A Renormalizable SYK-type Tensor Field Theory

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joint work with V. Rivasseau (LPTO, Univ. Paris Saclay)

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Quantum Spacetime and the Renormalization Group

Phyzikzentrum Bad-Honnef

Outline

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1. Introduction: From Tensor Models to Group/Tensor Field Theories

2. Orthogonal invariants as tensor graphs

3. Renormalizing a SYK-type TFT

How did we get here? Tensor Models - Group Field Theory - Tensor Field Theory

• Tensor models (TM) as a Quantum Gravity (QG) approach [Ambjørn, Durhuus, Jónsson, 91'] attempt for extending the results of Matrix Models.

• Boulatov model [MPLA, '93]: Combining lattice gauge field theory with gauge goup SU(2) and TM to describe an Euclidean gravity model; notion of holonomy captures the notion of curvature.

• Birth of Group Field Theory [Oriti '06–∞] connection with Loop Quantum Gravity - spin foams.

• Huge efforts to understand GFT amplitudes ('08-'10): wild topologies and divergences.

• Colored GFT and Tensor models (Gurau, AHP, '10): Discovery of the large N limit

→ Dominant graphs at large N: Melonic graphs (super-planar)

→ Universality in colored TM: Continuum limit branched polymer-like geometry. [Gurau, Ryan, '13]

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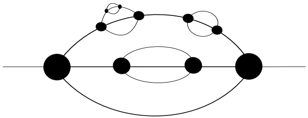

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Tensor Field Theories (TFT)

What if $N = \infty$ from the beginning (infinite number of degrees of freedom)?

- TFT [BG, Rivasseau, '11]

(0) keeps tensor fields do not use gauged fields (not lattice gauge theories like GFT)

(1) introduces non trivial propagators (to address divergences)
$\rightarrow$ the inception of scales in TM.
Attention: large and small scales is associated with distances on the space of the tensor indices.

(2) Morphs the $1/N$ expansion into a power-counting theorem preparing a Renormalization Group (RG) analysis:
$\rightarrow$ N becomes the cut-off Λ
$\rightarrow$ search for fixed points and phase transition

$\rightarrow$ Some TFTs are perturbatively renormalizable at all orders, and are asymptotically free or safe. This extends to some GFTs.

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A connection between colored TM and Sachdev-Ye-Kitaev (SYK) model

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SYK and TM

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Goal

Exhibit the first SYK-type TFT which is perturbatively renormalizable at all orders of perturbation in the UV.

Outline

Constructing interactions: Orthogonal invariants as graphs (Carozza, Tanasa, '15)

(Back to pure tensors, 0-Dimension)

• A covariant real tensor $T_{p_1,\dots,p_d}$ with transformation rule

$$T_{p_1,\dots,p_d} = \sum_{q_k} O^{(1)}_{p_1 q_1} \dots O^{(d)}_{p_d q_d} T_{q_1,\dots,q_d} \,, \qquad O^{(a)} \in O(N_a) \tag{1}$$

• Tensor contractions = orthogonal invariants $S^{\text{int}}_{\mathbf{b}}(T) = \mathrm{Tr}_{\mathbf{b}}(T \cdot T \dots T \cdot T)$

• Coding orthogonal invariants: **b** colored graphs

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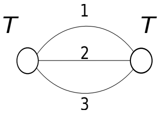

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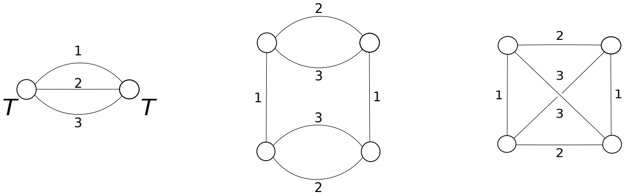

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The model: Fields

• To built a new TFT: use the Carrozza-Tanasa interaction with a true condensed matter model encompassing the Klebanov-Tarnopolski model

• Consider a pair of rank 3 Majorana tensor fields $\{\chi(t, \vec{x}, \sigma)\}$, $\sigma = 1, 2$

• Coordinates $\vec{x} = (x_1, x_2, x_3) \in \mathbb{R}^3$, $U(1)^3$ replace the three $O(N)^3$ tensor indices.
• The time $t \in [-\frac{\beta}{2}, \frac{\beta}{2}]$, $\beta = \frac{1}{kT}$, this thermal circle becomes large at low temperature.

