Main questions

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Quadratic-in-curvature terms in the action

\[ S_{\text{quad}} = \int d^4x \sqrt{-g} \mathcal{L}_{\text{quad}}, \quad \mathcal{L}_{\text{quad}} = \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} \]

are **unavoidably** generated by matter loops, such as

\[
(4\pi)^2 \frac{d\alpha}{d\ln \bar{\mu}} = \frac{N_V}{15} + \frac{N_f}{60} + \frac{N_s}{180} - \frac{(\delta_{ab} + 6\xi_{ab})(\delta_{ab} + 6\xi_{ab})}{72} \]

\[
(4\pi)^2 \frac{d\beta}{d\ln \bar{\mu}} = -\frac{N_V}{5} - \frac{N_f}{20} - \frac{N_s}{60} \]

\(N_V, N_f, N_s\) are the numbers of vectors, Weyl fermions and real scalars \(\phi_a\) with non-minimal couplings \(\xi_{ab}\) (that is \(\mathcal{L} \supset -\xi_{ab}\phi_a\phi_b R/2\))
Quadratic gravity
Adding the quadratic terms makes gravity (and all other interactions) renormalizable
[Stelle (1977)]

Intuitive reason: in the UV the theory is the most general dimensionless one

The general quadratic gravity (QG) Lagrangian:

\[ \mathcal{L} = \frac{R^2}{6f_0} - \frac{W^2}{2f_2} + \mathcal{L}_{EH} + \mathcal{L}_{SM} + \mathcal{L}_{BSM} \]

where

- \( W^2 \equiv W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma} \)
- \( \mathcal{L}_{EH} \) is the Einstein-Hilbert piece plus a cosmological constant
- \( \mathcal{L}_{SM} \) is the SM \( \mathcal{L} \) (plus \(-\xi_H|H|^2 R\)):
- \( \mathcal{L}_{BSM} \) describes BSM physics.
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- $\mathcal{L}_{BSM}$ describes BSM physics.

It is also possible to generate scales dynamically

The dimensionful terms (the Planck mass, the electroweak scale and the cosmological constant) can all be dynamically generated through dimensional transmutation (Agravity) [Salvio, Strumia (2014)]
Condition to solve the Higgs hierarchy problem

The theory is renormalizable

\[ \delta M_h^2 \sim \frac{\tilde{M}_{\text{Pl}}^2 f^4}{(4\pi)^2} \]
Condition to solve the Higgs hierarchy problem

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\[ \delta M_h^2 \sim \frac{\bar{M}_{P1}^2 f^4}{(4\pi)^2}, \quad \text{naturalness} \rightarrow f_2 \sim \sqrt{\frac{4\pi M_h}{\bar{M}_{P1}}} \sim 10^{-8} \]

(for such tiny couplings the Higgs field acquires a shift symmetry that protects \( M_h \))
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(for such tiny couplings the Higgs field acquires a shift symmetry that protects \( M_h \))

\[ M_2 = \frac{1}{\sqrt{2}} f_2 \bar{M}_{\text{Pl}} \sim 10^{10} \text{GeV} \]

[Salvio, Strumia (2014)]
Quadratic gravity is a realization of “softened gravity”

(Einstein) gravitational interactions increase with energy

Idea (softened gravity):
consider theories where the increase of the gravitational coupling → stops at some $\Lambda_G \ll M_{Pl}$.
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**Idea (softened gravity):**

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The gravitational contribution to the Higgs mass is

$$\delta M_h^2 \approx \frac{G_N \Lambda_G^4}{(4\pi)^2}$$

Requiring $\delta M_h \sim M_h \rightarrow \Lambda_G \lesssim 10^{11}$ GeV [Giudice, Isidori, Salvio, Strumia (2014)]

In quadratic gravity $\Lambda_G \sim M_2$
The Ostrogradsky theorem

*Classical* Lagrangians that depend non-degenerately on the second derivatives have Hamiltonians unbounded from below [Ostrogradsky (1848)]

Indeed, looking at the degrees of freedom (expanding around the flat spacetime):

(i) massless graviton

(ii) scalar $\omega$ with squared mass $M_0^2 \sim \frac{1}{2} f_0^2 \tilde{M}_{P1}^2$

