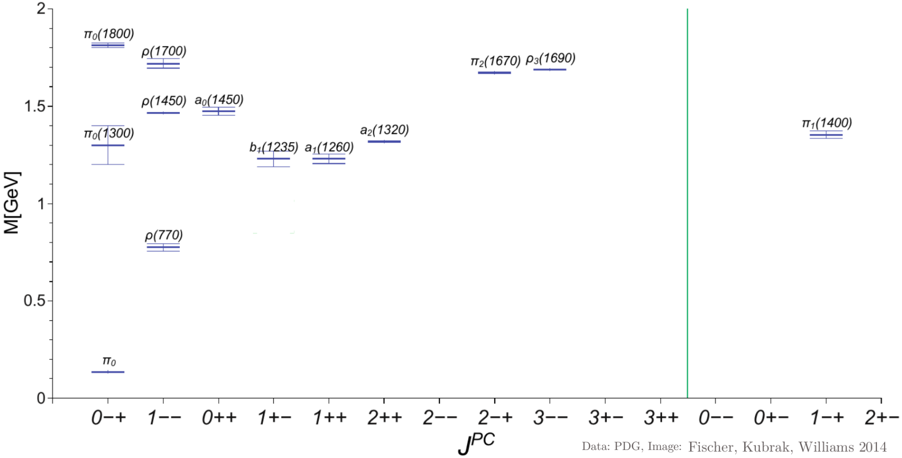


dominant SS-coupling:



Wolfgang Gradl, BESIII, St Goar 2015

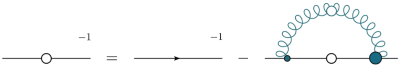
Light meson spectrum

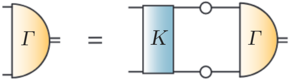


Data: PDG, Image: Fischer, Kubrak, Williams 2014

Our goals:

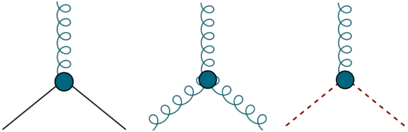
- Reproducing the experimental spectrum from fundamental equations of QCD

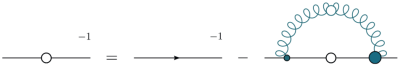


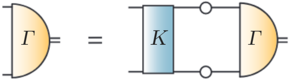


Our goals:

- Reproducing the experimental spectrum from fundamental equations of QCD
- Understanding the nature of QCD interactions

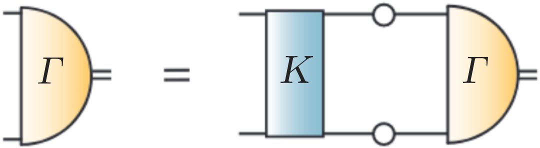






Overview

The Bethe-Salpeter equation



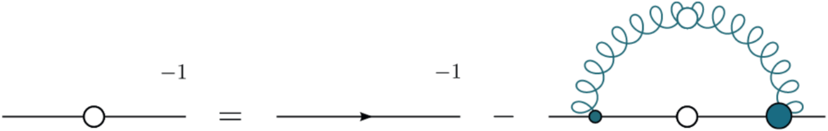
$$[\Gamma(p,P)]_{tu} = \int đ^4 q [S(q_+)\Gamma(q,P)S(q_-)]_{sr} K^{rs}_{tu}(q,p,P)$$

We need as input:

- Quark propagator $S(p)$
- Scattering kernel $K(p,q,P)$

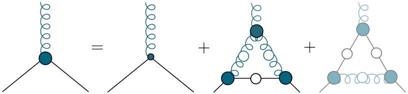
Obtaining the quark propagator

The Dyson-Schwinger equation



$$S^{-1}(p) = Z_2 S_0^{-1}(p) + Z_{1F} g^2 C_f \int đ^4 q \gamma_\mu S(q) \Gamma_\nu(q,p) D^{\mu\nu}(p-q)$$

More DSEs. . .



