Quantum critical transport in the unitary Fermi gas

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ERG 2012, Aussois, 3 September 2012

Enss, PRA 86, 013616 (2012)
Enss, Küppersbusch, Fritz, PRA 86, 013617 (2012)
Enss and Haussmann, arXiv:1207.3103
Unitary Fermi gas

- Fermi gas with contact interaction

\[ S = \int d^d x \, d\tau \sum_{\sigma = \uparrow, \downarrow} \psi^*_\sigma \left[ \partial_\tau - \frac{\nabla^2}{2m} - \mu_\sigma \right] \psi_\sigma + g \psi^*_\uparrow \psi^*_\downarrow \psi_\downarrow \psi_\uparrow \]

- scattering amplitude (3d)

\[ f(k) = \frac{1}{-1/a - i k + r_e k^2/2} \]

- strong scattering in unitary limit

\[ 1/a = 0 : f(k \to 0) = \frac{i}{k} \]

- universal for dilute system (broad resonance)

\[ r_e \ll n^{-1/3} \]

[Sa de Melo, Physics Today 2008]
Renormalization group

- vacuum (T=0, n=0): **exact** beta function  [Nikolic, Sachdev; Diehl et al.]
  \[
  \frac{dg}{d\ell} = (2 - d)g - \frac{g^2}{2}
  \]

- 2<d<4: unstable fixed point g* (unitarity, Feshbach resonance)

- detuning 1/a is relevant perturbation

\[d > 2\]

\[d < 2\]

\[u^*\]

\[0\]

\[u\]

[Nikolic, Sachdev 2007]
Quantum critical point

- resonant fixed point is Quantum critical point (QCP) [Nikolic, Sachdev]

\begin{align*}
\text{dilute classical gas} & \quad \text{vacuum} \\
\text{quantum critical regime} & \quad \text{super fluid} \\
\text{QCP} & \quad \text{vacuum for } T=0, \mu<0
\end{align*}

\[ T_c \approx 0.4\mu \]

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Universal properties

- thermodynamic functions depend only on $\mu/T$ ("angle")

  e.g. equation of state $n = \lambda_T^{-3} f_n(\mu/T)$

- measured by Zwierlein group, Science 2012, computed using Bold Diagrammatic MC:

  agreement on percent level (benchmark)

- open: contact, imbalance, superfluid (in progr.)

  challenge: transport

  focus on quantum critical regime: $\lambda_T \approx n^{-1/3}$

quantum and thermal fluctuations equally important

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Effective action

• Hubbard-Stratonovich transformation in **Cooper channel**: exchange of virtual molecules (T-matrix)

\[
S = \int d^d x d\tau \sum_{\sigma = \uparrow, \downarrow} \psi^*_{\sigma} \left[ \partial_\tau - \frac{\nabla^2}{2m} - \mu_\sigma \right] \psi_{\sigma} - \frac{1}{g_0} |\phi|^2 - h \psi^*_{\uparrow} \psi^*_{\downarrow} \phi - h \phi^* \psi_{\downarrow} \psi_{\uparrow}
\]

(positive mass!)

• integrate out fermions: bosonic effective action

\[
\text{tr} \ln G_f[\phi] = b_2(q, \omega) |\phi|^2 + b_4(q_i, \omega_i) |\phi|^4 + \cdots
\]
Vacuum (at QCP)

- 3d, T=μ=0: $b_2(q, \omega) \simeq \sqrt{\frac{q^2}{4m}} - \omega - i0$

- anomalous dimension:

$$\eta_\phi = 4 - d > 0, \quad \text{dim}[\phi] = \frac{d + \eta_\phi}{2} = 2 \quad (2 < d < 4)$$

- n-point functions $b_n \sim q^{5-2n}$ all scale marginally [Enss 2012]

- no simple $\phi^4$ theory (Hertz-Millis)

- in vacuum no feedback on 2-point function, dg/dl remains exact but possible limit cycles in 3-body sector can change ground state (Efimov physics, Richard Schmidt’s talk on Friday) [Floerchinger, Schmidt, Moroz, Wetterich]
Finite density

