Mini-jet quenching in non-equilibrium quark-gluon plasma

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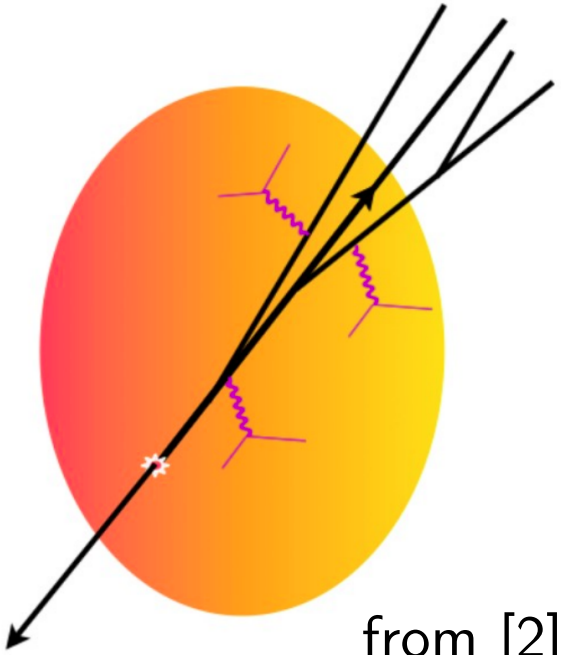
Motivation



from [2]

- interactions between Quark Gluon plasma (QGP) & hard partons $\rightarrow$ jet quenching
- study far-from-equilibrium evolution
- kinetic theory serves as bridge between initial state and hydrodynamics
- promising tool to describe the physics in the hard as well as in the soft sector

Goal: describe parton energy loss in an expanding plasma

Outline

Overview

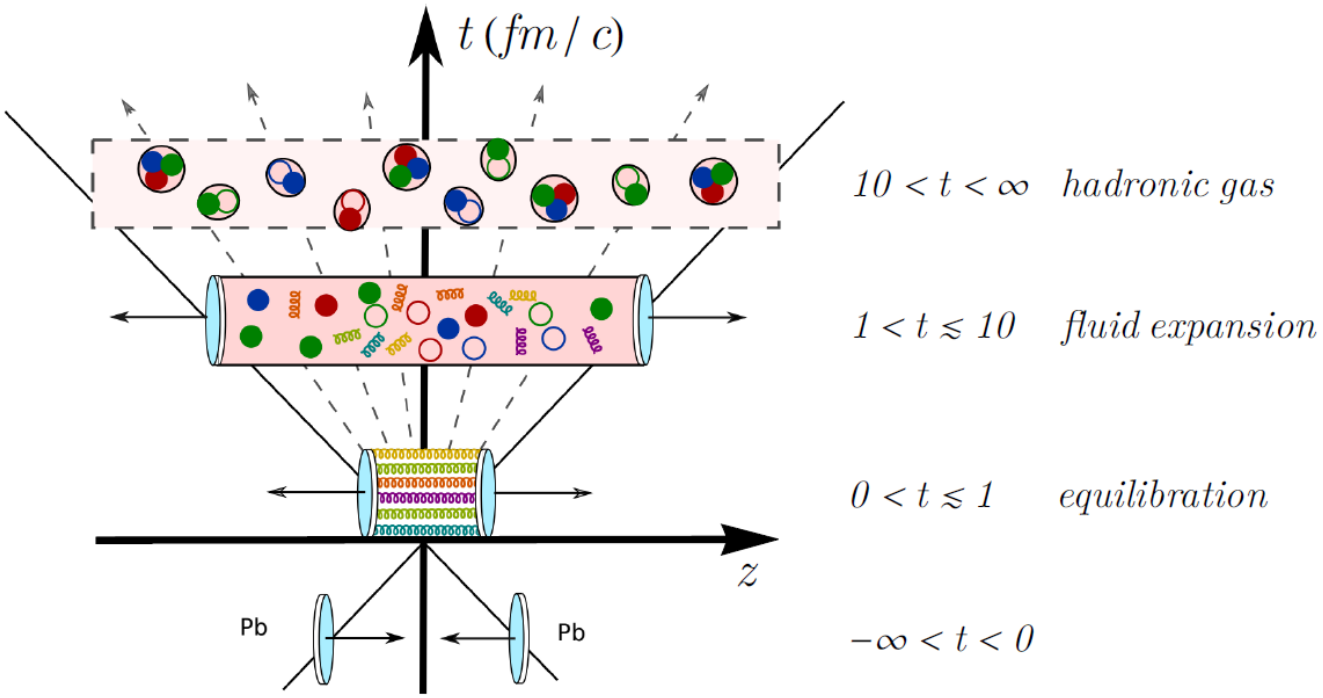


Figure 1: Main features of the evolution of a heavy ion collision, [4]

- study pre-equilibrium phase with an effective kinetic theory (EKT)

Framework

- EKT for high temperature gauge theories
  Arnold, Moore, Yaffe (2003))

  phase space distribution $f(\tau, \mathbf{x}, \mathbf{p})$
- weak coupling $\alpha_s \ll 1$
- dilute system $f \ll \alpha_s^{-1}$
- mean free path $\gg$ De-Broglie wavelength

  $\rightarrow$ quasi-particle picture

$$\Rightarrow (\partial_\tau + \mathbf{v} \cdot \nabla_{\mathbf{x}}) f(\tau, \mathbf{x}, \mathbf{p}) = -C[f] \tag{1}$$

proper time $\tau = \sqrt{t^2 - z^2}$

Boltzmann Equation

$$(\partial_\tau + \mathbf{v} \cdot \nabla_{\mathbf{x}}) f(\tau, \mathbf{x}, \mathbf{p}) = -C[f] \tag{2}$$

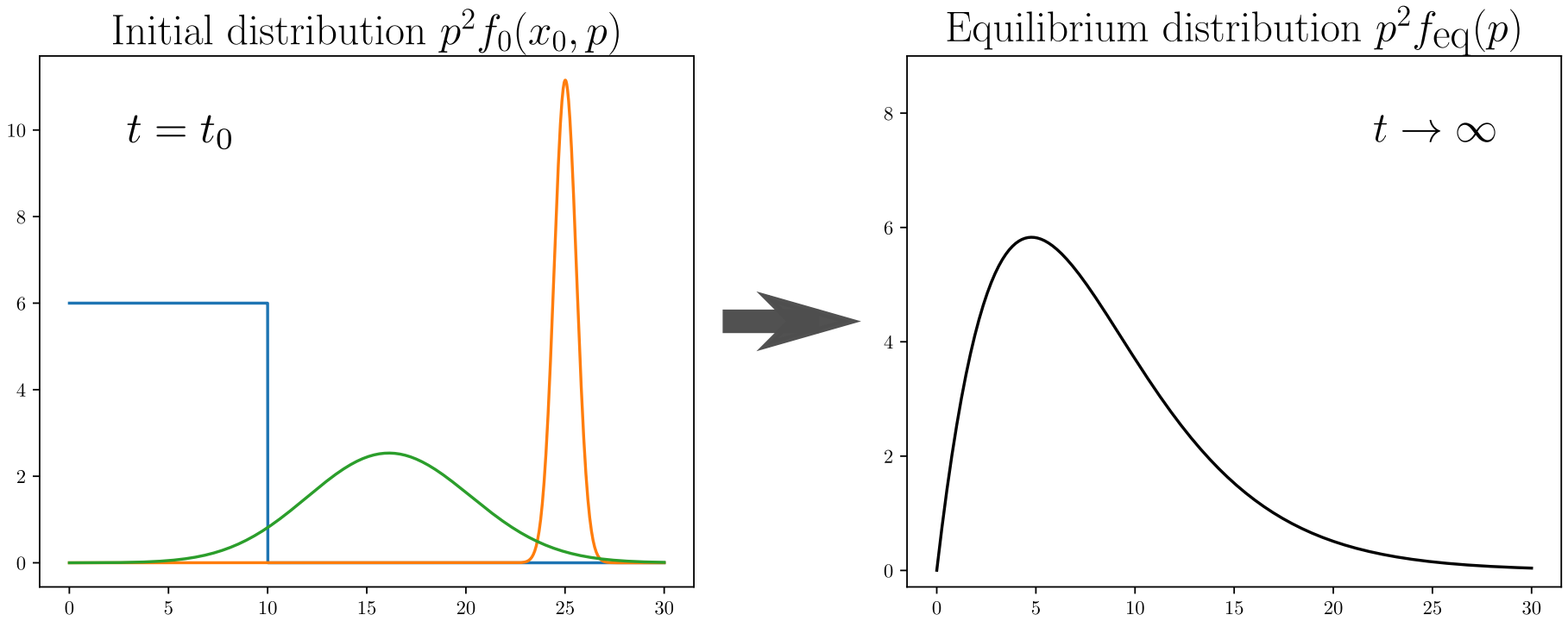
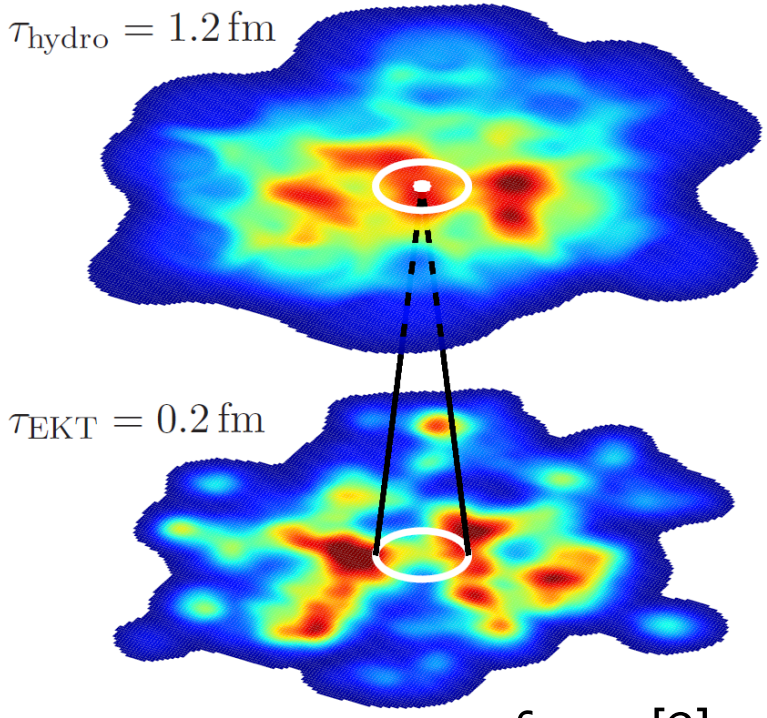


