Stable and unstable perturbations in universal scaling phenomena far from equilibrium

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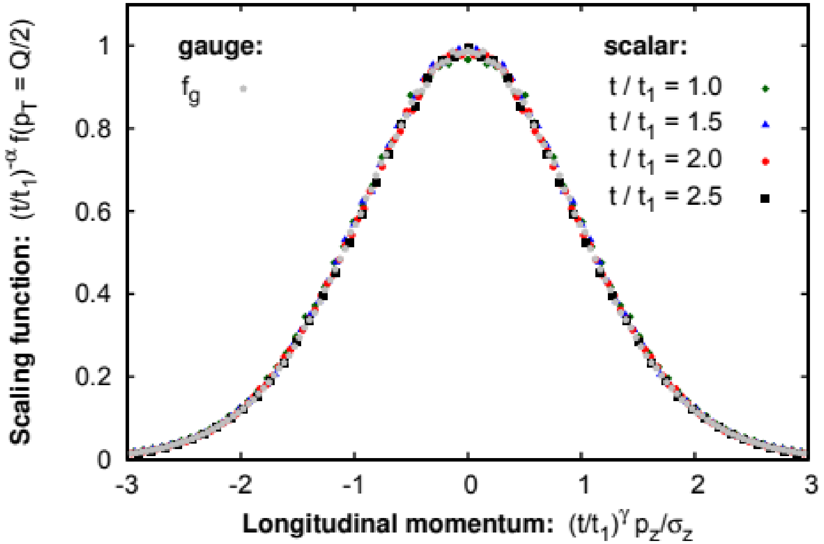
STRUCTURES CLUSTER OF EXCELLENCE

Nonthermal fixed points

Universal self-similar scaling far from equilibrium:

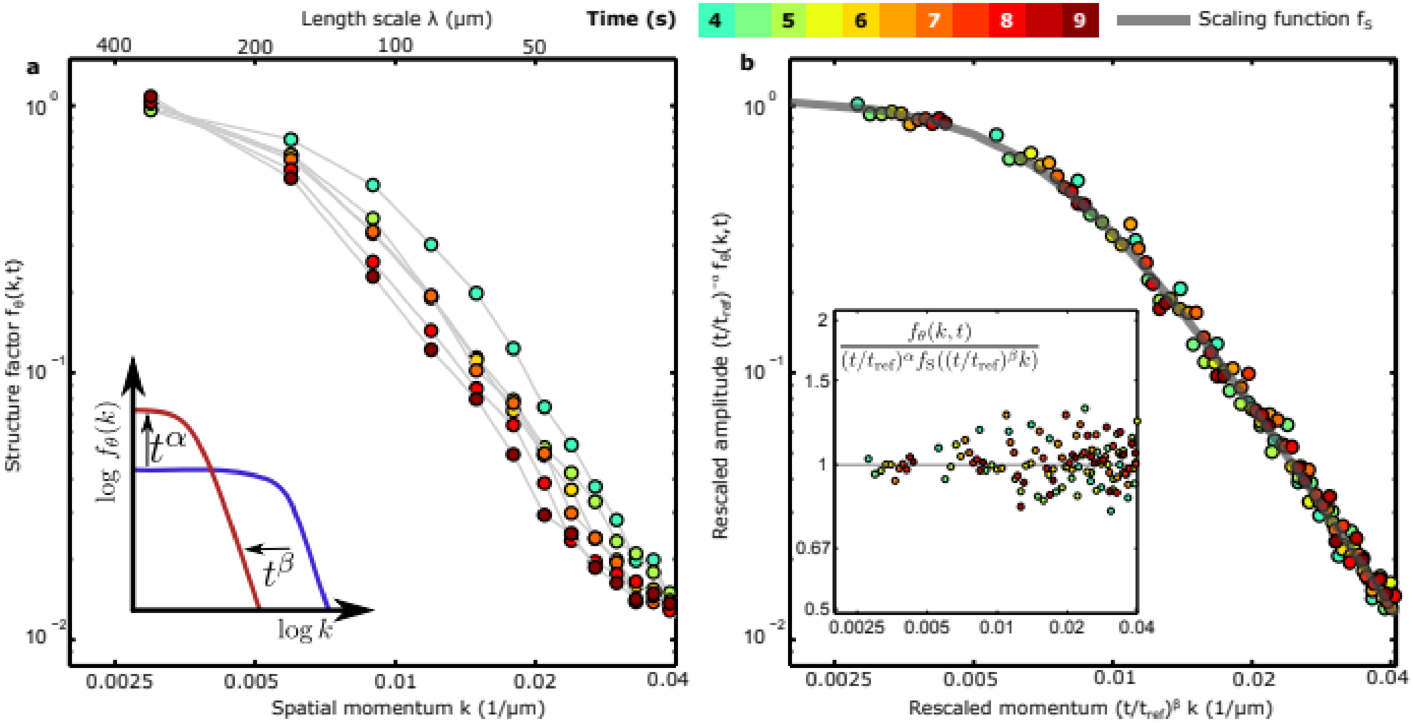
$$f(t,\mathbf{p}) = (t/t_{\mathrm{ref}})^{\alpha} f_S((t/t_{\mathrm{ref}})^{\beta}\mathbf{p})$$

Berges et al., PRL 114, 061601 (2015)



See also: Kurkela et al., PRL 115, 182301 (2015), Micha et al., PRL 90, 121301 (2003), Berges et al., PRL 101, 041603 (2008).

Prüfer et al., Nature 563, 217 (2018)

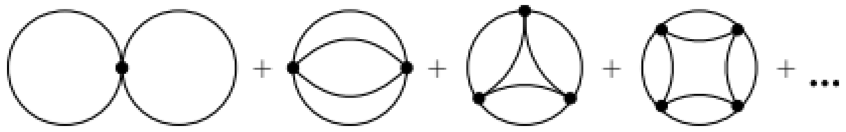


See also: Erne et al., Nature 563, 225 (2018), Glidden et al., Nature Phys. 17, 457 (2021).