(iii) massive spin-2 ghost with squared mass $M_2^2 = \frac{1}{2} f_2^2 \tilde{M}_{P1}^2$

(a manifestation of the Ostrogradsky theorem)

It is associated with $\frac{W^2}{2f_2^2}$

By linearizing the theory one finds the spin-2 classical Hamiltonians [Salvio (2017)]

$$H_{\text{graviton}} = \sum_{\lambda=\pm 2} \int d^3 q \left[ P_{\lambda}^2 + q^2 Q_{\lambda}^2 \right]$$

$$H_{\text{ghost}} = - \sum_{\lambda=\pm 2, \pm 1, 0} \int d^3 q \left[ \tilde{P}_{\lambda}^2 + (q^2 + M_2^2) \tilde{Q}_{\lambda}^2 \right]$$
Proceeding perturbatively

Let us split the metric $g_{\mu\nu}$ as follows:

$$g_{\mu\nu} = g^{\text{cl}}_{\mu\nu} + \hat{h}_{\mu\nu}$$

- $g^{\text{cl}}_{\mu\nu}$ is a classical background that solves the classical EOMs
- $\hat{h}_{\mu\nu}$ is a *quantum* deviation
Classical theory

Can we avoid the possible Ostrogradsky instabilities?
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- Recall that in the **free-field limit**

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This argument can be made precise in quadratic gravity. The whole cosmology can only involve energies below this threshold and avoid runaways

→ “metastability in quadratic gravity”

[Salvio (2019)]
Two-derivative formulation

To show this it is useful to separate the two-derivative d.o.f.: graviton, $\omega$ and ghost
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First perform the field redefinition $g_{\mu\nu} \rightarrow \bar{M}_{\text{Pl}}^2 f g_{\mu\nu}$, $f \equiv \bar{M}_{\text{Pl}}^2 - \frac{2R}{3f_0^2} > 0$,

(where the Ricci scalar above is computed in the Jordan frame metric) that gives

$$S = \int d^4 x \sqrt{-g} \left( -\frac{W^2}{2f_0^2} - \frac{\bar{M}_{\text{Pl}}^2}{2} R + \mathcal{L}_m^E \right)$$

“Einstein frame action”

The Einstein-frame matter Lagrangian, $\mathcal{L}_m^E$, also contains an effective scalar $\omega$, which corresponds to the $R^2$ term in the Jordan frame:

$$\mathcal{L}_\omega^E = \frac{(\partial \omega)^2}{2} - U(\omega), \quad U(\omega) = \frac{3f_0^2 \bar{M}_{\text{Pl}}^4}{8} \left(1 - e^{-2\omega/\sqrt{6}\bar{M}_{\text{Pl}}} \right)^2$$
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\[
S = \int d^4 x \sqrt{-g} \left( -\frac{W^2}{2f_0^2} - \frac{\bar{M}_{\text{Pl}}^2}{2} R + \mathcal{L}^E_m \right) \quad \text{“Einstein frame action”}
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\]

To make the ghost explicit consider an auxiliary field \( \gamma_{\mu\nu} \):

\[
S = \int d^4 x \sqrt{-g} \left[ \frac{\bar{M}_{\text{Pl}}^2}{8} \left( \gamma_{\mu\nu} \gamma_{\mu\nu} - \gamma^2 \right) - \frac{\bar{M}_{\text{Pl}}^2}{2} G_{\mu\nu} \gamma_{\mu\nu} - \frac{\bar{M}_{\text{Pl}}^2}{2} R + \mathcal{L}^E_m \right]
\]

where \( G_{\mu\nu} \) is the Einstein tensor and \( \gamma \equiv \gamma_{\mu\nu} g^{\mu\nu} \).