Figure 2: Out-of-equilibrium initial state is transported to equilibrium

Expanding QGP



from [3]

- longitudinal expansion
- approximate boost invariance
- homogeneity in the transverse plane

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right) f(\tau, \mathbf{p}) = -C[f] \tag{3}$$

- leading order elastic and inelastic scattering processes

$$C[f](\mathbf{p}) = C_{2\leftrightarrow 2}[f](\mathbf{p}) + C_{1\leftrightarrow 2}[f](\mathbf{p}) \tag{4}$$

$2 \leftrightarrow 2$

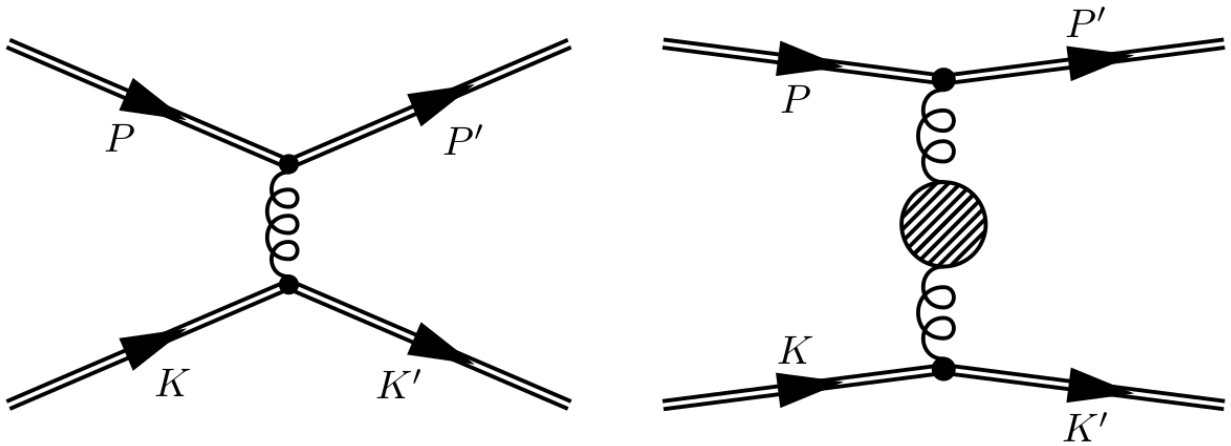


Figure 3: Hard (left) and soft (right) medium regulated scattering

$$C_{2\leftrightarrow 2}[f](\mathbf{p}) = \frac{1}{4p\nu} \int\limits_{\mathbf{k},\mathbf{p}',\mathbf{k}'} (2\pi)^4 \delta^{(4)} \left(p^\mu + k^\mu - p'^\mu - k'^\mu\right)$$
$$\times |\mathcal{M}|^2 \{ \underbrace{f_{\mathbf{p}} f_{\mathbf{k}} \left(1 \pm f_{\mathbf{p}'}\right) \left(1 \pm f_{\mathbf{k}'}\right)}_{\text{loss}} - \underbrace{f_{\mathbf{p}'} f_{\mathbf{k}'} \left(1 \pm f_{\mathbf{p}}\right) \left(1 \pm f_{\mathbf{k}}\right)}_{\text{gain}} \} \quad (5)$$

$2 \leftrightarrow 2$

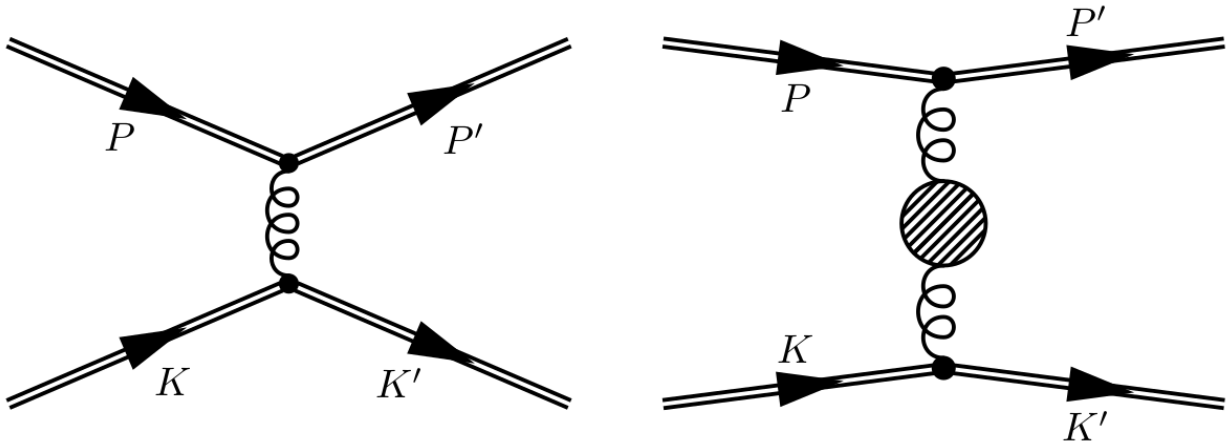


Figure 4: Hard (left) and soft (right) medium regulated scattering

$$|\mathcal{M}| = 2\lambda^2\nu\left(9 + \frac{(s-t)^2}{u^2} + \frac{(u-s)^2}{t^2} + \frac{(t-u)^2}{s^2}\right) \tag{6}$$

- small momentum transfer $q = |\mathbf{p}' - \mathbf{p}| \ll 1$ regulated by

$$\frac{1}{q^2} \to \frac{1}{q^2 + m_{\text{eff}}^2} \tag{7}$$

$1 \leftrightarrow 2$

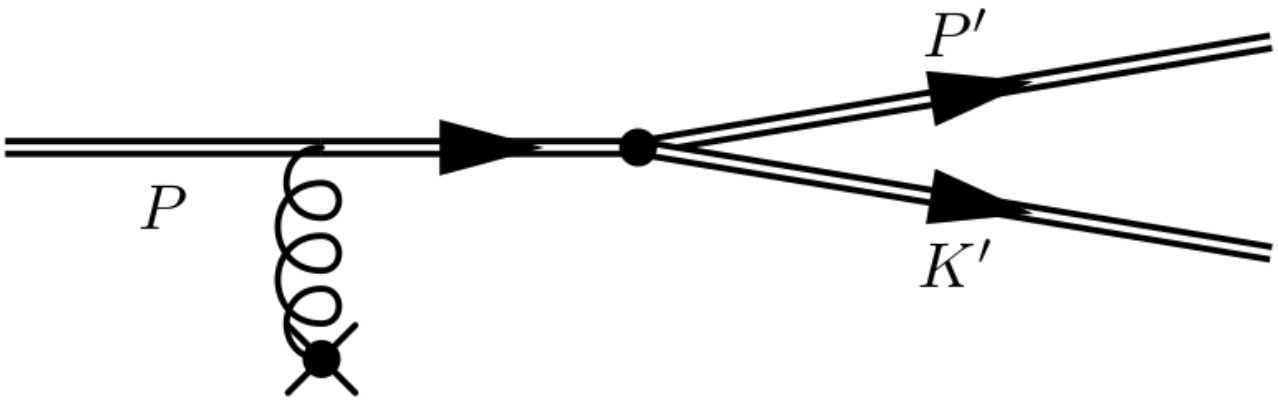


Figure 5: effective $1 \leftrightarrow 2$ process

$$C_{1\leftrightarrow 2}[f](\mathbf{p}) = \frac{1}{2}\frac{1}{\nu}(2\pi)^3 \int\limits_{\tilde{\mathbf{p}},\mathbf{p}',\mathbf{k}'} (2\pi)^4 \delta^{(4)}\left(\tilde{p}^\mu - p'^\mu - k'^\mu\right)$$
$$\times \left[\delta^{(3)}(\mathbf{p}-\tilde{\mathbf{p}}) - \delta^{(3)}\left(\mathbf{p}-\mathbf{p}'\right) - \delta^{(3)}\left(\mathbf{p}-\mathbf{k}'\right)\right]$$
$$\times \gamma\{\underbrace{f_{\mathbf{p}}\left(1 \pm f_{\tilde{\mathbf{p}}'}\right)(1 \pm f_{\mathbf{k}'})}_{\text{loss}} - \underbrace{f_{\mathbf{p}'} f_{\mathbf{k}'}\left(1 \pm f_{\tilde{\mathbf{p}}}\right)}_{\text{gain}}\} \quad (8)$$

