Exploring the QCD phase diagram at finite temperature and density

Martin Pospiech
TU Darmstadt

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The (conjectured) QCD phase diagram

[FAIR @ GSI]

[Bailin, Love '84; Alford, Rajagopal '98; Rapp, Schäfer, Shuryak, Velkovsky '98; Berges, Rajagopal '99; Son '99; Pisarski, Rischke '00, ...]
“Tool time”: The functional RG

Idea: Wilsonian momentum shell integration

\[ \partial_t \Gamma_k[\Phi] = -\frac{1}{2} \text{Tr} \left\{ \left[ \Gamma_k^{(1,1)}[\Phi] + R_k^{\psi} \right]^{-1} \cdot (\partial_t R_k^{\psi}) \right\} \]

[Wetterich '93]
(Some) Symmetries of hot and dense matter

- Color: $SU(N_c)$
- Chiral: $SU_L(2) \otimes SU_R(2)$
- Vector (baryon conservation): $U_V(1)$

- Axial (anomaly):
  - $\mu > 0$
  - Integration over space-time
  - $C$, Poincaré, $U_A(1)$
NJL-model as a low-energy QCD model

[Nambu, Jona-Lasinio '61]

gluon interactions induce four-quark interactions, e.g.

\[
\begin{array}{c}
g & g \\
\hline
\hline
\hline
\hline
n & n
\end{array}
\quad \rightarrow \quad \begin{array}{c}
\lambda \\
\hline
\hline
\hline
\hline
\end{array}
\]

2-flavor NJL “classical” action (vacuum):

\[
S = \int d^4x \left\{ \bar{\psi} i\gamma_\mu \partial^\mu \psi + \frac{1}{2} \lambda_{(\sigma-\pi)} \left[ (\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\tau_1\psi)^2 \right] \right\}
\]

chiral condensate oder parameter: \( \langle \bar{\psi}\psi \rangle \neq 0 \)
Spontaneous chiral symmetry breaking

\[ m^2 > 0 \quad \sigma \sim \bar{\psi}\psi \quad \begin{array}{c} \underline{\bar{\pi} \sim \bar{\psi}\gamma_5 \bar{\tau}\psi} \\
\end{array} \]

\[ m^2 < 0 \]

\[ \begin{array}{c}
m^2 \sim \lambda_{(\sigma-\pi)}^{-1}
\end{array} \]

ground state in symmetric phase

ground state in broken phase

partially bosonized action:

\[
S = \int d^4x \left\{ \bar{\psi}i\gamma^\mu \partial_\mu \psi + \bar{\psi} (\sigma + i\gamma_5 \bar{\tau} \cdot \bar{\pi}) \psi + \lambda_{(\sigma-\pi)}^{-1} (\sigma^2 + \bar{\pi}^2) \right\}
\]

diverging four-quark coupling: onset \( \chi_{SB} \)
Fierz-complete NJL-model set up

ansatz for effective action:

\[
\Gamma_{LO}[\bar{\psi}, \psi] = \int_0^\beta d\tau \int d^3x \left\{ \bar{\psi} \left( i\gamma_0 \partial_0 + i\gamma_i \partial_i - i\mu \gamma_0 \right) \psi + \frac{1}{2} \sum_{j \in B} \tilde{\lambda}_j \mathcal{L}_j \right\}
\]

all four-quark interactions compatible with symmetries

cautions: Fierz ambiguity!

\[
(O^A)_{ab}(O^A)_{cd} = \sum_B C^{AB}(O^B)_{ad}(O^B)_{cb}
\]

e.g. \[
(\bar{\psi} \psi)^2 = a_1(\bar{\psi} \psi)^2 + a_2(\bar{\psi} \gamma_5 \psi)^2 + a_3(\bar{\psi} \gamma_\mu \psi)^2 + \ldots
\]

challenge: find minimal set of four-quark interactions
Fierz-complete NJL-model set up

\[ \mathcal{L}_{(V+A)_\parallel}, \mathcal{L}_{(V+A)_\perp}, \mathcal{L}_{(V-A)_\parallel} \]
\[ \mathcal{L}_{(V-A)_\perp}, \mathcal{L}_{(V+A)_{\text{adj}}}, \mathcal{L}_{(V-A)_{\text{adj}}} \]

- 4 channels:
  - \( SU(N_c) \) × \( U_V(1) \)
  - \( SU_L(2) \otimes SU_R(2) \) × \( U_A(1) \)

- 6 channels:
  - \( SU(N_c) \) × \( U_V(1) \)
  - \( SU_L(2) \otimes SU_R(2) \) × \( U_A(1) \)

