

# *Entanglement dynamics of hadronization*

Stefan Flörchinger (Heidelberg U.)

Quantum Entanglement at Collider Energies, CFNS Stony Brook,  
10/09/2018.

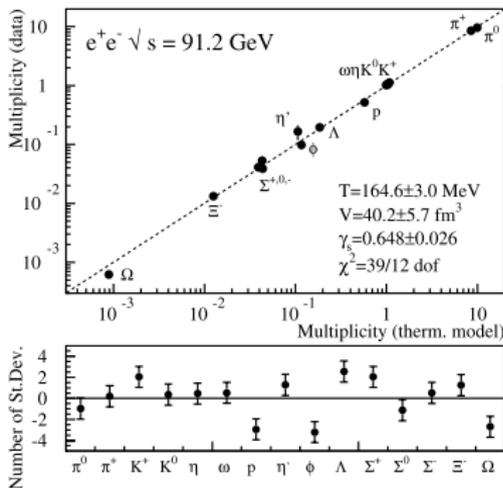


UNIVERSITÄT  
HEIDELBERG  
ZUKUNFT  
SEIT 1386



## The thermal model puzzle

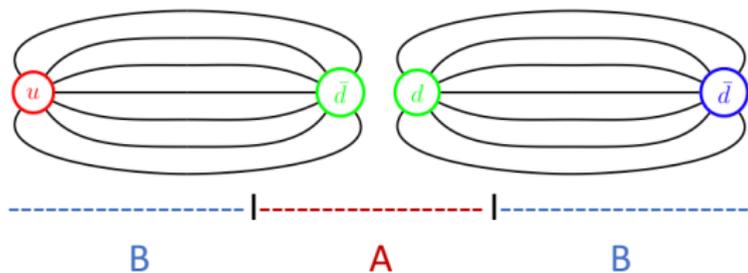
- elementary particle collision experiments such as  $e^+ e^-$  collisions show some thermal-like features
- particle multiplicities well described by thermal model



[Becattini, Casterina, Milov & Satz, EPJC 66, 377 (2010)]

- conventional thermalization by collisions unlikely
- more thermal-like features difficult to understand in PYTHIA [Fischer, Sjöstrand (2017)]
- alternative explanations needed

## *QCD strings*



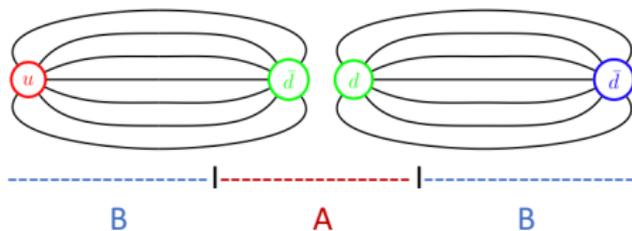
- particle production from QCD strings
- Lund string model (e. g. PYTHIA)
- different regions in a string are entangled
- subinterval  $A$  is described by reduced density matrix

$$\rho_A = \text{Tr}_B \rho$$

- reduced density matrix is of mixed state form
- could this lead to thermal-like effects?

## Entropy and entanglement

- consider a split of a quantum system into two  $A + B$



- reduced density operator for system  $A$

$$\rho_A = \text{Tr}_B\{\rho\}$$

- entropy associated with subsystem  $A$ : **entanglement entropy**

$$S_A = -\text{Tr}_A\{\rho_A \ln \rho_A\}$$

- globally pure** state  $S = 0$  can be **locally mixed**  $S_A > 0$
- coherent information**  $I_{B \rangle A} = S_A - S$  can be **positive**

## Microscopic model

- QCD in 1+1 dimensions described by 't Hooft model

$$\mathcal{L} = -\bar{\psi}_j \gamma^\mu (\partial_\mu - ig \mathbf{A}_\mu) \psi_j - m \bar{\psi}_j \psi_j - \frac{1}{2} \text{tr} \mathbf{F}_{\mu\nu} \mathbf{F}^{\mu\nu}$$

- fermionic fields  $\psi_j$  with sums over flavor species  $j = 1, \dots, N_f$
- $SU(N_c)$  gauge fields  $\mathbf{A}_\mu$  with field strength tensor  $\mathbf{F}_{\mu\nu}$
- gluons are not dynamical in two dimensions
- gauge coupling  $g$  has dimension of mass
- non-trivial, interacting theory, cannot be solved exactly
- spectrum of excitations known for  $N_c \rightarrow \infty$  with  $g^2 N_c$  fixed  
[ 't Hooft (1974) ]

