Anisotropy induces non-Fermi-liquid behavior and nemagnetic order in 3D Luttinger semimetals

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Simon Fraser U

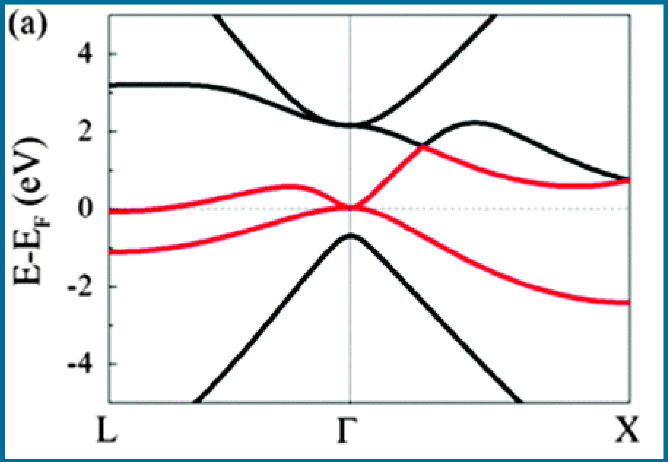
Vancouver

Joint work with Igor Herbut

IB, Herbut, PRB 93, 205138 (2016)
IB, Herbut, PRB 95, 075149 (2017)