- 10 channels (Fierz complete!):
  - \( SU(N_c) \) × \( U_V(1) \)
  - \( SU_L(2) \otimes SU_R(2) \) × \( U_A(1) \)

"chiral" order-parameter
condensate: \( \langle \bar{\psi} \psi \rangle \)

"diquark" channel
condensate: \( \langle \imath \psi^T C \gamma_5 \epsilon_f \epsilon^l_c \psi \rangle \)

(Fierz complete!)
NJL-model $\beta$ - functions

$\partial_t \lambda = 2 \frac{\lambda}{\lambda} - \alpha$

$T/k = 0$
$\mu/k = 0$

diverging coupling

initial UV value
NJL-model $\beta$ - functions

\[
\partial_t \lambda = 2 \lambda - \alpha
\]

\[
\frac{T}{k} > 0 \quad \mu/k = 0
\]
NJL-model $\beta$ - functions

\[ \partial_t \lambda = 2 \lambda - \alpha \]

\[ T/k = 0 \]
\[ \mu/k > 0 \]
NJL-model $\beta$ - functions

\[ \partial_t \lambda = \frac{T}{\hbar} \frac{\lambda}{T} - 2 \lambda + b \]

$T/k = 0$

$\mu/k > 0$

$k \to \infty$ BCS-type behavior!
NJL-model $\beta$ - functions

\[ \partial_t \lambda = 2 \lambda_i - \sum_{j,k} a_{jk} \lambda_j \lambda_k \]

\[ T/k = 0 \]
\[ \mu/k = 0 \]
Scale fixing procedure

$\frac{h_\pi^2}{2m_\pi^2}$ - channel dominates low-energy regime

choice: $\lambda^{(UV)}_{(\sigma-\pi)} \neq 0$, else $\lambda^{(UV)}_i = 0$ ($\Lambda = 1\text{GeV}$)

map one-channel RG flow on mean-field gap equation (vacuum, one-channel): [Braun '11]

$m_\psi \approx 0.3\text{GeV}$ $\quad \leftrightarrow \quad k_0 \approx 0.48\text{GeV}$ (scale $\chi_{SB}$)

tune $\lambda^{(UV)}_{(\sigma-\pi)}$ such that $k_{cr} \approx k_0$
One-channel: phase diagram

\[ T_{cr}(\mu = 0) \approx 0.19 \text{GeV} \]
Fierz-complete: analytical methods

$n$ channel problem: which kind of condensate is created $\langle ? \rangle$

- check relative “dominance” of a channel [e.g. Braun, Gies, Janssen, Roscher ‘14]

\[ \mu = 0 \]
\[ T \simeq T_{cr} \simeq 0.19 \text{GeV} \]

“dominance” of $(\sigma - \pi)$ - channel
Fierz-complete phase diagram

\[ T/k_0 \]

\[ \mu/k_0 \]

\[ \mu_{cr} \approx 0.36 \text{GeV} \]

\[ \langle i\psi^T C\gamma_5 \epsilon_f \epsilon^I_c \psi \rangle \neq 0 \]

“mechanism” for diquark condensation

[Braun, Leonhardt, MP ‘18]
Two channels: Fixed Points & Phase Structure

\[ \Gamma_{LO}^{\bar{\psi}, \psi} = \int_0^\beta d\tau \int d^3x \left\{ \bar{\psi} \left( i \gamma_0 \partial_0 + i \gamma_i \partial_i - i \mu \gamma_0 \right) \psi + \frac{1}{2} \sum_{j \in \{(\sigma-\pi),csc\}} \bar{\lambda}_j L_j \right\} \]

scale-fixing analogue to Fierz-complete calculation

\[ \lambda^{(UV)}_{(\sigma-\pi)} \neq 0, \text{ else } \lambda^{(UV)}_{csc} = 0 \quad (\Lambda = 1\text{GeV}) \]
Two channels: Finite temperature

\[ \mu/k = 0 \]
\[ T/k = 0 \]
Two channels: Finite temperature

\[ \frac{\mu}{k} = 0 \]

\[ \frac{T}{k} = 0.1 \]
Two channels: Finite temperature

\[ \mu/k = 0 \]
\[ T/k = 0.2 \]
Two channels: Finite temperature

Separatrix is shifted away from Gaussian fixed-point!