Williams, Fischer, Heupel 2016

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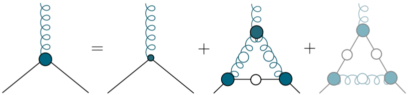

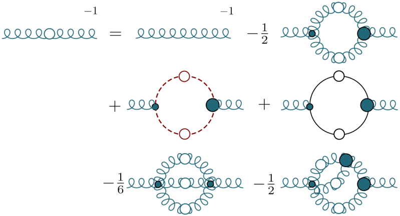

Williams, Fischer, Heupel 2016

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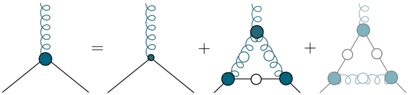

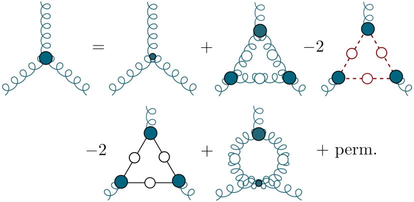

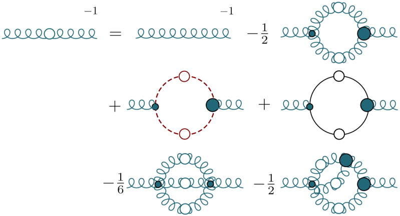

Williams, Fischer, Heupel 2016

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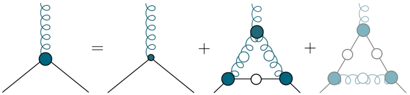
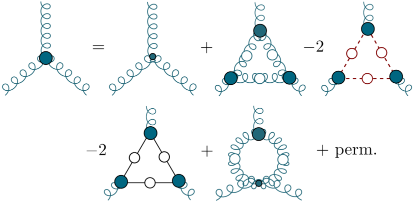
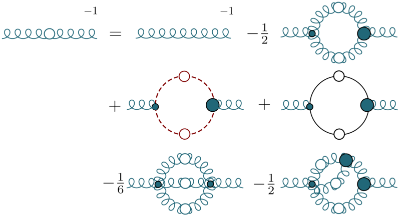
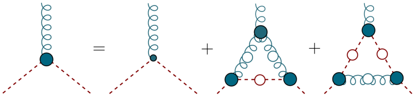








Williams, Fischer, Heupel 2016

More DSEs. . .

And so on. . .

More DSEs. . .

And so on. . .

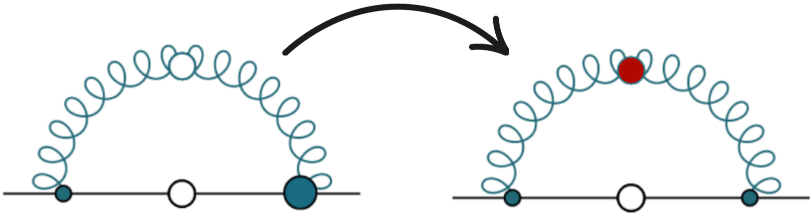
. . . forever.

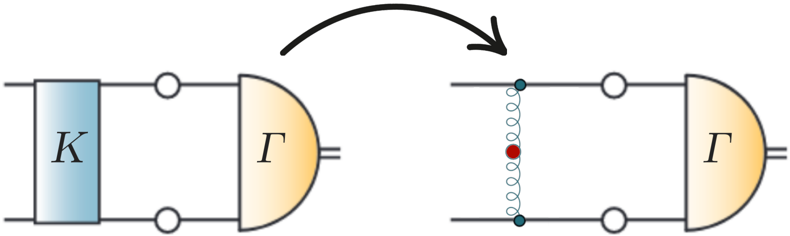
More DSEs. . .

Solution: Truncation!

The rainbow-ladder truncation

The rainbow-ladder truncation





General idea

General form of the vertex:

$$\Gamma^\mu(p,q) \in \{\gamma^\mu, p^\mu, q^\mu\} \otimes \left\{\mathbb{1}, \not{p}, \not{q}, \left[\not{p}, \not{q}\right]_-\right\}$$

Truncated form:

$$\Gamma^\mu(p,q) \propto \gamma^\mu$$

General idea

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Truncated form:

$$\Gamma^{\mu}(p,q) \propto \gamma^{\mu}$$

General form of the gluon propagator (in Landau gauge):

$$D^{\mu\nu}(k) = \frac{Z(k^2)}{k^2}\left(\delta^{\mu\nu} - k^{\mu}k^{\nu}/k^2\right)$$