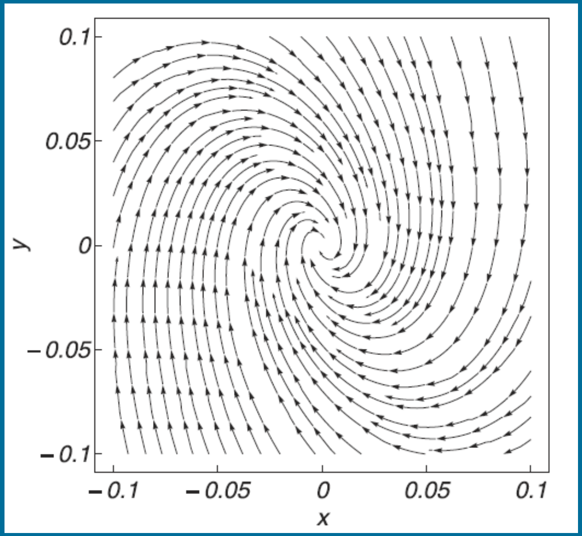
Deutsche Forschungsgemeinschaft

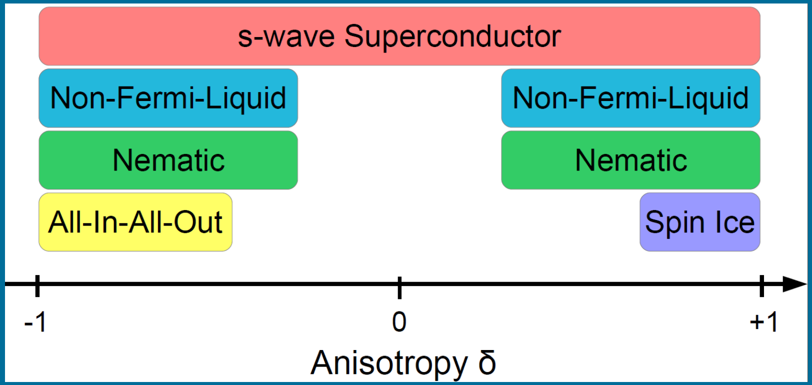
Outline



Quadratic band touching

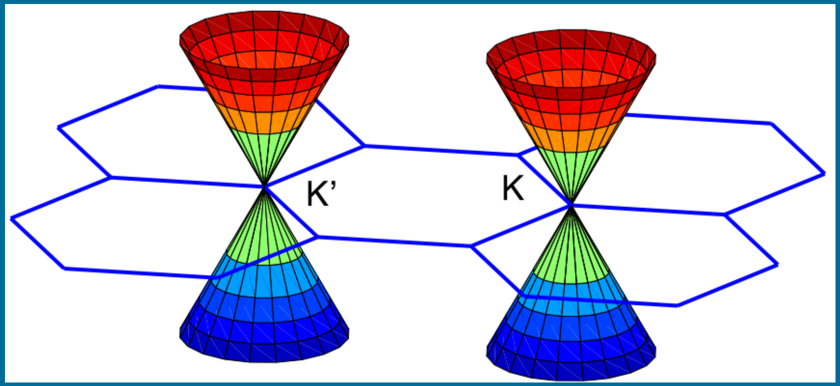
g < 0 : Superconductivity

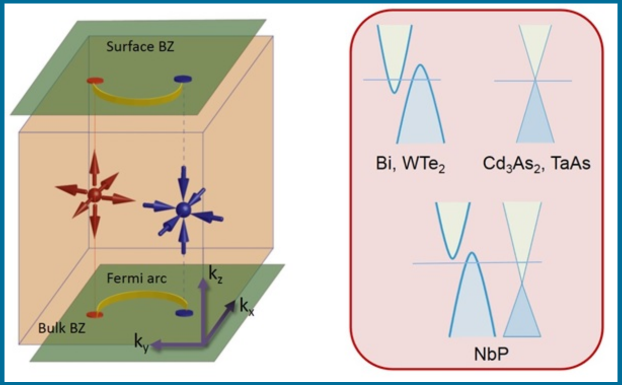




g > 0 : NFL and tensor order

Quadratic band touching

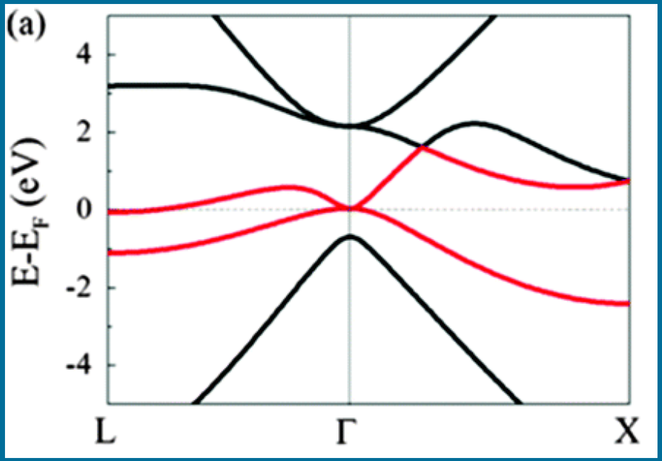


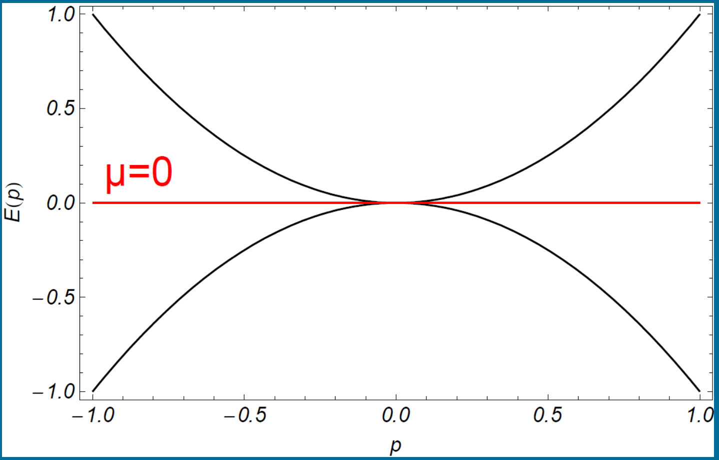


Pics: MPIKS Dresden

Dirac semimetals

Weyl semimetals



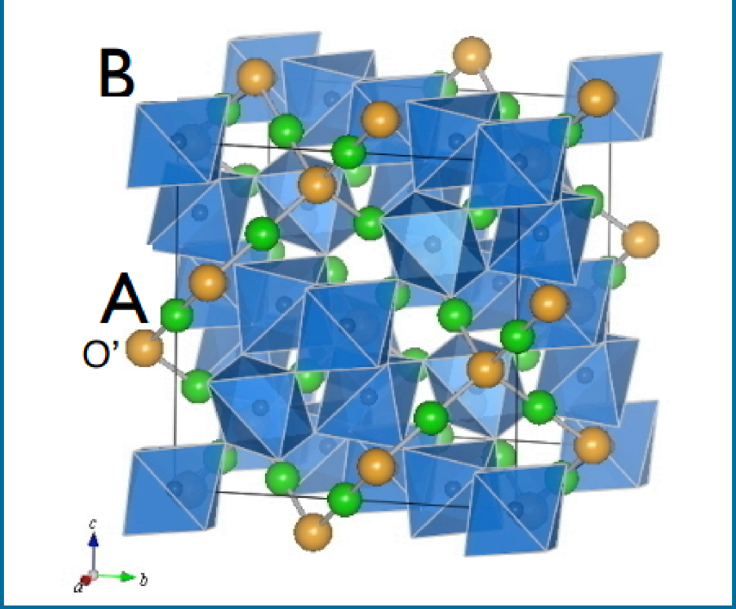


Luttinger Hamiltonian: Luttinger semimetals

Murakami, Nagaosa, Zhang

Quadratic band touching

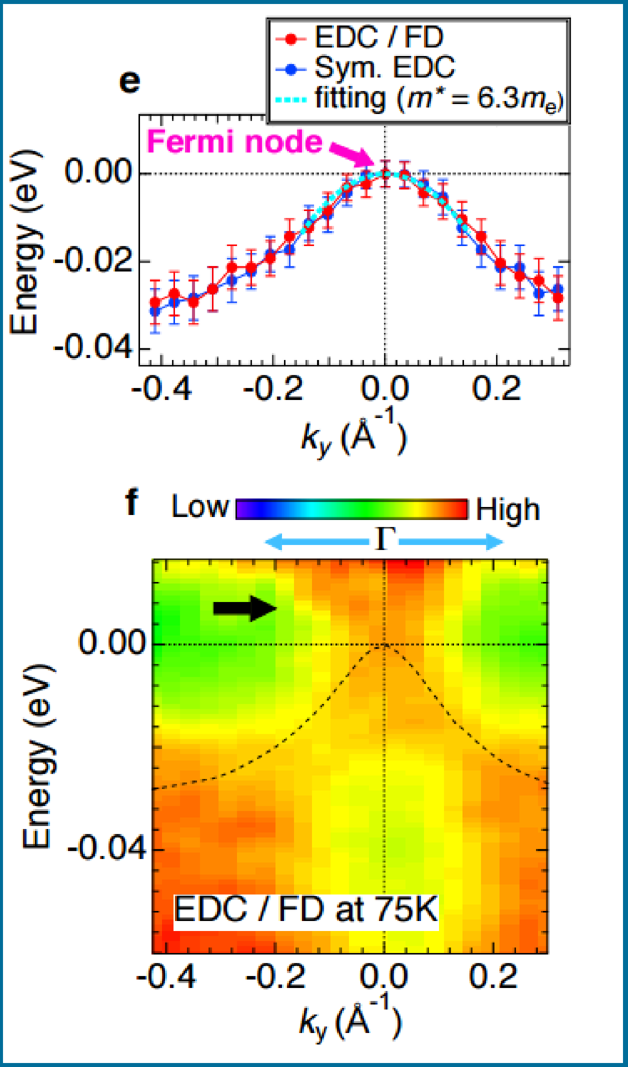
Pyrochlore iridates $R_2Ir_2O_7$



Balents, Pesin, Witczak-Krempa, Chen, Kim

Pr-227

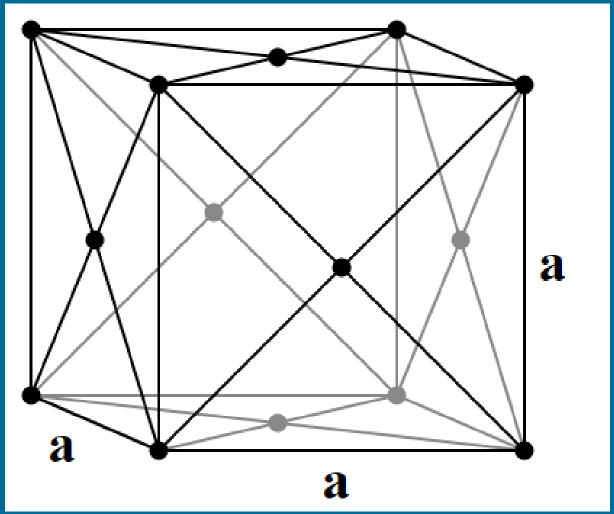
Kondo et al,
Nat. Comm. 6,
10042 (2015)



Nd-227: Nakayama et al
PRL 117, 056403 (2016)

