

*Ultracold fermions with three components:
Cooper-pairs, molecules and trions*

Stefan Flörchinger (CERN)

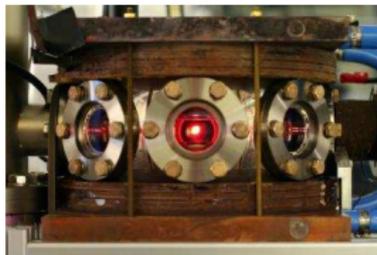
work done in Heidelberg in collaboration with

Richard Schmidt, Sergej Moroz and Christof Wetterich

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Cold atoms in a laser trap



- typical density
 - particle number $N = 10^6$
 - cloud volume $V = 10^{-9} \text{ cm}^3$
 - interparticle distance $d = 0.1 \mu\text{m}$
- typical temperature
 - temperature $T = 10^{-6} \text{ K}$
 - thermal de-Broglie length $\lambda_T = 1 \mu\text{m}$
- typical interaction parameters
 - interaction range $\lambda_{\text{vdW}} = 10^{-4} \mu\text{m}$
 - scattering length $a = (0 \dots \infty) \mu\text{m}$

Theoretical challenge

- quantum effects are important
- many particles / nonzero density
- nonzero temperature
- large interaction strength
- possibly non-equilibrium dynamics
- similar problems as in QCD matter: Heavy ion collisions, Neutron stars, ...
- advantage for cold quantum gases: very well controlled, experiments on a table-top

Complexity problem with strong interaction

- Strong interactions lead to strong effects. Qualitative features of a theory can change!
- Physical properties can become universal! Microscopic details become irrelevant.
- Strong interaction effects lead to fast Equilibration: Dynamics can be described by Close-to-Equilibrium methods.

Fermi gases with different physics

- 1 component Fermi gas - no s-wave interaction
- 2 component Fermi gas - BCS-BEC crossover
- 3 component Fermi gas - ??

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Fermi gases with different physics

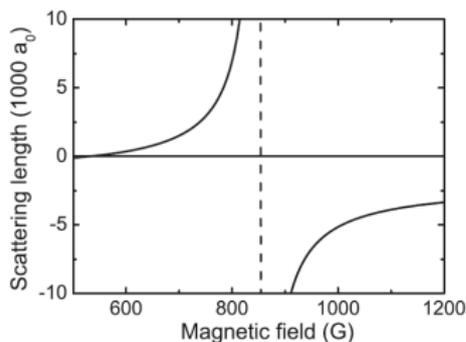
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 - On the lattice: Trion formation
(Rapp, Zarand, Honerkamp, and Hofstetter, PRL **98**, 160405 (2007),
Rapp, Hofstetter and Zarand, PRB **77**, 144520 (2008).)

Single component Fermi gas

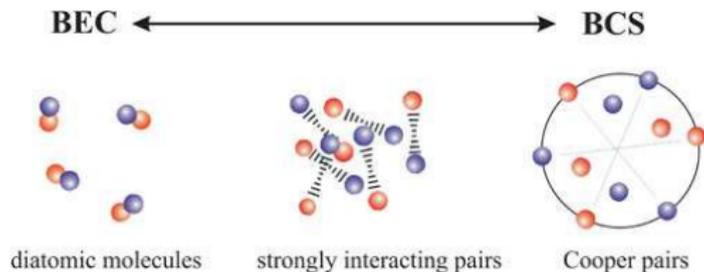
- Most properties of dilute ultracold quantum gases are dominated by s -wave interactions.
- For identical fermions (only one spin component) wavefunction has to be antisymmetric in position space.
- s -wave interaction suppressed by Pauli blocking.
- Behaves like ideal Fermi gas in many respects.

Two component Fermi gas

- Two spin (or hyperfine-spin) components ψ_1 and ψ_2 .
- For equal mass $M_{\psi_1} = M_{\psi_2}$, density $n_{\psi_1} = n_{\psi_2}$ etc. SU(2) spin symmetry
- s -wave interaction measured by scattering length a .
- Repulsive microscopic interaction: Landau Fermi liquid.
- Attractive interaction leads to many interesting effects!
- Scattering length can be tuned experimentally with Feshbach resonances.