Far-from-equilibrium quantum fields

$$\hat{H}(t) = \int d^3x \left[\frac{1}{2}\left(\partial_t \hat{\Phi}_a(t,\mathbf{x})\right)^2 + \frac{1}{2}\left(\nabla_{\mathbf{x}} \hat{\Phi}_a(t,\mathbf{x})\right)^2 + \frac{\lambda}{4!N}\left(\hat{\Phi}_a(t,\mathbf{x})\hat{\Phi}_a(t,\mathbf{x})\right)^2\right]$$

$\langle \hat{H}(t) \rangle \longrightarrow$



Solve quantum evolution equations of

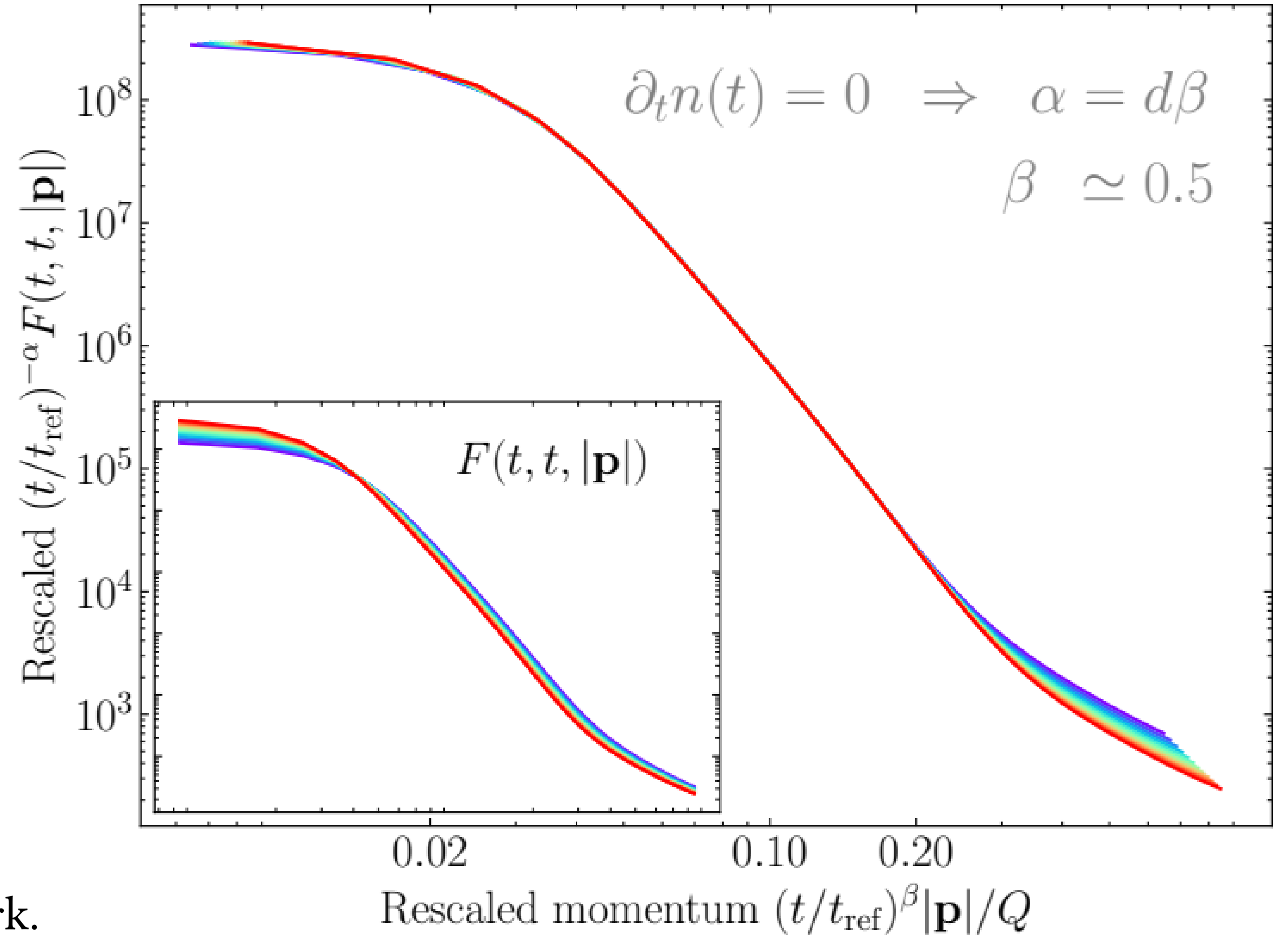
$$F_{ab}(t,t',\mathbf{x}-\mathbf{x}') = \frac{1}{2}\langle\{\hat{\Phi}_a(t,\mathbf{x}), \hat{\Phi}_b(t',\mathbf{x}')\}\rangle - \langle\hat{\Phi}_a(t,\mathbf{x})\rangle\langle\hat{\Phi}_b(t',\mathbf{x}')\rangle$$

$$\rho_{ab}(t,t',\mathbf{x}-\mathbf{x}') = i\langle[\hat{\Phi}_a(t,\mathbf{x}), \hat{\Phi}_b(t',\mathbf{x}')]\rangle$$

$$F(t_0,t_0,\mathbf{p}) = \frac{1}{\sqrt{\mathbf{p}^2+m^2}}\left[\left(\frac{Nn_0}{\lambda}\right)\theta(Q-|\mathbf{p}|) + \frac{1}{2}\right]$$

We consider $\lambda = 0.01, N = 4$ and $n_0 = 25$ in this work.

$$F(t,t,|\mathbf{p}|) = (t/t_{\mathrm{ref}})^{\alpha} F_S\left((t/t_{\mathrm{ref}})^{\beta}|\mathbf{p}|\right)$$