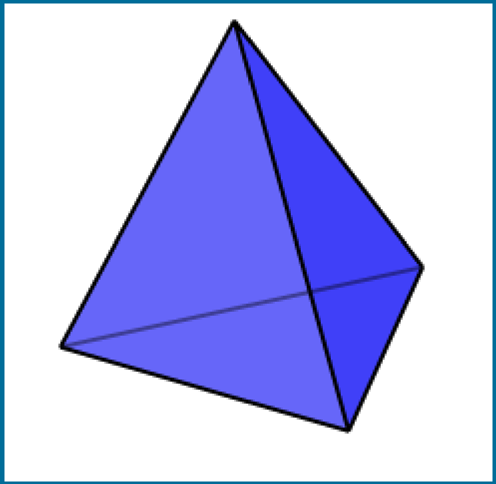
Quadratic band touching

Pyrochlore lattice: corner-sharing tetrahedra



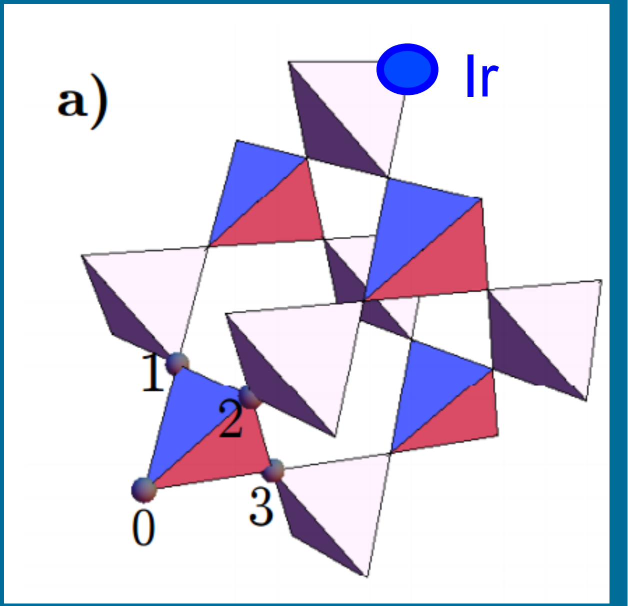
fcc cubic lattice

+



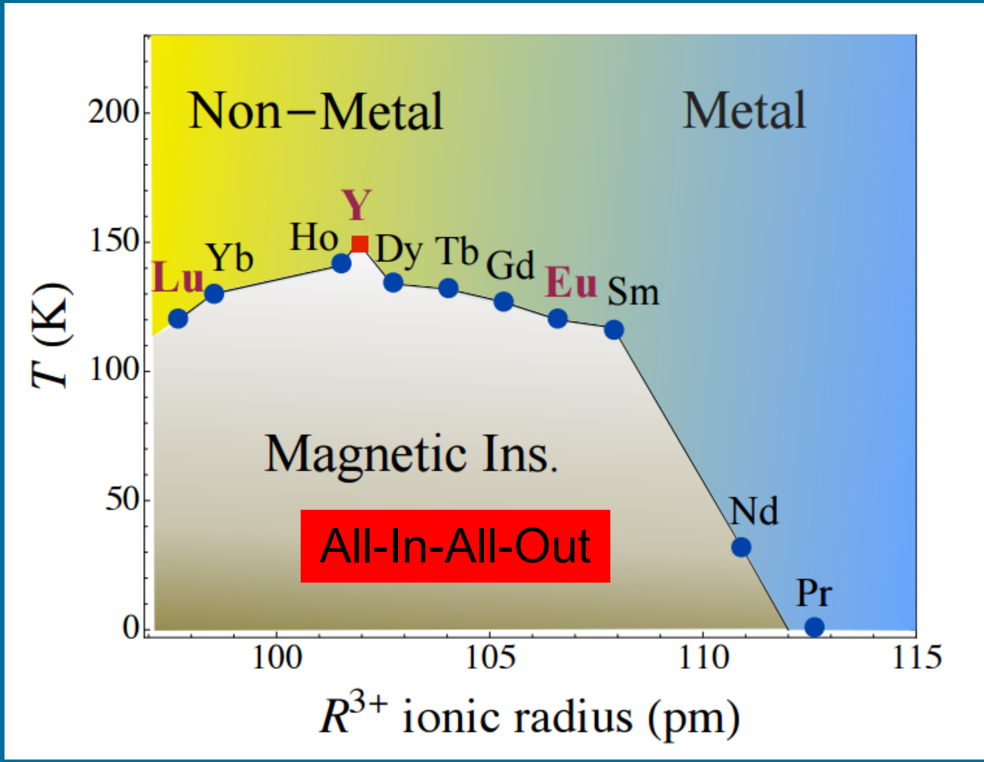
tetrahedra

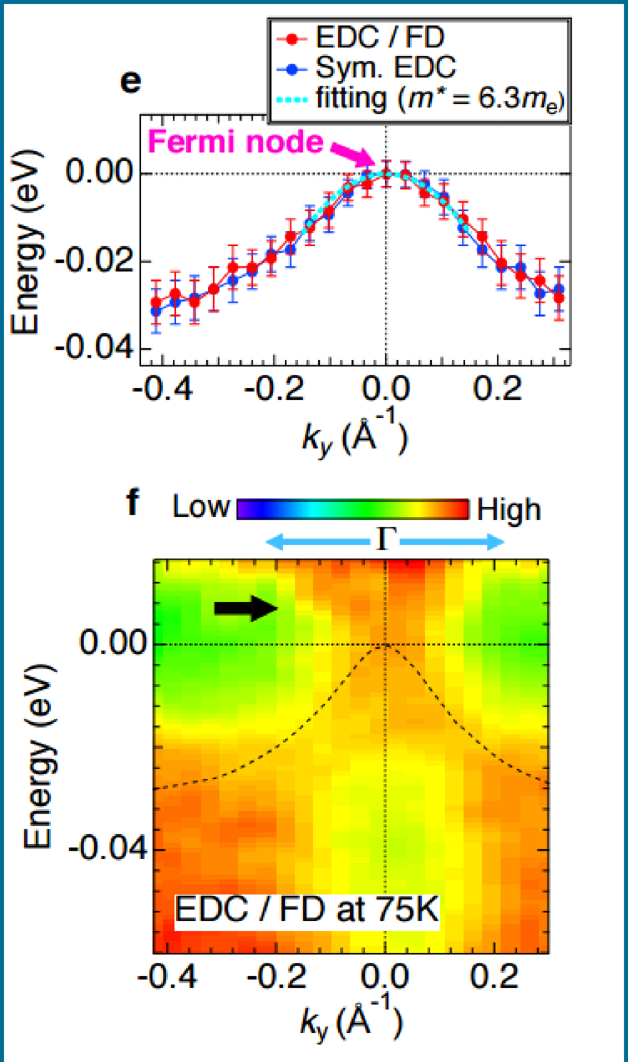
=



Quadratic band touching

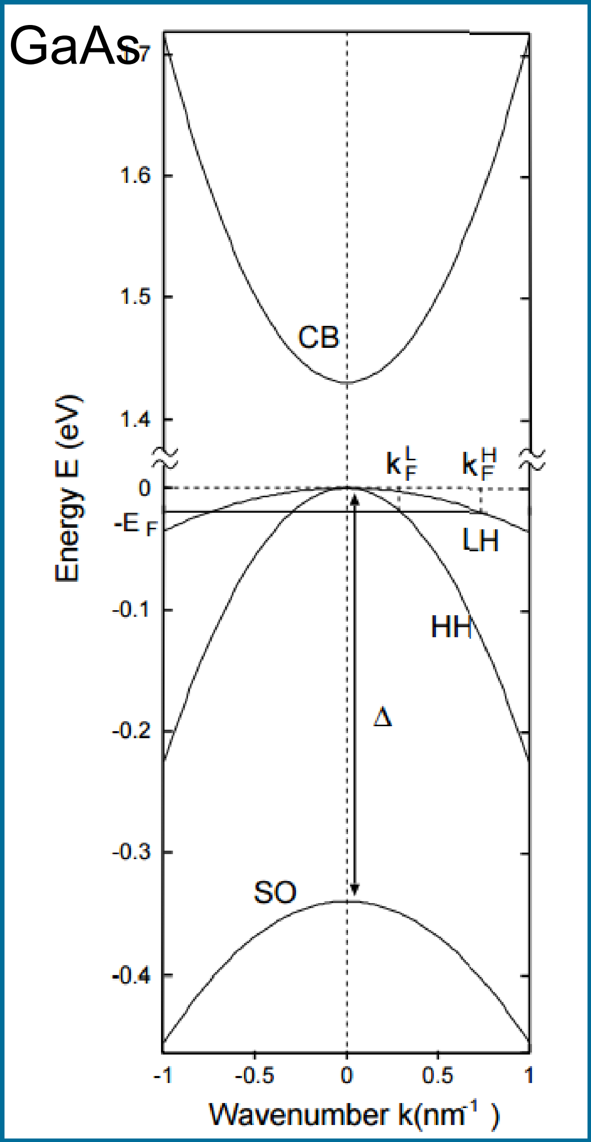
Pyrochlore iridates $R_2Ir_2O_7$





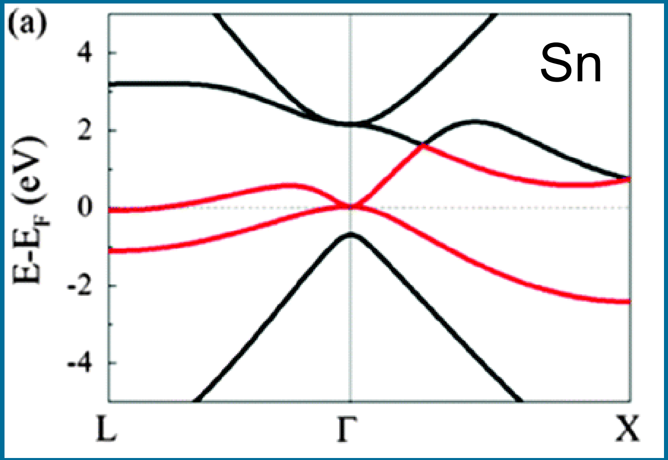
Witczak-Krempa, Chen, Kim, Balents,
Ann. Rev. of Cond. Mat. Phys.,Vol. 5: 57-82 (2014)

Quadratic band touching



4 x 4 Luttinger Hamiltonian

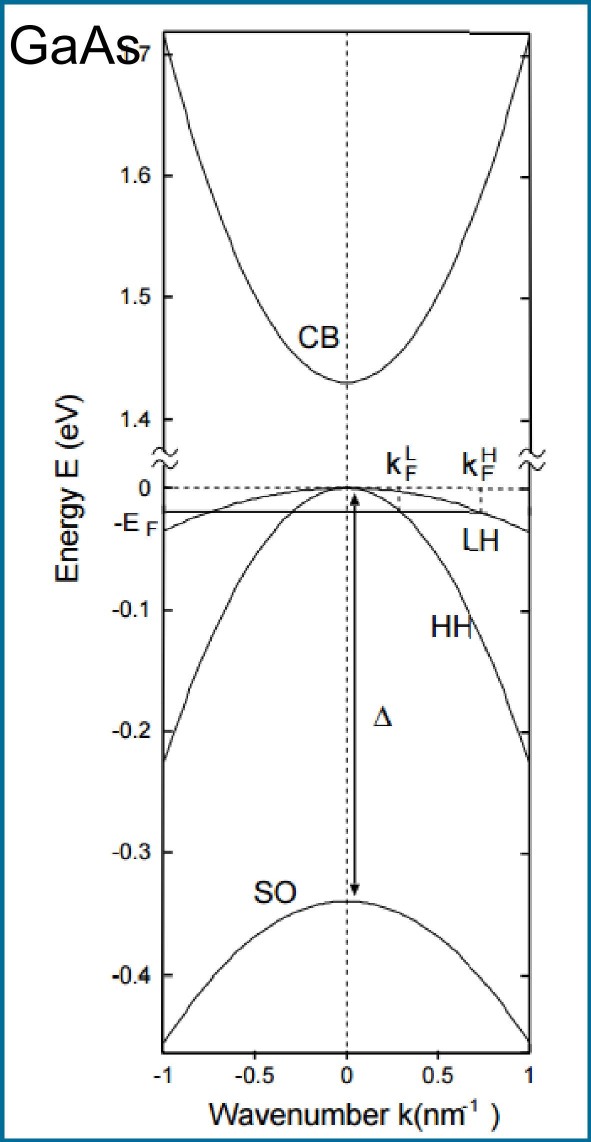
$$H = \frac{\hbar^2}{2m^*}\Big[\Big(\alpha_1 + \frac{5}{2}\alpha_2\Big)p^2 1_4 - 2\alpha_3(\vec{p}\cdot\vec{J})^2 + 2(\alpha_3 - \alpha_2)\sum_{i=1}^{3} p_i^2 J_i^2\Big]$$