The model interaction

$$\alpha(k^2) = \alpha_{\mathsf{IR}}(k^2) + \alpha_{\mathsf{UV}}(k^2)$$

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$$\alpha_{\mathrm{IR}}(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right)^2 e^{-\eta^2 k^2/\Lambda^2}$$

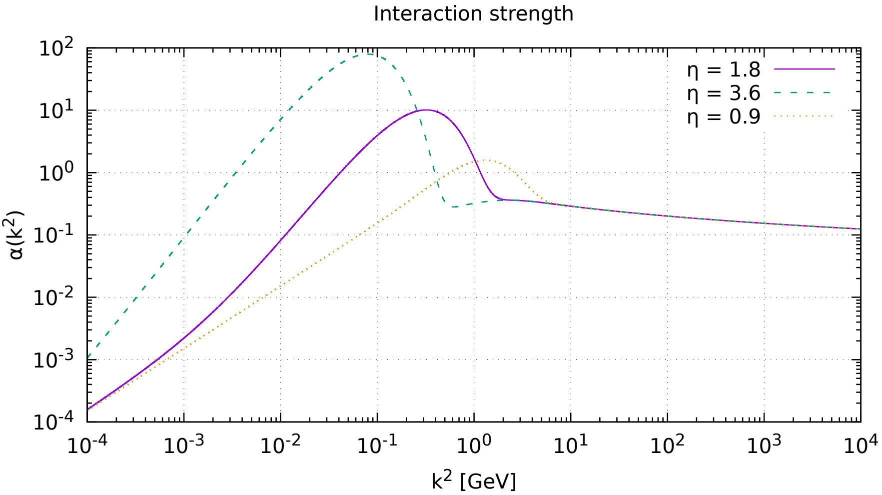
The model interaction

$$\alpha(k^2) = \alpha_{\mathrm{IR}}(k^2) + \alpha_{\mathrm{UV}}(k^2)$$

$$\alpha_{\mathrm{IR}}(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right)^2 e^{-\eta^2 k^2/\Lambda^2}$$

$$\alpha_{\mathrm{UV}}(k^2) = \frac{\pi\gamma_m \left(1 - e^{-k^2/\Lambda_0^2}\right)}{\ln\sqrt{e^2 - 1 + \left(1 + k^2/\Lambda_{\mathrm{QCD}}^2\right)^2}}$$

The model interaction



Solving the DSE

General form of the quark propagator:

$$S^{-1}(p) = i\not{p}A(p^2) + B(p^2)$$

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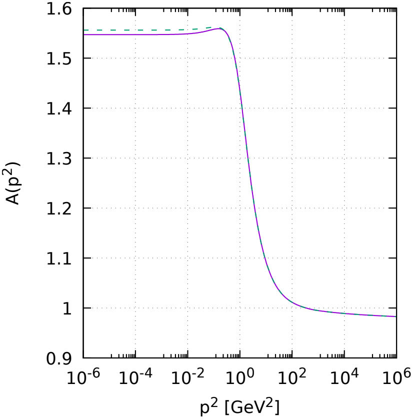
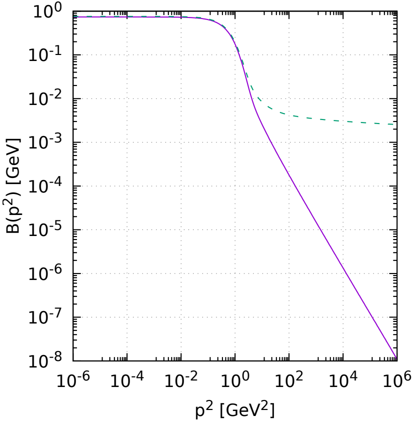
Coupled integral equations for $A(p^2)$ and $B(p^2)$

$$A(p^2) = Z_2 + \Sigma_A(p^2)$$
$$B(p^2) = Z_2 Z_m m_c + \Sigma_B(p^2)$$

Numerical results



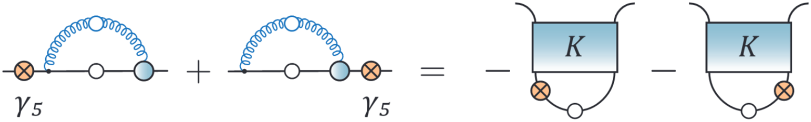


$$M_{\text{eff}} = 488 \text{ MeV}$$

Back to the Bethe-Salpeter equation

Axialvector Ward-Takahashi identity

Kernel cannot be chosen arbitrarily,
Symmetries must be conserved



Gell-Mann-Oakes-Renner relation:

$$f_\pi^2 m_\pi^2 = -2 m_c \left\langle \overline{\psi}\psi \right\rangle / N_f + O(m_c^2)$$