Closed time path for initial value problems

Expectation values require only the initial density matrix via the von Neumann equation

$$\langle \mathcal{O}(t) \rangle = \frac{\mathrm{Tr}\left[\rho_D(t)\mathcal{O}\right]}{\mathrm{Tr}[\rho_D(t)]}$$

$$\partial_t \rho = -i[H, \rho]$$

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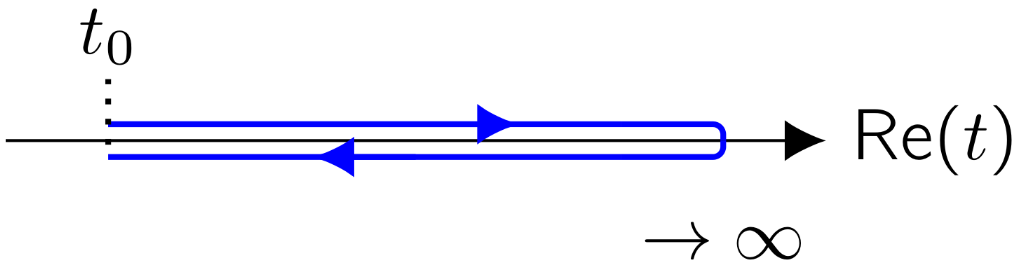
$$\rho_D(t) = U_{t,t_0}\rho_D(t_0)U_{t_0,t}$$



$$\langle \mathcal{O}(t) \rangle = \frac{\mathrm{Tr}\left[\rho_D(t_0)U_{t_0,t}\mathcal{O}U_{t,t_0}\right]}{\mathrm{Tr}[\rho_D(t_0)]}$$

Nonequilibrium generating functional for a Gaussian density matrix at initial time: Berges AIP (2004)

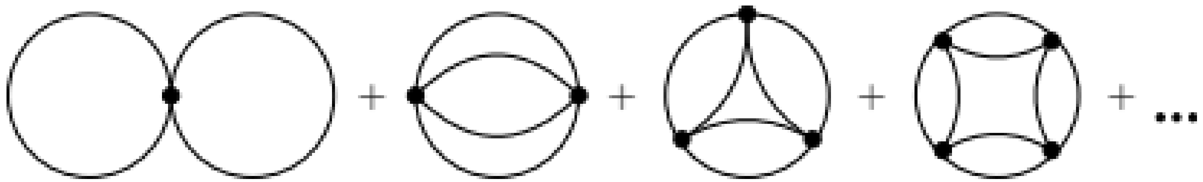
$$Z[J, R; \rho_D^{\mathrm{Gauss}}] = \int \mathcal{D}\varphi e^{iS[\varphi]+iJ\cdot\varphi+\frac{i}{2}\varphi\cdot R\cdot\varphi}$$

2PI Effective action

Classical action

$$S[\varphi], Z[J, R; \rho_D^{\text{Gauss}}]$$

Legendre transformation



2PI effective action

$$\Gamma[G]$$ = 1-loop quantum corrections + 2PI diagrams

Evolution equation via stationarity principle:

$$\frac{\delta \Gamma}{\delta G} = 0$$

How to describe real-time processes ?

2PI considers a self-consistent resummation of the propagator Schwinger-Dyson equation

$$G^{-1} = G_0^{-1} - \Sigma \qquad \text{Schwinger-Dyson equation}$$

$$G = G_0 + G_0\Sigma[G_0]G \qquad \text{Dyson series}$$

$$G_0^{-1}G = \mathbb{I} + \Sigma[G]G \qquad \text{self-consistent 2PI,}$$

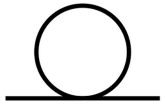
which allows for the description of nonequilibrium phenomena by capturing the long time evolution.

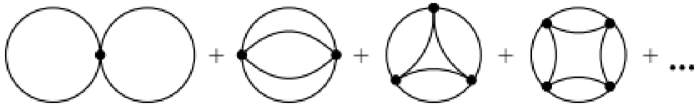
2PI Equations of motion

The 2PI equations for spectral and statistical dynamics read via $G(x,y) = F(x,y) - \frac{i}{2}\mathrm{sgn}_{\mathcal{C}}(x^0 - y^0)\rho(x,y)$

$$\left[\Box_x \delta_{ac} + M^2_{ac}(x)\right] F_{cb}(x,y) = -\int_{t_0}^{x^0} \mathrm{d}^{\mathrm{d}+1} z \Sigma^{\rho}_{ac}(x,z) F_{cb}(z,y) + \int_{t_0}^{y^0} \mathrm{d}^{\mathrm{d}+1} z \Sigma^{F}_{ac}(x,z) \rho_{cb}(z,y)$$

$$\left[\Box_x \delta_{ac} + M^2_{ac}(x)\right] \rho_{cb}(x,y) = -\int_{y^0}^{x^0} \mathrm{d}^{\mathrm{d}+1} z \Sigma^{\rho}_{ac}(x,z) \rho_{cb}(z,y)$$

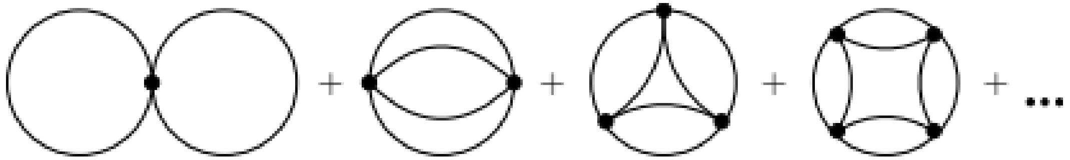
With $\int_{y^0}^{x^0} \mathrm{d}^{\mathrm{d}+1} z = \int_{y^0}^{x^0} \mathrm{d}z^0 \int_{\mathbb{R}^{\mathrm{d}}} \mathrm{d}^{\mathrm{d}} z$, effective mass $M^2(x) = m^2 +$ 

and self-energies expanded to NLO in 1/N: 