J_x, J_y, J_z

spin 3/2 matrices

Quadratic band touching



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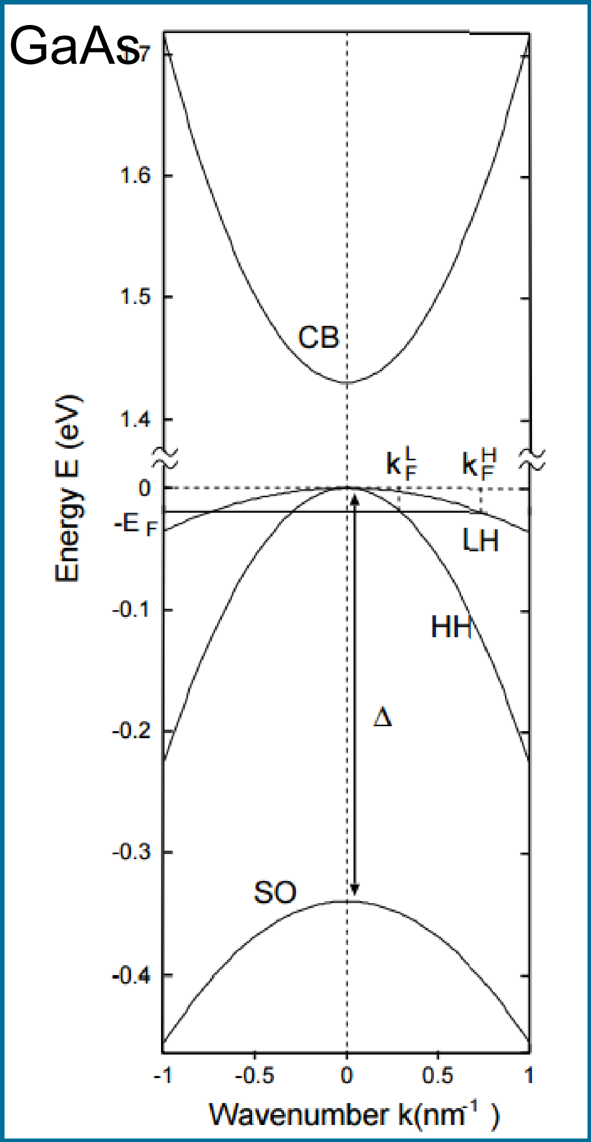
rotation invariant SO(3)

cubic invariant Oh
≈ permutations of x,y,z

Quadratic band touching



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$$x = -\frac{\alpha_1}{\alpha_2 + \alpha_3}$$

particle-hole asymmetry
diminishes under RG -> 0

$$\delta = -\frac{\alpha_2 - \alpha_3}{\alpha_2 + \alpha_3}$$

spatial anisotropy
approximately constant -> 0

Part I

Superconductivity

relevant materials
e.g. half-Heuslers YPtBi

Superconductivity

$$L = \psi^\dagger(\partial_\tau + H)\psi, \quad H = \sum_{a=1}^{5} d_a(\vec{p})\gamma_a, \quad H^2 = p^4 1$$