Rainbow-ladder kernel

$$K_{abcd}(p, q, P) = -C_f Z_2^2 4\pi T_{\mu\nu}(k)\gamma^{\mu}_{ab}\gamma^{\nu}_{cd}\frac{\alpha(k^2)}{k^2}$$

Rainbow-ladder kernel

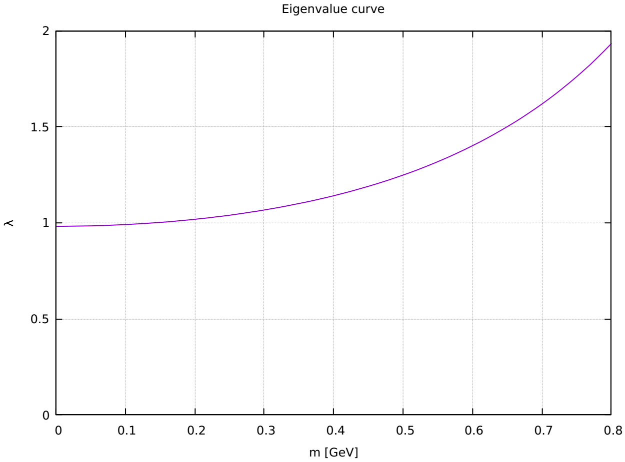
$$K_{abcd}(p, q, P) = -C_f Z_2^2 4\pi T_{\mu\nu}(k)\gamma^{\mu}_{ab}\gamma^{\nu}_{cd}\frac{\alpha(k^2)}{k^2}$$

Eigenvalue equation of the form:

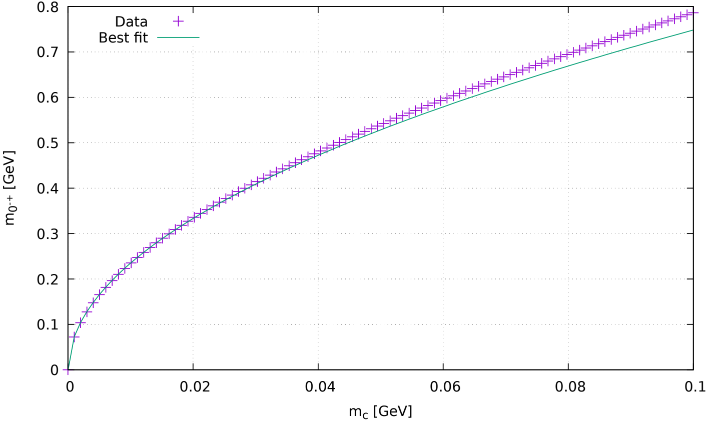
$$(KG_0)(m) \cdot \Gamma(m) = \lambda(m)\Gamma(m)$$

$\lambda(m) = 1$ at the physical meson mass.