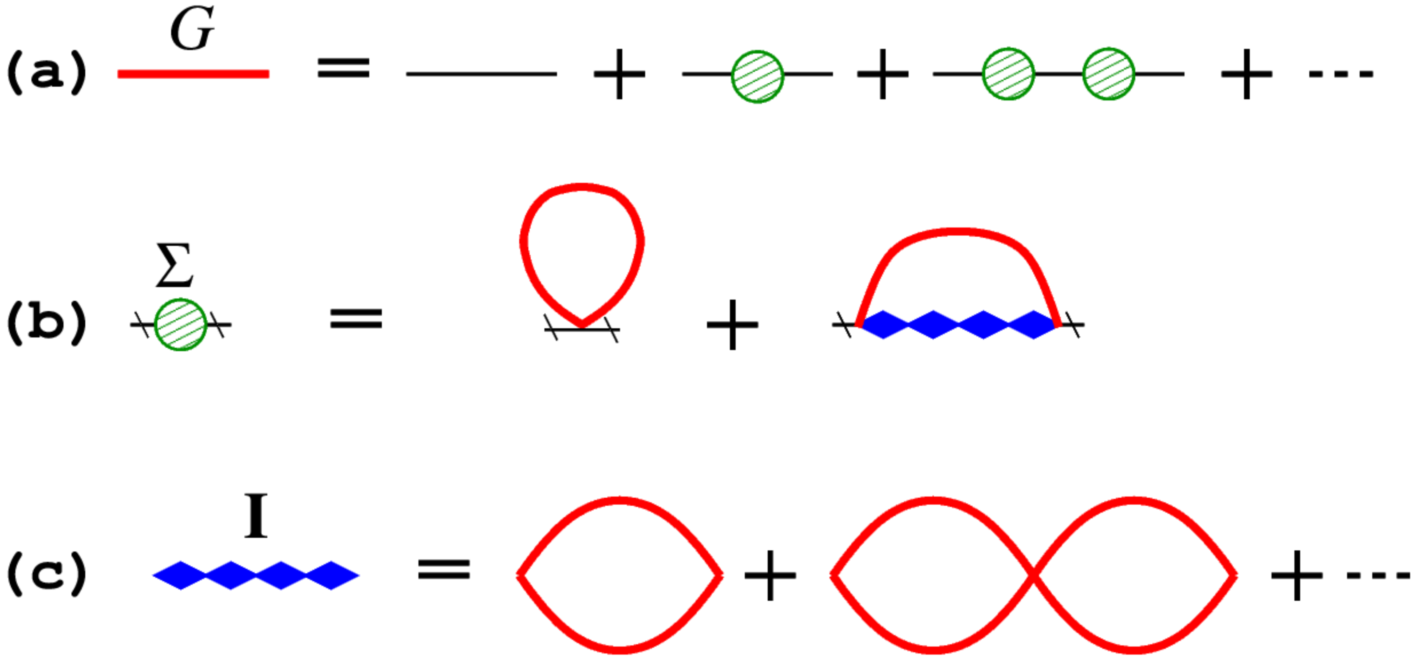
Large-N expansion

$$\mathrm{Tr}_{O(\mathrm{N})}\{G^n\} \sim N, \qquad n \leq N$$

$$\Gamma[G] = \frac{i}{2}\mathrm{Tr}_{\mathcal{C}} \log G^{-1} + \frac{i}{2}\mathrm{Tr}_{\mathcal{C}} G_0^{-1} G + \Gamma_2[G]$$



$\sim N$ $\qquad \sim N^0$

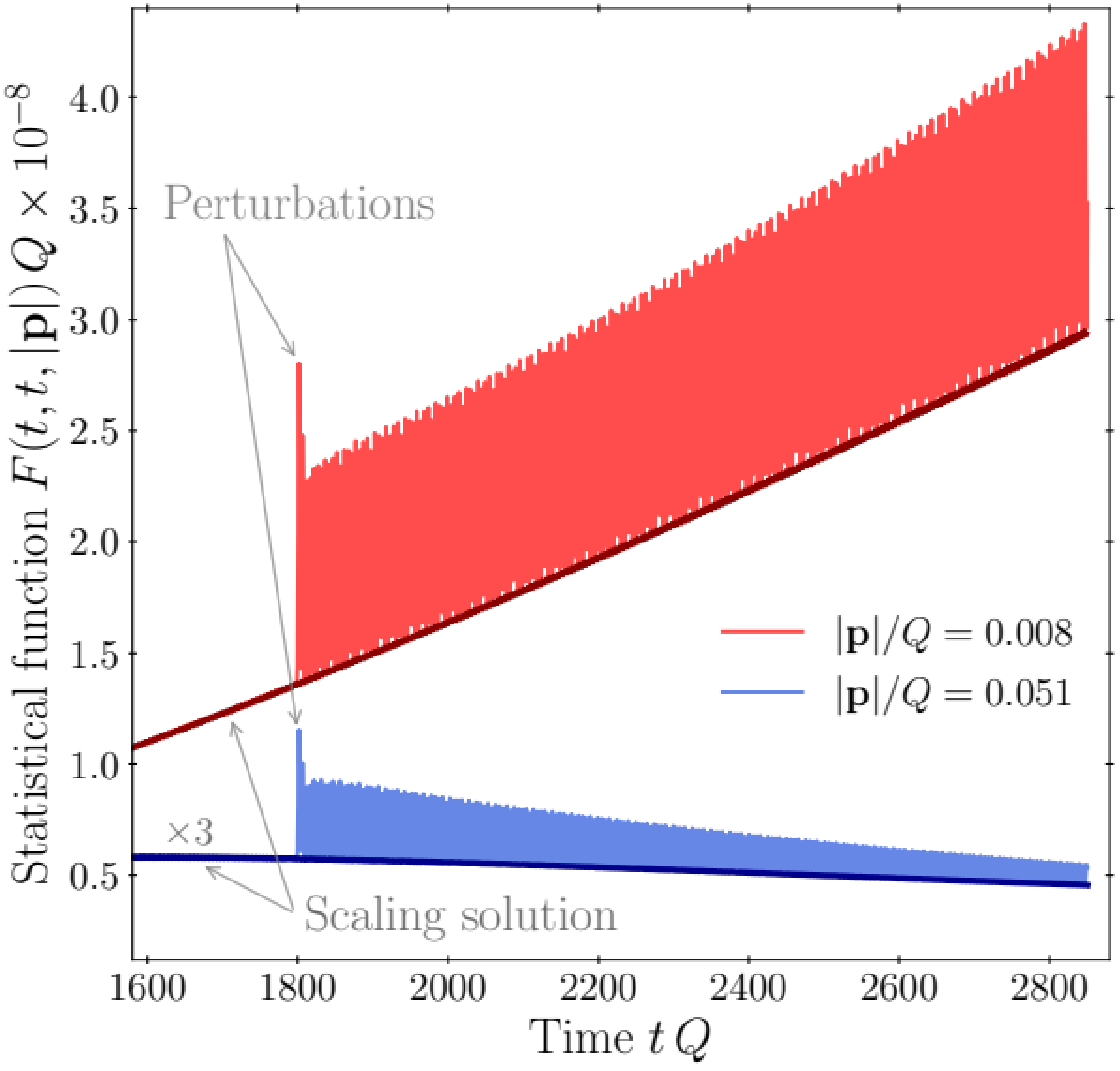


$$\begin{aligned}\Sigma(x,y) &= 2i\frac{\delta\Gamma_2}{\delta G(x,y)} \\ &\equiv -i\Sigma^{(0)}(x)\delta_{\mathcal{C}}(x-y) \\ &+ \Sigma_F(x,y) - \frac{i}{2}\mathrm{sgn}_{\mathcal{C}}(x^0 - y^0)\Sigma_\rho(x,y)\end{aligned}$$