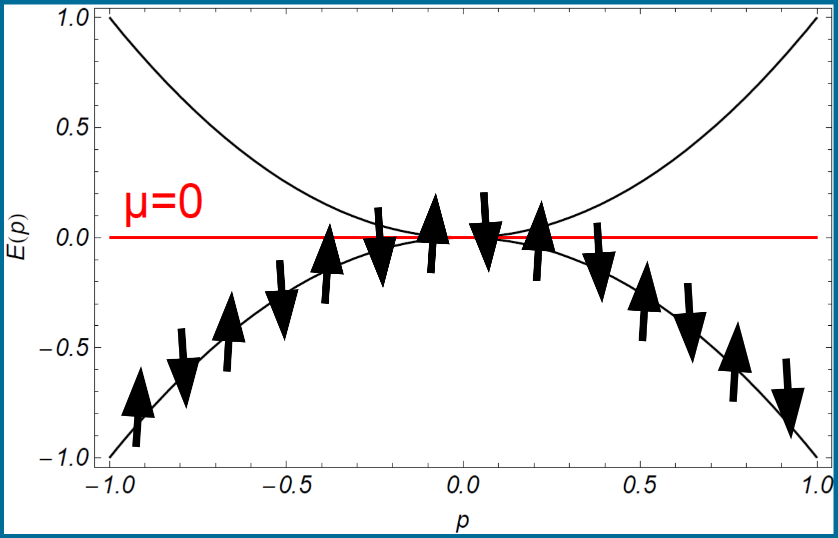
L=2 spherical harmonics

$$d_2(\vec{p}) = \frac{1}{2}(2p_z^2 - p_x^2 - p_y^2)$$

$$d_5(\vec{p}) = \sqrt{3}p_x p_y$$

4x4 gamma matrices

$$\{\gamma_a, \gamma_b\} = 2\delta_{ab}1$$



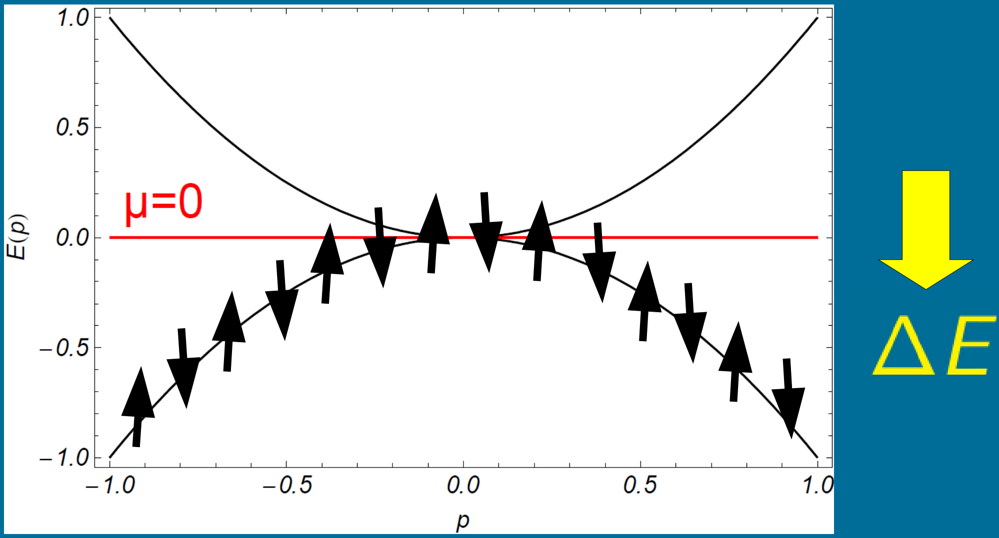
Superconductivity

$$L = \psi^\dagger(\partial_\tau + H)\psi, \quad H = \sum_{a=1}^{5} d_a(\vec{p})\gamma_a, \quad H^2 = p^4 1$$

Ground state?

Push down filled states?!

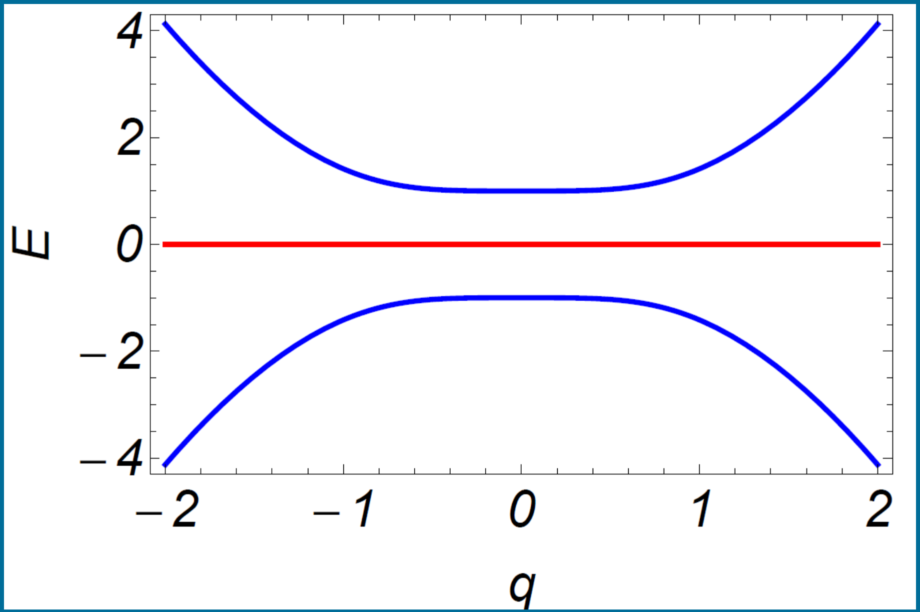
How to get full gap?