$1 \leftrightarrow 2$

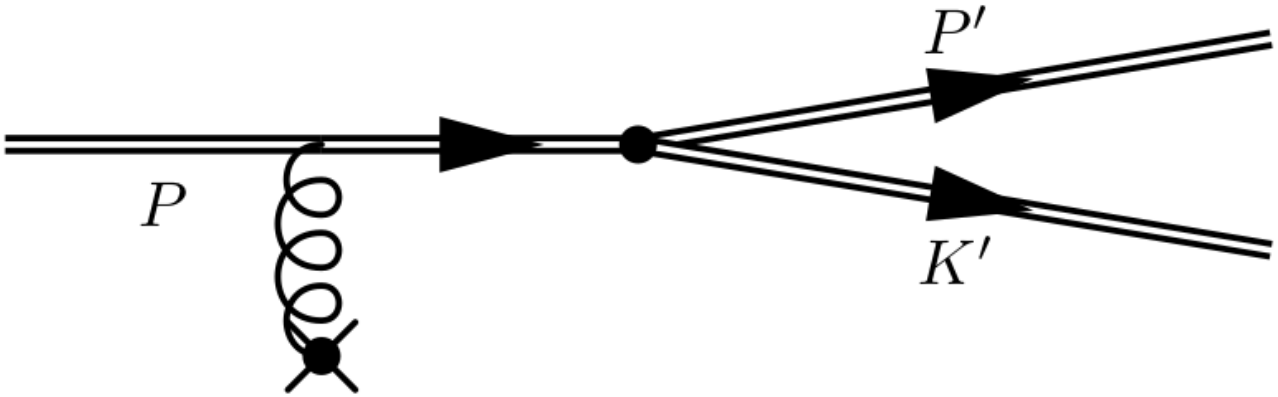


Figure 6: effective $1 \leftrightarrow 2$ process

- LO $\rightarrow$ strictly collinear
- medium induced radiation of gluons
- $N+1 \leftrightarrow N+2$ effectively $1 \leftrightarrow 2$

$1 \leftrightarrow 2$

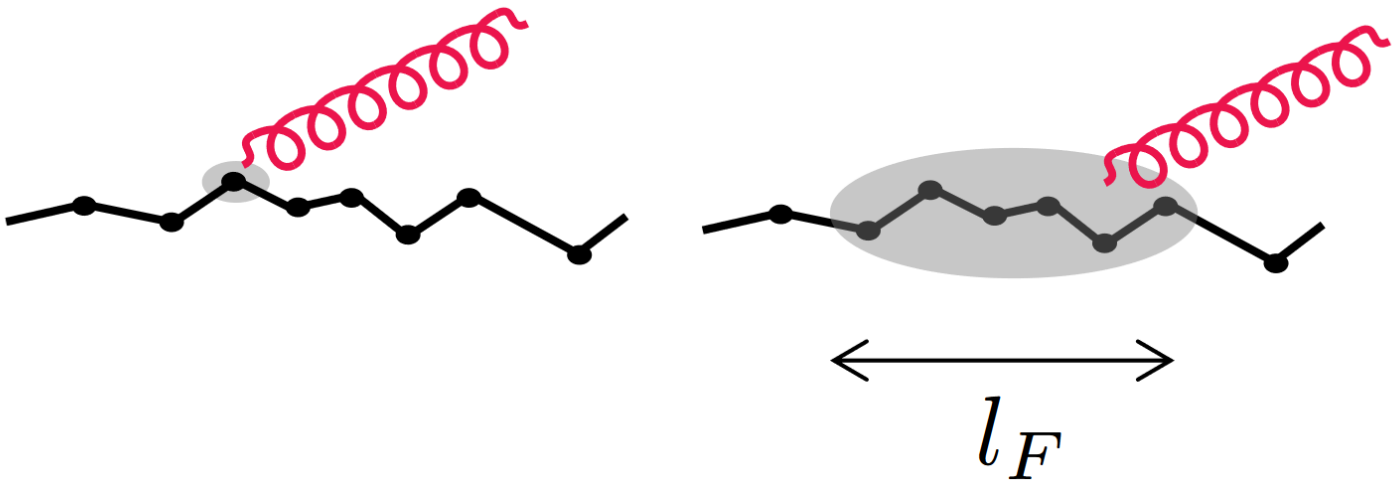


Figure 7: Parton going through the medium, Figure from [2]

- hard parton receiving multiple kicks
- formation time $\tau_f \sim E$
- BH: $l_f \ll l_{\mathrm{mfp}}$, independent emissions
- LPM: $l_f \sim l_{\mathrm{mfp}}$, destructive interference $\rightarrow$ suppression

Bottom-up thermalization

Baier, Müller, Schiff, Son (2001)

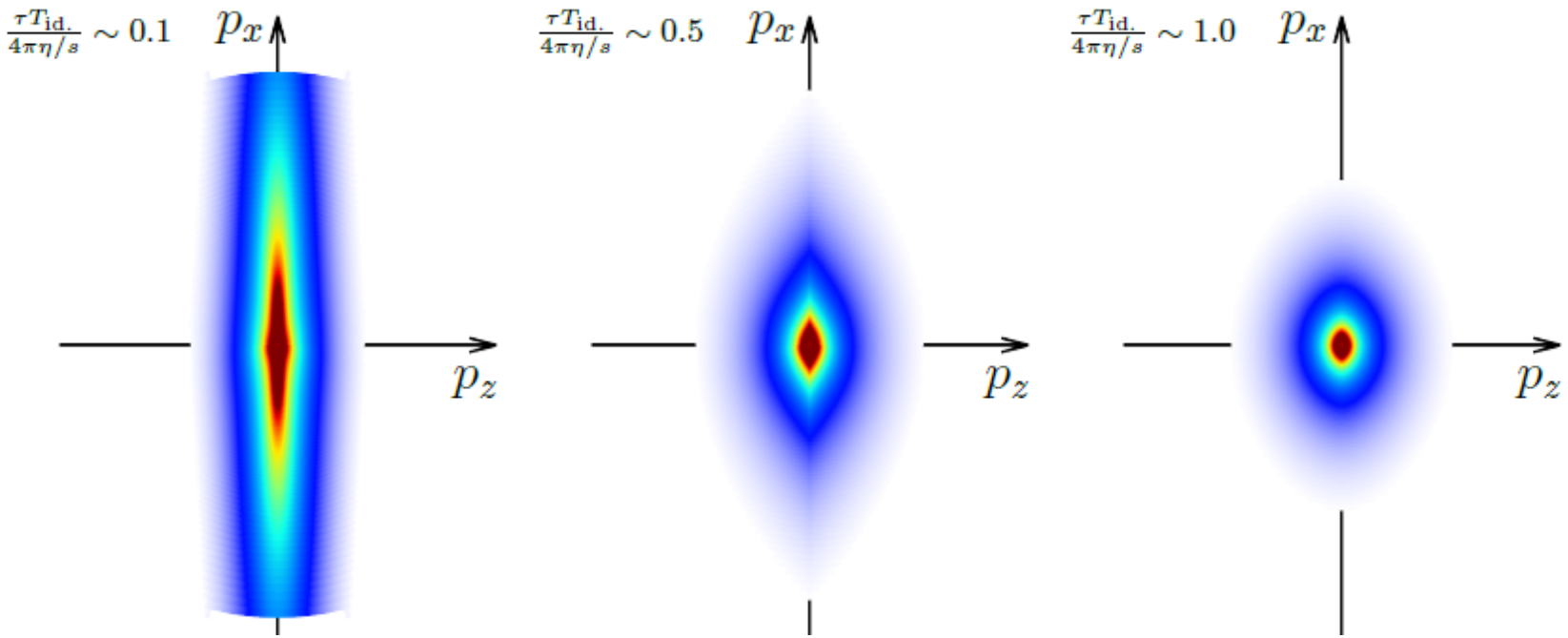


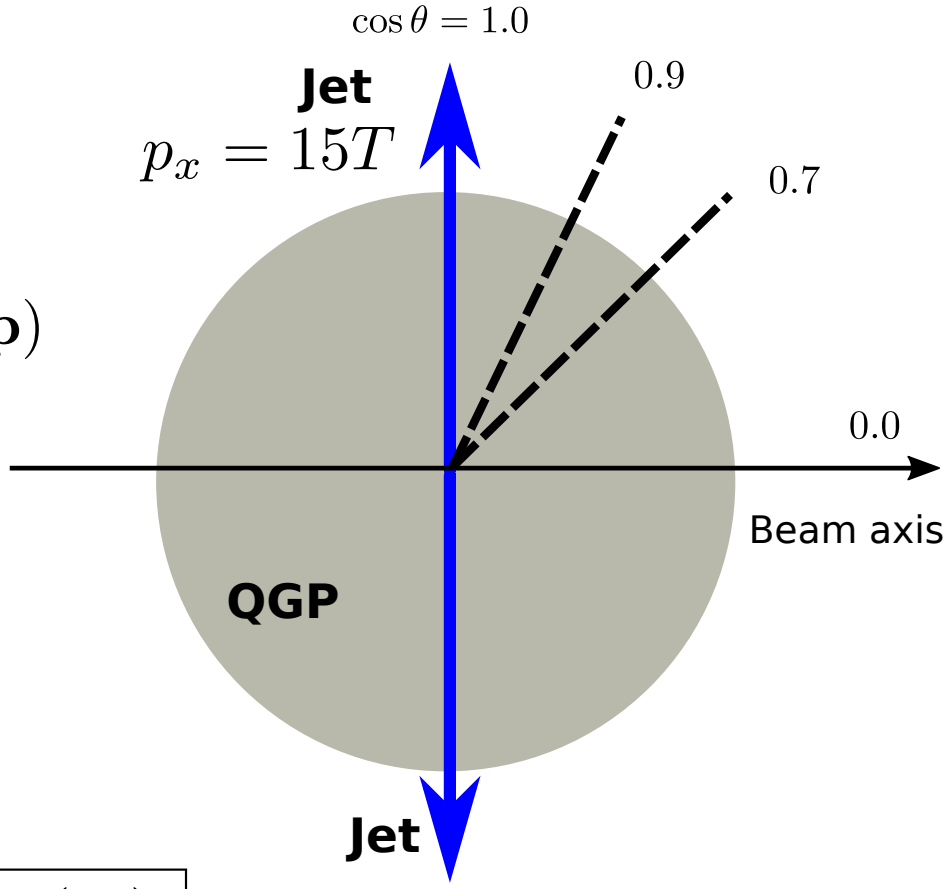
Figure 8: Isotropization of the distribution, Figure from [3]

- 1: overoccupied system getting more anisotropic
- 2: population of soft gluons
- 3: inverse energy cascade

Perturbative treatment

- background + perturbation
  $f(\tau, \mathbf{p}) = \bar{f}(\tau, \mathbf{p}) + \delta f_{\mathrm{Jet}}(\tau, \mathbf{p})$
- thermal medium
  $\bar{f}(\tau_0, \mathbf{p}) = n_{\mathrm{BE}}(\mathbf{p})$
- want to study

$$\boxed{\delta f_{\mathrm{Jet}}(\tau_0, \mathbf{p}) \to \delta f_{\mathrm{eq}}(p)}$$



Momentum space!

