Polarons in ultracold atomic gases

- flowing spectral functions -

Tilman Enss (ITP, Uni Heidelberg)
Richard Schmidt (TU München)
Ville Pietilä (Harvard)
Eugene Demler (Harvard)
Marianne Bauer (Cambridge)
Meera Parish (Cambridge)

ERG 2014, Lefkada, 26 September 2014
The Fermi polaron

- impurity coupled to environment, fundamental condensed matter problem
  Feynman 1955; Anderson 1967

- here: single mobile ↓ fermion in ↑ Fermi sea:  
  \textit{ferromagnet:} Nagaoka 1966

- strong polarization limit of:
  BEC-BCS crossover (attractive)
  Stoner ferromagnetism (repulsive)

- \textit{ultracold atoms:}
  Chevy 2006 (variational);
  Combescot et al. 2007 (T-matrix);
  Prokof’ev & Svistunov 2008 (diagMC);
  Punk, Dumitrescu & Zwerger 2009 (var)

- \textit{cf.} Sarma phase  \textit{Strack+ 1311.4885, Boettcher+ 1409.5232}
Ultracold atoms

- contact interactions, s-wave scattering length $a$

- two-body problem:

\[ \epsilon_B = \frac{\hbar^2}{ma^2} \]
Polaron to molecule transition

• ground state properties well understood (variational, Monte Carlo, experiment)
  Chevy 2006; Prokof’ev & Svistunov 2008; Schirotzek et al. 2009

• here: dynamical properties, decay rates, linear response (more involved)
The model

- two-component Fermi gas with contact interaction, \textbf{microscopic action}

\[ S = \int_P \sum_{\sigma = \uparrow, \downarrow} \psi^*_\sigma [-i\omega + p^2 - \mu_\sigma] \psi_\sigma + g \int_X \psi^*_\uparrow \psi^*_\downarrow \psi_\downarrow \psi_\uparrow \]

- Hubbard-Stratonovich transformation in \textbf{Cooper channel:}
  exchange of virtual molecules (T-matrix)

  \[ S = \int_P \sum_{\sigma = \uparrow, \downarrow} \psi^*_\sigma [-i\omega + p^2 - \mu_\sigma] \psi_\sigma + \phi^* G_{\phi,\Lambda}^{-1} \phi + h \int_X (\psi^*_\uparrow \psi^*_\downarrow \phi + h.c.) \]
Functional Renormalization Group

• include quantum and thermal fluctuations successively:

\[ \partial_k \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \left( \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_k R_k \right) = \frac{1}{2} \]

Functional renormalization group equation  Wetterich 1993

• compute two processes:
  (a) \( \Sigma\downarrow(k, \omega) \): fermions scatter off virtual molecules
  (b) \( \Sigma\phi(k, \omega) \): molecules excite virtual fermion pairs

• need arbitrary frequency/momentum dependence; use cubic splines to get smooth right-hand side

UV: \( \Gamma_{k=\Lambda} = S \)
Need for full frequency dependence

\[ S = \int_P \sum_{\sigma = \uparrow, \downarrow} \psi^*_\sigma [-i\omega + p^2 - \mu_\sigma] \psi_\sigma + \phi^* G_{\phi, \Lambda}^{-1} \phi + \hbar \int_X (\psi^*_\uparrow \psi^*_\downarrow \phi + h.c.) \]

**derivative expansion:** \( \Gamma_{k, \phi, \text{kin}} = \int_{p,\omega} \phi^* [-iA_k \omega + B_k p^2 + C_k] \phi \)

- single coherent quasi-particle excitation
- no anomalous dimension

**analytical solution for zero density:**

\[ G_\phi(\omega, p) \sim \frac{1}{-\alpha^{-1} + \sqrt{-\omega/2 + p^2/4 - \mu - i0^+}} \]

\[ A_\phi(\omega, p = 0) \]

coherent molecule peak

continuum states

incoherent background

gap/binding energy

not captured in expansion:

- most weight in continuum
- anomalous dimension \( \eta = 1 \)
Flowing spectral functions

\[ \Gamma_k = \int_{\mathbf{p}, \omega} \left\{ \psi_{\uparrow}^*[-i\omega + \mathbf{p}^2 - \mu_{\uparrow}]\psi_{\uparrow} + \psi_{\downarrow}^*G^{-1}_{\downarrow, k}(\omega, \mathbf{p})\psi_{\downarrow} + \phi^*G^{-1}_{\phi, k}(\omega, \mathbf{p})\phi \right\} \]

+ \int_{\vec{x}, \tau} \left[ h(\psi_{\uparrow}^*\psi_{\downarrow}^*\phi + h.c.) \right]

(1) sharp momentum cutoff:

\[ G_{\downarrow, k}^c(\omega, \mathbf{p}) = \frac{\theta(|\mathbf{p}| - k)}{P_{\downarrow, k}(\omega, \mathbf{p})}, \quad G_{\phi, k}^c(\omega, \mathbf{p}) = \frac{\theta(|\mathbf{p}| - k)}{P_{\phi, k}(\omega, \mathbf{p})}, \quad G_{\uparrow, k}^c(\omega, \mathbf{p}) = \frac{\theta(|\mathbf{p}^2 - \mu_{\uparrow}| - k^2)}{P_{\uparrow, k}(\omega, \mathbf{p})}. \]

(2) reconstruct \( P_k(i\omega, \mathbf{p}) \) from bicubic spline interpolation of \( P_{k_{ij}} = P_k(i\omega_i, \mathbf{p}_j) \)

(3) analytical continuation \( P_k(i\omega, \mathbf{p}) \to P_k(\omega + i0, \mathbf{p}) \) for spectral function at scale \( k \)

(4) compute smooth RHS of flow equation, \( \tilde{\partial}_k P_{k_{ij}} \) and integrate flow down to IR
Excitation spectrum

polaron $A_{\downarrow}(\omega, p = 0)$

molecule $A_{\phi}(\omega, p = 0)$

polaron has three characters:
repulsive polaron  attractive polaron  bound molecule

Schmidt & Enss 2011
Excitation spectrum

\[ (\omega - \mu_1) / \epsilon_F \]

polaron

crossover of quasiparticle weight

\[ Z_{\text{rep}} \]
\[ Z_{\text{att}} \]

\[ 1 / (k_F a) \]

characters:

- attractive polaron
- bound molecule

Schmidt & Enss 2011
Excitation spectrum

polaron $\mathcal{A}_\downarrow(\omega, p = 0)$

molecule $\mathcal{A}_\phi(\omega, p = 0)$

- polaron to molecule transition at $(k_Fa_c)^{-1} = 0.904(5)$
  cf. bold diagMC $(k_Fa_c)^{-1} = 0.90(2)$

- $E_{\text{rep}} > E_F$: ferromagnetism favored

Schmidt & Enss 2011
Polaron decay

- strong binding: stable repulsive branch

- intermediate binding \((k_F a)^{-1} < 0.6\):
  \[ E_{\text{rep}} > E_F \]: onset of ferromagnetism
  \[ \Gamma_{\text{rep}} > 0.2 E_F \]: molecule formation

- competition of dynamical phenomena
  Jo et al. 2009; Pekker et al. 2011; Cui & Zhai 2010

Schmidt & Enss 2011
Molecule decay

- leading **3-body** process (incl. in fRG)
- molecule stable: \( \Gamma \propto \Delta \omega^{9/2} \)
  Bruun & Massignan 2010
- 1st order transition
Radio-frequency response

rf protocol

linear response

bubble + vertex corrections,
cf. transport Enss, Haussmann, Zwerger 2011
but for polaron they vanish

rf current: decay rate of rf photons by coupling to atoms,
imaginary part of photon self-energy:

\[ I_{\text{rf}}(\omega) = \frac{\pi \Omega_{\text{rf}}^2}{2} \int \frac{dp}{(2\pi)^2} A_{\downarrow}(p, \omega + \varepsilon_p - \mu_\downarrow) n_F(\varepsilon_p - \mu_2) \]