Figure adapted from Alford (2004)

Perturbations around the universal scaling solution

$$F(t,t,|\mathbf{p}|) = (t/t_{\mathrm{ref}})^{\alpha} F_s((t/t_{\mathrm{ref}})^{\beta}|\mathbf{p}|) + \delta F(t,t,|\mathbf{p}|)$$



Far-from-equilibrium quasi-particles

Wigner transform with $\tau = \frac{1}{2}(t + t')$ and $\Delta t = t - t'$

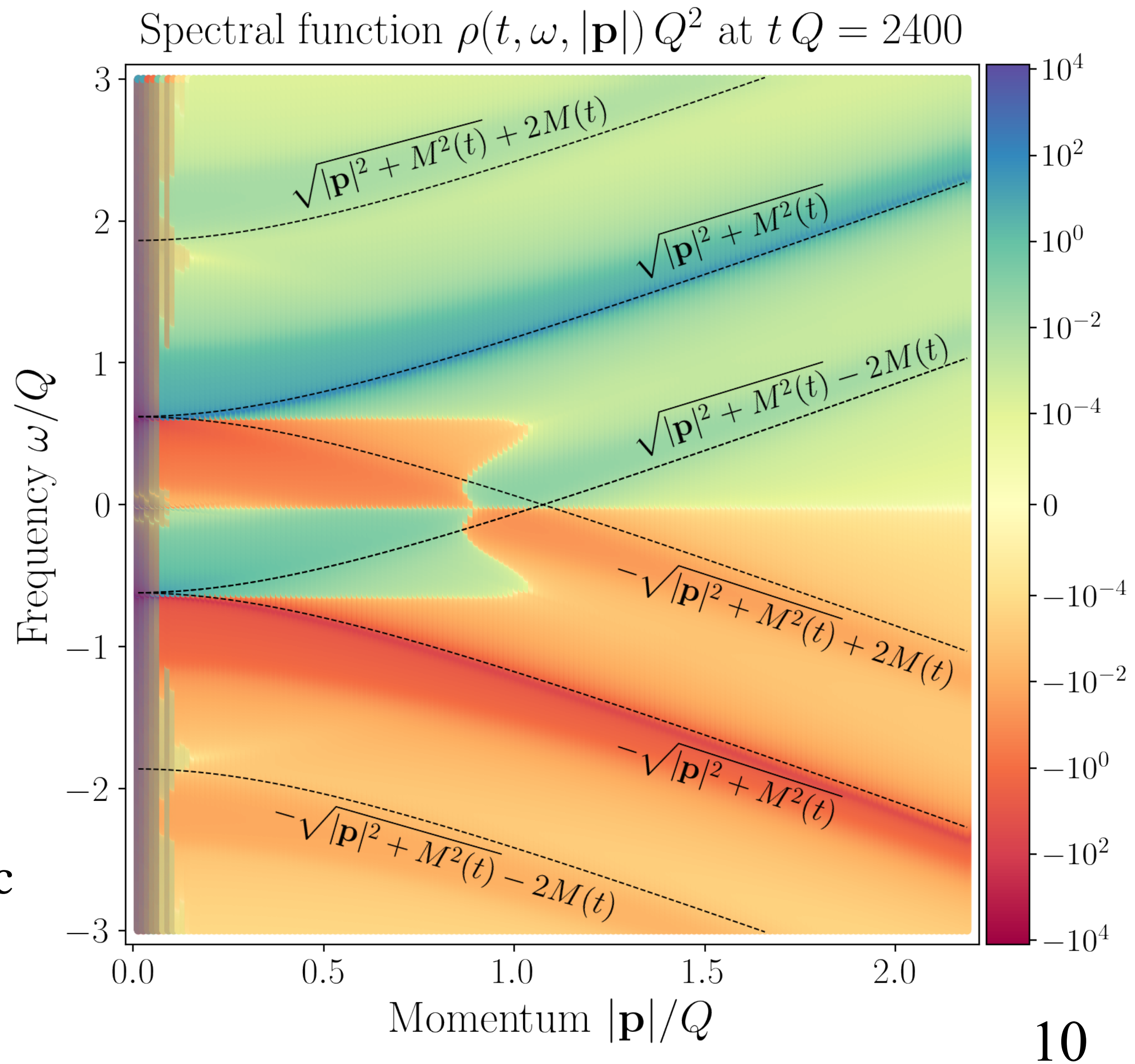
$$\rho(\tau, \omega, |\mathbf{p}|) = -i \int_{-2\tau}^{2\tau} d\Delta t \, e^{i\omega \Delta t} \rho(\tau + \tfrac{\Delta t}{2}, \tau - \tfrac{\Delta t}{2}, |\mathbf{p}|)$$

Tower of quasi-particles

$$\omega(t, \mathbf{p}) = \sqrt{\mathbf{p}^2 + M^2(t)}$$
$$\omega^{\pm}(t, \mathbf{p}) = 2M(t) \pm \omega(t, \mathbf{p})$$

Interpretation as phase fluctuations of a macroscopic zero mode rotating in N-component field space.

