Physics and Neurophysiology of Hearing

H.G. Dosch, Inst. Theor. Phys. Heidelberg

- I Signal and Percept
- II The Physics of the Ear
- III From the Ear to the Cortex
- IV Electrophysiology

Part I: Signal and Percept

1)The time scales in hearing2)Relation between Physics and Sensation (Psychophysics)

- a) Propagation and Production of Sound
- b) The principal relations and caveats
- c) A Byte of Signal processing (mainly Fourier etc)
- d) Application to acoustics: Ohms Law
- e) Theory of Musical Consonance
- f) Fusion of Harmonics and Fundamental Tracking

Sound pressure curve of the sentence:

each acoustical signal can be mapped unto a sound pressure curve



Typical time scale of variation 0.1 s and larger

sound-pressure-example.wav in praat1.collection

selection of the previous sentence: mapped



Typical time scale of variation: 10 ms

sound-pressure-selection.wav in praat1.collection

Amplitude-modulated sinusoidal tone with carrier frequency ν_0 and modulation frequency ν_m .

$p(t) = \sin(2\pi\nu_m t)\,\sin(2\pi\nu_0 t)$





$\nu_m = 10 \text{ Hz}$



 $\nu_m = 100 \text{ Hz}$

101praat: sinfreqmod10.wav sinfreqmod100.wav



Time scales in visual processes



Original collage by Helmholtz to illustrate the optical and acoustical spectrum.

The red has faded away

The reception of sound waves in the ear is achieved by mechanical means on the basilar membrane,

the reception of light by chemical processes in the retina.

In the following only concerned on the time variations in the ms range.



 $R_2^2 = \ell^2 + R_1^2 - 2\ell R \cos \phi$ $d = R_2 - R_1 \approx \ell \cos \phi$ $\delta T = \frac{\delta d}{c_L} \approx -\frac{\ell}{c} \sin \phi \,\delta \phi$

time reolution for angular resolution of 10 degrees

$$\frac{\ell}{c}\sin\phi\,\delta\phi\approxrac{0.2}{330}\cdotrac{2\pi}{360}\cdot10pprox10^{-4}\,\mathrm{s}$$

shorter than tact of neurons

101praat gap01-005.wav in 1praat

2 Relation between Physics and Sensation Signal-Percept

Psychophysics

Principal perceived properties of sound:

Loudeness (volume)

Pitch

Timbre



g. 2. frigure



Newton's Law in our case Conservation of matter Compressibility (adiabatic)

$$m\dot{v} = K$$

$$\rho \dot{v} = -\partial_x p$$

$$\partial_t \rho = -\partial_x (\rho v)$$

$$\frac{d\rho}{dp} = \frac{1}{c^2}$$

c turns out to be the speed of sound in the material.

For sound:

 the variations of density and and pressure are small compared to the prevailing values:

 $\Delta p \ll p_0, \ \Delta \rho \ll \rho_0$ (e.g. atmospheric conditions).

2)
$$\dot{v} = \partial_t v + \partial_x v \, v \approx \partial_t v$$

Then we obtain:

$$\partial_t \rho = \frac{d\rho}{dp} \partial_t p = -\rho_0 \partial_x v$$

$$\frac{1}{c^2}\partial_t^2 p = -\rho_0 \partial_t \partial_x v = \partial_x^2 p$$

$$\frac{1}{c^2}\partial_t^2 p - \partial_x^2 p = 0$$

(Wave equation in 1+1 dimensions)

General solution (D'Alembert, ca 1750) :

$$p(x,t) = p_+(x + xct) + p_-(x - ct)$$

Also valid for strings, but there *c* related to the properties of the string (tension, diameter etc).

Solution with harmonic time dependence, $\sim e^{i2\pi\nu t}$, without boundary conditions

$$p(x,t) = Ae^{-2\pi i \nu t + ik x + i\delta}$$

 ν Frequency, $k = 2\pi/\lambda = 2\pi\nu/c$) wave number (λ wave length).

With boundary conditions, e.g. $\rho(0,t) = \rho(L,t) = 0$ special solution

$$p(x,t) = \sin(\frac{n\pi}{L}x)\cos(\frac{n\pi c}{L}t)$$

organ pipe with two open ends or vibrating string

relation between variation of p and v for the wave solution:

$$\omega \rho_0 v = -k p \text{ or } p = -Z v$$

 $Z = \rho_0 c$ is called the impedance (wave resistance) of the medium (cf with U = IR).