Solving the BSE

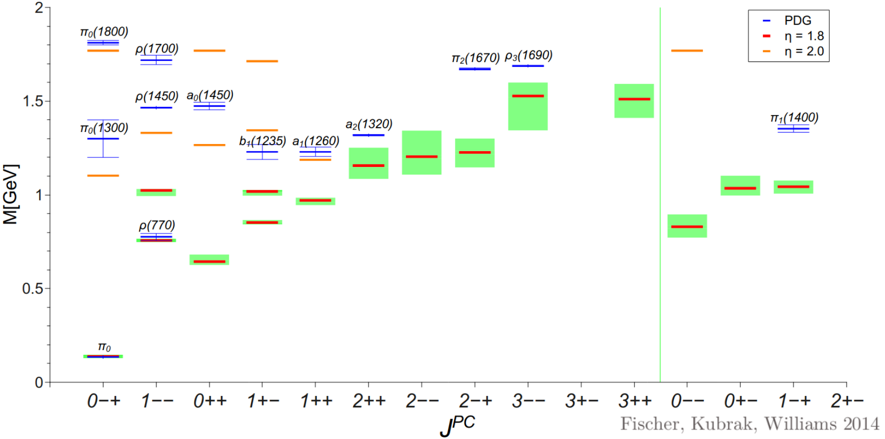


Goldstone's theorem



$$f_\pi^2 m_\pi^2 = -2 m_c \left\langle \overline{\psi}\psi \right\rangle / N_f + O(m_c^2)$$

Results



M[GeV]
2
1.5
1
0.5
0
PDG
η = 1.8
η = 2.0
π0(1800)
ρ(1700)
π2(1670)
ρ3(1690)
ρ(1450)
a0(1450)
π0(1300)
b1(1235)
a1(1260)
a2(1320)
π1(1400)
ρ(770)
π0
0−+
1−−
0++
1+−
1++
2++
2−−
2−+
3−−
3+−
3++
0−−
0+−
1−+
2+−
JPC
Fischer, Kubrak, Williams 2014

Overview

Usual approach

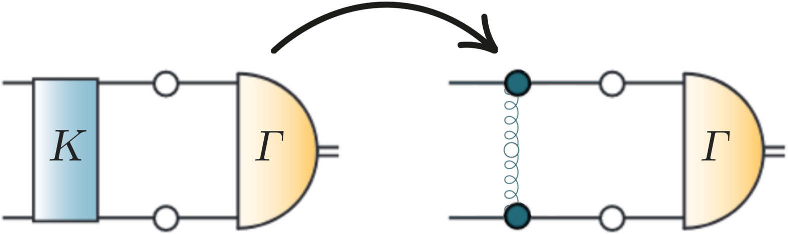
- Start with classical action $S[\Phi]$
- Transform to effective action $\Gamma[\Phi]$
- Take first order derivative and evaluate at $\Phi = \Phi_0$ to obtain self-energy
- Take second order derivative and evaluate at $\Phi = \Phi_0$ to obtain the scattering kernel

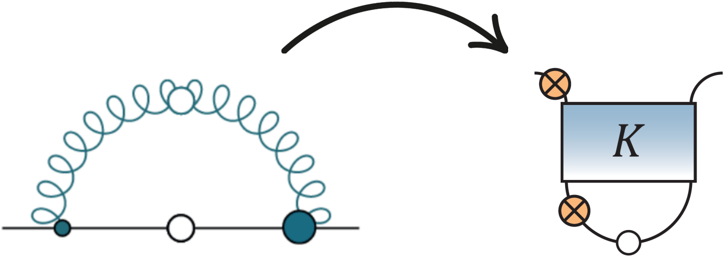
Usual approach

- Start with classical action $S[\Phi]$
- Transform to effective action $\Gamma[\Phi]$
- Take first order derivative and evaluate at $\Phi = \Phi_0$ to obtain self-energy
- Take second order derivative and evaluate at $\Phi = \Phi_0$ to obtain the scattering kernel

But we can go the other way around!

The kernel-first truncation





The basic idea

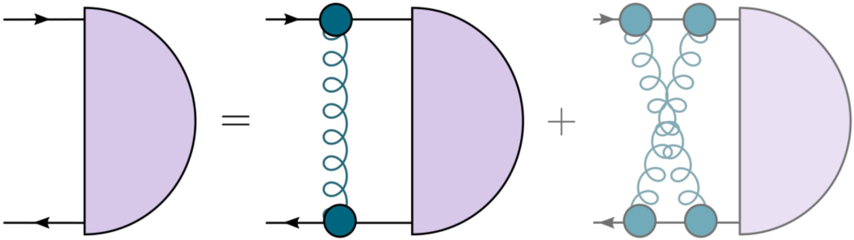
- Specify a kernel
- Use the AVWTI to construct a self-energy