## Schwinger model

- QED in 1+1 dimension

$$\mathcal{L} = -\bar{\psi}_j \gamma^\mu (\partial_\mu - iqA_\mu) \psi_j - m \bar{\psi}_j \psi_j - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- geometric confinement
- U(1) charge related to string tension  $q = \sqrt{2\sigma}$
- for single fermion one can **bosonize theory** exactly  
[Coleman, Jackiw, Susskind (1975)]

$$S = \int d^2x \sqrt{g} \left\{ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} M^2 \phi^2 - \frac{m q e^\gamma}{2\pi^{3/2}} \cos(2\sqrt{\pi}\phi + \theta) \right\}$$

- Schwinger bosons are dipoles  $\phi \sim \bar{\psi}\psi$
- scalar mass related to U(1) charge by  $M = q/\sqrt{\pi} = \sqrt{2\sigma/\pi}$
- massless Schwinger model  $m = 0$  leads to free bosonic theory

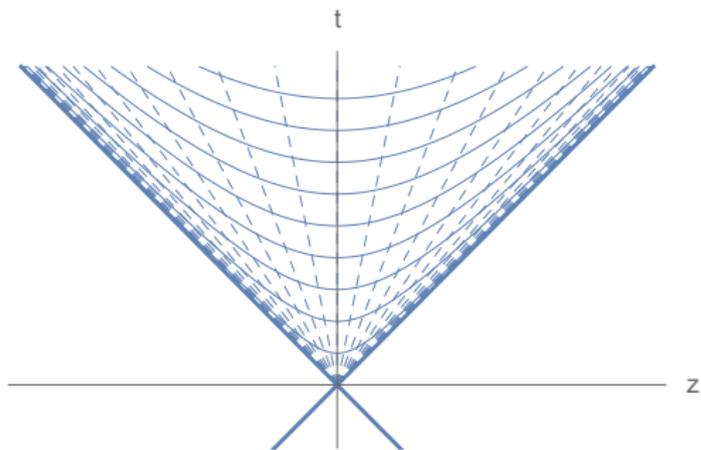
## Transverse coordinates

- so far dynamics strictly confined to 1+1 dimensions
- transverse coordinates may fluctuate, can be described by Nambu-Goto action ( $h_{\mu\nu} = \partial_\mu X^m \partial_\nu X_m$ )

$$S_{\text{NG}} = \int d^2x \sqrt{-\det h_{\mu\nu}} \{-\sigma + \dots\}$$
$$\approx \int d^2x \sqrt{g} \left\{ -\sigma - \frac{\sigma}{2} g^{\mu\nu} \partial_\mu X^i \partial_\nu X^i + \dots \right\}$$

- two additional, massless, bosonic degrees of freedom corresponding to transverse coordinates  $X^i$  with  $i = 1, 2$

## Expanding string solution 1



- external quark-anti-quark pair on trajectories  $z = \pm t$
- coordinates: Bjorken time  $\tau = \sqrt{t^2 - z^2}$ , rapidity  $\eta = \text{arctanh}(z/t)$
- metric  $ds^2 = -d\tau^2 + \tau^2 d\eta^2$
- symmetry with respect to longitudinal boosts  $\eta \rightarrow \eta + \Delta\eta$

## Expanding string solution 2

- Schwinger boson field depends only on  $\tau$

$$\bar{\phi} = \bar{\phi}(\tau)$$

- equation of motion

$$\partial_\tau^2 \bar{\phi} + \frac{1}{\tau} \partial_\tau \bar{\phi} + M^2 \bar{\phi} = 0.$$

- Gauss law: electric field  $E = q\phi/\sqrt{\pi}$  must approach the U(1) charge of the external quarks  $E \rightarrow q_e$  for  $\tau \rightarrow 0_+$

$$\bar{\phi}(\tau) \rightarrow \frac{\sqrt{\pi}q_e}{q} \quad (\tau \rightarrow 0_+)$$

- solution of equation of motion [Loshaj, Kharzeev (2011)]

$$\bar{\phi}(\tau) = \frac{\sqrt{\pi}q_e}{q} J_0(M\tau)$$

## *Gaussian states*

- theories with quadratic action often have Gaussian density matrix
- fully characterized by field expectation values

$$\bar{\phi}(x) = \langle \phi(x) \rangle, \quad \bar{\pi}(x) = \langle \pi(x) \rangle$$

and connected two-point correlation functions, e. g.

$$\langle \phi(x)\phi(y) \rangle_c = \langle \phi(x)\phi(y) \rangle - \bar{\phi}(x)\bar{\phi}(y)$$

- if  $\rho$  is Gaussian, also reduced density matrix  $\rho_A$  is Gaussian

## Functional representation

- Schrödinger functional representation of quantum field theory
- pure state  $|\Psi\rangle$  has functional

$$\Psi[\phi] = \langle \phi | \Psi \rangle$$

with field “positions”  $\phi_n$

- density matrix

$$\rho[\phi_+, \phi_-] = \langle \phi_+ | \rho | \phi_- \rangle$$

- fields and conjugate momenta

$$\phi_m, \quad \pi_m = -i \frac{\delta}{\delta \phi_m}$$

- canonical commutation relation

$$[\phi_m, \pi_n] = i\delta_{mn}$$

## *Symplectic transformations*

- combined field

$$\chi = \begin{pmatrix} \phi \\ \pi^* \end{pmatrix}, \quad \chi^* = \begin{pmatrix} \phi^* \\ \pi \end{pmatrix}$$

- commutation relation as symplectic metric

$$[\chi_m, \chi_n^*] = \Omega_{mn}, \quad \Omega = \Omega^\dagger = \begin{pmatrix} 0 & i\mathbb{1} \\ -i\mathbb{1} & 0 \end{pmatrix},$$

- symplectic transformations  $S_{mn}$

$$\chi_m \rightarrow S_{mn}\chi_n, \quad \chi_m^* \rightarrow \chi_n^*(S^\dagger)_{nm}, \quad S\Omega S^\dagger = \Omega,$$

have unitary representations on Gaussian states

## Williamson's theorem and entropy

- Covariance matrix

$$\Delta_{mn} = \frac{1}{2} \langle \chi_m \chi_n^* + \chi_n^* \chi_m \rangle_c$$

transforms as

$$\Delta \rightarrow S \Delta S^\dagger \neq S \Delta S^{-1}$$

- Williamson's theorem: can find  $S_{mn}$  such that

$$\Delta \rightarrow \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_1, \lambda_2, \dots),$$

- symplectic eigenvalues  $\lambda_j > 0$
- Heisenberg's uncertainty principle:  $\lambda_j \geq 1/2$
- von Neumann entropy

$$S = \sum_j \left\{ \left( \lambda_j + \frac{1}{2} \right) \ln \left( \lambda_j + \frac{1}{2} \right) - \left( \lambda_j - \frac{1}{2} \right) \ln \left( \lambda_j - \frac{1}{2} \right) \right\}$$

- pure state:  $\lambda_j = 1/2$ ,  $S = 0$

## Entanglement entropy for Gaussian state

- entanglement entropy of Gaussian state in region  $A$   
[Berges, Floerchinger, Venugopalan, JHEP 1804 (2018) 145]

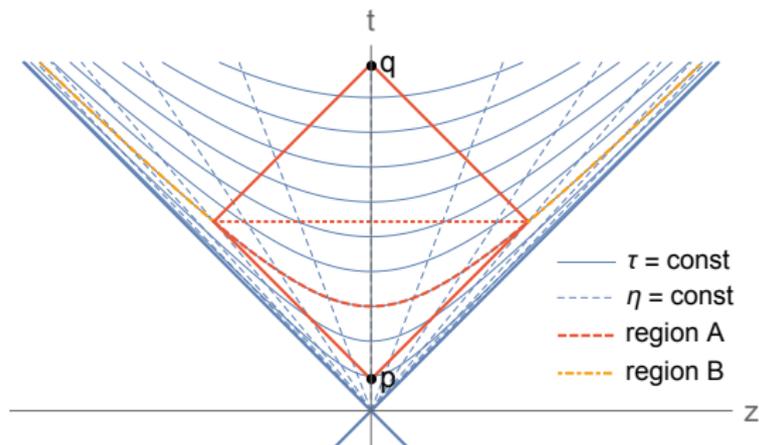
$$S_A = \frac{1}{2} \text{Tr}_A \{ D \ln(D^2) \}$$

- operator trace over region  $A$  only
- matrix of correlation functions

$$D(x, y) = \begin{pmatrix} -i\langle \phi(x)\pi(y) \rangle_c & i\langle \phi(x)\phi(y) \rangle_c \\ -i\langle \pi(x)\pi(y) \rangle_c & i\langle \pi(x)\phi(y) \rangle_c \end{pmatrix}$$

- involves connected correlation functions of field  $\phi(x)$  and canonically conjugate momentum field  $\pi(x)$
- expectation value  $\bar{\phi}$  does not appear explicitly
- coherent states and vacuum have equal entanglement entropy  $S_A$