One has a mixing between \( h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu} \) and \( \gamma_{\mu\nu} \). The tensors \( \bar{h}_{\mu\nu} = h_{\mu\nu} + \gamma_{\mu\nu} \) and \( \gamma_{\mu\nu} \) represent the graviton and the ghost.
Interactions of the ghost and energy thresholds

The two-derivative formulation is good to understand the ghost interactions. First, one can show that the ghost interactions are suppressed by $f_2$.
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Next, $\frac{M_2^2}{8} (\gamma_{\mu\nu} \gamma^{\mu\nu} - \gamma^2)$ leads to mass and interaction terms of the schematic form

$$\frac{M_2^2}{2} \left( \phi_2^2 + \frac{\phi_2^3}{M_{P1}} + \frac{\phi_2^4}{M_{P1}^2} + \ldots \right),$$

($\phi_2$ represents the canonically normalized spin-2 fields: graviton and ghost)

The mass term has the same order of magnitude of the interactions for $\phi_2 \sim \bar{M}_{P1}$, which gives $M_2^2 \phi_2^2/2 = M_2^4/f_2^2 \equiv E_2^4$, where

$$E_2 \equiv \frac{M_2}{\sqrt{f_2}} = \sqrt{\frac{f_2}{2}} \bar{M}_{P1}$$

**For energies $E \ll E_2$ the Ostrogradsky instabilities are avoided**

This bound applies to the derivatives of the spin-2 fields.
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Analogously, one can show that the energy $E$ in the matter sector must satisfy

$E \ll E_m$ \quad $E_m \equiv \sqrt[4]{f_2 M_{P1}}$ (matter sector)
Relations with chaotic inflation [Linde (1983)]

For a natural Higgs mass \( f_2 \sim 10^{-8}, M_2 \sim 10^{10} \text{ GeV} \)

\[
E_2 \sim 10^{-4} \bar{M}_{Pl}, \quad E_m \sim 10^{-2} \bar{M}_{Pl}
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*It is clear that inflation (and the preceding epoch) is the only stage of the universe that can provide us information about such high scales.*
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But we live in one of those patches where the energy scales of inhomogeneities \((1/L)\) and anisotropies \((A)\) were small enough:

\[
\frac{1}{L} \ll |U'(\phi)/\phi|^{1/2}, \quad A \ll H
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these conditions justify the use of homogeneous and isotropic solutions to describe the classical part of inflation (Linde’s idea)

The chaotic theory automatically ensures that the conditions to avoid runaway solutions are satisfied (verified for Starobinsky inflation, hilltop inflation and other models).
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The chaotic theory automatically ensures that the conditions to avoid runaway solutions are satisfied (verified for Starobinsky inflation, hilltop inflation and other models). The fatal runaways above the energy thresholds give an (anthropic) rational for a homogeneous and isotropic universe
General check of the ghost metastability: linear analysis

Check that \textit{all} linear modes around dS are bounded (for a fixed initial condition) for any wave number $q$. 

Scalar modes:

\begin{align*}
g_B(\eta,q) & \equiv H \sqrt{2q^3 + 3i\eta q - \eta^2} e^{-iq\eta} + R_{-\text{term}}
\end{align*}

where $\eta$ is the conformal time ($a^2 d\eta^2 = dt^2$, $\eta < 0$).

Vector and tensor modes:
General check of the ghost metastability: linear analysis

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**Scalar modes:** they are like in GR plus an gravity-isocurvature mode:

$$g_B(\eta, q) \equiv \frac{H}{\sqrt{2q}} \left( \frac{3}{q^2} + \frac{3i\eta}{q} - \eta^2 \right) e^{-iq\eta} + \mathcal{R} \text{ term}$$

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**Vector and tensor modes:**
Let us go back to the following metric splitting

\[ g_{\mu \nu} = g_{\mu \nu}^{\text{cl}} + \hat{h}_{\mu \nu} \]

- \( g_{\mu \nu}^{\text{cl}} \) is a classical background that solves the classical EOMs.
- \( \hat{h}_{\mu \nu} \) is a quantum deviation

Do quantum fluctuations lead to tunneling beyond the energy threshold?
Can a different quantization help?

Recall that the classical Dirac theory of fermions has arbitrarily negative energies and the problem is solved by a different quantization. Can we hope that something similar happens for gravitons? Yes, renormalizability implies that the quantum Hamiltonian governing \( \hat{h}^{\mu
u} \) is bounded from below [Stelle (1977)].