Perturbative treatment

- linearized Boltzmann equation (for YM)

$$\partial_\tau \delta f(\tau, \mathbf{p}) = -\delta C[\bar{f}, \delta f] \tag{9}$$

- $\delta f(\tau_0, \mathbf{p})$ Gaussian at $E = 15T$

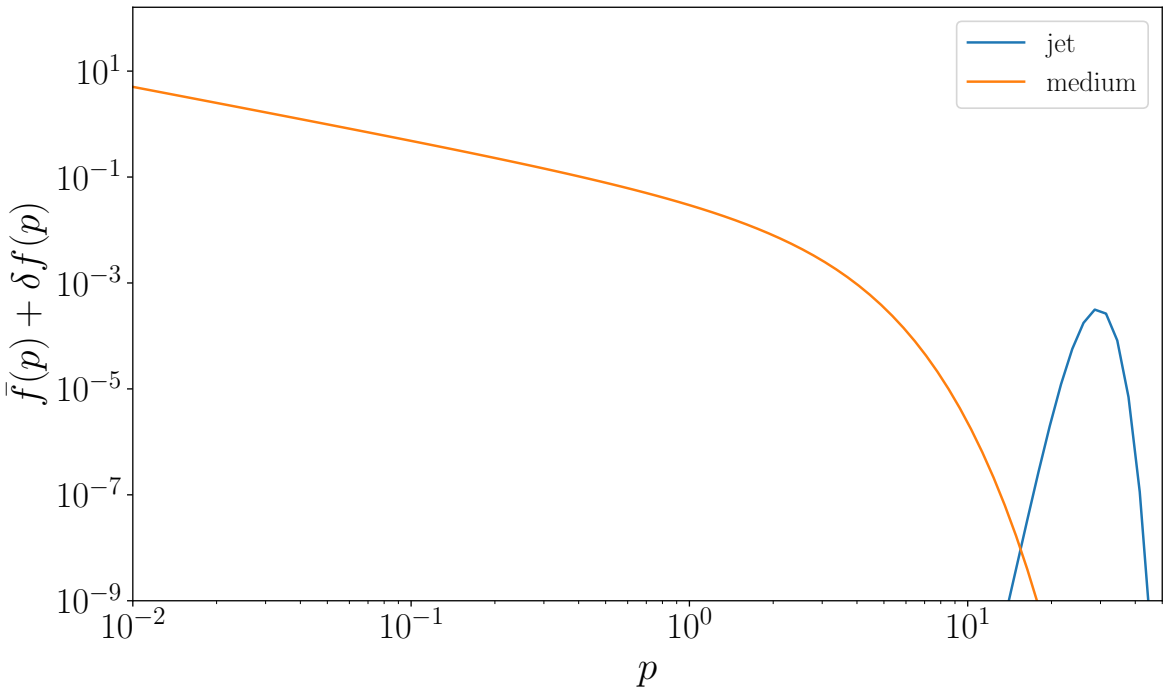


Figure 9: Initial condition for the jet on top of a thermal background

Inverse energy cascade

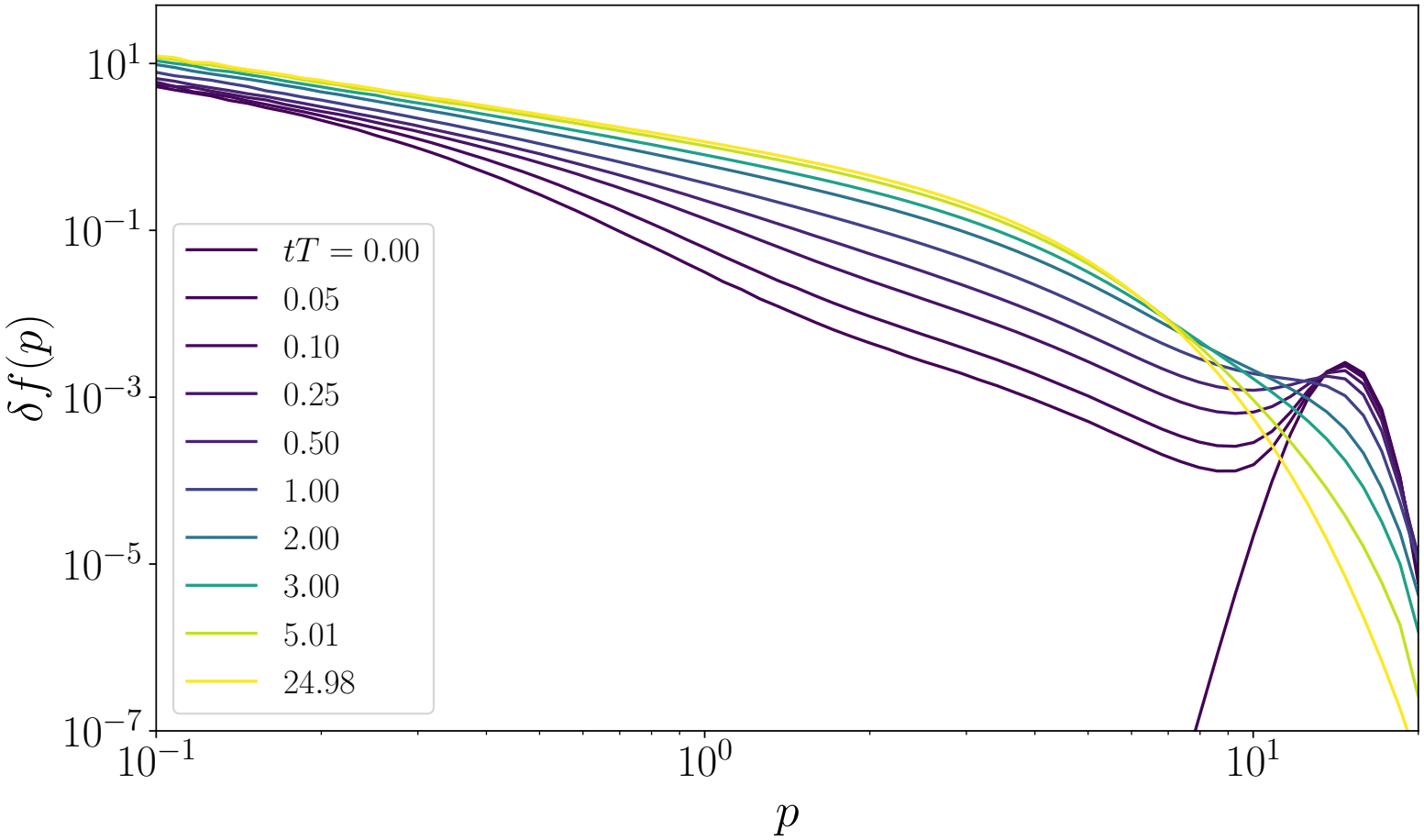


Figure 10: 1) build up of soft bath 2) transport of energy
Kurkela and Lu (2014)

- Now: study angular (θ) dependence!

Radiation vs. elastic scattering

- particle number $n = \int_{\mathbf{p}} f(\tau, \mathbf{p}) \to C_{1\leftrightarrow 2}[f]$
- transverse pressure $P_T = \frac{1}{2}\int_{\mathbf{p}} p_\perp^2/p f(\tau, \mathbf{p}) \to C_{2\leftrightarrow 2}[f]$