Boguslavski et al., PRD 101, 091902 (2020)



Linear response around the universal scaling solution

Perturbations described by time-dependent response rate:

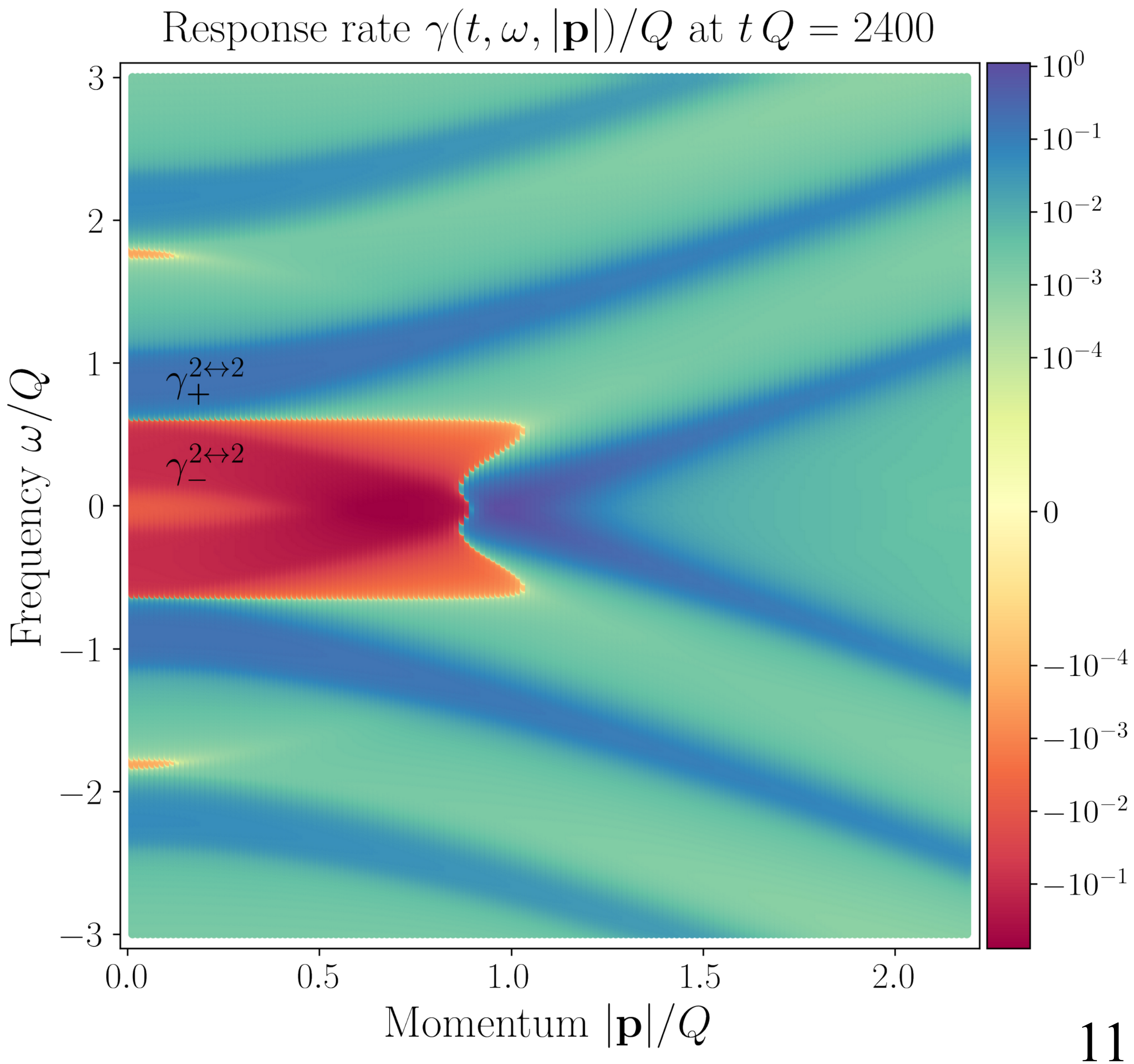
$$\frac{\partial_t \delta F(t,\omega,|\mathbf{p}|)}{\delta F(t,\omega,|\mathbf{p}|)} = -\gamma(t,\omega,|\mathbf{p}|)$$

where

$$\gamma(t,\omega \simeq M, |\mathbf{p}| \simeq 0) < 0$$

To leading order in gradients, the diagonal contribution to the response rate is

$$\gamma(t,\omega,|\mathbf{p}|) = -\frac{\Sigma_\rho(t,\omega,|\mathbf{p}|)}{2\omega}$$



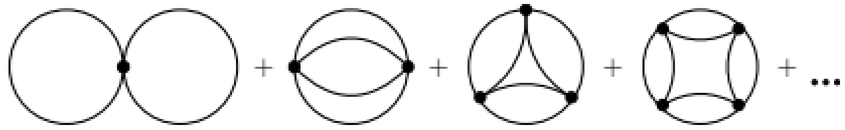
Interpretation via scattering processes

Elastic scatterings describe the leading contribution to the on-shell linear response rate (with $p = (p^0, \mathbf{p})$)

$$\gamma^{2\leftrightarrow 2}(t,p) = -\frac{\lambda^2}{36Np^0}\int \frac{d^3q}{(2\pi)^3}\frac{d^3l}{(2\pi)^3}\frac{d^3r}{(2\pi)^3}(2\pi)^4$$
$$\times \int_0^\infty \frac{dq^0 dl^0 dr^0}{(2\pi)^3}\delta^{(4)}(p+l-q-r)\rho(t,q)\rho(t,l)\rho(t,r)$$
$$\times \left[v_{\text{eff}}(t,p+l) + v_{\text{eff}}(t,p-q) + v_{\text{eff}}(t,p-r)\right]$$
$$\times \left[(f_l(t)+1)f_q(t)f_r(t) - f_l(t)(f_q(t)+1)(f_r(t)+1)\right]$$

with $\quad F(t,p) = \left[f_p(t) + \frac{1}{2}\right]\rho(t,p)$

$$v_{\text{eff}}(t,p) = \left|1 + \frac{\lambda}{3}\int \frac{d^4k}{(2\pi)^4}F(t,p-k)G^R(t,k)\right|^{-2}$$

s-channel: 

$$\partial_t \delta f_p(t) = \delta C^{2\leftrightarrow 2}(t,p)\delta f_p(t)$$
$$= -2\omega\gamma^{2\leftrightarrow 2}(t,p)\ \rho(t,p)\delta f_p(t)$$