\[ \frac{\mu}{k} = 0 \]

\[ \frac{T}{k} = 0.3 \]
Two channels: Finite temperature

\[ \mu/k = 0 \]
\[ T/k = 0.4 \]
Two channels: Finite temperature

\[\frac{\mu}{k} = 0\]
\[\frac{T}{k} = 0.5\]
Two channels: Finite density

\[ \lambda_{\text{CSC}} \]

\[ \lambda(\sigma-\pi) \]

\[ \mu/k = 0 \]

\[ T/k = 0 \]
Two channels: Finite density

\[ \frac{\mu}{k} = 0.3 \]
\[ \frac{T}{k} = 0 \]

fixed-points become real!
Two channels: Finite density

\[ \mu/k = 0.35 \]
\[ T/k = 0 \]
Two channels: Finite density

\[ \mu/k = 0.4 \]
\[ T/k = 0 \]
Two channels: Finite density

\[ \frac{\mu}{k} = 0.45 \]

\[ \frac{T}{k} = 0 \]
Two channels: Finite density

\[ \mu/k = 0.5 \]
\[ T/k = 0 \]

fixed-point annihilation
Two channels: Finite density

\[ \frac{\mu}{k} = 0.55 \]
\[ \frac{T}{k} = 0 \]
Two channels: Finite density

\[ \frac{\mu}{k} = 0.65 \]

\[ \frac{T}{k} = 0 \]
Two channels: Finite density

\[ \mu/k = 0.75 \]
\[ T/k = 0 \]
Two channels: Finite density

\[ \frac{\mu}{k} = 0.85 \]
\[ \frac{T}{k} = 0 \]
Two-channel phase diagram

[Braun, Leonhardt, MP ‘18]
Fierz-complete large $\mathcal{N}_c$

\[
\frac{\partial_t}{\lambda_{(\sigma-\pi)}} = 2 \frac{\lambda_{(\sigma-\pi)}}{\lambda_{(\sigma-\pi)}} - \sum_{jk} a_{jk} \frac{\lambda_j}{\lambda_j} \frac{\lambda_k}{\lambda_k}
\]

\[
\frac{\partial_t}{\lambda_i} = 2 \frac{\lambda_i}{\lambda_i} - \sum_{jk} a_{jk} \frac{\lambda_j}{\lambda_j} \frac{\lambda_k}{\lambda_k}
\]

using: $\lambda_{(\sigma-\pi)}^{(UV)} \neq 0$, else $\lambda_i^{(UV)} = 0$ ($\Lambda = 1\text{GeV}$)
Fierz-complete large $N_c$

\[ \partial_t \lambda_{(\sigma-\pi)} = 2 \lambda_{(\sigma-\pi)} - \alpha \lambda_{(\sigma-\pi)} \lambda_{(\sigma-\pi)} \]

\[ \partial_t \lambda_i = 0 \]

\[ \text{using: } \lambda_{(\sigma-\pi)}^{(UV)} \neq 0 \text{, else } \lambda_i^{(UV)} = 0 \text{ (} \Lambda = 1 \text{GeV)} \]

decoupling of $(\sigma-\pi)$ - channel $\rightarrow$ one-channel phase diagram!
Beyond NJL: Dynamical Gauge Fields
Beyond NJL: Dynamical Gauge Fields

\[ \partial_t \lambda \]

\[ \lambda \]

\[ \partial_t \simeq 2 \quad \sim \lambda^2 \]
Beyond NJL: Dynamical Gauge Fields

[Gies, Jaeckel '05; Braun, Gies '05/06]

\[ \partial_t \lambda \]

\[ g = g_{cr} \]

\[ g > 0 \]

\[ g > g_{cr} \]

\[ \sim \lambda^2 \]

\[ \sim g^4 \]

\[ \sim g^2 \lambda \]
Beyond NJL: Scale fixing

\[ \alpha_s^{(\text{UV})} = 0.22 \]
\[ \lambda_i^{(\text{UV})} = 0 \]
\[ \rightarrow T_{cr} \simeq 0.15 \text{ GeV} \]

\[ T_{cr} \simeq 0.13 \text{ GeV} \]

exp. value: \[ \alpha_s(k \simeq 10 \text{ GeV}) = 0.16 \pm 0.048 \]

\[ T_{cr} \simeq 0.01 \text{ GeV} \]

running gauge coupling taken from background-field study

\[ \partial_t \simeq 2 \]

\[ \sim \lambda^2 \]

\[ \sim g^4 \]

\[ \sim g^2 \lambda \]
Preliminary phase diagram

[Braun, Leonhardt, MP in prep]
Summary and Outlook

• Fierz-complete study (10 channels)
  – indications for diquark condensation at high density
  – significant increase of $T_{cr}$ at high density
  – analysis of fixed-point structure and symmetry breaking mechanisms

• QCD study
  – preliminary phase diagram from gauge dynamics
  – improve quark content
  – further improvements for $g(k)$
Thank you for your attention!