Starting with $K(p, q, P)$, we get

$$B(p^2) = Z_2 Z_m m_c - \frac{1}{4}\,\mathrm{tr}\left[\gamma^5_{ab}\int đ^4 q K_{bcde}(p, q, P)\gamma^5_{cd} B(q^2) d(q^2)\right]$$

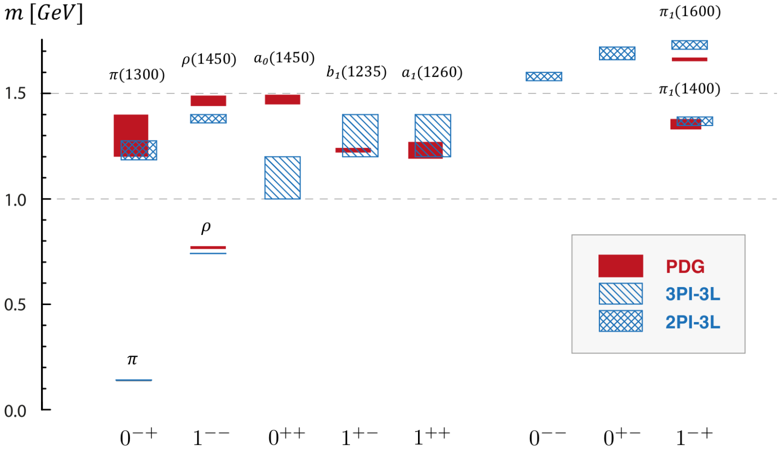
Reminder: $S^{-1}(p) = i\not{p}A(p^2) + B(p^2)$

Goal: 3PI kernel



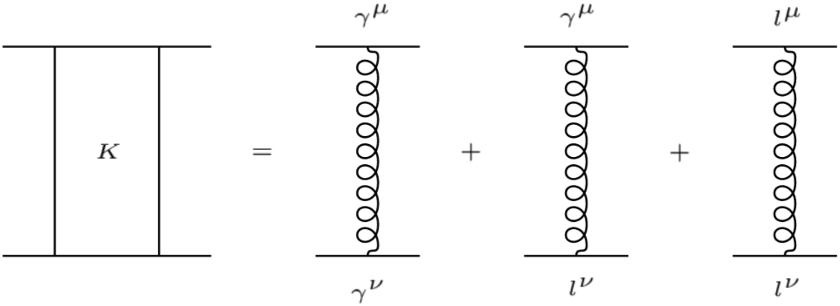
Williams, Fischer, Heupel 2016

3PI kernel results



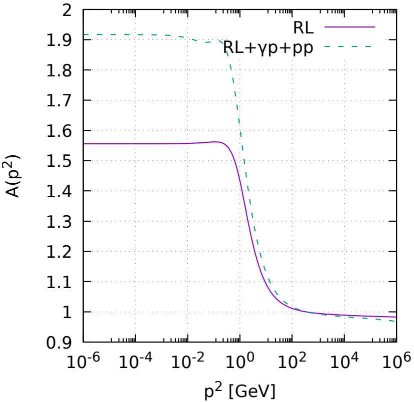
Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer 2016

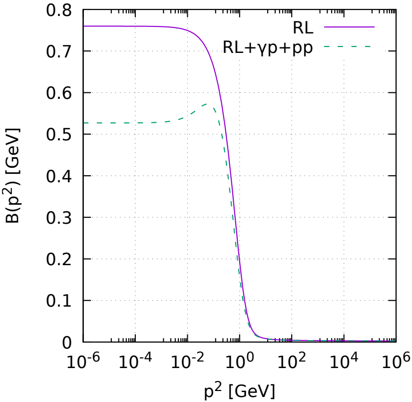
First steps: A toy kernel



Results (Toy kernel)