However, the space of states must be endowed with an indefinite metric (with respect to which the "position" \( q \) and momentum \( p \) operators are self-adjoint).
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Yes, renormalizability implies that the quantum Hamiltonian governing $\hat{h}_{\mu\nu}$ is bounded from below [Stelle (1977)]

However, the space of states must be endowed with an indefinite metric (with respect to which the “position” $q$ and momentum $p$ operators are self-adjoint)
The presence of an indefinite metric leads to the question:

How can we define probabilities consistently?
A derivation of probability

- Define observable any operator $A$ with complete eigenstates $\{ |a\rangle \}$ [Salvio (2018)]: for any state $|\psi\rangle$ there is a decomposition

$$|\psi\rangle = \sum_a c_a |a\rangle$$

One can show that the basic operators $q, p$ and $H$ have complete eigenstates at any order in perturbation theory.

- Interpret $|a\rangle$ as the state where $A$ assumes certainly the value $a$ (call it the deterministic Born rule).
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Experimentalists prepare a large number $N$ of times the same state, so consider

$$|\Psi_N\rangle \equiv \nu^N |\psi\rangle \cdots |\psi\rangle = \sum_{a_1 \cdots a_N} \nu^N c_{a_1} \cdots c_{a_N} |a_1\rangle \cdots |a_N\rangle, \quad \nu \equiv \frac{1}{\sqrt{\sum_b |c_b|^2}}$$

Define a frequency operator $F_a$ which counts the number $N_a$ of times there is the value $a$ in the state $|a_1\rangle \cdots |a_N\rangle$:

$$F_a |a_1\rangle \cdots |a_N\rangle = \frac{N_a}{N} |a_1\rangle \cdots |a_N\rangle$$
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$$F_a |a_1\rangle ... |a_N\rangle = \frac{N_a}{N} |a_1\rangle ... |a_N\rangle$$

Strumia (2017) showed that

$$\lim_{N \to \infty} F_a |\Psi_N\rangle = B_a |\Psi_N\rangle, \quad B_a \equiv \frac{\sum |c_a|^2}{\sum |c_b|^2}$$

(all coefficients in the basis $|a_1\rangle ... |a_N\rangle$ converge to the same quantities)
The emergent norms to compute probabilities

\{\ket{a}\} is complete so we can \textbf{define} a “norm” operator \( P_A \):

\[
\langle a' | P_A | a \rangle \equiv \delta_{aa'}
\]

where for any pair of states \( \ket{\psi_1}, \ket{\psi_2} \), we denote the indefinite metric with \( \langle \psi_2 | \psi_1 \rangle \).

The definition above provides a positive metric (a norm):

\[
\langle \psi_2 | \psi_1 \rangle_A \equiv \langle \psi_2 | P_A | \psi_1 \rangle_A = \sum_a c_{a2}^* c_{a1}
\]

(which is positive for \( \ket{\psi_1} = \ket{\psi_2} \))

\[
B_a = \frac{|c_{a2}|^2}{\sum_b |c_{b2}|^2} = \frac{|\langle a | \psi \rangle_A|^2}{\langle \psi | \psi \rangle_A}
\]

We recover the full probabilistic Born rule, but expressed in terms of the \textit{positive norm} not in terms of the indefinite one

\begin{itemize}
\item All probabilities are positive
\item The probabilities sum up to one at any time (the theory is unitary)
\end{itemize}
Observational consequences
Observational consequences: \( M_2 > H \)

No differences compared to GR
Observational consequences: $M_2 < H$

The modifications:

- $r$ gets suppressed

\[ r \rightarrow \frac{r}{1 + \frac{2H^2}{M_2^2}} \]

models that are excluded for a large $r$ (e.g. quadratic inflation) become viable

- There is an isocurvature mode (which fulfills the observational bounds) corresponding to the scalar component of the spin-2 ghost (the vector components and the other tensor component decay with time)

Indeed,

- $P_R$ is not changed by the ghost (so $n_s$ is not changed either)
- while the tensor power spectrum is modified:

\[ P_t \rightarrow \frac{P_t}{1 + \frac{2H^2}{M_2^2}} \]