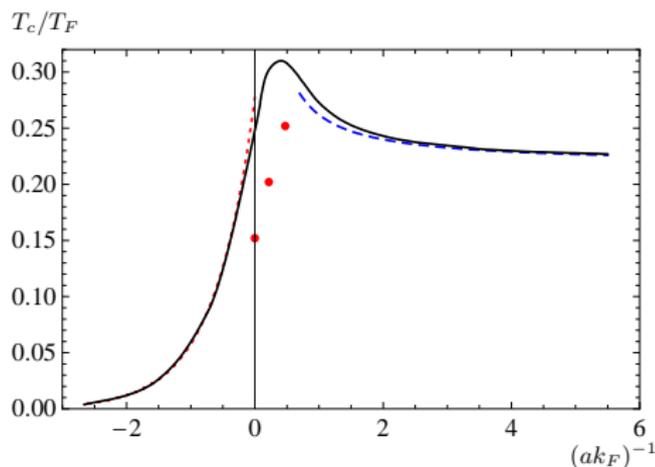
BCS-BEC Crossover



- Small negative scattering length $a \rightarrow 0_-$
 - Formation of Cooper pairs in momentum space
 - BCS-theory valid
 - superfluid at small temperatures
 - order parameter $\varphi \sim \psi_1\psi_2$
- Small positive scattering length $a \rightarrow 0_+$
 - Formation of dimers or molecules in position space
 - Bosonic mean field theory valid
 - superfluid at small temperatures
 - order parameter $\varphi \sim \psi_1\psi_2$
- Between both limits: Continuous *BCS-BEC Crossover*
 - scattering length becomes large: strong interaction
 - superfluid, order parameter $\varphi \sim \psi_1\psi_2$ at small T

Phase diagram BCS-BEC Crossover

- Crossover best parameterized by $c^{-1} = (ak_F)^{-1}$.
- Different methods give phase diagram
- Result of renormalization group study:



(Floerchinger, Scherer and Wetterich, PRA **81**, 063619 (2010).)

- More complicated phase diagram with population imbalance

Three component Fermi gas

- For equal masses, densities etc. global SU(3) symmetry

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \rightarrow u \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}, \quad u \in \text{SU}(3).$$

Similar to flavor symmetry in the Standard model!

- For small scattering length $|a| \rightarrow 0$
 - BCS ($a < 0$) or BEC ($a > 0$) superfluidity at small T.
 - order parameter is conjugate triplet $\bar{\mathbf{3}}$ under SU(3)

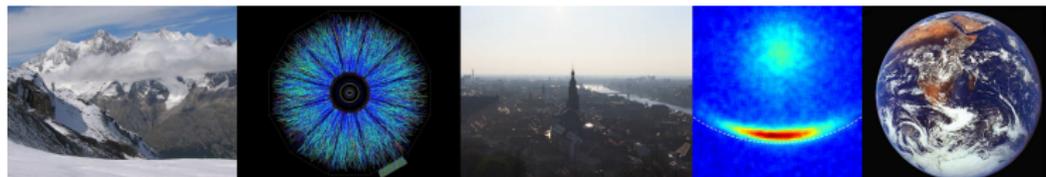
$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} \sim \begin{pmatrix} \psi_2\psi_3 \\ \psi_3\psi_1 \\ \psi_1\psi_2 \end{pmatrix}.$$

- SU(3) symmetry is broken spontaneously for $\varphi \neq 0$.
- What happens for large $|a|$?

How should we describe the world?



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- There are many different phenomena.

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- We look for a unified description.

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QUANTUM FIELD THEORY.

Classical field theory

- Describes electro-magnetic fields, waves, ... ($\hbar \rightarrow 0$).
- Crucial object: classical action

$$S[\phi] = \int dt \int d^d x \mathcal{L}(\phi, \partial_t \phi, \vec{\nabla} \phi, \dots)$$

- Classical field equations from $\frac{\delta S}{\delta \phi} = 0$.
- Symmetries of S lead to conserved currents.
- All physical observables are easily obtained from S .