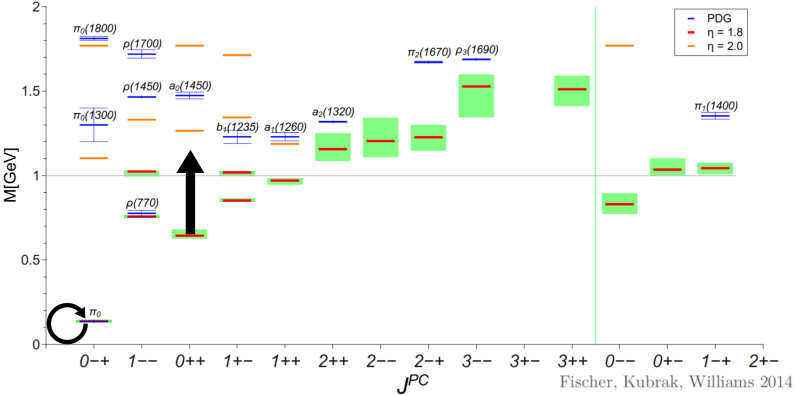
Quark propagator (Toy kernel)





$$M_{\text{eff}} = 275 \text{ MeV}$$

Meson spectrum (Toy kernel)



Goldstone’s theorem and Gell-Mann-Oakes-Renner relation are conserved!

The equation for $A(p^2)$

WORK IN PROGRESS

Axialvector Ward-Takahashi identity gives equation for $B(p^2)$

$$Q^\mu \Gamma_5^\mu(k,Q) + 2m_c \Gamma_5(k,Q) = S^{-1}(k_+) i\gamma^5 + i\gamma^5 S^{-1}(k_-)$$

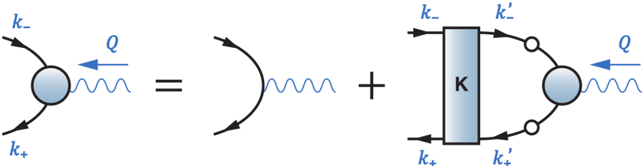
$$\Rightarrow B(p^2) = Z_2 Z_m m_c - \frac{1}{4}\,\mathrm{tr}\left[\gamma^5_{ab} \int \text{đ}^4 q K_{bcde}(p,q,P)\gamma^5_{cd} B(q^2) d(q^2)\right]$$

Use **Vector** Ward-Takahashi identity to get an equation for $A(p^2)$

$$Q^\mu \Gamma^\mu(k,Q) = S^{-1}(k_+) - S^{-1}(k_-)$$

$$\Rightarrow A(p^2) = ?$$

The Quark-Photon vertex



Photon is off-shell ⇒ Inhomogeneous BSE needs to be iterated!

$$\Gamma^\mu(k,Q) = \sum_{j=1}^{12} a_j\left(k^2,z,Q^2\right) i\tau_j^\mu(k,Q)$$

$$a_i\left(k^2,z,Q^2\right) = Z_2 a_i^0 + \sum_{j=1}^{12}\int_{k'} K_{ij}\left(k^2,k'^2,z,z',y,Q^2\right) b_j\left(k'^2,z',Q^2\right)$$

$$b_i\left(k^2,z,Q^2\right) = \sum_{j=1}^{12} G_{ij}\left(k^2,z,Q^2\right) a_j\left(k^2,z,Q^2\right)$$

The Quark-Photon vertex beyond rainbow-ladder

WORK IN PROGRESS

Rainbow-ladder:
$$K = \begin{pmatrix} S_{8\times8}(k^{(\prime)}, z^{(\prime)}) & \mathbf{0}_{8\times4} \\ \mathbf{0}_{4\times8} & S_{4\times4}(k^{(\prime)}, z^{(\prime)}) \end{pmatrix}$$

Beyond rainbow-ladder:
$$K = \begin{pmatrix} D_{8\times8}(k^{(\prime)}, z^{(\prime)}, Q^2) & \mathbf{0}_{8\times4} \\ \mathbf{0}_{4\times8} & D_{4\times4}(k^{(\prime)}, z^{(\prime)}, Q^2) \end{pmatrix}$$

$S_{n\times n}$: Sparse, real valued $n \times n$ matrix.
$D_{n\times n}$: Dense, complex valued $n \times n$ matrix.

Conclusion & outlook

Conclusion & Outlook

- We derived a self energy from a general kernel
- We can use an arbitrary scattering kernel
- We improved the predicted scalar meson mass with a toy kernel
- Meson calculations are very time efficient

- Use data from 3PI calculations for quark-gluon vertex
- Calculate more channels ((axial-)vector, tensor mesons)
- Apply to a wider range of problems
  - Heavy mesons
  - Form factors, $(g-2)_\mu$
  - Tetraquarks
  - ...