Superconductivity

no anti-commutating matrix α left: gap has nodes

$$(H + m\alpha)^2 = H^2 + m\underbrace{\{H, \alpha\}}_{0} + m^2 \overset{!}{=} (p^4 + m^2)1$$



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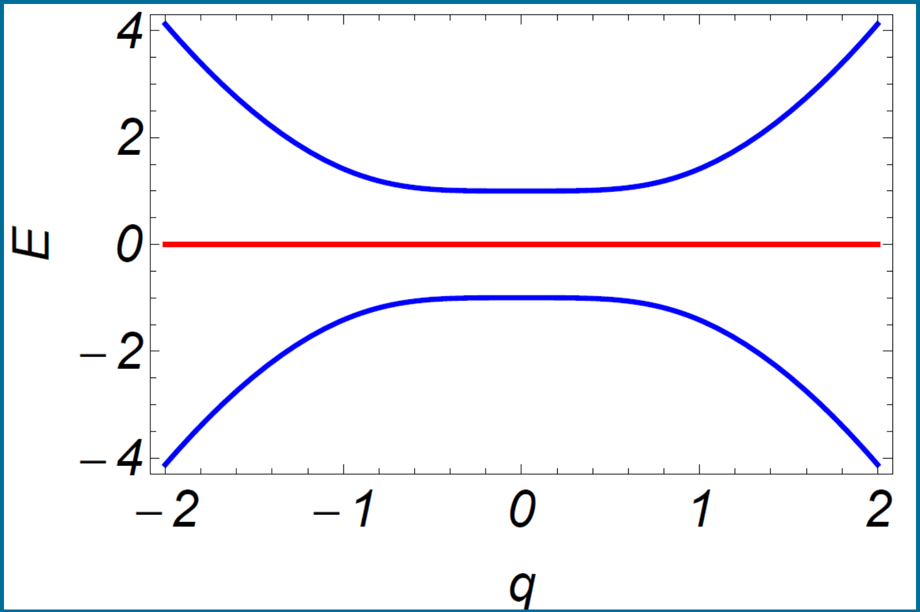


Majorana mass term

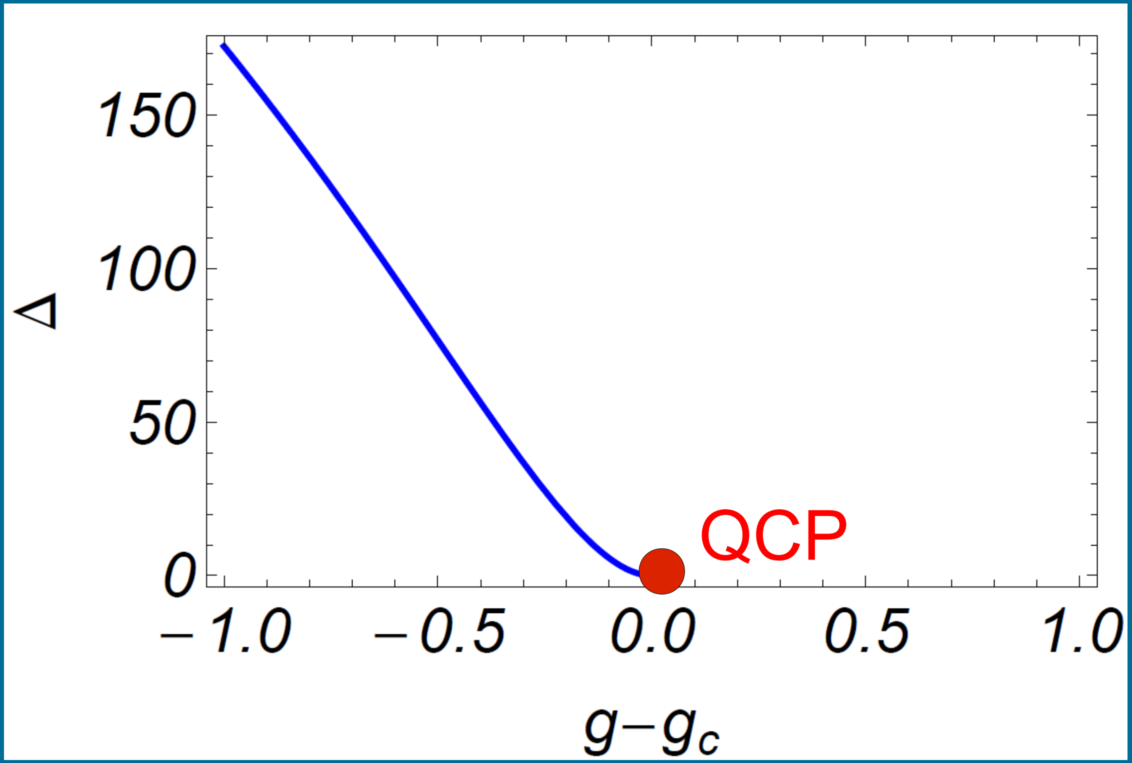
$$L \sim m\ \psi^\dagger \sigma_2 \psi^*$$

s-wave superconducting gap

$$\bar{\phi} = \langle \psi^\dagger \gamma_{45} \psi^* \rangle$$

$$L = \psi^\dagger(\partial_\tau + H)\psi + r\phi^*\phi + g[\ \phi(\psi^\dagger\gamma_{45}\psi^*) + \text{h.c.}\ ]$$

Superconductivity

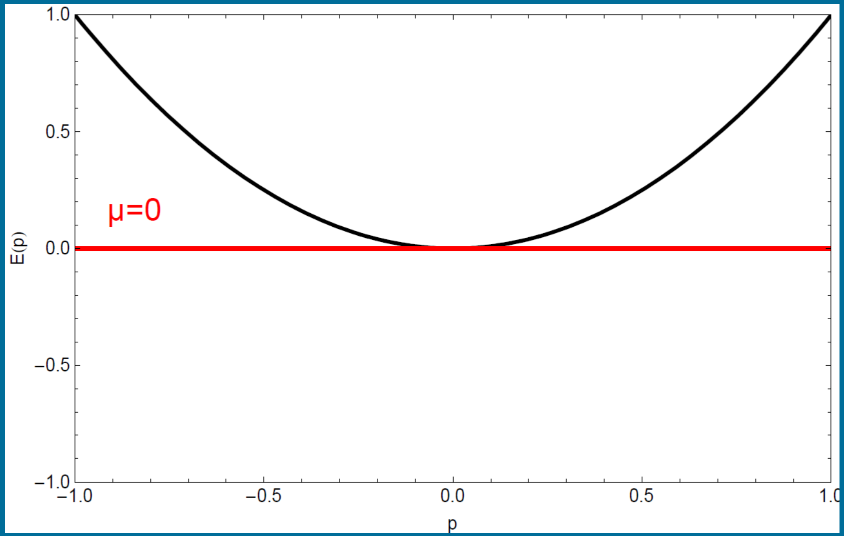


$$L_{\rm int} = g(\psi^\dagger\psi)^2 \sim g(\psi^\dagger\gamma_{45}\psi^*)(\psi^{\rm T}\gamma_{45}\psi)$$

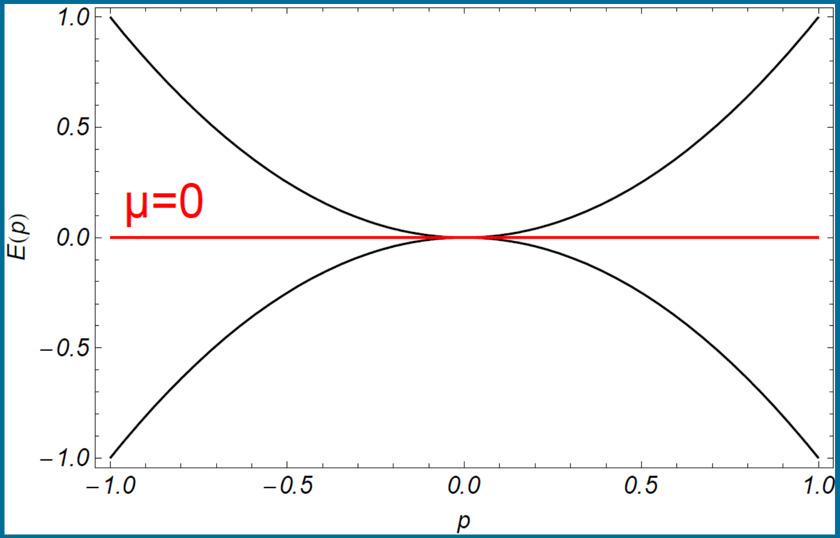
Attractive density-density interactions (e.g. phonon mediated)