Special solution, without boundary conditions (propagation of sound in space): $\rho(x,t) = A/c^2 \sin(2\pi\nu t - kx)$ ν Frequency, $k = 2\pi/\lambda = 2\pi\nu/c$) wave number (λ wave length). Wave propagating to the right.



$$\partial_t p = c^2 \rho_0 \partial_x v = -c^2 \rho_0 \frac{k}{\nu} \partial_t v = -Z \partial_t v$$

 $Z = \rho_0 c$ is called the impedance (wave resistance) of the medium . (cf with U = IR). With boundary conditions, e.g. $\rho(0,t) = \rho(L,t) = 0$ special solution

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organ pipe with two open ends or vibrating string

101praat sin440

2b Relation between acoustical signal (sound pressure) and perception (to be refined later)

Amplitude of sp Loudeness

If the change of the sensation, ds is proportinal to the relative change of the physical stimulus R

$$ds = \frac{dR}{R}$$
 then $s = \log \frac{R}{R_0}$

 R_0 is the threshold of sensation.

Therefore in acoustics the measure of the energy of sound pressure level is the decibel (dB).

If R_0 is amplitude (actually rms) of the sound pressure level at hearing threshold (at 2000 Hz) then dB (SPL) is given as

$$10 \log[10, \frac{R^2}{R_0^2}] = 20 \log[10, \frac{R}{R_0}]$$

101praat: chain leise-mittel-laut

Relation Energy-loudeness not so simple

101praat: karm1,harm3,harm7



again: real life is a bit more complicated

101praat: sound-440_3-5, 366_4_6

¹⁰¹praat: tief-hoch.wav

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¹⁰¹praat: sound440- sound 13_75c (pitch-range.collection)



Most complicated sensation: timbre

not (approximately) 1 dimensional like volume and pitch but many properties:

full, thin, soft, harsh

general relation:

form of the SP curve timbre

A very important distinction is the different timbre of different vowels

The human vocal tract



Abb. 12.1 Zum Stimmtrakt gehören Nasen- und Mundhöhle und der Rachen sowie Bestandteile, die sich bewegen, etwa Zunge, Lippen und Stimmbänder.

The vowels sound very different, though the sp curves look similar:



101praat: e_reg - u_reg



The sound pressure curves look different, but the sounds are practically indistinguishable





Fourier and wavelet transforms

Fourier transform:

$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} dt \ f(t) e^{i\omega t}$$

magic formula

$$\int \frac{d\omega}{2\pi} e^{i\omega\alpha} e^{-i\omega t} = \delta(t-\alpha)$$

$$f(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \tilde{f}(\omega)e^{-i\omega t}$$

The squared modulus
 $|\tilde{f}(\omega)|^2$
is called *power spectrum*.

The Fourier transform is linear: $\alpha f(t) + \beta g(t) \rightarrow \alpha \tilde{f}(\omega) + \beta \tilde{g}(\omega)$

If the function f(t) is real, i.e. if $f(t) = f^*(t)$, then $\tilde{f}(\omega) = \tilde{f}^*(-\omega)$.

Convolution:

$$(f * g)(\tau) = \int_{-\infty}^{\infty} dt dt' \ f(t)g(t')\delta(t+t'-\tau) = \int_{-\infty}^{\infty} dt \ f(t)g(\tau-t) = \int_{-\infty}^{\infty} dt \ f(\tau-t)g(t)$$
$$\int_{-\infty}^{\infty} dt \ f(t) \ g(t)e^{i\omega t} = \frac{1}{2\pi}(\tilde{f} * \tilde{g})(\omega)$$

Signal Processing

t time; $\omega = 2 \pi \nu$ circular frequency; ν frequency

p(t) Signal (e.g. sound pressure as function of time)

 $\tilde{p}(\nu) = \int dt \, p(t) \, e^{i2\pi\nu t}$ spectral function

 $|\tilde{p}(\nu)|^2$ (power) spectrum, normally displayed in logarithmic scale (dB) 10 db a factor 10 in power ($\sqrt{10}$ in amplitude)

Essential Theorem:

If the signal is periodic with period T, then the spectral function is different from zero only at n/T, n integer. Harmonic spectrum



$$p(t) = p(t+T) \implies \tilde{p}(\nu) = \sum_{n} c_n \delta(\nu - \frac{n}{T})$$

Proof:

 $p(t) = \int dt \tilde{p}(2\pi\nu) e^{i2\pi\nu t} = \int dt \tilde{p}(2\pi\nu) e^{i2\pi\nu (t+T)}$

$$2\pi \nu T = n$$

Only valid if $\nu T = n$ (integer).

signal

spectral function

 $p(t) = \cos(2\pi\nu_0 t)$ $p(t) = \sin(2\pi\nu_0 t)$

$$\tilde{p}(\nu) = \pi \,\delta(\nu - \nu_0) + \pi \,\delta(\nu + \nu_0)$$
$$\tilde{p}(\nu) = i \,\pi \,\delta(\nu - \nu_0) - i \,\pi \,\delta(\nu + \nu_0)$$

 $\nu_0 = 100 \text{ Hz}$



Fourier transform







signal spectral function

 $p(t) = \theta(t - c + b)$ $\theta(c+b-t) \qquad \qquad \tilde{p}(\nu) = e^{i2\pi\nu c} \, 2\frac{\sin(b2\pi\nu)}{2\pi\nu}$

c=0 b=0.5 b=2





spectral function

power spectrum

Windowed Fourier Analysis, Wavelet analysis

Fourier analysis over a time window, g(t):

$$\tilde{f}(t,\omega) := \int_{-\infty}^{\infty} dt' g(t-t') f(t') \exp[-i\omega t']$$

possible time windows rectangular Gaussian window

$$g(t - t') = \theta(t - t')\theta(\Delta - (t - t')).$$
$$g(t - t') = \frac{1}{\Delta\sqrt{\pi}} \exp\left[-\frac{(t - t')^2}{\Delta^2}\right].$$

Power spectrum of $sin(2\pi 440 t)$ for rectangular and Gaussian window ($\Delta = 0.5s$)



Representaion windowed Fourier transform by Spectrogramm:

time as x-Axes, frequency as y axis and intensity as gray value

Example: tone with modulated frequency: modulation frequency 3 Hz sin(2*pi*100*x-4*cos(2*pi*3*x))



Spectrogram and spectrum of

``Each acoustical signal can be mapped onto a sound pressure curve"



Spectrogram and spectrum taken at 2.1 s with gaussian window 0.05 s



Wavelet analysis

wavelet analysis is a windowed Fourier analysis, where the time window g(t) depends on the frequency, such that the number of wiggles is always the same:

$$\tilde{p}(t,\nu) = \int dt' g((t-t')\nu,\nu) e^{i 2\pi \nu (t-t')} p(t')$$

$$\tilde{p}(t,\nu) = \int du \, h(u) p(u + \frac{t}{2\pi\nu})$$

 $u = 2\pi\nu(t'-t), \quad h(u) = g(u)e^{iu}$

Examples for wavelet: Gaussian damped hamonics:

 $h(u) = e^{iu - u}$ $\nu = 200/(2\pi)$ $\nu = 100/(2\pi)$ Imh(u)Imh(u)0.8 0.8 t 0.6 0.6 0.4 0.4 0.2 0.2 0/01 0.02 0.03 0.04 0**b**1 0.03 0.04 02 -0.2 -0.2 -0.4 -0.4

h(u) is called a ``note" :

102praat note.wav





Frequency (Hz)





102praat vokale-hgd rauschen fuer rauschvokale

а	3	1000	1400
e	è	500	2300
i		320	3200
C)	500	1000
ι	I	320	800











App. of Helmholtz to produce artificially vowels sponsored by Ludwig II von Bayern









Sound pressure level (dB/Hz)

40-

20

0

O





but the power spectra are up to a scale identical





Frequency (Hz)

3000

102

First refinement:

102praat cymbal.wav, noise.wav

Timbre determined by the power spectrum





Few spectral lines (few harmonics) soft tone many harmonics sharp tone

102praat flute.wav spinett.wav i-reg.wav cymbal.wav noise.wav click.wav

Ohm's Law of Acoustics:

Ohm 1841, Helmholtz 1863

1) Ear performs a (windowed) Fourier analysis

- 2) Phases play no role.
- 3) The different Harmonics of a periodic tone are fused
- 4) Pitch of a sound determined by lowest harmonic,

roughly: long time scales (>0.1 s) processed as temporal properties, short time scales (< 0.1 s) as power spectral properties



Independence of phases:



schroederplus10, sin10

Apparatus of helmholtz to test the independence on phases (Tonempf.)



$$p(t) = \sum_{n=1}^{100} \sin(2\pi (100 + n)t) \quad p(t) = \sum_{n=1}^{100} \sin(2\pi (100 + n)t + 2\pi r[n])$$



Fourier transform should only differ in phase, i.e. identical power spectrum but: spectra over finite time interval look also different



Our ear performs a windowed Fourier analysis, since we can perceive directly time variations of ca 0.1 s, this should be the maximal width of the window.