Quantum field theory

- Describes electrons, atoms, quarks, gluons, protons,...
...and cold quantum gases
- Crucial object: quantum effective action

$$\Gamma[\phi] = \int dt \int d^d x U(\phi) + \dots$$

- Quantum field equations from $\frac{\delta\Gamma}{\delta\phi} = 0$
- Symmetries of Γ lead to conserved currents
- All physical observables are easily obtained from Γ
- Γ is generating functional of 1-PI Feynman diagrams and depends on external parameters like T, μ , or \vec{B}

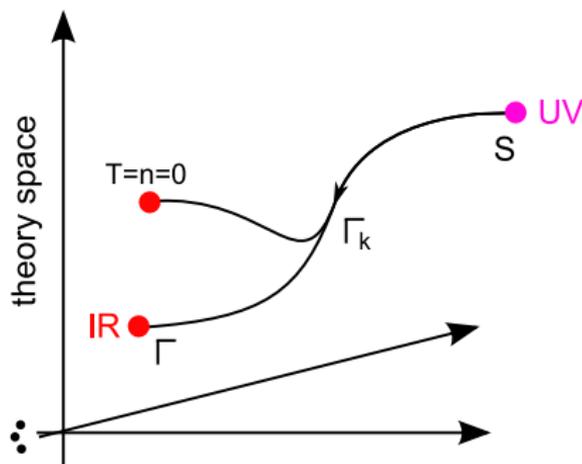
The renormalization group

- very important in modern understanding of quantum field theory
- describes how (effective) theories evolve to other (effective) theories at smaller energy/momentum scales
- makes a simple, efficient and intuitive description of complex phenomena possible

How do we obtain the quantum effective action $\Gamma[\phi]$?

Idea of functional renormalization: $\Gamma[\phi] \rightarrow \Gamma_k[\phi]$

- k is additional infrared cutoff parameter.
- $\Gamma_k[\phi] \rightarrow \Gamma[\phi]$ for $k \rightarrow 0$.
- $\Gamma_k[\phi] \rightarrow S[\phi]$ for $k \rightarrow \infty$.
- Dependence on T, μ or \vec{B} trivial for $k \rightarrow \infty$.



How the flowing action flows

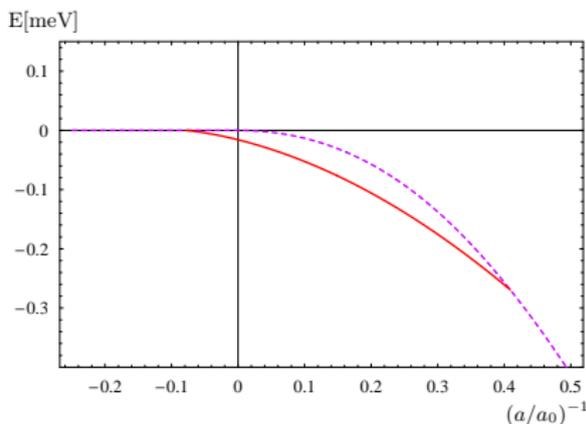
Simple and exact flow equation (WETTERICH 1993)

$$\partial_k \Gamma_k[\phi] = \frac{1}{2} \text{STr} \left(\Gamma_k^{(2)}[\phi] + R_k \right)^{-1} \partial_k R_k.$$

- Differential equation for a functional.
- For most cases not solvable exactly.
- Approximate solutions can be found from Truncations.
 - Ansatz for Γ_k with a finite number of parameters.
 - Derive ordinary differential equations for this parameters or couplings from the flow equation for Γ_k .
 - Solve these equations numerically.