• Dual momentum variables $(p_0, \vec{p})$,
$\to p_0 \sim \omega$ is an (Euclidean) Matsubara frequency,

$$p_0 = \frac{2\pi}{\beta}(n + \frac{1}{2})$$

(because χ is Fermionic, take anti-periodic conditions)
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The model: Propagator

- Matsubara formalism:

$$C(p_0, \vec{p}) = \frac{1}{ip_0 + e(p)} = \frac{-ip_0 + e(p)}{p_0^2 + e^2(p)} \qquad e(p) = \frac{p^2}{2m} - \mu \ , \tag{2}$$

Simplifying we put $2m = \mu = 1$, so that $e(p) = p^2 - 1$.

- Adjusting the formalism: Matrix covariance rules

$$\Big(\langle \chi_\sigma(p_0, \vec{p}) \chi_{\sigma'}(p_0', \vec{p}\,') \rangle\Big)_{\sigma\sigma'} = \Big(C_{\sigma\sigma'}(p_0, \vec{p}) \delta(p_0 - p_0') \delta(\vec{p}, \vec{p}\,') \Big)_{\sigma\sigma'}$$
$$= \left[\frac{-ip_0}{p_0^2 + e^2(p^2)} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{e(p^2)}{p_0^2 + e^2(p^2)} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right] \delta(p_0 - p_0') \delta(\vec{p}, \vec{p}\,') , \tag{3}$$

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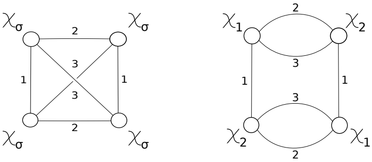
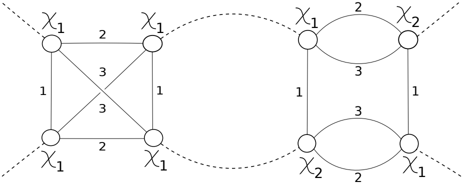
The model: interactions and graphs

• The tensor interaction CT $O(N)^3$-invariant + KT local part in t (the model is non-local in $\vec{p}$ but local in p_0)

$\rightarrow O(N)$ invariants as interactions



$\rightarrow$ A Feynman graph in the theory



Power counting theorem via multi-scale analysis

- Slice decomposition of the propagator

$$\hat{C}_{\sigma\sigma'}(p_0,\vec{p}) = \sum_{i=1}^{\infty} C_{\sigma\sigma';\, i}(p_0,\vec{p})\,,$$
$$C_{\sigma\sigma';\, i}(p_0,\vec{p}) = \int_{M^{-2i}}^{M^{-2(i-1)}} \mathrm{d}\alpha\, (-ip_0\delta_{\sigma\sigma'} + \varepsilon_{\sigma\sigma'}e(p))e^{-\alpha(p_0^2+e^2(p))} \tag{4}$$

- Bound on a slice

$$|C_{\sigma\sigma';\, i}(p_0,\vec{p})| \leq KM^{-i}\; e^{-\delta M^{-i}(|p_0|+p^2)}\,,$$
$$|p_0| \sim M^i\,, \qquad p^2 \sim M^i \tag{5}$$

Power counting theorem

Theorem (Power counting)

Let $\mathcal{G}$ be a connected graph of the model with Gaussian measure determined by the covariance (3). *Considering $A_{\mathcal{G};\boldsymbol{\mu}}$ the amplitude associated with $\mathcal{G}$ at index assignment $\boldsymbol{\mu}$, there exists some large constant K such that*

$$|A_{\mathcal{G};\boldsymbol{\mu}}| \leq K^{V(\mathcal{G})} \prod_{(i,k)\in\mathbb{N}^2} M^{\omega_{\deg}(\mathcal{G}^i_{(k)})}, \tag{6}$$

where $\mathcal{G}^i_{(k)}$ are the quasi-local subgraphs and the divergence degree is given by

$$\begin{aligned} \omega_{\deg}(\mathcal{G}) &= -(V(\mathcal{G})-1) + \frac{1}{2}F_{int}(\mathcal{G}) \\ &= -\frac{1}{2}[\omega(\mathcal{G}_{\rm color}) - V_+ - g_{\partial\mathcal{G}} + (C_{\partial\mathcal{G}} - 1)] - \frac{1}{2}(N_{ext} - 4) \end{aligned} \tag{7}$$