atom inserted in polaron state
atom removed from state 2
Radio-frequency response

rf protocol

rf current: decay rate of rf photons by coupling to atoms, imaginary part of photon self-energy:

\[ I_{\text{rf}}(\omega) = \frac{\pi \Omega_{\text{rf}}^2}{2} \int \frac{d\mathbf{p}}{(2\pi)^2} \rho(\omega - \omega_{\text{rf}} - \mathbf{p}) \]

rf spectrum for $^6\text{Li}$

- attractive polaron
- repulsive polaron

\[ (k_P a_{1})^{-1} = -1.88 \]
\[ (k_P a_{2})^{-1} = 0.0 \]
\[ (k_P a_{3})^{-1} = 0.39 \]

\[ (k_P a_{1})^{-1} = -1.80 \]
\[ (k_P a_{2})^{-1} = 0.2 \]
\[ (k_P a_{3})^{-1} = 0.58 \]

\[ (k_P a_{1})^{-1} = -1.70 \]
\[ (k_P a_{2})^{-1} = 0.5 \]
\[ (k_P a_{3})^{-1} = 0.86 \]

\[ (k_P a_{1})^{-1} = -1.62 \]
\[ (k_P a_{2})^{-1} = 0.7 \]
\[ (k_P a_{3})^{-1} = 1.04 \]
Experimental confirmation: Innsbruck group

Metastability and coherence of repulsive polarons in a strongly interacting Fermi mixture

C. Kohstall1,2, M. Zaccanti1, M. Jag1,2, A. Trenkwalder1, P. Massignan3, G. M. Bruun4, F. Schreck1 & R. Grimm1,2

quasiparticle weight

energy spectrum agrees with theory for narrow resonance (Richard Schmidt, unpubl.)
Fermi polarons in two dimensions

Schmidt, Enss, Pietilä & Demler, PRA 85, 021602(R) (2012)
2D scattering

Experimental setup: quasi-2D “pancakes”

Longitudinal motion frozen if $k_B T, E_F \ll \hbar \omega_0$

Exact 2D scattering amplitude:

$$3D: \ f(k) = \frac{1}{-1/a_{3D} - ik}$$

$$2D: \ f(k) = \frac{1}{\ln(1/k^2 a_{2D}^2) + i\pi}$$

Quasi-2D scattering:

$$\varepsilon_B = 0.905 (\hbar \omega_0 / \pi) \exp(-\sqrt{2\pi} \ell_0 / |a_{3D}|)$$

Determines $a_{2D}$ from $a_{3D}$

$\varepsilon_B$ determines from $a_{2D}$

2D: always bound state

$$\varepsilon_B = \frac{\hbar^2}{ma_{2D}^2}$$

Adhikari 1986

Petrov & Shlyapnikov 2001
Many-body T-matrix

\[ T = \sum + T \]

Nozières & Schmitt-Rink 1985; 2D: Engelbrecht & Randeria 1990
Many-body T-matrix

**step 1: compute many-body T-matrix**

two-body T-matrix: \[ T_0(E) = \frac{4\pi/m}{\ln(\varepsilon_B/E) + i\pi} \]

many-body: finite density medium scattering

\[
T^{-1}(\mathbf{q}, \omega) = T_0^{-1}(\omega + i0 + \mu_\uparrow + \mu_\downarrow - \varepsilon_\mathbf{q}/2) + \int \frac{d^2k}{(2\pi)^2} \frac{n_F(\varepsilon_\mathbf{k} - \mu_\uparrow) + n_F(\varepsilon_{\mathbf{k}+\mathbf{q}} - \mu_\downarrow)}{\omega + i0 + \mu_\uparrow + \mu_\downarrow - \varepsilon_\mathbf{k} - \varepsilon_{\mathbf{k}+\mathbf{q}}} 
\]

we find compact solution

\[
T(\mathbf{q}, \omega) = T_0\left( \frac{1}{2}z \pm \frac{1}{2}\sqrt{(z - \varepsilon_\mathbf{q})^2 - 4\varepsilon_F\varepsilon_\mathbf{q}} \right) \quad z = \omega + i0 - \varepsilon_F + \mu_\downarrow
\]
Many-body T-matrix