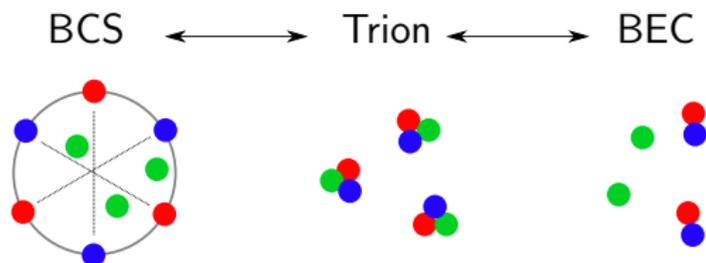
Binding energies

- Vacuum limit $T \rightarrow 0$, $n \rightarrow 0$.



- Binding energy per atom for
 - molecule or dimer φ (dashed line)
 - trion or trimer χ (solid line)
- For large scattering length a trion is energetically favorable!
- Three-body bound state even for $a < 0$.

Quantum phase diagram

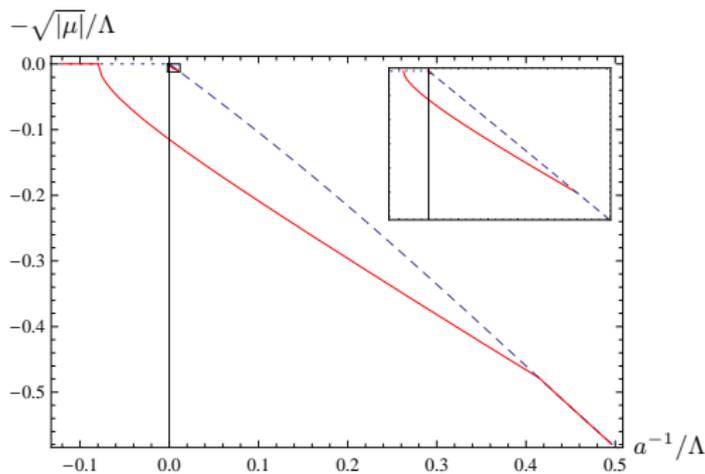


- BCS-Trion-BEC transition

(Floerchinger, Schmidt, Moroz and Wetterich, PRA **79**, 013603 (2009)).

- $a \rightarrow 0_-$: Cooper pairs, $SU(3) \times U(1) \rightarrow SU(2) \times U(1)$.
 - $a \rightarrow 0_+$: BEC of molecules, $SU(3) \times U(1) \rightarrow SU(2) \times U(1)$.
 - $a \rightarrow \pm\infty$: Trion phase, $SU(3)$ unbroken.
- Quantum phase transitions
 - from BCS to Trion phase
 - from Trion to BEC phase.

Efimov effect



- Self-similarity in energy spectrum.
- Efimov trimers become more and more shallow. At $a = \infty$

$$E_{n+1} = e^{-2\pi/s_0} E_n.$$

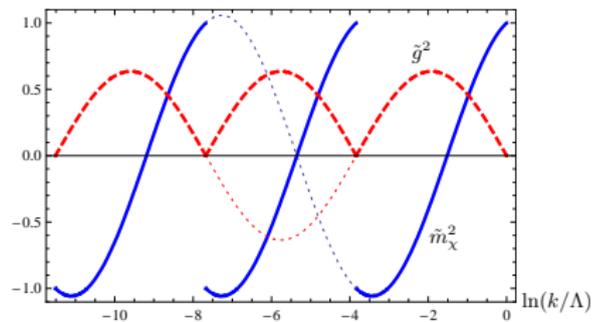
- Simple truncation: $s_0 \approx 0.82$.
- Advanced truncation: $s_0 \approx 1.006$ (exact result)
(Moroz, Floerchinger, Schmidt and Wetterich, PRA **79**, 042705 (2009).)