- The isocurvature power spectrum $P_B$ is the same as the tensor power spectrum in Einstein's gravity, except that it is smaller by a factor of $3/16 \approx 1/5$:

\[ P_B = \frac{3}{2M_{Pl}^2} \left( \frac{H}{2\pi} \right)^2 \]

and the correlation $P_{RB}$ is highly suppressed

[Ivanov, Tokareva (2016)], [Salvio (2017)]
Ghost-isocurvature power spectrum \((M_2 < H)\)

\[ q_1 = 0.002 \, \text{Mpc}^{-1} \text{ and } q_2 = 0.1 \, \text{Mpc}^{-1}. \]

The strongest constraints from \textit{Planck (2018)} have been taken

\cite{Salvio2017}, \cite{Salvio2019}
What happens above $\mathcal{M}_{P1}$?
Asymptotic freedom and stability

\[(4\pi)^2 \frac{df_2^2}{d\ln \bar{\mu}} = -f_2^4 \left( \frac{133}{10} + \frac{N_V}{5} + \frac{N_f}{20} + \frac{N_s}{60} \right)\]

\[(4\pi)^2 \frac{df_0^2}{d\ln \bar{\mu}} = \frac{5}{3} f_2^4 + 5 f_2^2 f_0^2 + \frac{5}{6} f_0^4 + \frac{f_0^4}{12} (\delta_{ab} + 6\xi_{ab})(\delta_{ab} + 6\xi_{ab})\]

\[\downarrow\]

- \(f_2^2\) is asymptotically free for \(f_2^2 > 0\) (no problem with \(f_2^2 > 0\))
- \(f_0^2\) is asymptotically free only for \(f_0^2 < 0\)
Asymptotic freedom and stability

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\]

\[
(4\pi)^2 \frac{df_0^2}{d\ln \bar{\mu}} = \frac{5}{3} f_2^4 + 5 f_2^2 f_0^2 + \frac{5}{6} f_0^4 + \frac{f_0^4}{12} (\delta_{ab} + 6 \xi_{ab})(\delta_{ab} + 6 \xi_{ab})
\]

\[\downarrow\]

- \(f_2^2\) is asymptotically free for \(f_2^2 > 0\) (no problem with \(f_2^2 > 0\))
- \(f_0^2\) is asymptotically free only for \(f_0^2 < 0\)

\(f_0^2 < 0\) corresponds to a tachyonic scalar \(\omega\) with squared mass \(M_0^2 \sim f_0^2 M_{\text{Pl}}^2\)

The potential of the effective scalar \(\omega\) that corresponds to the term \(R^2/6f_0^2\):

![Potential diagrams](image-url)
Asymptotic safety can save us

*Strumia and AS (2017)* showed that, when $f_0 \to \infty$ in the infinite energy limit, $f_0$ does not hit any Landau pole, provided that

- All scalars have asymptotically Weyl-invariant ($\xi_{ab} = -\delta_{ab}/6$) couplings
- All other couplings approach fixed points
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So QG can flow to a Weyl-invariant theory, a.k.a. conformal gravity, at infinite energy.
Are there viable models of the early universe that are quasi conformal, i.e.

\[ \frac{1}{f_0} \approx 0, \quad \xi_{ab} \approx -\delta_{ab}/6 \quad ? \]
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Two options were found:

- The inflaton is a fundamental scalar with a quasi-conformal non-minimal coupling to the Ricci scalar. In this case the field excursion must not exceed the Planck mass by far. An example is hilltop inflation.

- The inflaton is a pseudo-Goldstone boson (natural inflation). In this case one can elegantly obtain a UV completion within an asymptotically free QCD-like theory.

\[
V_N(\phi) = \Lambda^4 \left( 1 + \cos \left( \frac{N\phi}{f} \right) \right)
\]

The conditions to avoid the Ostrogradsky instability are satisfied
**Natural quasi-conformal inflation**

![Graphs showing the evolution of slow-roll parameters and the tensor-to-scalar ratio over different values of $f$.]