Superconducting quantum criticality

s-wave particle-particle pairing



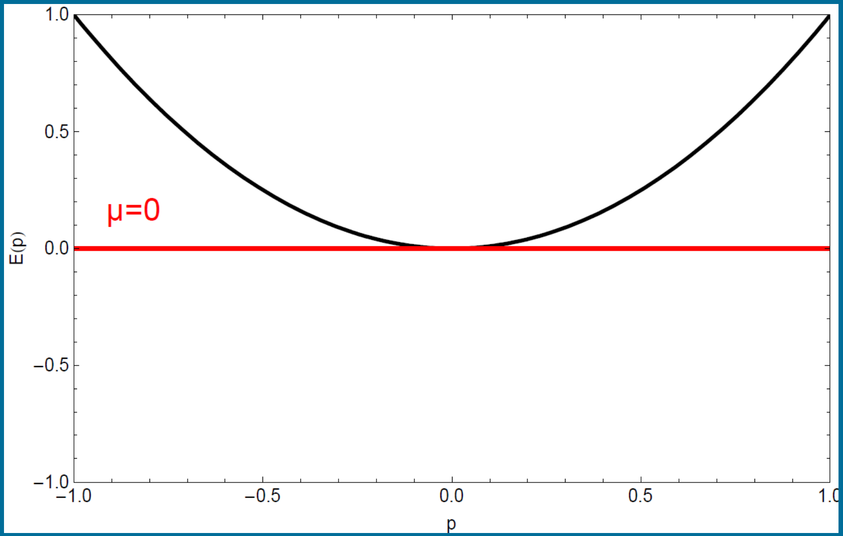
3D ultracold atoms at
a Feshbach resonance

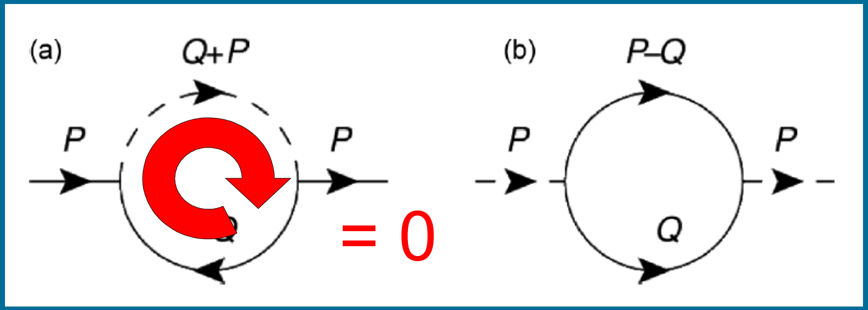


3D Luttinger semimetals
at a superconducting QCP

Superconducting quantum criticality

s-wave particle-particle pairing





3D ultracold atoms at
a Feshbach resonance

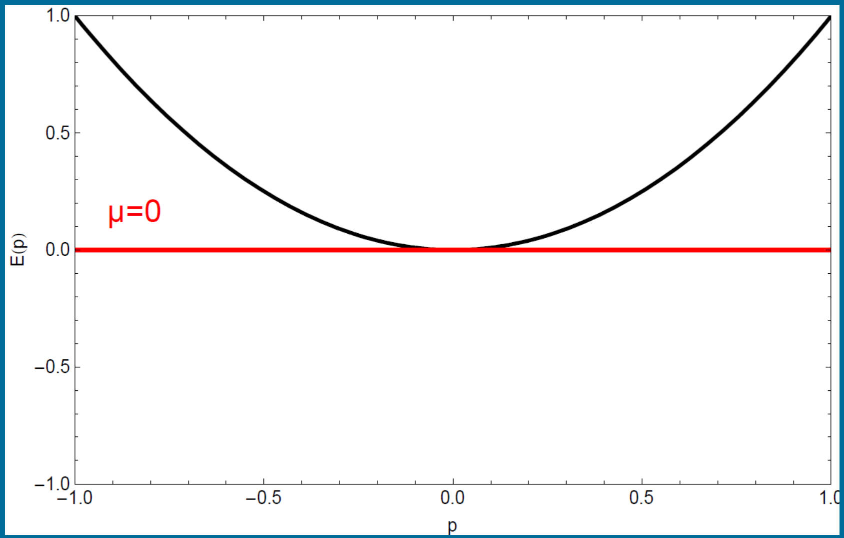
$$\eta_\phi = 1,$$

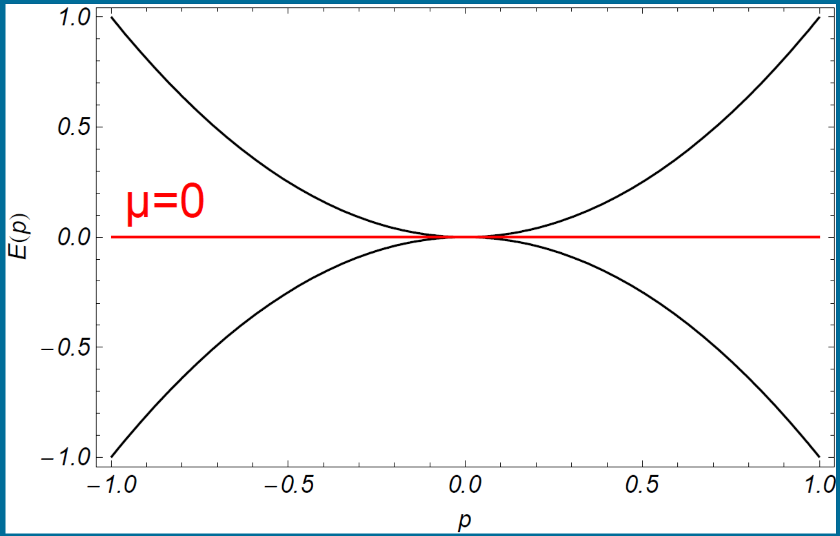
$$\eta_\psi = 0, \;\; z = 2$$

Diehl, Wetterich; Sachdev, Nikolic

Superconducting quantum criticality

s-wave particle-particle pairing





3D ultracold atoms at
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3D Luttinger semimetals
at a superconducting QCP