Renormalization group limit cycle

- For $\mu = 0$ and $a^{-1} = 0$ flow equations for rescaled couplings

$$k \frac{\partial}{\partial k} \begin{pmatrix} \tilde{g}^2 \\ \tilde{m}_\chi^2 \end{pmatrix} = \begin{pmatrix} 7/25 & -13/25 \\ 36/25 & 7/25 \end{pmatrix} \begin{pmatrix} \tilde{g}^2 \\ \tilde{m}_\chi^2 \end{pmatrix}.$$

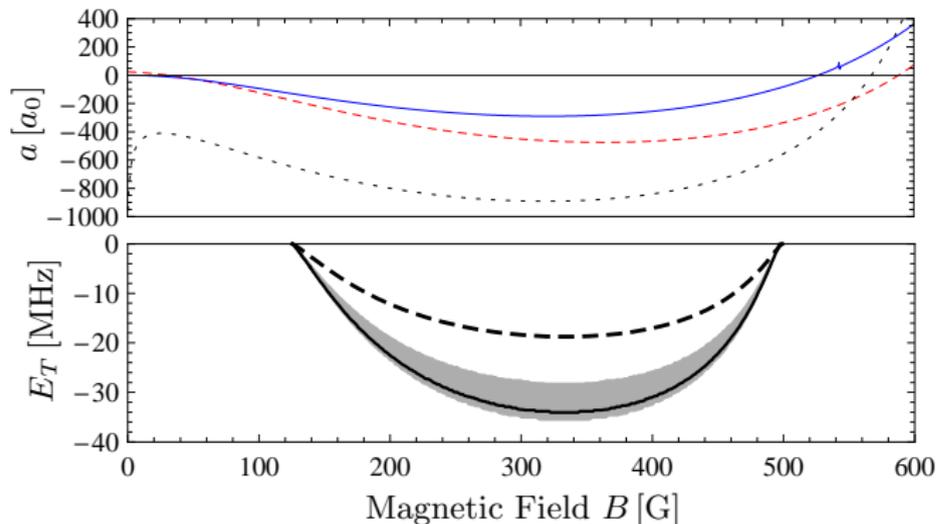
- Solution is log-periodic in scale.



- Every zero-crossing of \tilde{m}_χ^2 corresponds to a new bound state.
- For $\mu \neq 0$ or $a^{-1} \neq 0$ limit cycle scaling stops at some scale k . Only finite number of Efimov trimers.

Contact to experiments

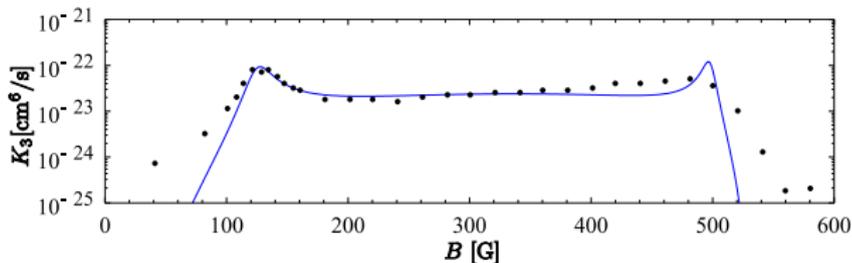
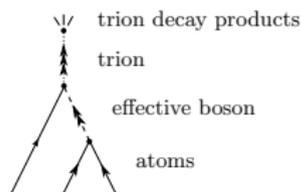
- Model can be generalized to case without SU(3) symmetry (Floerchinger, Schmidt and Wetterich, PRA A **79**, 053633 (2009)).
- Hyperfine states of ${}^6\text{Li}$ have large scattering lengths.



- Binding energies might be measured using RF-spectroscopy.
- Lifetime is quite short $\sim 10\text{ns}$.

Three-body loss rate

- Three-body loss rate measured experimentally (Ottenstein et al., PRL **101**, 203202 (2008); Huckans et al., PRL **102**, 165302 (2009))



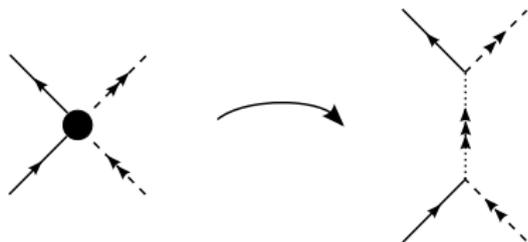
- Trion may decay into deeper bound molecule states
- Calculate B -field dependence of loss process above.
- Left resonance (position and width) fixes model parameters.
- Form of curve for large B is prediction.
- Similar results obtained by other methods (Braaten, Hammer, Kang and Platter, PRL **103**, 073202 (2009); Naidon and Ueda, PRL **103**, 073203 (2009).)