**Figure:** We have set $\Lambda \approx 6 \times 10^{-3} \bar{M}_{P1}$ to fit the observed value of the curvature power spectrum $P_R$ and chosen $N = 1$ and $f_2 = 10^{-8}$ (the value of $f_2$ influences the plot of $r$ only). The bounds from the latest Planck analysis from 2018 are also shown.
Summary

- Quadratic gravity is renormalizable and can reach infinite energy (if all matter couplings have UV fixed points)

- The classical runaway solutions can be avoided even at energies exceeding the ghost mass $M_2$. The runaways occurring above an energy threshold ($\gg M_2$) can explain homogeneity and isotropy for $f_2 \sim 10^{-8}$

- We have provided a possible way of quantizing the theory, which maintains unitarity. All probabilities are positive and they sum up to 1

- For $f_2 \sim 10^{-8}$ quadratic gravity leads to testable predictions for the inflationary observables and the EW scale

- The theory can be UV complete provided that all couplings flow to a UV fixed point such that $1/f_0 \to 0$ and $\xi_{ab} \to -\delta_{ab}/6$. Viable quasi-conformal models of the early universe (i.e. $1/f_0 \approx 0$ and $\xi_{ab} \approx -\delta_{ab}/6$) have been shown.
Thank you very much for your attention!
Extra slides
A possible way to avoid the ghost

Weinberg (1979) proposed to add all (infinite) terms:

\[ \mathcal{L}_W = -\Lambda - \frac{\bar{M}_P^2}{2} R + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} + \alpha_3 R^3 + \beta_3 R_{\mu\nu} R^{\mu\nu} R + \ldots, \]

including all the (perturbatively) renormalizable and nonrenormalizable terms in the matter sector.

- **IF** all couplings \( g = \{\alpha_i, \beta_i, \ldots\} \) flow to a **finite dimensional** surface
- **IF** all these infinite terms do not develop ghost d.o.f. (possible because Ostrogradsky theorem applies to a finite number of higher derivatives)

Then we would have a UV-complete relativistic field theory of all forces without ghosts

(So far), however, this possibility is unfortunately **uncomputable**. Requiring computability we go back to quadratic gravity
Probability: previous attempts

1. Associating the negative probabilities to undetectable events:

   - Dirac (1941): “Negative probability should not be considered as nonsense ... they should be considered simply as things which do not appear in experimental results”
   - Feynman (1987): “… the final probability of a verifiable event is positive. On the other hand, probabilities of imagined intermediate states may be negative”
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The ghost is unstable and the negative probability of its creation is compensated by the negative probability of its decay:

This is the Lee-Wick approach: The ghost is unstable and, therefore, is not an asymptotic state [Lee, Wick (1960)], [Salvio, Strumia, Veermae (2018)]
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   The ghost is unstable and the negative probability of its creation is compensated by the negative probability of its decay:

   ![Diagram of scattering and decay](image)

   This is the Lee-Wick approach: The ghost is unstable and, therefore, is not an asymptotic state [Lee, Wick (1960)], [Salvio, Strumia, Veermae (2018)]

   **Problems:** - there are other applications of QM beyond scattering;  
               - how do you define the decay rate of the ghost?
Probability: previous attempts

2 Eliminating the ghost from the physical spectrum

*Anselmi and Piva (2017)* showed that the $S$-matrix elements between asymptotic states (no ghost) forms a unitary matrix if the ghost propagator is prescribed appropriately:

\[
\text{Euclidean propagator} = \lim_{\varepsilon \to 0} \frac{p^2 + M_2^2}{(p^2 + M_2^2)^2 + \varepsilon^4}
\]

Intuitively this works because

\[
\text{Minkowsky propagator} = \lim_{\varepsilon \to 0} \frac{-p^2 + M_2^2}{(-p^2 + M_2^2)^2 + \varepsilon^4}
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which vanishes on-shell
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Problem: this is developed only for the flat and to some extent for the Friedmann-Robertson-Walker (FRW) metrics [Anselmi (2019)]. What about cosmological perturbations?
Trading negative energies with negative norm

Diagonalization of the Hamiltonian

For $V = 0$ the Hamiltonian is

$$H = \omega_1 \left( -\tilde{a}^\dagger_1 \tilde{a}_1 + \frac{1}{2} \right) + \omega_2 \left( \tilde{a}^\dagger_2 \tilde{a}_2 + \frac{1}{2} \right)$$