Example beats and roughness.

$$e^{i2\pi\nu_{1}t} + e^{i2\pi\nu_{2}t}$$
$$= e^{i2\pi\frac{\nu_{1}+\nu_{2}}{2}t} (e^{i2\pi\frac{\nu_{1}-\nu_{2}}{2}t} + e^{-i2\pi\frac{\nu_{1}-\nu_{2}}{2}t})$$

$$= 2\cos\left(2\pi \frac{\nu_1 - \nu_2}{2}t\right)e^{i2\pi \frac{\nu_1 + \nu_2}{2}t}$$

We expect: $1/(\nu_1 - \nu_2) \ll 0.1$ s

two tones

$$1/(\nu_1 - \nu_2) \gg 0.1$$
 s beats

$$\nu_1 = 2000; \ \nu_2 = 2500$$

102praat beats-third.wav beats-10.wav beats-rough.wav

 $\nu_1 = 2000; \ \nu_2 = 2010$

$$\nu_1 = 2000; \ \nu_2 = 2120$$

 $1/(\nu_1-\nu_2)>\sim 0.1$ s beats

$u_2/ u_1 > \sim 1.2 \text{ two tones}$

else rough. maximal roughness at $\nu_2/\nu_1 \sim 1.06 - 1.12$ based on pysiology of the ear

Theory of harmony Pythagoras - Helmholtz

	c(530)	530	1060	1590	2120	2650	3180	3710	4240
9:8	d(590)	590	1180	1770	2360	2950	3540	4130	4720
3:2	g(790)	790		1580		2370	3160	3950	
15:8	h(1000)		1000		2000		3000		4000
2:1	c(1060)		1060		2120		3180		4240

live-music



Calculations of Helmholtz for the degree of roughness for a violin



Interval od roughness larger at low frequencies:





Beetvoven, Appasionata



Example: "pure tone" $p(t) = sin(2\pi 220 t)$

complex tone
$$p(t) = \sum_{n=1}^{5} \sin(2\pi n 220 t)$$

But one can also hear the components, some people better, some worse.



Fusion of tones

A tone with several harmonics: is it a single tone or a collection of tones?

both:

Helmholtz:

we perceive synthetically (perzipieren) the whole tone we can perceive analytically (apperzipieren) the components of the tone.



The missing fundamental We now take off components from below

102praat chain-fundamental-tracking.collection



We take subsequently one harmonic tone out of the complex tone



This happens also in musical instruments

103praat 27_5Hz_A2_real_steinway.wav 110Hz-A_real



Explanation until ca 1940 : Difference tones

If a sound is processed linearly, nothing happens to the spectrum, except linear scaling.

But if the function is not linear, we observe additional spectral lines (partial tones)

 $(2a\cos(\omega_1 t) + 2b\cos(\omega_2 t))^n$

$$= \left(a(e^{i\omega_{1}t} + e^{-i\omega_{1}t}) + b(e^{i\omega_{2}t} + e^{-i\omega_{2}t}) \right)^{n}$$

$$= (a b)^{n} (e^{ni\omega_{1}t} + e^{i((n-1)\omega_{1} + \omega_{2})t} + \dots e^{i((n-1)\omega_{1} - \omega_{2})t} +$$

$$n = 2$$
: $2\omega_1, \ \omega_1 + \omega_2, \ \omega_1 - \omega_2, \ 2\omega_2$

n = 3: $3\omega_1$, $2\omega_1 - \omega_2$,...

. . .

For a harmonic tone $p(t) = \sum_{k=1}^{n} \cos(2\pi k\nu_0)$

the difference ton $\omega_{k+1} - \omega_k = 2\pi (k+1-k) \nu_0 = 2\pi \nu_0$ is the fundamental tone



Spectrogram of the rcorded tone, consisting of two components at 1000 and 1220 Hz. At the high level the nonlinear distortions are clearly visible.

Schouten:

The periodicity of the tone is essential for the pitch of the ``residue", that is the perceived, but not present fundamental tone.

Indeed sp curves show this periodicity:



The difference tone was excluded by van Schouten by a series of ingenious experiments. A particular simple one is the shifted hamonic tone.



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The difference tone was excluded by van Schouten by a series of ingenious experiments. A particular simple one is the shifted hamonic tone with missing



The fact that the shifted tone has a ditincly different pitch is a sure sign that the difference tone is not reponsible for fundamental tracking, since the difference tone is unaffected by the shift

The question what determines the pitch of a complex tone is still controversial. We shall come back to it after looking closer into the physics of the ear and the auditory pathway.