Conclusions

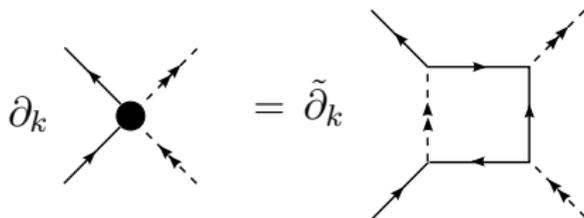
- Physics of ultracold fermions with three components quite interesting.
- Few-body physics (Efimov effect) well described by functional renormalization.
- Many-body physics shows parallels to QCD
 - BCS – “Color” – superfluidity for small negative a .
 - Trion – “Hadron” – phase for large $|a|$.
 - BEC – “Color” – superfluidity for small positive a .
- Very nice playground for renormalization group methods.
- Experimental tests seem possible.

“Refermionization”

- Trion field is introduced via a generalized Hubbard-Stratonovich transformation



- Fermion-boson coupling is regenerated by the flow



- Express this again by trion exchange
(Gies and Wetterich, PRD **65**, 065001 (2002),
Floerchinger and Wetterich, PLB **680**, 371 (2009).)

Truncations

For many purposes *derivative expansions* are suitable approximations. For example we use for the BCS-BEC Crossover

$$\Gamma_k = \int_{\tau, \vec{x}} \left\{ \psi^\dagger (\partial_\tau - \vec{\nabla}^2 - \mu) \psi + \varphi^* (Z_\varphi \partial_\tau - A_\varphi \frac{1}{2} \vec{\nabla}^2) \varphi - h(\varphi^* \psi_1 \psi_2 + h.c.) + \frac{1}{2} \lambda_\psi (\psi^\dagger \psi)^2 + U_k(\varphi^* \varphi, \mu) \right\}$$

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- The coefficients Z_φ , A_φ , λ_ψ , h and the effective potential U_k are scale-dependent.
- The effective potential U_k contains no derivatives - describes homogeneous fields.
- Wave-function renormalization and self-energy corrections for fermions can be included as well.

The effective potential

- We use a Taylor expansion around the minimum ρ_0

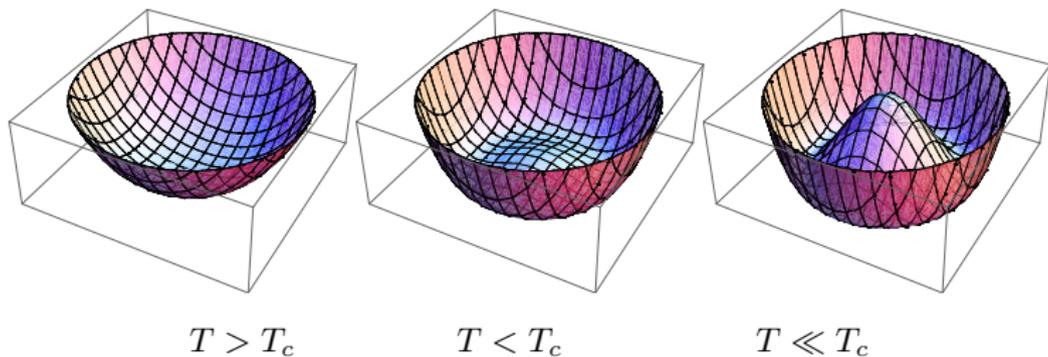
$$U_k(\varphi^* \varphi) = -p + m^2 (\varphi^* \varphi - \rho_0) + \frac{1}{2} \lambda (\varphi^* \varphi - \rho_0)^2.$$

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- Symmetry breaking:



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- Typical flow:

