PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS

Large N fermionic tensor models in d = 2

Sylvain Carrozza

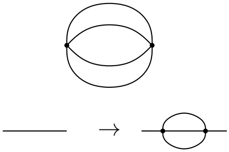
"Quantum spacetime and the Renormalization Group" 2018 – Bad Honnef

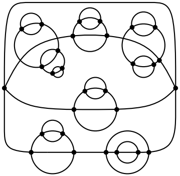
Context

- early '10s: universal **large N expansion** for tensor models

  [Gurau, Bonzom, Rivasseau,...]

  Main feature: dominated by **melon diagrams**.





Primary applications:

- **Random geometry** in $d \geq 3$ [Talk by T. Koslowski]
- **Tensor Field Theory** [Talk by J. Ben Geloun]
- **Group Field Theory (renormalization)** [Talk by D. Oriti]

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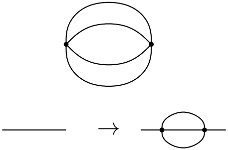

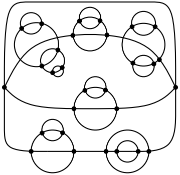

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- '16 '17: applications to **holography** [Witten; Klebanov, Tarnopolsky; ...]

  and more generally to local QFTs on fixed space-time backgrounds.

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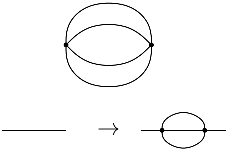

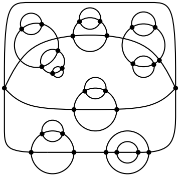

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In this talk: **large N QFT in $d = 2$**

arXiv:1710.10253 JHEP 2018

with D. Benedetti (Paris Sud), R. Gurau (X-Polytechnique) and A. Sfondrini (ETH).

Outline

1. SYK, tensor quantum mechanics and holography

2. Tensor field theory in d=2

SYK-like tensor models

- **Sachdev-Ye-Kitaev** models = disordered systems of N Majorana fermions

  [Sachdev, Ye, George, Parcollet '90s...; Kitaev '15, Maldacena, Stanford, Polchinski, Rosenhaus...]

$$H_{\rm int} \sim J_{i_1 i_2 i_3 i_4} \, \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4} \, , \qquad \left\langle J_{i_1 i_2 i_3 i_4} \right\rangle \sim 0 \, , \quad \left\langle J^2_{i_1 i_2 i_3 i_4} \right\rangle \sim \frac{J^2}{N^3}$$

- Many interesting properties:
  - **solvable at large N**
  - emergent **conformal symmetry** at **strong coupling**
  - maximal **quantum chaos**
  - holography in low dimension: "$NAdS_2/NCFT_1$"

- Same **melonic large N limit** as tensor models [Witten '16]

→ **SYK-like quantum-mechanical models**:
- same qualitative properties at large N and strong coupling;
- **no disorder**.

→ New class of **QFTs** with solvable large N limits.

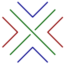
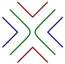
[Gurau, Bonzom, Rivasseau,... 10s; Tanasa, SC '15]

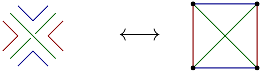
- Statistical model for $T_{i_1 i_2 i_3}$ transforming under $\mathrm{O}(N)^3$ as

$$T_{i_1 i_2 i_3} \to O^{(1)}_{i_1 j_1} O^{(2)}_{i_2 j_2} O^{(3)}_{i_3 j_3} \, T_{j_1 j_2 j_3}$$

- Invariant action:

$$S(T) = \frac{1}{2} T_{i_1 i_2 i_3} T_{i_1 i_2 i_3} + \frac{\lambda_1}{N^{3/2}} T_{i_6 i_2 i_3} T_{i_1 i_4 i_3} T_{i_6 i_4 i_5} T_{i_1 i_2 i_5} + \frac{\lambda_2}{N^2} T_{i_6 i_2 i_3} T_{i_1 i_2 i_3} T_{i_6 i_4 i_5} T_{i_1 i_4 i_5} + \cdots$$

$= \frac{1}{2}$  $+ \frac{\lambda_1}{N^{3/2}}$  $+ \frac{\lambda_2}{N^2}$  $+ \cdots$



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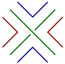
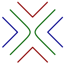
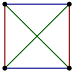
$= \frac{1}{2}$  $+ \frac{\lambda_1}{N^{3/2}}$  $+ \frac{\lambda_2}{N^2}$  $+ \cdots$

 $\longleftrightarrow$ 

- **Large N expansion** indexed by **Gurau degree** ω

$$\mathcal{F}_N := \ln \int [dT] e^{-S(T)} = \sum_{\omega \in \mathbb{N}/2} N^{3-\omega} \mathcal{F}_\omega$$

and dominated by **melon diagrams** ($\omega = 0$).

- Tensor quantum mechanics of N^3 **Majorana fermions**: [Klebanov, Tarnopolsky '16...]

$$S = \int dt \left(\frac{\mathrm{i}}{2} \psi_{i_1 i_2 i_3} \partial_t \psi_{i_1 i_2 i_3} + \frac{\lambda}{4 N^{3/2}} \psi_{i_1 i_2 i_3} \psi_{i_4 i_5 i_3} \psi_{i_4 i_2 i_6} \psi_{i_1 i_5 i_6} \right)$$

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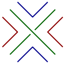
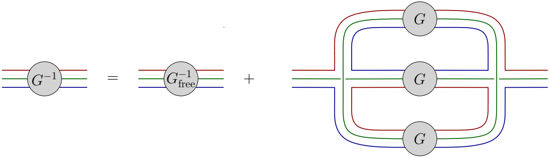


- **Melonic dominance** at large $N \Rightarrow$ closed Schwinger-Dyson equation:



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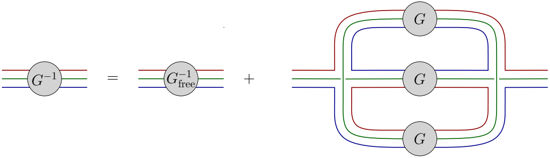


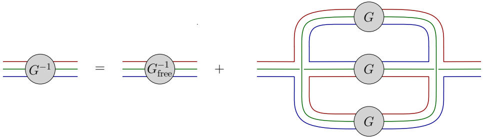
- **Strong coupling** $\Rightarrow$ neglect G_{free}:

$$\int dt' \, G(t,t') \Sigma(t',t'') = -\delta(t-t') \,, \qquad \Sigma(t,t') = \lambda^2 \, [G(t,t')]^3$$

$\Rightarrow$ **emergent reparametrization symmetry**

$$t \to f(t) \,, \qquad G(t,t') \to [f'(t) f'(t')]^{1/4} \, G(f(t), f(t'))$$

Symmetry breaking



- Strong coupling **conformal solution**:

$$G(t_2, t_1) \sim \frac{\mathrm{sgn}(t_2 - t_1)}{|t_2 - t_1|^{1/2}}$$

- Spontaneous breaking to $\mathrm{SL}(2, \mathbb{R}) \Rightarrow$ **Goldstone modes** $\phi \in \mathrm{Diff}(\mathbb{R})/\mathrm{SL}(2, \mathbb{R})$?

- G_{free} term $\Rightarrow$ explicit breaking $\Rightarrow$ non-zero **Schwarzian effective action**

$$S_{\text{eff}}[\phi] \sim \frac{1}{\lambda} \int \mathrm{d}t \, \{\phi, t\}$$

($\{\phi, t\}$ is the *Schwarzian derivative*) [Maldacena, Stanford '16]

  - Effective action common to SYK and tensor quantum mechanics.
  - Additional soft contributions are present in tensor models e.g. $O(N)^3$ sigma model.

[Minwalla et al. '17; Benedetti, Gurau '18]

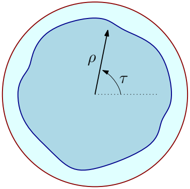
- Near-horizon geometry of near-extremal Reissner–Nordström BHs $\sim AdS_2 \times S_2$

- Near-horizon dynamics captured by **dilaton Jackiw-Teitelboim gravity**

$$S \sim \phi_0 \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|h|} R_h + 2\phi_0 \int_{\partial\mathcal{M}} K \tag{1}$$

$$+ \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|h|}\, \phi \,(R_h + 2) + 2 \int_{\partial\mathcal{M}} \phi_b K \tag{2}$$

Euclidean solutions = **cut-outs from the hyperbolic plane**



$$ds^2 = d\rho^2 + \sinh^2\rho\, d\tau^2$$

- Non-trivial zero modes of (1) $\Rightarrow \mathrm{Diff}(\mathbb{R})/\mathrm{SL}(2,\mathbb{R})$ symmetry.

- Action (2) explicitly breaks the symmetry $\Rightarrow$ **Schwarzian effective action**!

[Maldacena, Stanford, Yang '16]

Nice review: Gábor Sárosi arXiv:1711.08482

Motivations

Unlike SYK, tensor models naturally fit in the framework of **local quantum field theory**.

Natural research programme:

Investigate the properties of melonic large N QFTs in $d \geq 2$.

Bosonic: [Giombi, Klebanov, Tarnopolsky '17]

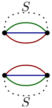
Fermionic: [Prakash, Sinah '17]

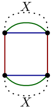
[Benedetti, SC, Gurau, Sfondrini '17]

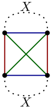
Action

Tensorial Gross-Neveu model, with e.g. $O(N)$ **Majorana** fermions.

$$S_N = \frac{1}{2}\int d^2x\, \psi_{i_1 i_2 i_3} \partial\!\!\!/ \psi_{i_1 i_2 i_3}$$

$$- \frac{\lambda_0}{4N^3}\int d^2x$$ 

$$- \sum_{X=S,V,P} \frac{\lambda_1^X}{4N^2}\int d^2x$$ 

$$- \sum_{X=S,V,P} \frac{\lambda_2^X}{4N^{3/2}}\int d^2x$$ 

Spinorial contractions with: $\mathbf{1}$ (S), γ_μ (V) or γ_5 (P) insertions.

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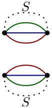
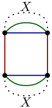
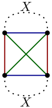
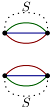
$$- \frac{\lambda_0}{4N^3} \int d^2x$$ 

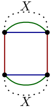
$$- \sum_{X=S,V,P} \frac{\lambda_1^X}{4N^2} \int d^2x$$ 

$$- \sum_{X=S,V,P} \frac{\lambda_2^X}{4N^{3/2}} \int d^2x$$ 

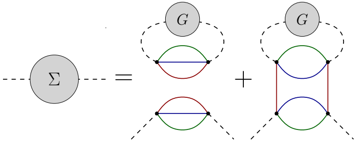
Spinorial contractions with: $\mathbf{1}$ (S), γ_μ (V) or γ_5 (P) insertions.

1. $\lambda_0 > 0$, $\lambda_1^S > 0$, $\lambda_2 = 0$:
   - Tadpole diagrams $\Rightarrow$ non-perturbative **generation of mass** in the IR
   - **Asymptotic freedom**.
2. $\lambda_0 = \lambda_1^S = 0$ and $\lambda_2 \neq 0$:
   - Melonic Schwinger-Dyson equation
   - Free part still relevant at strong coupling $\Rightarrow$ **no IR conformal regime**

λ_0 

$+ \lambda_1^X$ 

$\Rightarrow$ Spontaneous generation of **mass** and **asymptotic freedom**



- Large-N Schwinger-Dyson equations:

$$G^{-1}(x,x') = G_0^{-1}(x,x') - \Sigma(x,x')\,, \qquad \Sigma(x,x') = -(\lambda_0^S + \lambda_1^S)\mathrm{Tr}[G(x,x)]\delta(x,x')$$

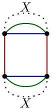
- **Gap equation**:

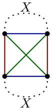
$$m = 2m(\lambda_0^S + \lambda_1^S)\int_{q\leq\Lambda} \frac{d^2q}{(2\pi)^2}\frac{1}{q^2+m^2} \approx m(\lambda_0^S + \lambda_1^S)\frac{1}{\pi}\ln\frac{\Lambda}{m}$$

- Non-perturbative solution:

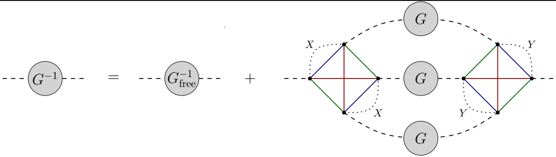
$$m = \Lambda \exp\left(-\frac{\pi}{\lambda_0^S + \lambda_1^S}\right)$$

- Callan-Symanzik $\Rightarrow$ asymptotic freedom: $\beta_1^S = -1/\pi(\lambda_1^S)^2 < 0$

$\sum_{X \neq S} \lambda_1^X$ 

$+ \lambda_2^X$ 

$\Rightarrow$ **melonic Schwinger-Dyson equation**



- Same **melonic-like equation** as in $d = 1$.
- Main difference: G_{free} **cannot be neglected** in the IR limit.
- Is there a non-trivial IR fixed point nonetheless?

- Strategy:
  1. Ansatz: $\hat{G}(p)^{-1} \propto \mathrm{i}\not{p} + m$.
  2. Solve gap equation and find conditions to cancel m.
  3. Impose stability of $\lambda_0^S = \lambda_1^S = 0$ and other conditions under radiative correction.
  4. Impose that the remaining coupling(s) have 0 beta function.

Results

1. Ansatz: $\hat{G}(p)^{-1} \propto \mathrm{i}\not{p} + m$.

2. Solve gap equation and find conditions to cancel m:

$$-(\lambda_2^S)^2 + (\lambda_2^P)^2 - 4\lambda_2^V \lambda_2^P = 0$$

3. Impose stability of $\lambda_0^S = \lambda_1^S = 0$ and other conditions under radiative correction:

$$(\lambda_2^{S,1})^2 = (\lambda_2^{P,1})^2 = \lambda^2 \neq 0 \qquad \text{and} \qquad \lambda_2^{V,1} = 0\ ,$$

   or

$$(\lambda_2^{S,1})^2 = (\lambda_2^{P,1})^2 = 0 \qquad \text{and} \qquad \lambda_2^{V,1} = \lambda \neq 0\ .$$

   Note: satisfied by models invariant under continuous chiral symmetry.

4. Impose that the remaining coupling(s) have 0 beta function:
   non-trivial wave-function renormalization $\Rightarrow$

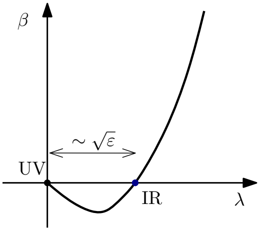
$$\beta_\lambda = \frac{3}{\pi^2}\lambda^3$$

   (perturbative calculation)

Weakly interacting fixed point in $d = 2 - \varepsilon$

- Very interesting feature in $d = 2 - \varepsilon$:

$$\beta = \frac{3}{\pi^2}(\lambda)^3 \quad \rightarrow \quad \beta^{(\varepsilon)} = -\varepsilon\lambda + \frac{3}{\pi^2}(\lambda)^3$$



$\rightarrow$ **weakly interacting IR fixed point** $\lambda_2 \sim \sqrt{\varepsilon}$, analogous to Wilson-Fisher in bosonic $\varphi^4_{4-\varepsilon}$.

Conjecture: becomes strongly coupled and governs the conformal regime of SYK in the limit $\varepsilon \rightarrow 1$.

$\rightarrow$ **ε-expansion** for SYK

Conclusion and outlook

- Recent developments in tensor models beyond the intended applications to random geometry / discrete quantum gravity:
  - **tensor quantum mechanics** reproducing salient features of the **SYK many-body system**
  - unanticipated **connections to other quantum gravity approaches**: $\mathrm{AdS}_2/\mathrm{CFT}_1$, black holes, higher spins...
- Motivates a more systematic investigation of such large N QFTs
  - higher dimensions
  - **(anti)symmetrized tensors** [Klebanov, Tarnopolsky '17; Benedetti, SC, Gurau, Sfondrini '17]
  - multi-matrix models [Ferrari '17;...]
  - condensed matter propagators [Ben Geloun, Rivasseau '17]
  - hierarchy of Schwinger-Dyson equations [Perez-Sanchez, Wulkenhaar '17]

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- "Controversial" statement: holography in $1+1$ dimensions is very interesting.

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  - hierarchy of Schwinger-Dyson equations [Perez-Sanchez, Wulkenhaar '17]
- "Controversial" statement: holography in $1+1$ dimensions is very interesting.
- Provocative question: convergence between tensor models for discrete quantum gravity and holographic tensor models?

Conclusion and outlook

- Recent developments in tensor models beyond the intended applications to random geometry / discrete quantum gravity:
  - **tensor quantum mechanics** reproducing salient features of the **SYK many-body system**
  - unanticipated **connections to other quantum gravity approaches**: $\mathrm{AdS}_2/\mathrm{CFT}_1$, black holes, higher spins...
- Motivates a more systematic investigation of such large N QFTs
  - higher dimensions
  - **(anti)symmetrized tensors** [Klebanov, Tarnopolsky '17; Benedetti, SC, Gurau, Sfondrini '17]
  - multi-matrix models [Ferrari '17;...]
  - condensed matter propagators [Ben Geloun, Rivasseau '17]
  - hierarchy of Schwinger-Dyson equations [Perez-Sanchez, Wulkenhaar '17]
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Thank you for your attention !