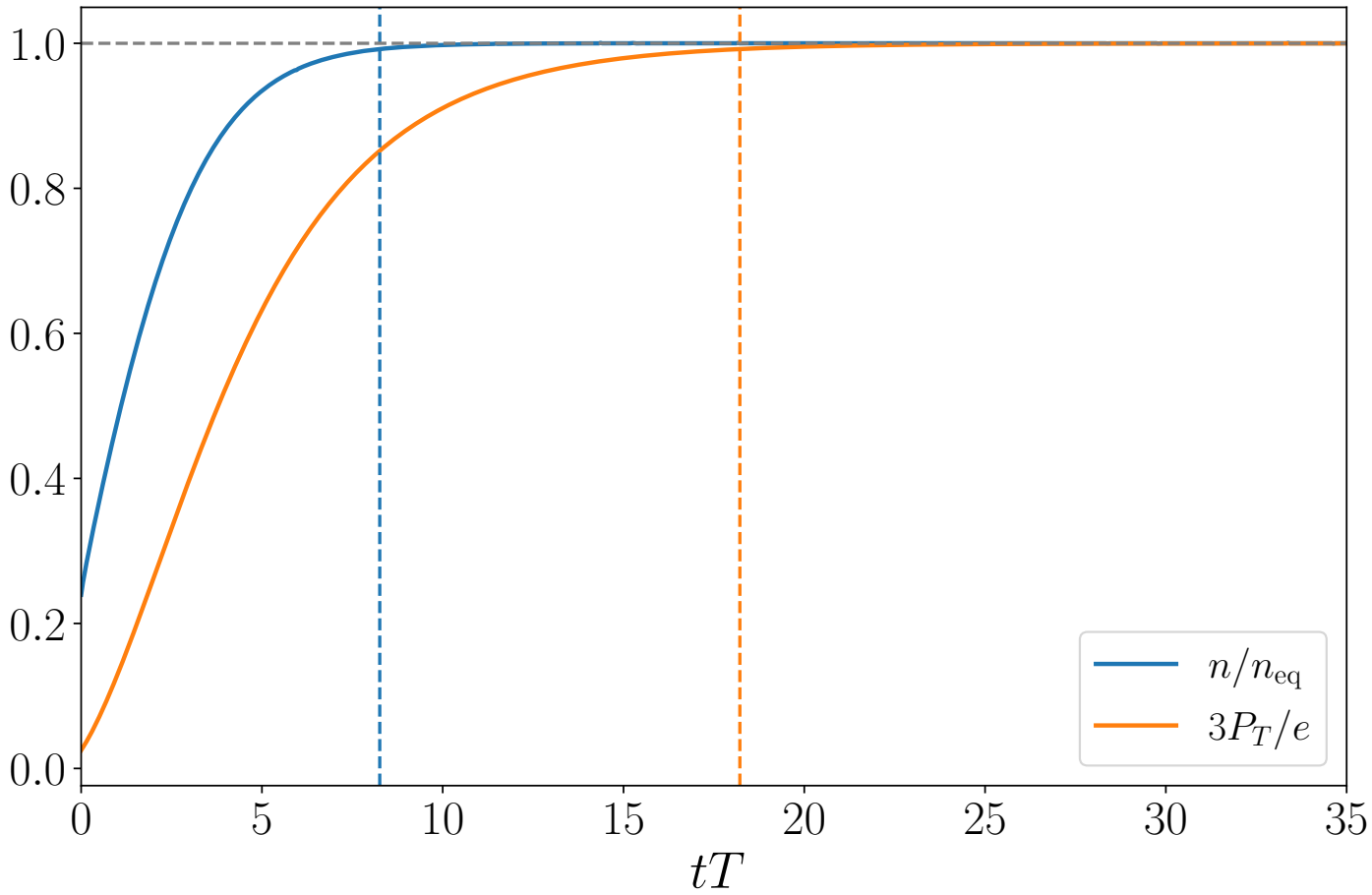


Figure 11: Equilibration along each θ-slice?

Jet distributions

- equilibrated jet $\rightarrow$ increase in temperature

$$\delta f_{\rm eq}(p) = n_{\rm BE}\left(\frac{p}{T+\delta T}\right) - n_{\rm BE}\left(\frac{p}{T}\right) = \partial_T n_{\rm BE}\left(\frac{p}{T}\right)\delta T \quad (10)$$

time $\longrightarrow$

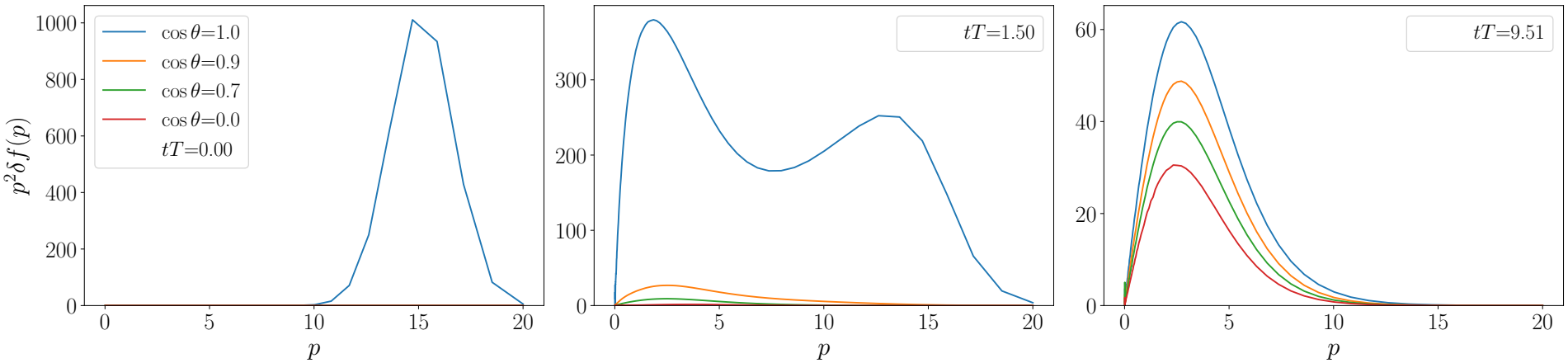


Figure 12: Distributions at different θ as a function of t

Jet distributions

- equilibrated jet $\rightarrow$ increase in temperature

$$\delta f_{\rm eq}(p) = n_{\rm BE}\left(\frac{p}{T+\delta T}\right) - n_{\rm BE}\left(\frac{p}{T}\right) = \underbrace{\partial_T n_{\rm BE}\left(\frac{p}{T}\right)}_{\text{eq. distr.}} \delta T \quad (11)$$

- Normalize $\int dp p^2 \delta f(p,\theta) = 1$

time $\longrightarrow$

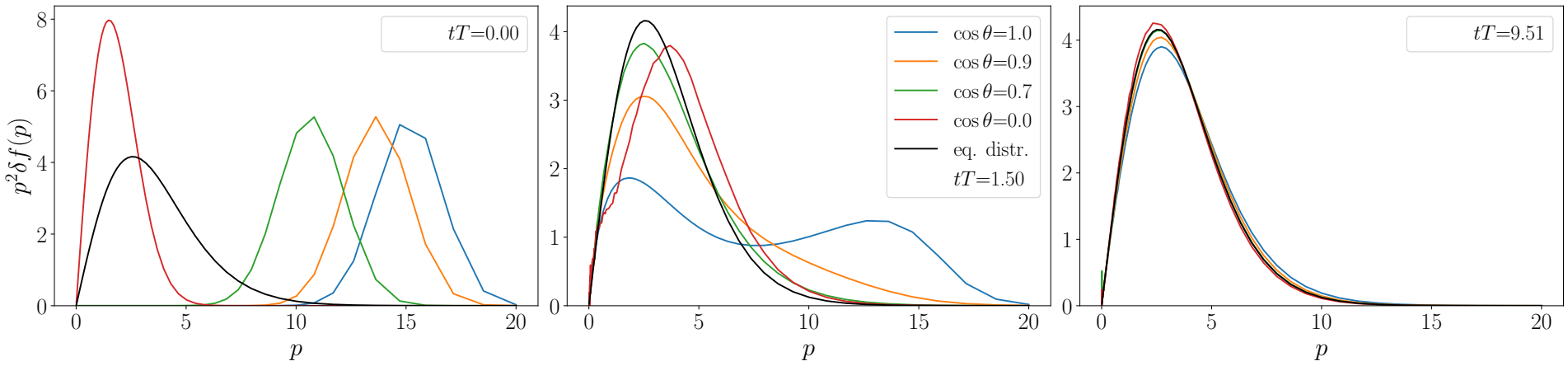


Figure 13: Equilibrium distributions with temperatures $\delta T(\theta)$

Expanding system

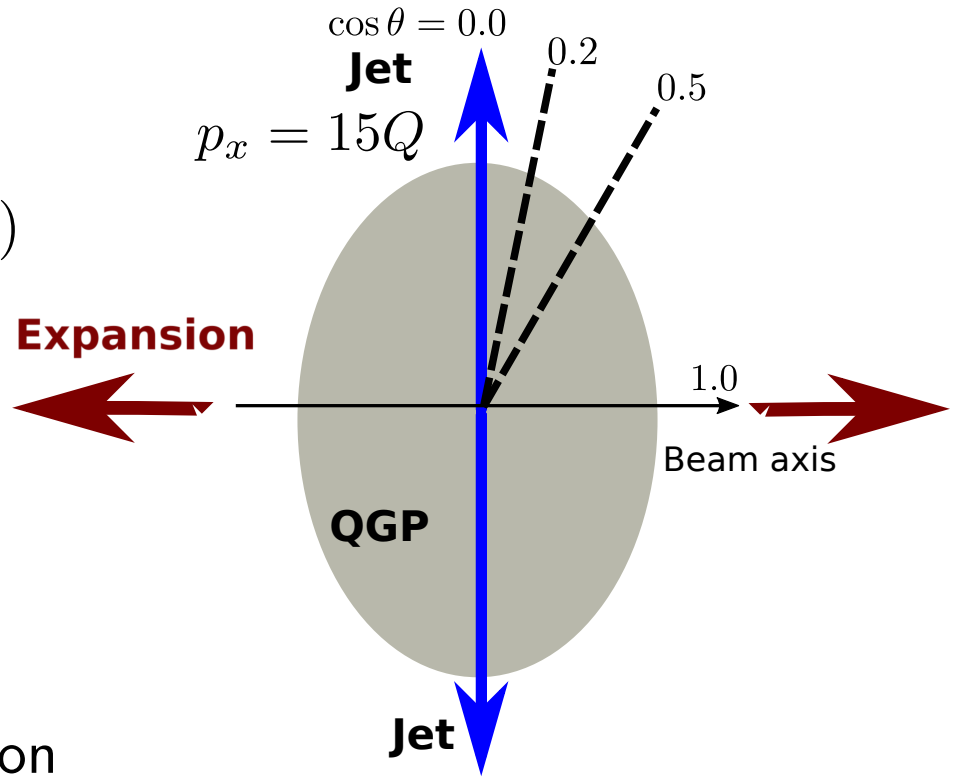
$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right) f(\tau, \mathbf{p}) = -C[f]$$

$$f(\tau, \mathbf{p}) = \bar{f}(\tau, \mathbf{p}) + \delta f_{\mathrm{Jet}}(\tau, \mathbf{p})$$

- non-thermal medium

$$\bar{f}(\tau, \mathbf{p}) \propto \exp\left(-\frac{2}{3}\frac{p_\perp^2 + \xi^2 p_z^2}{Q^2}\right)$$

- Now: study hydrodynamization



Momentum space!

$$\boxed{\delta f_{\mathrm{Jet}}(\tau_0, \mathbf{p}) \to \delta f_{\mathrm{hydro}}(\mathbf{p}, \tau)}$$

How to study hydrodynamization of jets?

$$\boxed{\delta f_{\mathrm{Jet}}(\tau_0, \mathbf{p}) \to \delta f_{\mathrm{hydro}}(\tau, \mathbf{p})}$$