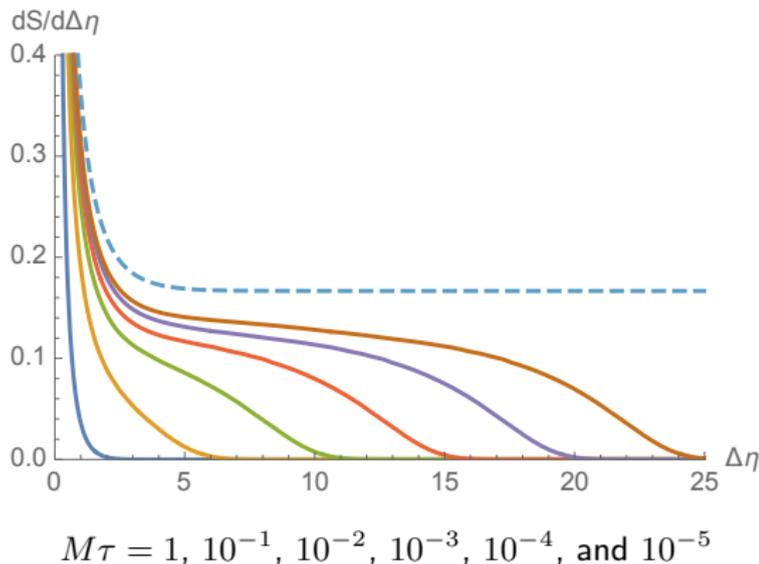
## Rapidity interval



- consider rapidity interval  $(-\Delta\eta/2, \Delta\eta/2)$  at fixed Bjorken time  $\tau$
- entanglement entropy does not change by unitary time evolution with endpoints kept fixed
- can be evaluated equivalently in interval  $\Delta z = 2\tau \sinh(\Delta\eta/2)$  at fixed time  $t = \tau \cosh(\Delta\eta/2)$
- need to solve eigenvalue problem with correct **boundary conditions**

## Bosonized massless Schwinger model

- entanglement entropy understood numerically for free massive scalars [Casini, Huerta (2009)]
- entanglement entropy density  $dS/d\Delta\eta$  for bosonized massless Schwinger model ( $M = \frac{g}{\sqrt{\pi}}$ )



## Conformal limit

- For  $M\tau \rightarrow 0$  one has conformal field theory limit  
[Holzhey, Larsen, Wilczek (1994)]

$$S(\Delta z) = \frac{c}{3} \ln(\Delta z/\epsilon) + \text{constant}$$

with small length  $\epsilon$  acting as UV cutoff.

- Here this implies

$$S(\tau, \Delta\eta) = \frac{c}{3} \ln(2\tau \sinh(\Delta\eta/2)/\epsilon) + \text{constant}$$

- Conformal charge  $c = 1$  for free massless scalars or Dirac fermions.
- Additive constant not universal but entropy density is

$$\begin{aligned} \frac{\partial}{\partial \Delta\eta} S(\tau, \Delta\eta) &= \frac{c}{6} \coth(\Delta\eta/2) \\ &\rightarrow \frac{c}{6} \quad (\Delta\eta \gg 1) \end{aligned}$$

- Entropy becomes extensive in  $\Delta\eta$  !

## *Universal entanglement entropy density*

- for very early times “Hubble” expansion rate dominates over masses and interactions

$$H = \frac{1}{\tau} \gg M = \frac{q}{\sqrt{\pi}}, m$$

- theory dominated by free, massless fermions
- universal entanglement entropy density

$$\frac{dS}{d\Delta\eta} = \frac{c}{6}$$

with conformal charge  $c$

- for QCD in 1+1 D (gluons not dynamical, no transverse excitations)

$$c = N_c \times N_f$$

- from fluctuating transverse coordinates (Nambu-Goto action)

$$c = N_c \times N_f + 2 \approx 9 + 2 = 11$$

## Temperature and entanglement entropy

- for conformal fields, entanglement entropy has also been calculated at non-zero temperature.
- for static interval of length  $L$  [Korepin (2004); Calabrese, Cardy (2004)]

$$S(T, l) = \frac{c}{3} \ln \left( \frac{1}{\pi T \epsilon} \sinh(\pi L T) \right) + \text{const}$$

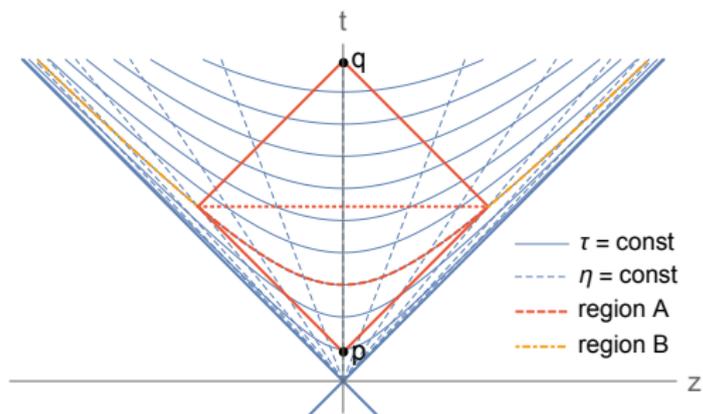
- compare this to our result in expanding geometry

$$S(\tau, \Delta\eta) = \frac{c}{3} \ln \left( \frac{2\tau}{\epsilon} \sinh(\Delta\eta/2) \right) + \text{const}$$

- expressions agree for  $L = \tau\Delta\eta$  (with metric  $ds^2 = -d\tau^2 + \tau^2 d\eta^2$ ) and time-dependent temperature

$$T = \frac{1}{2\pi\tau}$$

# Modular or entanglement Hamiltonian 1



- conformal field theory
- hypersurface  $\Sigma$  with boundary on the intersection of two light cones
- reduced density matrix [Casini, Huerta, Myers (2011), Arias, Blanco, Casini, Huerta (2017), see also Candelas, Dowker (1979)]

$$\rho_A = \frac{1}{Z_A} e^{-K}, \quad Z_A = \text{Tr} e^{-K}$$

- modular or entanglement Hamiltonian  $K$

## Modular or entanglement Hamiltonian 2

- modular or entanglement Hamiltonian is **local expression**

$$K = \int_{\Sigma} d\Sigma_{\mu} \xi_{\nu}(x) T^{\mu\nu}(x).$$

- energy-momentum tensor  $T^{\mu\nu}(x)$  of excitations
- vector field

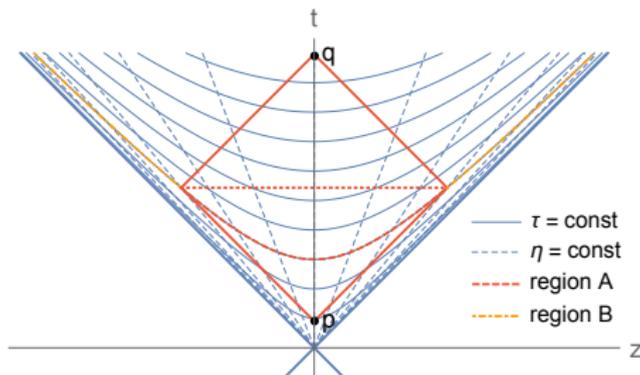
$$\xi^{\mu}(x) = \frac{2\pi}{(q-p)^2} [(q-x)^{\mu}(x-p)(q-p) \\ + (x-p)^{\mu}(q-x)(q-p) - (q-p)^{\mu}(x-p)(q-x)]$$

end point of future light cone  $q$ , starting point of past light cone  $p$

- inverse temperature and fluid velocity

$$\xi^{\mu}(x) = \beta^{\mu}(x) = \frac{u^{\mu}(x)}{T(x)}$$

## Modular or entanglement Hamiltonian 3



- for  $\Delta\eta \rightarrow \infty$ : fluid velocity in  $\tau$ -direction,  $\tau$ -dependent temperature

$$T(\tau) = \frac{\hbar}{2\pi\tau}$$

- **Entanglement between different rapidity intervals alone leads to local thermal density matrix at very early times !**
- Hawking-Unruh temperature in Rindler wedge  $T(x) = \hbar c/(2\pi x)$

## Alternative derivation: mode functions

- fluctuation field  $\varphi = \phi - \bar{\phi}$  has equation of motion

$$\partial_\tau^2 \varphi(\tau, \eta) + \frac{1}{\tau} \partial_\tau \varphi(\tau, \eta) + \left( M^2 - \frac{1}{\tau^2} \frac{\partial^2}{\partial \eta^2} \right) \varphi(\tau, \eta) = 0$$

- solution in terms of plane waves

$$\varphi(\tau, \eta) = \int \frac{dk}{2\pi} \{ a(k) f(\tau, |k|) e^{ik\eta} + a^\dagger(k) f^*(\tau, |k|) e^{-ik\eta} \}$$

- mode functions as Hankel functions

$$f(\tau, k) = \frac{\sqrt{\pi}}{2} e^{\frac{k\pi}{2}} H_{ik}^{(2)}(M\tau)$$

or alternatively as Bessel functions

$$\bar{f}(\tau, k) = \frac{\sqrt{\pi}}{\sqrt{2 \sinh(\pi k)}} J_{-ik}(M\tau)$$

## Bogoliubov transformation

- mode functions are related

$$\begin{aligned}\bar{f}(\tau, k) &= \alpha(k)f(\tau, k) + \beta(k)f^*(\tau, k) \\ f(\tau, k) &= \alpha^*(k)\bar{f}(\tau, k) - \beta(k)\bar{f}^*(\tau, k)\end{aligned}$$

- creation and annihilation operators are related by

$$\begin{aligned}\bar{a}(k) &= \alpha^*(k)a(k) - \beta^*(k)a^\dagger(k) \\ a(k) &= \alpha(k)\bar{a}(k) + \beta(k)\bar{a}^\dagger(k)\end{aligned}$$

- Bogoliubov coefficients

$$\alpha(k) = \sqrt{\frac{e^{\pi k}}{2 \sinh(\pi k)}} \quad \beta(k) = \sqrt{\frac{e^{-\pi k}}{2 \sinh(\pi k)}}$$

- vacuum  $|\Omega\rangle$  with respect to  $a(k)$  such that  $a(k)|\Omega\rangle = 0$  contains excitations with respect to  $\bar{a}(k)$  such that  $\bar{a}(k)|\Omega\rangle \neq 0$  and *vice versa*

## *Role of different mode functions*

- Hankel functions  $f(\tau, k)$  are superpositions of *positive* frequency modes with respect to Minkowski time  $t$
- Bessel functions  $\bar{f}(\tau, k)$  are superpositions of *positive and negative* frequency modes with respect to Minkowski time  $t$
- at very early time  $1/\tau \gg M, m$  conformal symmetry

$$ds^2 = \tau^2 [-d \ln(\tau)^2 + d\eta^2]$$

- Hankel functions  $f(\tau, k)$  are superpositions of *positive and negative* frequency modes with respect to conformal time  $\ln(\tau)$
- Bessel functions  $\bar{f}(\tau, k)$  are superpositions of *positive* frequency modes with respect to conformal time  $\ln(\tau)$

## Occupation numbers

- Minkowski space coherent states have two-point functions

$$\langle \bar{a}^\dagger(k) \bar{a}(k') \rangle_c = \bar{n}(k) 2\pi \delta(k - k') = |\beta(k)|^2 2\pi \delta(k - k')$$

$$\langle \bar{a}(k) \bar{a}(k') \rangle_c = \bar{u}(k) 2\pi \delta(k + k') = -\alpha^*(k) \beta^*(k) 2\pi \delta(k + k')$$

$$\langle \bar{a}^\dagger(k) \bar{a}^\dagger(k') \rangle_c = \bar{u}^*(k) 2\pi \delta(k + k') = -\alpha(k) \beta(k) 2\pi \delta(k + k')$$

- occupation number

$$\bar{n}(k) = |\beta(k)|^2 = \frac{1}{e^{2\pi k} - 1}$$

- Bose-Einstein distribution with excitation energy  $E = |k|/\tau$  and temperature

$$T = \frac{1}{2\pi\tau}$$

- off-diagonal occupation number  $\bar{u}(k) = -1/(2 \sinh(\pi k))$  make sure we still have pure state

## Local description

- consider now rapidity interval  $(-\Delta\eta/2, \Delta\eta/2)$
- Fourier expansion becomes discrete

$$\varphi(\eta) = \frac{1}{L} \sum_{n=-\infty}^{\infty} \varphi_n e^{in\pi \frac{\eta}{\Delta\eta}}$$

$$\varphi_n = \int_{-\Delta\eta/2}^{\Delta\eta/2} d\eta \varphi(\eta) \frac{1}{2} \left[ e^{-in\pi \frac{\eta}{\Delta\eta}} + (-1)^n e^{in\pi \frac{\eta}{\Delta\eta}} \right]$$

- relation to continuous momentum modes by integration kernel

$$\varphi_n = \int \frac{dk}{2\pi} \sin\left(\frac{k\Delta\eta}{2} - \frac{n\pi}{2}\right) \left[ \frac{1}{k - \frac{n\pi}{\Delta\eta}} + \frac{1}{k + \frac{n\pi}{\Delta\eta}} \right] \varphi(k)$$

- local density matrix determined by correlation functions

$$\langle \varphi_n \rangle, \quad \langle \pi_n \rangle, \quad \langle \varphi_n \varphi_m \rangle_c, \quad \text{etc.}$$

## *Emergence of locally thermal state*

- mode functions at early time

$$\bar{f}(\tau, k) = \frac{1}{\sqrt{2k}} e^{-ik \ln(\tau) - i\theta(k, M)}$$

- phase varies strongly with  $k$  for  $M \rightarrow 0$

$$\theta(k, M) = k \ln(M/2) + \arg(\Gamma(1 - ik))$$

- off-diagonal term  $\bar{u}(k)$  have factors strongly oscillating with  $k$

$$\begin{aligned} \langle \varphi(\tau, k) \varphi^*(\tau, k') \rangle_c &= 2\pi \delta(k - k') \frac{1}{|k|} \\ &\times \left\{ \left[ \frac{1}{2} + \bar{n}(k) \right] + \cos [2k \ln(\tau) + 2\theta(k, M)] \bar{u}(k) \right\} \end{aligned}$$

cancel out when going to finite interval !

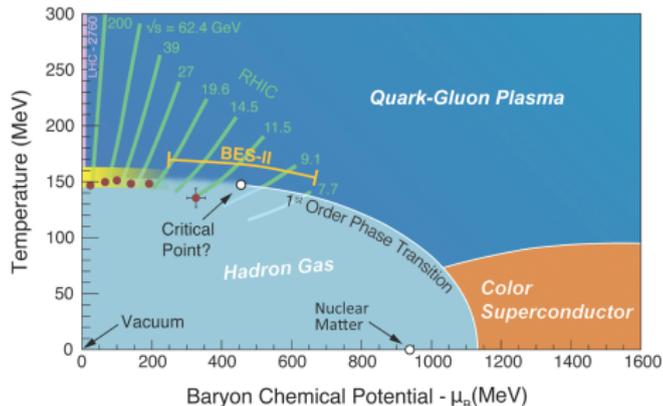
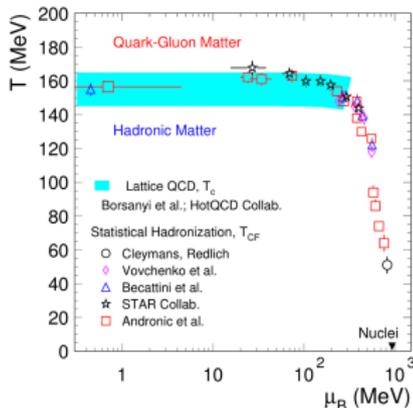
- only Bose-Einstein occupation numbers  $\bar{n}(k)$  remain

## *Physics picture*

- coherent state vacuum at early time contains entangled pairs of quasi-particles with opposite wave numbers
- on finite rapidity interval  $(-\Delta\eta/2, \Delta\eta/2)$  in- and out-flux of quasi-particles with thermal distribution via boundaries
- technically limits  $\Delta\eta \rightarrow \infty$  and  $M\tau \rightarrow 0$  do not commute
  - $\Delta\eta \rightarrow \infty$  for any finite  $M\tau$  gives pure state
  - $M\tau \rightarrow 0$  for any finite  $\Delta\eta$  gives thermal state with  $T = 1/(2\pi\tau)$

## The heavy ion limit

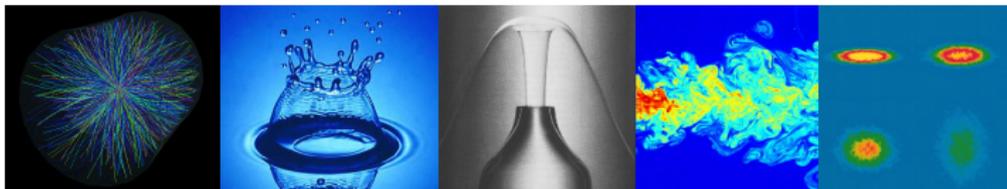
- high energy nuclear collisions create a dense medium close-to thermal equilibrium
- hadron ratios well described by thermal models



[Andronic, Braun-Munzinger, Redlich, Stachel (2017)]

- chemical freeze-out at small  $\mu_B$  close to chiral cross-over [Braun-Munzinger, Stachel, Wetterich (2004)]
- chemical freeze-out at large  $\mu_B$  *not* close to any phase transition [Floerchinger, Wetterich (2012)]
- what precisely triggers chemical freeze-out?

# Fluid dynamics



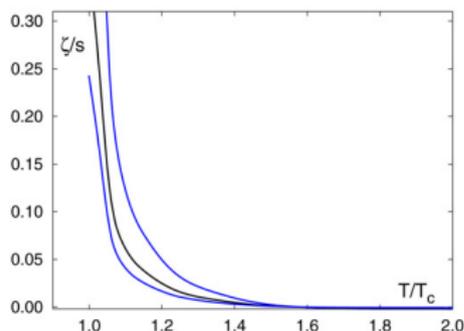
- long distances, long times or strong enough interactions
- quantum fields form a fluid!
- needs **macroscopic** fluid properties
  - equation of state  $p(T, \mu)$
  - shear viscosity  $\eta(T, \mu)$
  - bulk viscosity  $\zeta(T, \mu)$
  - heat conductivity  $\kappa(T, \mu)$
  - relaxation times, ...
- *ab initio* calculation of transport properties difficult but in principle fixed by **microscopic** properties encoded in lagrangian
- **relativistic** fluid dynamics describes high-energy nuclear collisions

## Bulk viscosity

- bulk viscous pressure is negative for expanding fluid

$$\pi_{\text{bulk}} = -\zeta \nabla_{\mu} u^{\mu} < 0$$

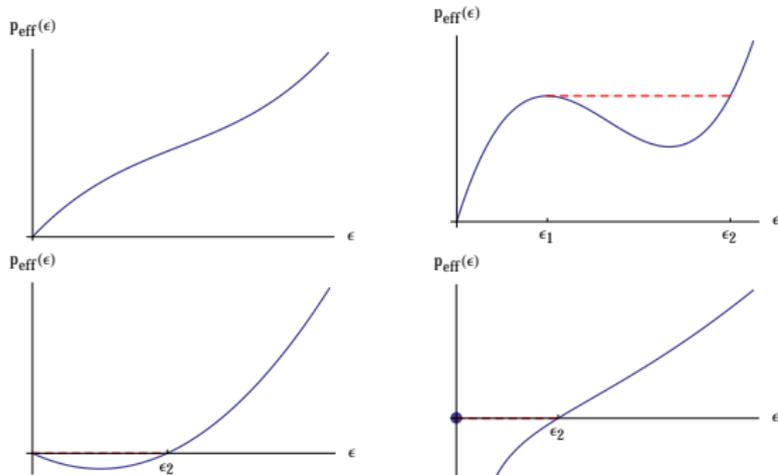
- effective pressure  $p_{\text{eff}} = p + \pi_{\text{bulk}}$
- bulk viscosity grows large at  $T_c$  [Karsch, Kharzeev, Tuchin (2008)]



- *cavitation*: possible instability for  $p_{\text{eff}} < 0$   
[Torrieri & Mishustin (2008), Rajagopal & Tripuraneni (2010), ...]
- what precisely happens at the instability?

# What happens at negative effective pressure?

- stability argument



- if there is a vacuum with  $\epsilon = p_{\text{eff}} = 0$ , phases with  $p_{\text{eff}} < 0$  cannot be mechanically stable (but could be metastable)
- non-equilibrium phase transition could trigger chemical freeze-out

## *Dissipation and entanglement*

- dissipation = entropy productions
- e.g. for relativistic Navier-Stokes

$$\nabla_{\mu} s^{\mu} = \frac{1}{T} [2\eta \sigma_{\rho\lambda} \sigma^{\rho\lambda} + \zeta (\nabla_{\rho} u^{\rho})^2]$$

- can  $s^{\mu}$  be understood as entanglement entropy current ?
- dissipation = entanglement generation
- *aim*: improved understanding of relativistic fluid dynamics based on underlying quantum dynamics

## Conclusions

- rapidity intervals in an expanding string are entangled
- at very early times theory effectively conformal

$$\frac{1}{\tau} \gg m, q$$

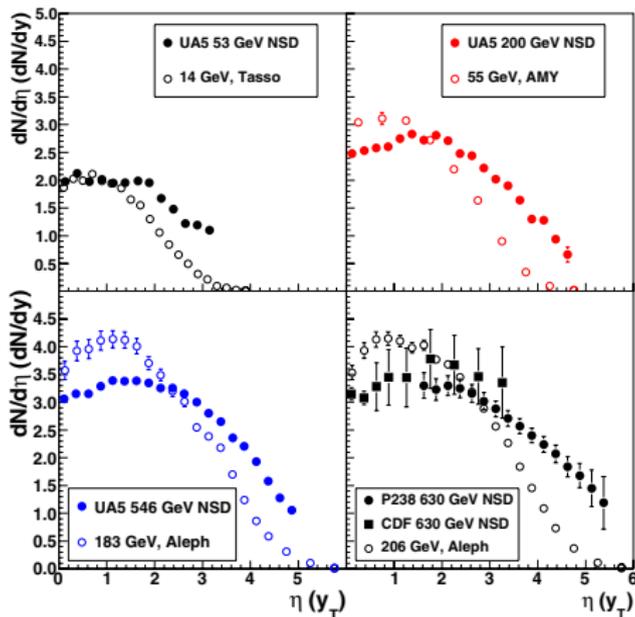
- entanglement entropy extensive in rapidity  $\frac{dS}{d\Delta\eta} = \frac{c}{6}$
- determined by conformal charge  $c = N_c \times N_f + 2$
- reduced density matrix for conformal field theory is of locally thermal form with temperature

$$T = \frac{\hbar}{2\pi\tau}$$

- entanglement could be important ingredient to understand apparent “thermal effects” in  $e^+e^-$  and other collider experiments
- entanglement could also help to better understand relativistic fluids

*Backup*

# Rapidity distribution



[open (filled) symbols:  $e^+e^-$  (pp), Grosse-Oetringhaus & Reygers (2010)]

- rapidity distribution  $dN/d\eta$  has plateau around midrapidity
- only logarithmic dependence on collision energy

## *Experimental access to entanglement ?*

- could longitudinal entanglement be tested experimentally?
- unfortunately entropy density  $dS/d\eta$  not straight-forward to access
- measured in  $e^+e^-$  is the number of charged particles per unit rapidity  $dN_{\text{ch}}/d\eta$  (rapidity defined with respect to the thrust axis)
- typical values for collision energies  $\sqrt{s} = 14 - 206$  GeV in the range

$$dN_{\text{ch}}/d\eta \approx 2 - 4$$

- entropy per particle  $S/N$  can be estimated for a hadron resonance gas in thermal equilibrium  $S/N_{\text{ch}} = 7.2$  would give

$$dS/d\eta \approx 14 - 28$$

- this is an upper bound: correlations beyond one-particle functions would lead to reduced entropy