We have       $[\tilde{a}_1, \tilde{a}^\dagger_1] = -1$,       $[\tilde{a}_2, \tilde{a}^\dagger_2] = 1$,       (all other commutators vanish)
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Onset of “negative norms”

As usual \([a_1, N_1] = a_1\) and \([a_2, N_2] = a_2\) by defining \(N_1 \equiv -\tilde{a}_1^\dagger \tilde{a}_1\) and \(N_2 \equiv \tilde{a}_2^\dagger \tilde{a}_2\)

The spectrum of \(N_1\) is bounded from below if you introduce an indefinite metric:

\[
-\nu_n n = \langle n | a_1^\dagger a_1 | n \rangle
\]
Trading negative energies with negative norm

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The spectrum of $N_1$ is bounded from below if you introduce an indefinite metric:

$$-\nu_n n = \langle n | a^\dagger_1 a_1 | n \rangle = |c|^2 \langle n - 1 | n - 1 \rangle = |c|^2 \nu_{n-1}$$
Classical dynamics: a simple scalar field example

To simplify consider

\[ \mathcal{L} = -\frac{1}{2} \phi \Box \phi - \frac{c_4}{2} \phi^2 \phi - V(\phi) \]

It is a toy version of our theory:

- \(-\frac{1}{2} \phi \Box \phi \) represents the Einstein-Hilbert part
- \(-\frac{c_4}{2} \phi^2 \phi \) represents the quadratic terms
- \(V \) is some interaction
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- \(V\) is some interaction

Two-derivative form

Add \(\frac{c_4}{2} \left( \Box \phi - \frac{A - \phi/2}{c_4} \right)^2\) (vanishes when the EOM of the auxiliary field \(A\) are used)

\[ \mathcal{L} = -\frac{1}{2} \phi_+ \Box \phi_+ + \frac{1}{2} \phi_- \Box \phi_- + \frac{m^2}{2} \phi_-^2 - V(\phi_+ + \phi_-) \]

where \(m^2 \equiv 1/c_4\) has to be taken positive to avoid tachyonics.
Classical dynamics: a simple scalar field example

The EOMs are

\[ \Box \phi_+ = -V'(\phi_+ + \phi_-), \quad \Box \phi_- = -m^2 \phi_-^2 + V'(\phi_+ + \phi_-). \]

For definiteness take \( V(\phi) = \lambda \phi^4/4 \), where \( \lambda > 0 \), which stabilizes \( \phi_+ \), while \( \phi_- \) feels

\[ v(\varphi) = \frac{m^2}{2} \varphi^2 - \frac{\lambda}{4} \varphi^4, \quad \varphi = \text{typical order of magnitude of field values} \]
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Ghost metastability

For

\[ \varphi \ll E_f \equiv \frac{m}{\sqrt{\lambda/2}} \]

and

\[ E \ll E_d \equiv \frac{m}{(4\lambda)^{1/4}} \]

(where \( E \) is the energy associated with the field derivatives)

the runaway solutions are avoided
Classical dynamics: a simple scalar field example

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the runaway solutions are avoided

Example in the figure: \( \lambda = 10^{-2} \), \( \phi_+(0) = 10^{-2} E_f \), \( \phi_-(0) = 10^{-2} E_f \), \( \dot{\phi}_+(0) = (1.5 \cdot 10^{-1} E_d)^2 \) and \( \dot{\phi}_-(0) = -(10^{-2} E_d)^2 \).
Explicit nonlinear calculations

\[ ds^2 = dt^2 - a(t)^2 \sum_{i=1}^{3} e^{2\alpha_i(t)} dx^i dx^i \]

\[ \alpha_1 \equiv \beta_+ + \sqrt{3}\beta_- , \quad \alpha_2 \equiv \beta_+ - \sqrt{3}\beta_- , \quad \alpha_3 = -2\beta_+ . \]