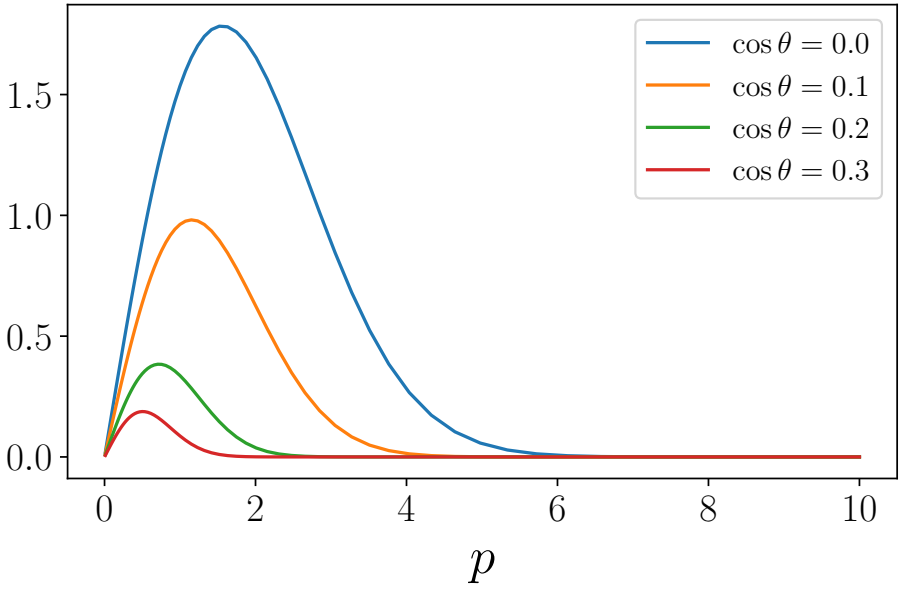
- thermal, non-expanding background: $\delta f_{\mathrm{eq}}(p) \to$ known analytic expression
- non-equilibrium, expanding background:

  no analytic expression for $\delta f_{\mathrm{hydro}}$
- compare with an arbitrary perturbation!

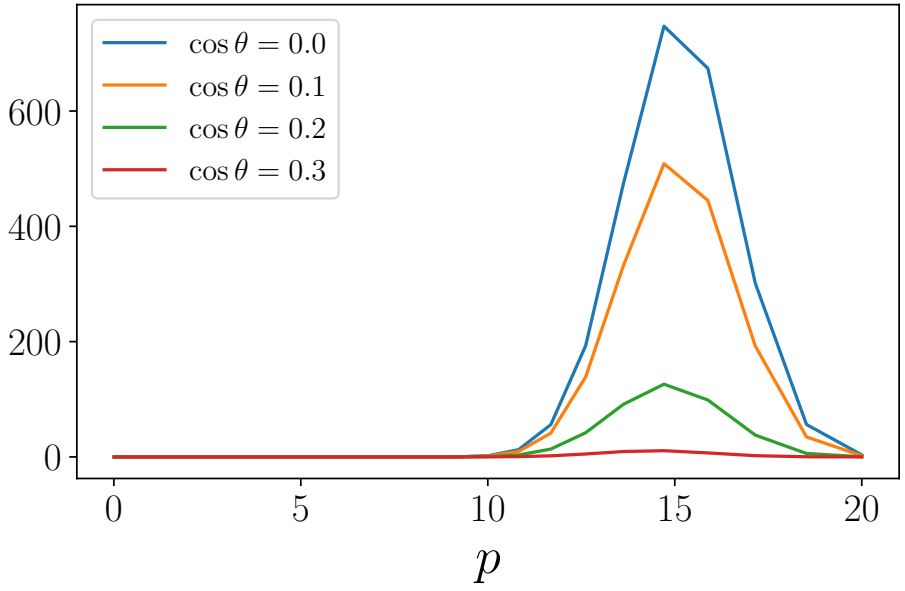
Comparison with background-like perturbation

- azimuthal symmetric (ϕ): $\delta f^{\rm az}_{\rm sym}(\tau_0, \mathbf{p}) = \epsilon \bar{f}(p, \theta)$

$$\bar{f} + \delta f^{\rm az}_{\rm sym} = (1+\epsilon)\bar{f} \tag{12}$$



(a) $p^2 \delta f^{\rm az}_{\rm sym}(\tau_0, p, \theta)$



(b) $p^2 \delta f_{\rm Jet}(\tau_0, p, \theta)$

- Hydrodynamization occurs, if both perturbations are indistinguishable

Hydrodynamization

- scaled time $\tilde{\omega} = \frac{\tau}{\tau_R}$

time $\longrightarrow$

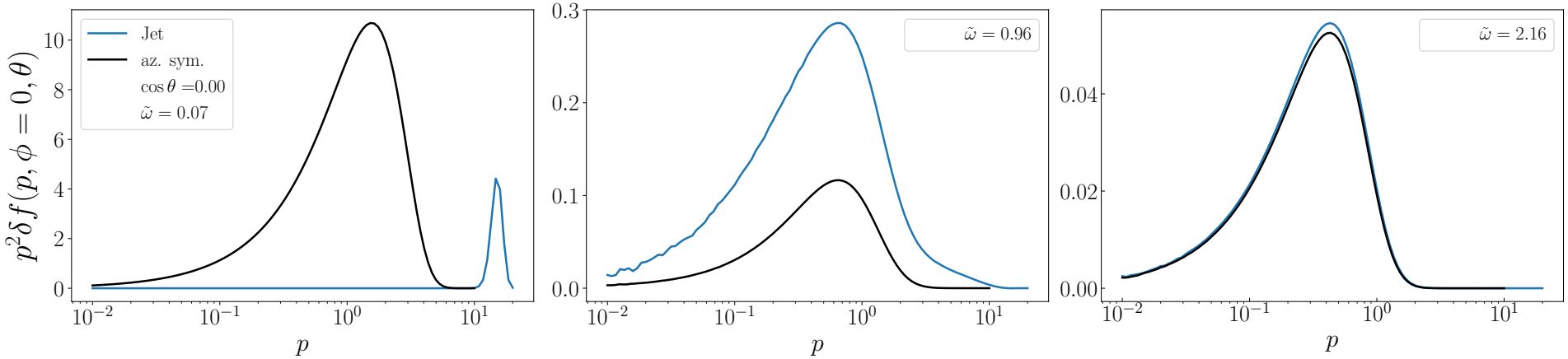


Figure 14: Agreement of $\delta f^{\mathrm{az}}_{\mathrm{sym}}$ and δf_{Jet} around $\tilde{\omega} \approx 2$

- loss of memory about the initial conditions

$\rightarrow$ **Hydrodynamization**!

Including fermions

- add quarks to $C[f]$

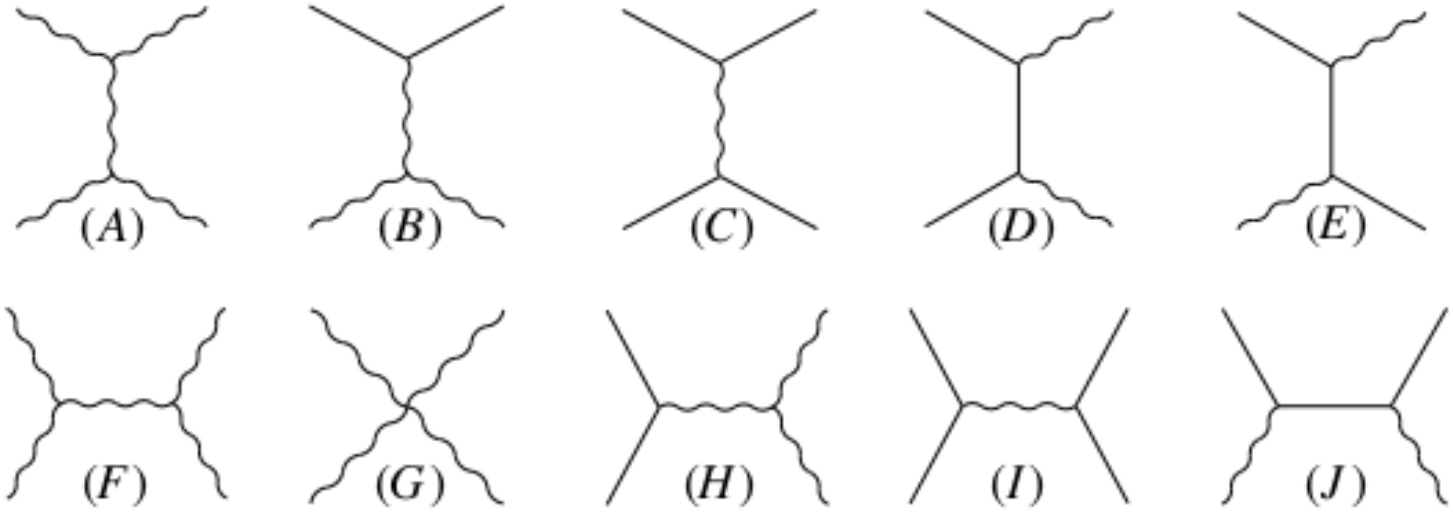
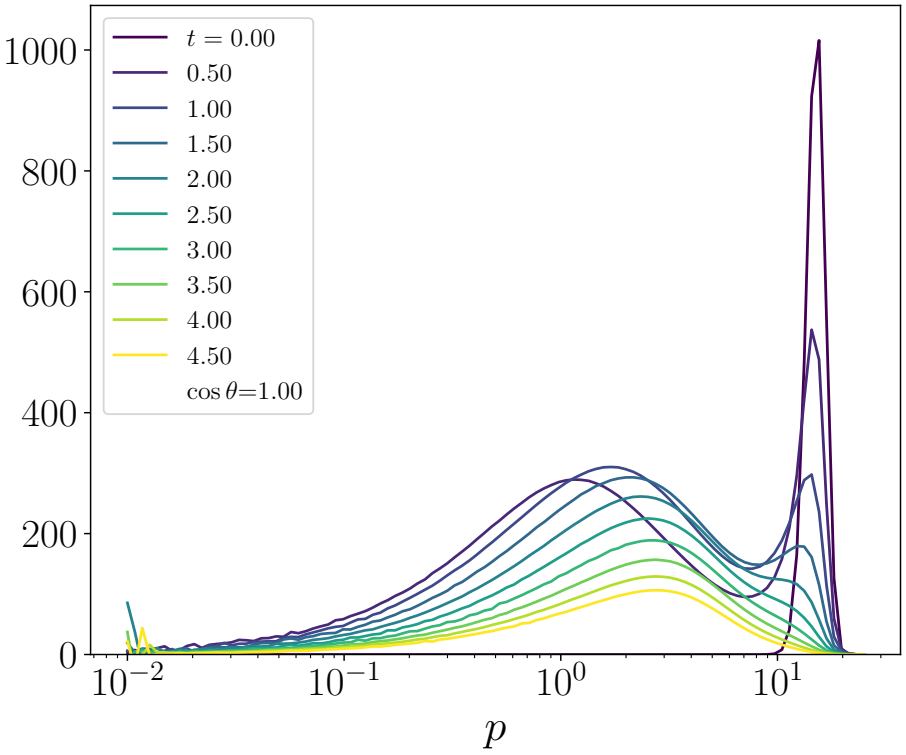