**Step 1: Compute Many-body T-matrix**

Two-body T-matrix:

\[
T_0(E) = \frac{4\pi/m}{\ln(\varepsilon_B/E)} + i\pi
\]

Many-body: finite density medium scattering

\[
T^{-1}(\mathbf{q}, \omega) = T_0^{-1}(\omega + i0 + \mu_{\uparrow} + \mu_{\downarrow} - \varepsilon_{\mathbf{q}}/2) + \int \frac{d^2k}{(2\pi)^2} \frac{n_F(\varepsilon_{\mathbf{k}} - \mu_{\uparrow}) + n_F(\varepsilon_{\mathbf{k}+\mathbf{q}} - \mu_{\downarrow})}{\omega + i0 + \mu_{\uparrow} + \mu_{\downarrow} - \varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{q}}}
\]

We find compact solution

\[
T(\mathbf{q}, \omega) = T_0 \left( \frac{1}{2} z \pm \frac{1}{2} \sqrt{(z - \varepsilon_{\mathbf{q}})^2 - 4\varepsilon_F\varepsilon_{\mathbf{q}}} \right)
\]

\[
z = \omega + i0 - \varepsilon_F + \mu_{\downarrow}
\]
Polaron self-energy

\[ \Sigma_{\downarrow}(\mathbf{p}, \omega) = \int_{k < k_F} \frac{d^2k}{(2\pi)^2} T(\mathbf{k} + \mathbf{p}, \varepsilon_{\mathbf{p}} - \mu_{\uparrow} + \omega) \]

**step 3: polaron spectral function**

\[ A_{\downarrow}(\mathbf{p}, \omega) = -2 \text{Im} \frac{1}{\omega + i0 + \mu_{\downarrow} - \varepsilon_{\mathbf{p}} - \Sigma_{\downarrow}(\mathbf{p}, \omega)} \]

contains full information about energy spectrum, quasiparticle weights, decay rates...
Polaron spectral function

Schmidt, Enss, Pietilä & Demler 2012

repulsive polaron

attractive polaron
Polaron spectral function

\begin{align*}
A_\downarrow(p, \omega) \\
A_\downarrow(p = 0, \omega)
\end{align*}

Schmidt, Enss, Pietilä & Demler 2012
Cambridge experiment

Attractive and repulsive Fermi polaron in two dimensions

LETTER
31 MAY 2012 | VOL 485 | NATURE | 619
doi:10.1038/nature11151

Attractive and repulsive Fermi polaron in two dimensions

Marco Koschorreck*, Daniel Pertot*, Enrico Vogt, Bernd Fröhlich, Michael Feld & Michael Köhl

energy spectrum

lifetime of rep. polaron

effective mass

confirms our prediction
BKT-BCS crossover in 2D Fermi gas

Spectral fct & pseudogap in balanced 2D gas

Bauer, Parish & Enss, PRL 112, 135302 (2014)
Conclusion & outlook

- full frequency/momentum dependence of self-energy and Cooper vertex
  - full self-energy feedback yields transition as accurately as diagMC
  - beyond quasiparticle picture, large anomalous dimension
  - resolve higher excited states, decay rates, power laws

- RG flow of spectral functions
  - see how many-body correlations emerge in spectrum

- predicted repulsive polaron, confirmed in experiment
  - inverse RF protocol to detect short-lived repulsive state

- outlook
  - fRG for 2D polaron to include self-energy feedback
  - interaction between impurities, finite impurity density
  - dynamical and transport processes

Schmidt & Enss, PRA 83, 063620 (2011)
Schmidt, Enss, Pietilä & Demler, PRA 85, 021602(R) (2012)
Bauer, Parish & Enss, PRL 112, 135302 (2014)