One can reduce the system to first-order equations through the definitions

\[ \gamma_\pm = \dot{\beta}_\pm , \quad \delta_\pm = \dot{\gamma}_\pm , \quad \epsilon_\pm = \dot{\delta}_\pm . \]
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Small initial values for the anisotropy

\[ (|\gamma_{\pm}(0)| \ll E_2, \sqrt{|\delta_{\pm}(0)|} \ll E_2, \sqrt{3}|\epsilon_{\pm}(0)| \ll E_2 \text{ and } \sqrt{M_{P1}H} \ll E_m ) \]

do not create problems: the anisotropy quickly goes to zero and one recovers the GR behavior

Example in the figure: \( f_2 = 10^{-8}, \ f_0 \approx 1.6 \cdot 10^{-5}, \ \phi(0) \approx 5.5M_{P1} \) and \( \sqrt{\pi\phi(0)} \approx 7.1 \cdot 10^{-6} M_{P1} \)

Possible to illustrate the argument in simple models
Explicit nonlinear calculations

$$ds^2 = dt^2 - a(t)^2 \sum_{i=1}^{3} e^{2\alpha_i(t)} dx^i dx^i$$

$$\alpha_1 \equiv \beta_+ + \sqrt{3}\beta_-, \quad \alpha_2 \equiv \beta_+ - \sqrt{3}\beta_-, \quad \alpha_3 = -2\beta_+.$$ 

One can reduce the system to first-order equations through the definitions

$$\gamma_\pm = \dot{\beta}_\pm, \quad \delta_\pm = \dot{\gamma}_\pm, \quad \epsilon_\pm = \dot{\delta}_\pm$$

The patches where those conditions are not satisfied quickly collapse:

The scale factor in the Jordan frame shrinks as shown in the figure →

Example in the figure: $\gamma_-(0) = 10^{-1} E_2$, $\delta_\pm(0) = 0$, $\epsilon_\pm(0) = 0$, $f_2 = 10^{-8}$, $f_0 \approx 1.6 \cdot 10^{-5}$, $R(0) \approx 1.3 \cdot 10^2 f_0^2 M_{\text{Pl}}^2$ and $H(0) = 1.2 E_2$. 

![Graph showing the scale factor in the Jordan frame](image)
Implications for BSM phenomenology
Given that gravity is now UV complete it makes sense to look for relativistic field theoretic Standard Model extensions that hold up to infinite energies.

Two options

- All couplings flow to zero at infinite energy: total asymptotic freedom

- (Some of) the couplings flow to an interacting UV fixed point (while the other ones flow to zero). *This option typically requires non perturbative methods (lattice?)*
Totally asymptotically free (TAF) phenomenology

TAF achieves total unification: all couplings flow to a common value in the UV (zero)

In order to eliminate the Landau poles or run into a non-perturbative regime so far we needed to avoid $U(1)$ gauge factors

$\Rightarrow$ explanation of the electric charge quantization
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⇒ explanation of the electric charge quantization

We find 2 options: \( \text{SU}(4)_{\text{PS}} \times \text{SU}(2)_L \times \text{SU}(2)_R \), \( \text{SU}(3)_C \times \text{SU}(3)_L \times \text{SU}(3)_R \)

that, unlike SU(5) and SO(10), are not severely constrained by proton decay

⇒ one can have \( M_{\text{NP}} \sim \text{TeV} \) (compatibly with naturalness)
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⇒ one can have $M_{NP} \sim \text{TeV}$ (compatibly with naturalness)

⇒ TAF models that predict new physics have been found

$W_R$, $Z'$, $H'$, etc

[Giudice, Isidori, Salvio, Strumia (2014); Pelaggi, Strumia, Vignali (2015)]
Because of quark-lepton unification in the Pati-Salam model, flavor bounds force the masses of the vectors in $\text{SU}(4)_{\text{PS}}$ to be larger than 100 TeV.
Because of quark-lepton unification in the Pati-Salam model, flavor bounds force the masses of the vectors in $SU(4)_{PS}/SU(3)_c$ to be larger than 100 TeV.
Another TAF model can be built by extending the minimal trinification.

Trinification does not predict quark-lepton unification and thereby is safer than Pati-Salam from the point of view of flavour bounds.

Higgs naturalness demands $M_{W_R} \leq 2 \text{ TeV} \sqrt{\Delta}$, where $\Delta \equiv \text{fine-tuning factor}$.