Figure 15: Leading order $2 \leftrightarrow 2$ diagrams, from [1]

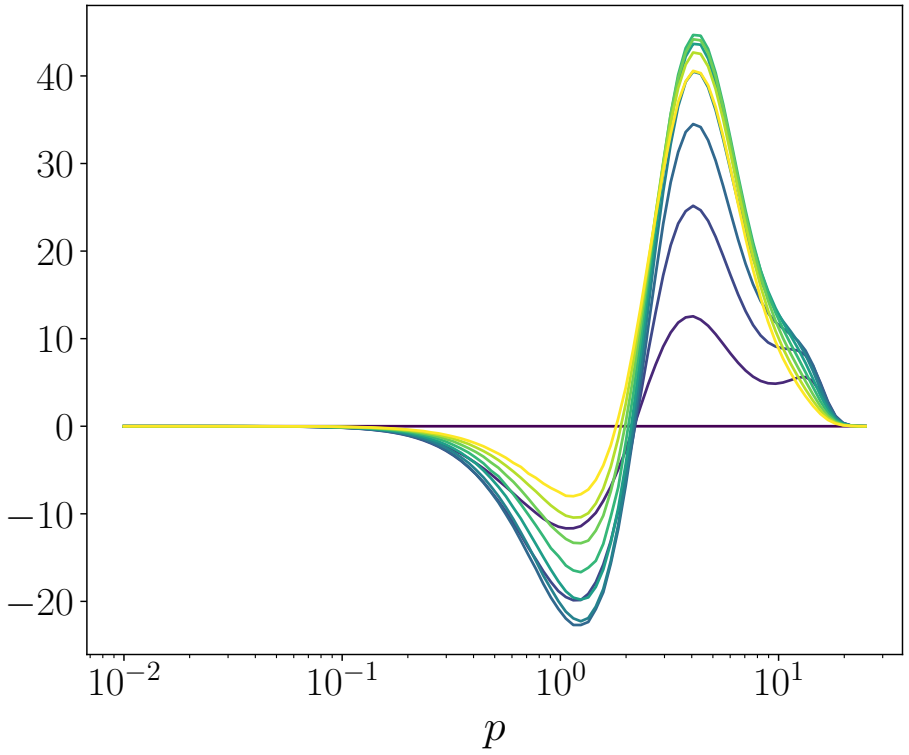
- solve 2 equations for $\delta f_g(\tau, \mathbf{p})$ and $\delta f_q(\tau, \mathbf{p})$

Including fermions

- initialize gluon jet $\delta f_g(\tau_0, \mathbf{p})$, quark jet $\delta f_q(\tau_0, \mathbf{p}) = 0$

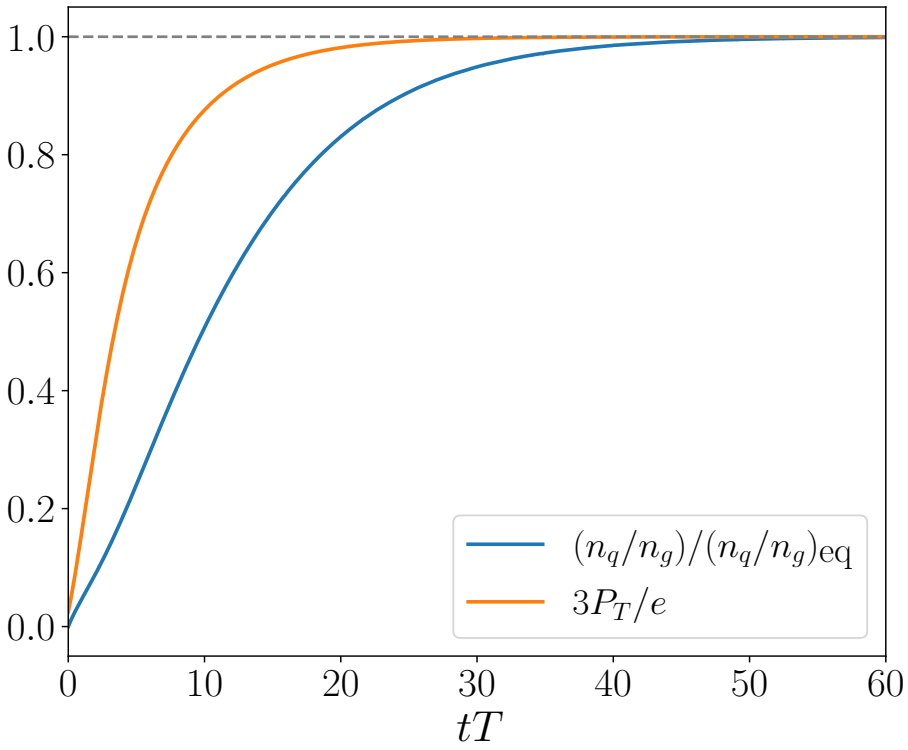


(a) Gluon distribution

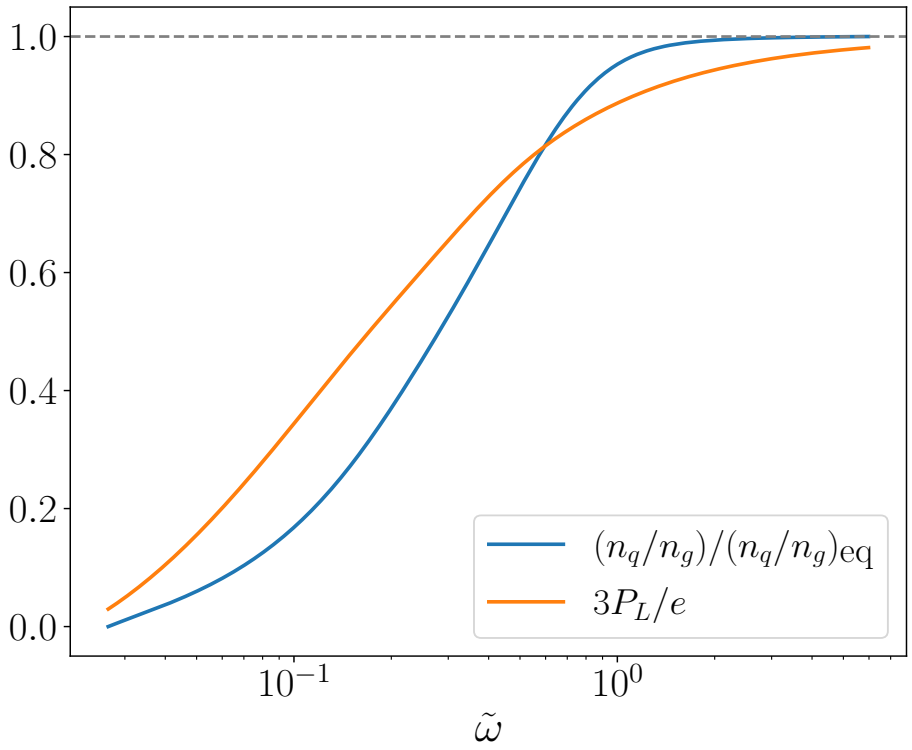


(b) Quark distribution

Chemical equilibration



(c) Thermal static background



(d) Non-equilibrium expanding background

- in chemical equilibrium more fermionic degrees of freedom
- chem. equilibration not affected by expansion

Summary & Outlook

static QGP:

- modelled a jet as a perturbation on top of a background
- two different time scales where inelastic and elastic processes are dominant
- thermal distribution with $T(\theta)$

expanding QGP:

- the minijet hydrodynamizes (later than the background)
- chemical equilibration before the system isotropizes

Outlook:

- get parametric estimates of equilibration time scales
- extract jet response functions $\rightarrow$ phenomenology
- include transverse dynamics $\rightarrow$ small systems

References

[1] Peter B Arnold, Guy D Moore, and Laurence G Yaffe. "Effective kinetic theory for high temperature gauge theories". In: *Journal of High Energy Physics* 2003.01 (2003), pp. 030–030.

[2] Jasmine Brewer. *Quark Matter 2022*.

[3] Aleksi Kurkela et al. "Effective kinetic description of event-by-event pre-equilibrium dynamics in high-energy heavy-ion collisions". In: *Physical Review C* 99.3 (2019).

[4] Aleksas Mauzeliaskas.