$$C^{2\leftrightarrow 2}[f](t,\mathbf{p}) = \frac{\lambda^2}{18N}\int \frac{d^3q}{(2\pi)^3}\frac{d^3l}{(2\pi)^3}\frac{d^3r}{(2\pi)^3}(2\pi)^3\delta^{(3)}(\mathbf{p}+\mathbf{l}-\mathbf{q}-\mathbf{r})$$
$$\times \frac{\delta(\omega_\mathbf{p}+\omega_\mathbf{l}-\omega_\mathbf{q}-\omega_\mathbf{r})}{2\omega_\mathbf{p}2\omega_\mathbf{l}2\omega_\mathbf{q}2\omega_\mathbf{r}}\left[v_{\text{eff}}(t,\mathbf{p}+\mathbf{l}) + v_{\text{eff}}(t,\mathbf{p}-\mathbf{q}) + v_{\text{eff}}(t,\mathbf{p}-\mathbf{r})\right]$$
$$\times \left[(f_\mathbf{p}(t)+1)(f_\mathbf{l}(t)+1)f_\mathbf{q}(t)f_\mathbf{r}(t) - f_\mathbf{p}(t)f_\mathbf{l}(t)(f_\mathbf{q}(t)+1)(f_\mathbf{r}(t)+1)\right]$$

Two-to-two scattering with

$$p^0 + \omega(t,\mathbf{l}) = \omega(t,\mathbf{q}) + \omega(t,\mathbf{r})$$

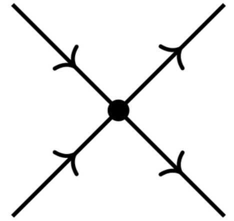
with contributions from

$p^0 \;=\; \omega(t,\mathbf{p})$ $\quad \gamma_+^{2\leftrightarrow 2}$

and

$p^0 = \omega^-(t,\mathbf{p}) = 2M(t) - \omega(t,\mathbf{p})$ $\quad \gamma_-^{2\leftrightarrow 2}$

$(p^0, \mathbf{p})$ $\quad \omega(t,\mathbf{q})$



$\omega(t,\mathbf{l})$ $\quad \omega(t,\mathbf{r})$

Comparison with on-shell linear response

Dynamics of perturbations are described by

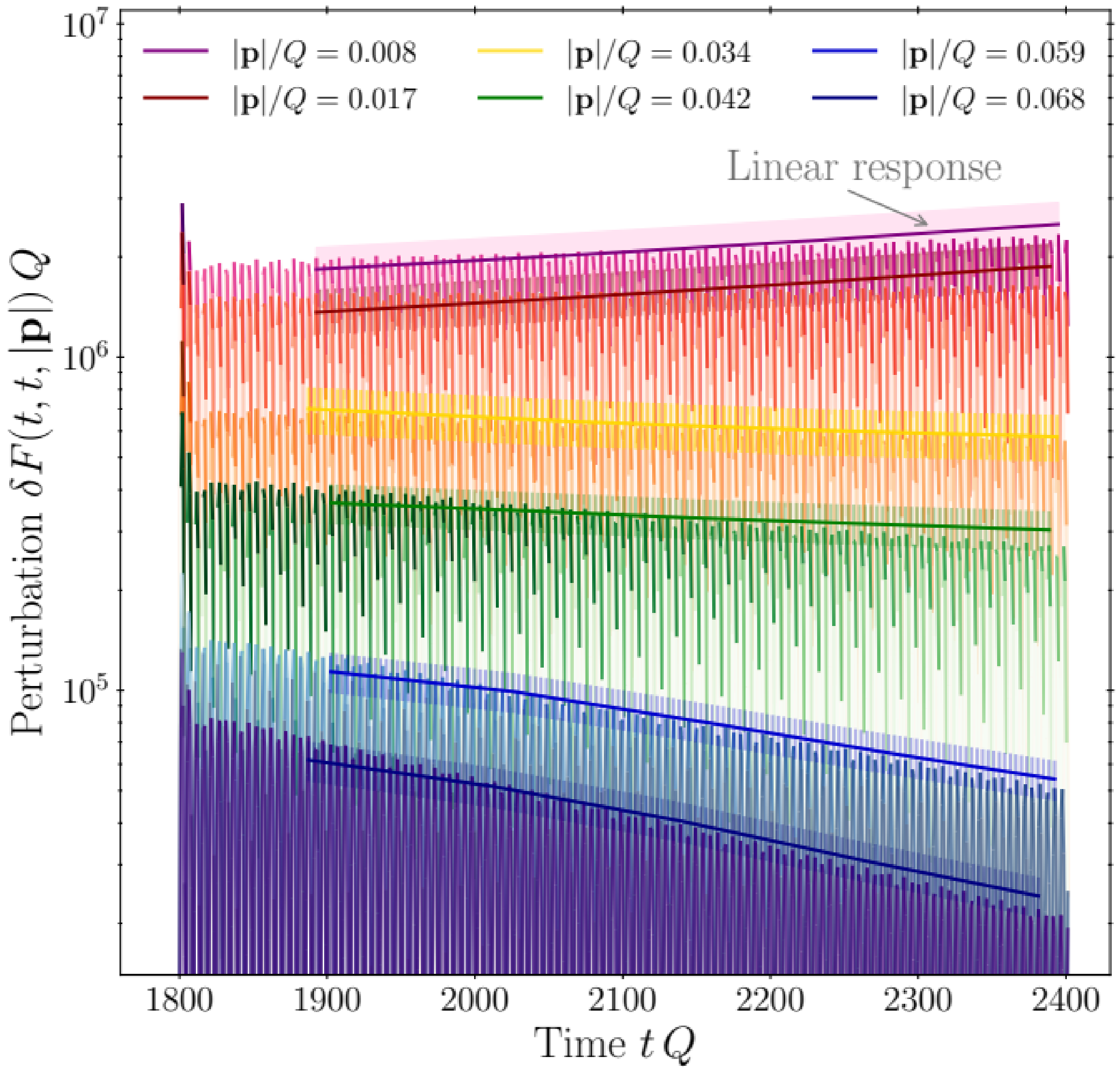
$$\frac{\delta F(t,\omega,|\mathbf{p}|)}{\delta F(t_i,\omega,|\mathbf{p}|)} = e^{-\Gamma(t,\omega,|\mathbf{p}|)}$$

with rate integral

$$\Gamma(t,\omega,|\mathbf{p}|) = \int_{t_i}^{t} dt' \; \gamma(t',\omega,|\mathbf{p}|)$$

