Pre-equilibrium photon production and the direct-photon puzzle
Theory part

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Contents

- Introduction
  - strong fields in the initial stage of heavy-ion collisions
- A short review of previous studies on the pre-equilibrium photon production
- Results from classical statistical lattice simulations
Initial stage of heavy-ion collisions: Glasma

Before a collision

color glass condensate
\[ E_T^a \perp B_T^a \]

Right after a collision

Glasma
\[ E_L^a \parallel B_L^a \]

non-Abelian Gauss’s laws
\[
\nabla \cdot E^a = -gf^{abc} A^b \cdot E^c \\
\nabla \cdot B^a = -gf^{abc} A^b \cdot B^c
\]
Initial stage of heavy-ion collisions: Glasma

Before a collision

\[ E_T^a \perp B_T^a \]

Right after a collision

\[ E_L^a \parallel B_L^a \]

strong color fields \( A \sim Q_s / g \)

strongly interacting system

- Nonperturbative treatments are necessary.
- In \( g \ll 1 \), the classical YM eq. gives the LO description.
- Classical statistical approach goes beyond the LO.

LO classical evolution

Initial stage of heavy-ion collisions: Magnetic fields

Strong magnetic field perpendicular to the reaction plane

\[ eB \sim 0.2 \text{ GeV}^2 \sim 10 m_\pi^2 \quad \text{for } b = 10 \text{ fm} \]

Photon production in magnetic fields

- Positive contributions to v2.
- Stronger for larger b.
Photon production in Glasma; Is it important?

- Small space-time volume
- QGP hadron pre-equilibrium
- Glasma
- Gluon
- Quark
- Thermal
- CSA
- Kinetic
Photon production in Glasma; Is it important?

small space-time volume may compensate?
high density

Glasma

1/\alpha_s
1/\alpha_s^{1/2}

\alpha_s

\gamma

hadron

QGP

pre-equilibrium

f

thermal

gluon

quark

CSA

kinetic
Photon production in Glasma; Is it important?

Initially, the system is almost purely gluonic. In perturbative calculations, chemical equilibrium between quarks and gluons is slow.
In strong gauge fields $A \sim 1/g$, quark production is an order one effect. The quark occupation number $\lesssim 1$ can be quickly developed at $\tau \sim 1/Q_s$. 

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QGP hadron small space-time volume may compensate?

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Flow is small at early time
Initial flow before hydro evolution? (e.g. IP-Glasma)

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In strong gauge fields, quark production is an order one effect. The quark occupation number $\lesssim 1$ can be quickly developed at $\tau \sim 1/Q_s$. Is it important?

Correlations specific to the early-time production? Can we ‘see’ Glasma?

Flow is small at early time
Initial flow before hydro evolution? (e.g. IP-Glasma)

Small space-time volume may compensate?
High density
The data points of different kinds of collisions and different energies are on the same line. The saturation scale $Q_s$ is the only relevant scale. Imply photon production at early times when other scales (system size, particle masses) are not yet important.
Photon production in Glasma


• Based on a simple model for the gluon and quark distribution functions

\[ f_g(E) = \frac{\kappa \Lambda_{\text{IR}}}{N_c \alpha_s \Lambda} \frac{1}{e^{E/\Lambda} - 1} \quad \quad f_q(E) = \frac{1}{e^{E/\Lambda} + 1} \]

Initially, \( \Lambda_{\text{IR}} = \Lambda \sim Q_s \)
Thermal distribution when \( \Lambda_{\text{IR}} = N_c \alpha_s \Lambda / \kappa \)

• Generalization of the LO rate (Compton and annihilation) in a thermal QGP

\[ E \frac{dN_{\text{Glasma}}}{d^4x d^3p} = \frac{5}{9} \frac{\alpha}{2\pi^2} \frac{\kappa}{N_c \alpha_s} \Lambda_{\text{IR}} \Lambda e^{-E/\Lambda} \ln \left( \frac{2.912}{\frac{E}{\Lambda_{\text{IR}}} \frac{N_c}{4\pi \kappa}} \right) \]

\( \text{can be parametrically large due to the factor } 1/\alpha_s \)

• No direct computation of the flow.

A typical emission time is delayed in the Glasma-thermal combined evolution compared with only thermal evolution.

Indicates larger flow
Effects of a non-thermal tail

L. McLerran, B. Schenke, NPA 946, 158 (2016).

- Replace the thermal distributions by power law distributions (Tsallis).
  \[ f_{g/q}(E) = \left[ \left( 1 + \frac{E}{\alpha T} \right)^a \mp 1 \right]^{-1} \quad \text{with} \quad \alpha = 6 \]

- Photon spectrum is drastically changed.

The shapes of the distributions of quarks and gluons are important. Relation to non-thermal fixed points?
Towards first-principle-based computations

- Ab initio computations of the quarks and gluons distributions
  
  The shapes of the distributions and their time evolution are important.
  An approach which does not require quasi-particle distributions is more favorable.

- Correctly take account of overoccupied gauge fields
  
  In the overoccupied regime $A \sim 1/g$, the kinetic approach or the naïve perturbative calculations are not valid.

Classical statistical approach can do
Other approach

**Boltzmann approach to multi parton scatterings**  Greif, Zhou, Greiner, Xu

Describe parton (quarks and gluons) scatterings by the Boltzmann equation

\[ p^\mu \partial_\mu f(x, p) = C_{22}[f] + C_{23}[f] \]

Photon production also by the Boltzmann

Compton  pair annihilation  Bremsstrahlung

Such a kinetic approach can be complementary to the classical statistical approach.
Classical statistical lattice simulations

Tanji, PRD 92, 125012 (2015); Berges, Garcia, Muller, Tanji, in progress.

- Solve classical YM eq. and Dirac eq. on real-time lattice
  - Classical-statistical approximation valid for strong gauge fields $A \gg 1$
  - No approximation for the quark fields
- Photons are treated perturbatively in $\alpha_{EM}$.

Photon spectrum can be computed from the current-current correlation:

$$\langle J_{\mu}(x) J_{\nu}(y) \rangle$$

quark propagator dressed by classical gauge fields
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Quark propagator dressed by classical gauge fields

One of characteristic features of a non-equilibrium state is nonzero current expectation.

\[
\langle J_\mu(x) \rangle = e \langle \overline{\psi}(x) \gamma_\mu \psi(x) \rangle \neq 0
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Gives the same order contribution in \( \alpha_{\text{EM}} \) as the connected one-loop.
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Glasma-like color fields

- Uniform in the z-direction
- Fluxtube-like configuration in the transverse plane

× Non-expanding fixed box
× Initially there are only electric fields

\[ E_z^a(t = 0, \mathbf{x}_\perp) = \frac{Q_s^2}{g} \sum_{j=1}^{N_{\text{tube}}} \exp \left( -Q_s^2 (\mathbf{x}_\perp - \xi_j)^2 \right) n_j^a \]

Transverse profile of the initial energy density

Time dependence of the gauge field energy density
Induction of the EM current

Plots of $|J^i_{EM}(x)|/(eQ_s^3)$

$m/Q_s = 0.1$

$Q_s t = 5$

x-component

y-component

z-component
Photon spectrum

Compute the spectrum of photons produced by classical processes

\[
\frac{dE}{dt} - \nabla \times \mathbf{B} = -\mathbf{J}_{EM}
\]

Solve the Maxwell equation

Photon energy spectrum

\[
\frac{dE_{ph}}{d^2p_Tdz} = \frac{1}{2(2\pi)^2} \left[ \left| \mathbf{E}(p_T) \right|^2 + \left| \mathbf{B}(p_T) \right|^2 \right]
\]

Photon number spectrum

\[
\frac{dN_{ph}}{d^2p_Tdz} = \frac{1}{\omega} \frac{dE_{ph}}{d^2p_Tdz} \quad \omega = p_T = \sqrt{p_x^2 + p_y^2}
\]
At $p_T/Q_s < 1$, the spectrum fluctuates largely run by run.

At $1 < p_T/Q_s < 4$, the spectrum can be fitted by $A \exp\left(-p_T/T_{eff}\right)$ with $T_{eff} \sim Q_s/3$, although the system is far away from thermal equilibrium.
Summary and outlook

- Study of the pre-equilibrium photon production is still in its infancy.
- Classical statistical approach enables first-principle-based studies.
- In the glasma-like color fields, the photon spectrum shows an exponential behavior in pT though the system is far from equilibrium.

- More realistic setup
- Two photon correlations
- Genuine quantum process