Equation of motion

- local homogeneity $\rightarrow$ relative coordinate $s^\mu = x^\mu - y^\mu$ and center coordinate $X^\mu = \frac{1}{2}(x^\mu + y^\mu)$
- gradient expansion in X^μ
- to lowest order, spectral function ρ is on shell $\rightarrow$ quasi-particle picture
- non-equilibrium distribution function $f(X,p)$:

$$F(X,p) = -i\left[\frac{1}{2} \pm f(X,p)\right]\rho(X,p) \tag{13}$$

$$\Rightarrow p^\mu \partial_\mu f(X,p) = -C[f] \tag{14}$$

Backup

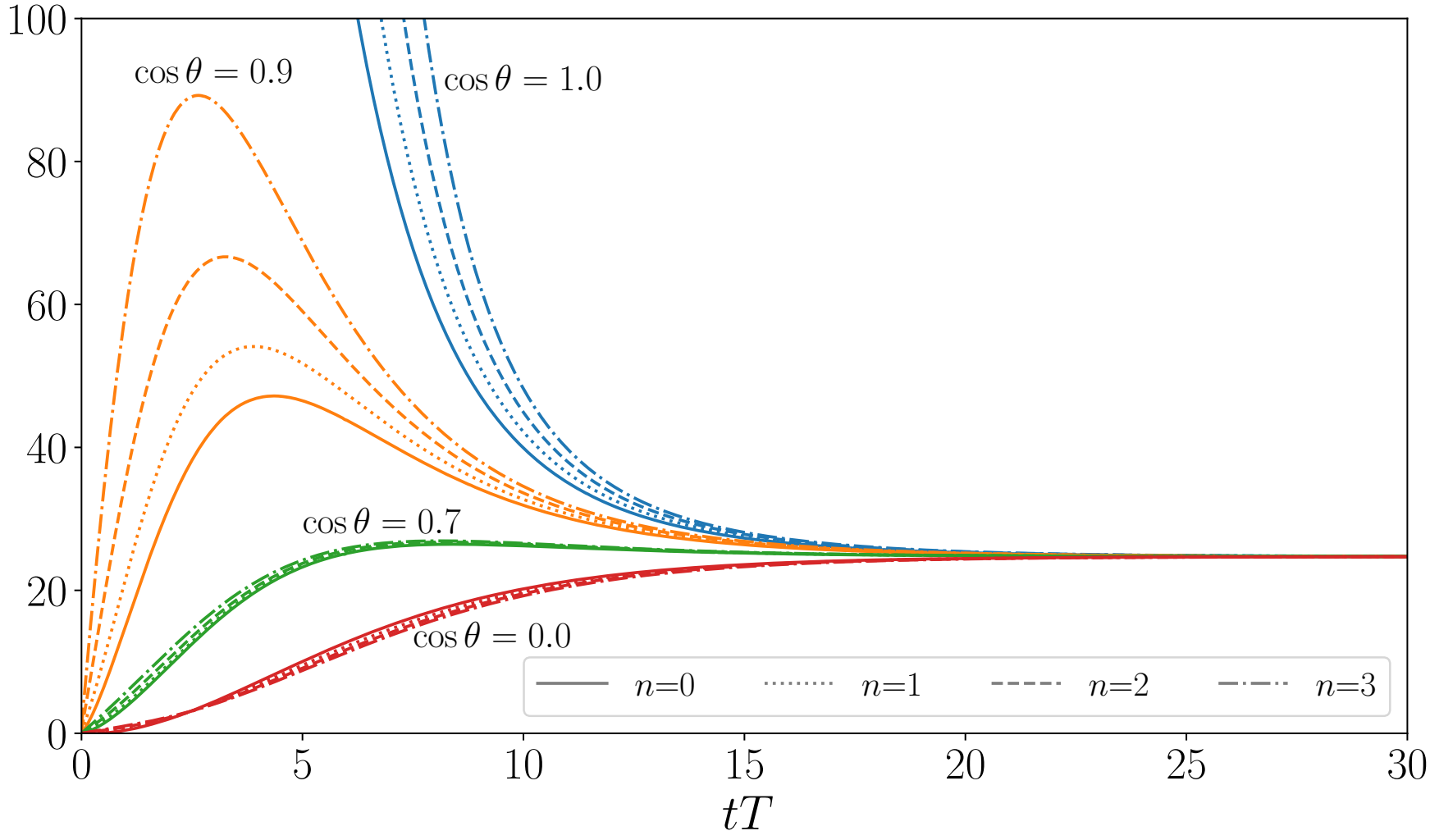


Figure 16: Moments of δf for different v_z static background

Backup

- scaled time

$$\tilde{w} = \frac{\tau \overline{T}}{4\pi\eta/s} \tag{15}$$

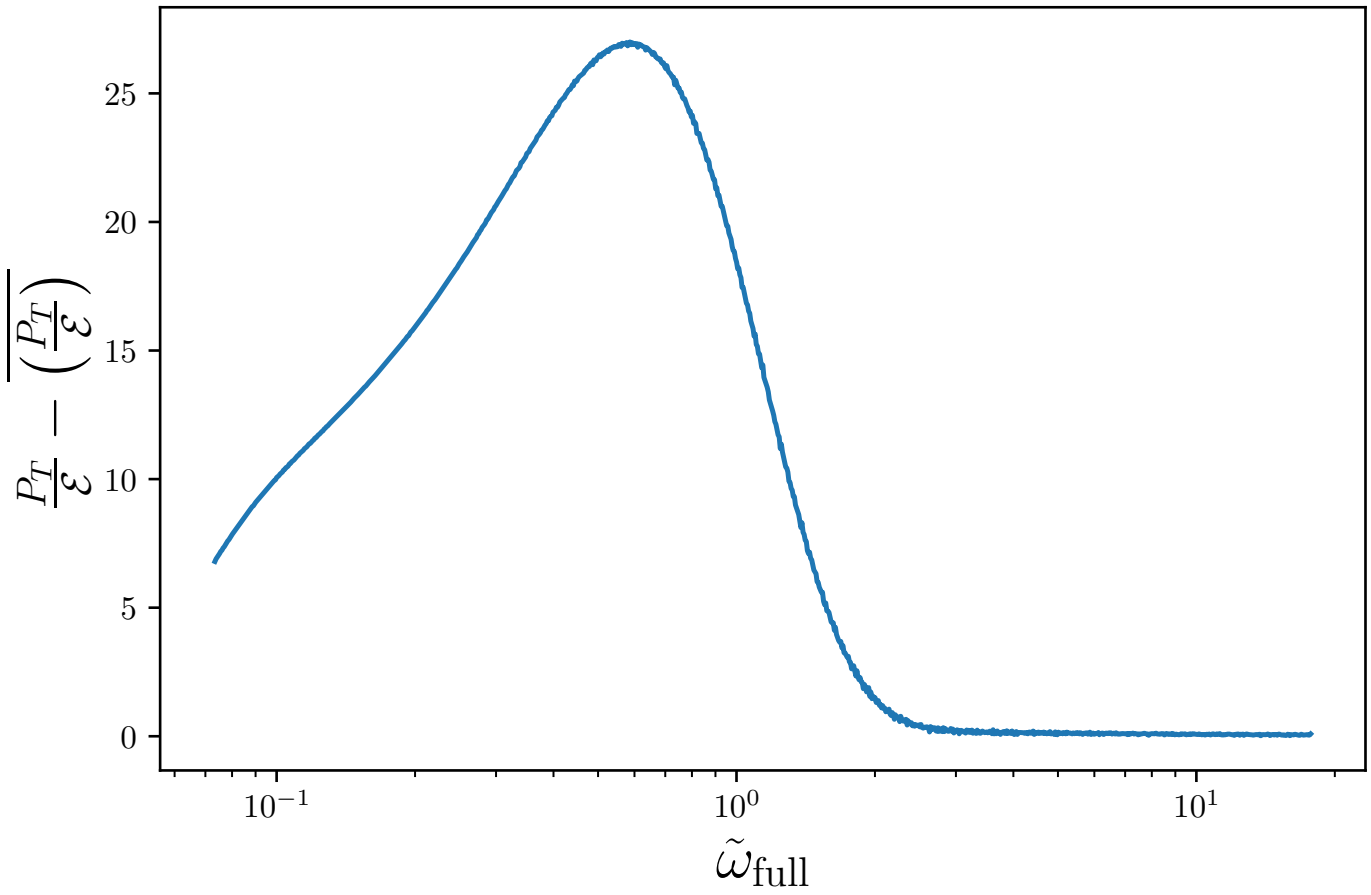


Figure 17: Jet thermalizing and becoming part of the medium

Backup

$$\left[iG^{-1,\mu\gamma}_{0,ac}(x;\mathcal{A})+\Pi^{(0),\mu\gamma}_{ac}(x)\right]\rho^{cb}_{\gamma\nu}(x,y)$$
$$=-\int_{y^0}^{x^0}dz\Pi^{(\rho),\mu\gamma}_{ac}(x,z)\rho^{cb}_{\gamma\nu}(z,y) \tag{16}$$

$$\left[iG^{-1,\mu\gamma}_{0,ac}(x;\mathcal{A})+\Pi^{(0),\mu\gamma}_{ac}(x)\right]F^{cb}_{\gamma\nu}(x,y)$$
$$=-\int_{t_0}^{x^0}dz\Pi^{(\rho),\mu\gamma}_{ac}(x,z)F^{cb}_{\gamma\nu}(z,y)+\int_{t_0}^{y^0}dz\Pi^{(F),\mu\gamma}_{ac}(x,z)\rho^{cb}_{\gamma\nu}(z,y) \tag{17}$$

$$\frac{\mathcal{P}_T}{\mathcal{P}_L}=\frac{\frac{1}{2}\int\frac{d^3p}{(2\pi)^3p}p_\perp^2 f^g_{\mathbf{p}}}{\int\frac{d^3p}{(2\pi)^3p}p_z^2 f^g_{\mathbf{p}}} \tag{18}$$

regulating IR-divergences with effective mass

$$m^2_{\text{eff}}=2g^2\int\frac{d^3\mathbf{p}}{(2\pi)^3p}\left[N_c f^g_{\mathbf{p}}+N_f f^q_{\mathbf{p}}\right] \tag{19}$$