Initial kick at $t_i Q = 1800$

$$\delta F(t_i,t_i,|\mathbf{p}|) = 10^{-3}(t_i/t_{\mathrm{ref}})^{\alpha} \times F_S((t_i/t_{\mathrm{ref}})^{\beta}|\mathbf{p}|)$$



Universal scaling of perturbations

The on-shell rate integral exhibits scaling

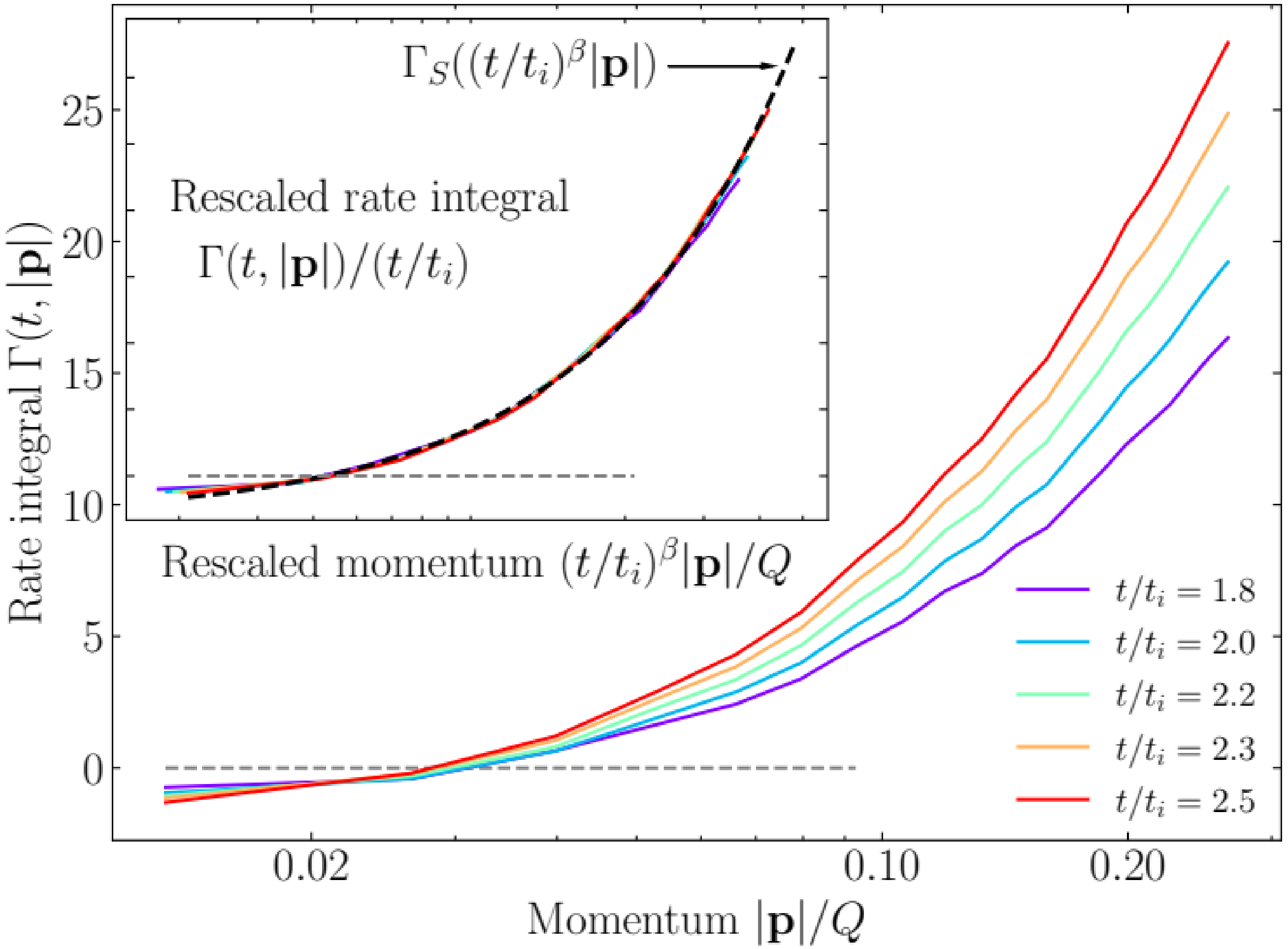
$$\Gamma(t, |\mathbf{p}|) = (t/t_{\text{ref}})\Gamma_S((t/t_{\text{ref}})^\beta |\mathbf{p}|)$$

where the scaling function can be described by

$$\Gamma_S((t/t_{\text{ref}})^\beta |\mathbf{p}|) = A\,(t/t_{\text{ref}})^\beta |\mathbf{p}|/Q - B$$

For a given momentum $|\mathbf{p}|$, the positive (stable) contribution $\sim A\,t^{\beta+1}|\mathbf{p}|$ will eventually outgrow the negative (unstable) $\sim B\,t$ term for all non-zero momenta in the scaling regime.

Scaling instability



Summary

based on 2209.14883

We studied the dynamics of perturbations around nonthermal fixed points, which revealed the presence of both stable and unstable perturbations.

Linear response theory allowed us to demonstrate that unstable dynamics arises from a competition between elastic scattering processes among different quasi-particle states.

We discovered the phenomenon of a scaling instability. As a consequence, the system shows attractor behavior after $t \sim 1/|\mathbf{p}|^{1/\beta}$ for any non-vanishing momentum $|\mathbf{p}|$.

Our results demonstrate from first principles that self-organized scaling (no fine-tuning) can be realized in the presence of both stable and unstable directions for the dynamics.

Outlook

Investigate this phenomenon for non-relativistic system, access via experimental platforms with ultra-cold quantum gases in the many-body regime with sufficiently large volumes.