• $T>0$, $n>0$: all higher $n$-point functions feed back into $\phi$ propagator, no obvious strategy to select diagrams (no small parameter)

• **strong coupling many-body problem:**
  - sample *all* diagrams (Bold Diagrammatic MC)
  - Luttinger-Ward (2PI): self-consistent propagators (1-loop skeleton diagrams)
  - functional RG: derivative expansion; full $\omega, q$ [Schmidt, Enss 2011]

• **large-N expansion:** [Nikolic, Sachdev]
  N flavors of $\uparrow \downarrow$ fermions
  fermion loops: factor $N$
  pair propagators: factor $1/N$ ($N=\infty$ free fermions)
  controlled expansion in orders $1/N$
  extrapolate to physical case $N=1$
  details: Enss, PRA 86, 013616 (2012)
Results: thermodynamics

• leading order large-N expansion (NSR) at μ=0: [Enss 2012]

<table>
<thead>
<tr>
<th></th>
<th>Experiment</th>
<th>Large-N</th>
<th>LuttWard</th>
<th>BoldDiagMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n\lambda_T^3$</td>
<td>2.966(35)</td>
<td>2.674</td>
<td>3.108</td>
<td>2.90(5)</td>
</tr>
<tr>
<td>$P [nk_B T]$</td>
<td>0.891(19)</td>
<td>0.928</td>
<td>0.863</td>
<td>0.90(2)</td>
</tr>
<tr>
<td>$s [nk_B]$</td>
<td>2.227(38)</td>
<td>2.320</td>
<td>2.177</td>
<td>2.25(5)</td>
</tr>
<tr>
<td>$C [k_F^4]$</td>
<td></td>
<td></td>
<td>0.0789</td>
<td>0.084</td>
</tr>
<tr>
<td>$\eta/s [\hbar/k_B]$</td>
<td>1.0(2)</td>
<td>0.741</td>
<td>0.708</td>
<td></td>
</tr>
</tbody>
</table>

• pressure and entropy within 3%, contact density within 1% of expt./BDMC:

$$C = m^2\langle\phi^*\phi\rangle = -\frac{m^2}{N} \int \frac{d^3k}{(2\pi)^3} \frac{d\omega}{\pi} b(\omega) \text{Im} T(k,\omega)$$

$$= 26.840 \frac{128\lambda_T^{-4}}{N}.$$  

• good and efficient approximations available!

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Results: transport

- Boltzmann equation justified in large-N expansion; need in-medium T-matrix for consistency!

- transport in quantum critical regime: scaling with temperature

\[ \eta \sim \hbar T^{3/2}, \quad s \sim k_B T^{3/2} \implies \frac{\eta}{s} = 0.74 \frac{\hbar}{k_B} \]

- viscosity \( \eta/s \) measures incoherent relaxation rate:

\[ \frac{\hbar}{\tau_\eta} = 0.54 k_B T \]

- large-N in 2d: viscosity [Enss, Küppersbusch, Fritz, PRA 86, 013617 (2012)]

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Results: transport

- Luttinger-Ward (2PI): self-consistent fermion and pair propagators

\[ \frac{\eta}{s} = \frac{\hbar}{k_B T} \]

viscosity \( \frac{\eta}{s} \):
Enss, Haussmann, Zwerger 2011

spin diffusion rate \( D_s \sim \frac{\hbar}{m} \)

comparison with experimental data:
Sommer et al. (Zwierlein group), Nature 2011
Conclusions

• phase diagram of unitary Fermi gas governed by quantum critical point

• scaling analysis of effective action:
  infinity number of marginal vertices, approximations not obvious

• comparison with benchmark:
  large-N expansion, Luttinger-Ward (2PI), functional RG work well

• lesson for functional RG:
  integrate out fermions and bosons simultaneously; full $\omega,q$ dependence helps

• large-N accurately determines pressure, entropy, contact; transport calculations can explain recent experiments:
  quantum limited spin diffusion