- $\omega(\mathcal{G}_{\rm color}) = \sum_{\tilde{J}} g_{\tilde{J}}$ *is the degree of Gurau*
- $g_{\partial\mathcal{G}}$ *the genus of the boundary graph $\partial\mathcal{G}$ (there is a unique map, or matrix graph that the external momentum faces of $\mathcal{G}$ can be mapped onto)*
- V_+ *is the number of vertices of the tetrahedric form*
- $C_{\partial\mathcal{G}}$ *number of connected component of the boundary graph*
- N_{ext} *number of external fields/legs of the graph*

Divergent graphs

Theorem

- *If* $N_{ext} \geq 6$

$$\omega_{\text{deg}}(\mathcal{G}) \leq -\frac{N_{ext}}{12} \qquad \textit{these functions are convergent} \tag{8}$$

- *If* $N_{ext} = 4$

$$\omega_{\text{deg}}(\mathcal{G}) \leq 0 \quad \textit{4pt-functions are at most log-divergent} \tag{9}$$

- *If* $N_{ext} = 2$

$$\omega_{\text{deg}}(\mathcal{G}) \leq 1 \qquad \textit{2pt-functions are at most linearly divergent} \tag{10}$$

Renorm

$A_{\mathcal{G};2}(\{p_{0;l_{ext}}\};\{p_f^{ext}\}) = \kappa(\lambda)\delta(\sum_{l_{ext}} p_{0;l_{ext}})\int[\prod_{l_{ext}\in\mathcal{L}_{ext}} d\alpha_{l_{ext}}\, e^{\alpha_{l_{ext}}}$ [redacted]

$\times[\prod_{l_{ext}\in\mathcal{L}_{ext}\cap\mathcal{L}_1}(-ip_{0;l_{ext}})][\prod_{l_{ext}\in\mathcal{L}_{ext}\cap\mathcal{L}_2} e(\sum_{f\in\mathcal{F}_{ext}} \epsilon_{l_{ext},f}$ [redacted]

$\times[\prod_{f\in\mathcal{F}_{ext}} e^{-(\alpha_{l_{ext}}+\alpha_{l'_{ext}})[(p_f^{ext})^4-2(p_f^{ext})^2]}][$ [redacted] $\prod_{\in\mathcal{L}_{ext}} e^{-\alpha_{l_{ext}} p^2_{0;l_{ext}}}]$

$\times[\prod_{f\in\mathcal{F}_{int}} e^{-(\sum_{\ell\in f}\alpha_\ell)[p_f^4-2p_f^2]}][$ [redacted]

$\times[\prod_{c\in\mathrm{Cycle}_{\mathcal{G}}} e^{-(\sum_{l\in\mathcal{L}_c}\alpha$ [redacted] $\sum_{\sigma=1,2} Q^{2;\sigma}+Q^3)$

$\times\{1-\sum_{f\in\mathcal{F}_{ext}}$ [redacted] $\sum_{f\in\mathcal{F}_{ext},f'\in\mathcal{F}_{int}}(Q^3_{ext;f,f'}+Q^{3'}_{ext;f,f'})$

$-\sum_{l_{ext}\in\mathcal{L}_{ext}}(Q^1_{l_{ext}}+$ [redacted] $\sum_{l_{ext},l'_{ext}\in\mathcal{L}_{ext},\ l_{ext}\neq l'_{ext}}(Q^3_{l_{ext},l'_{ext}}+Q^{3'}_{l_{ext},l'_{ext}})$

$+\sum(Q+Q)\cdot(Q+Q)+\dots$ [redacted] (21)

Conclusion and perspectives

• Summary:

→ SYK-type TFT can be made renormalizable at all order of perturbation theory.

→ Graphs with $V_+ > 0$ are suppressed in the UV and do not participate to the flow.

• Perspectives:

→ IR analysis: more subtle, involve an expansion around the Fermi surface.

→ What information can bring the dual geometrical picture of TFTs to the physics of SYK model ?

→ Vice versa: can the SYK model help in the search of nice geometries within the initial program of TM?