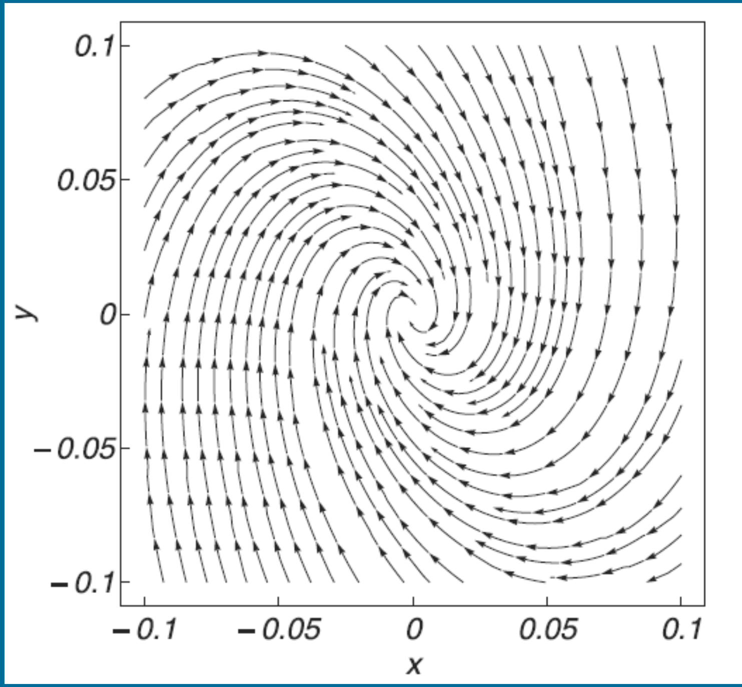
$$\eta_\phi = 1,$$
$$\eta_\psi = 0, \ z = 2$$

$$\eta_\phi = \frac{9}{11}\varepsilon = 0.82$$
$$\eta_\psi = \frac{2}{11}\varepsilon = 0.18, \ z = 2 - \frac{2}{11}\varepsilon = 1.82$$

IB, Herbut, PRB 93, 205138 (2016)

Superconducting quantum criticality

$$L = L_{\mathrm{kin}} + \phi^*(y\partial_\tau - \nabla^2)\phi + g\left(\phi\psi^\dagger\gamma_{45}\psi^* + \mathrm{h.c}\right)$$



$$x, y \to 0$$

IB, Herbut, PRB 93, 205138 (2016)

Superconducting quantum criticality

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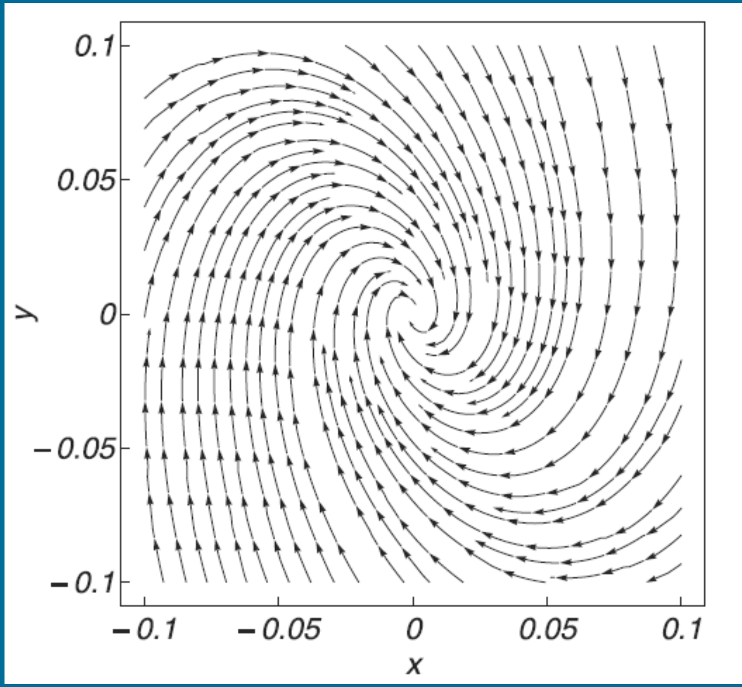
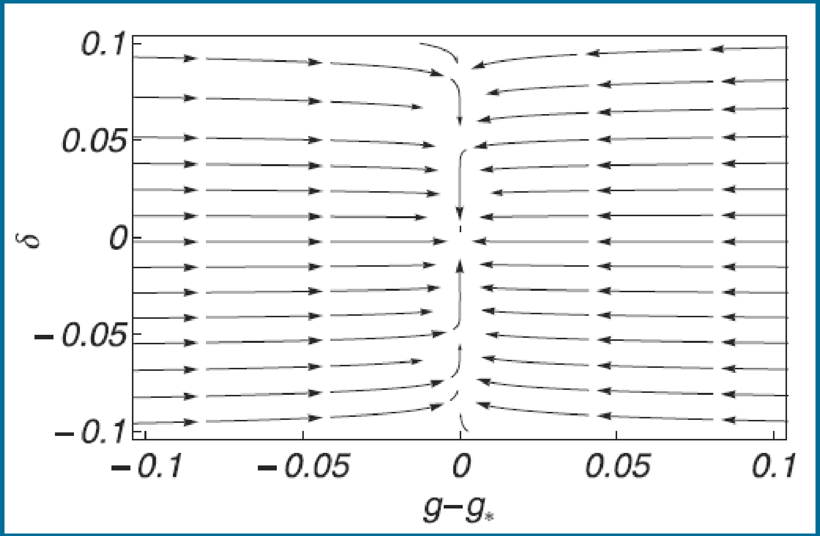


$$x, y \to 0$$



$$\delta \to 0 \qquad \dot{\delta} \simeq -\frac{2}{55}\varepsilon\,\delta$$

exceptionally slow!

IB, Herbut, PRB 93, 205138 (2016)

Part II

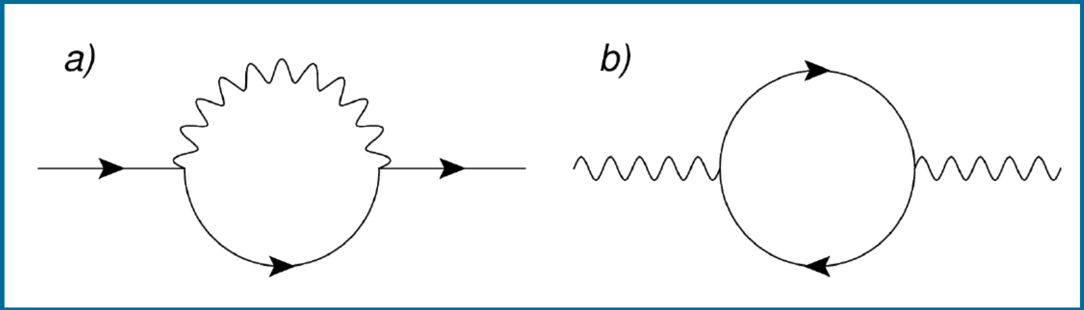
Coulomb interactions

relevant materials
e.g. Pyrochlore Iridates R-227

Abrikosov's NFL scenario

Quadratic band touching & Long-range Coulomb repulsion

$$L = \psi^\dagger(\partial_\tau + H + \mathrm{i}a)\psi + \frac{1}{2e^2}(\nabla a)^2$$



- charge renormalization
- non-Fermi liquid behavior

$$\frac{\mathrm{d}e^2}{\mathrm{d}\log b} = (z + 2 - d)e^2 - e^4$$

$$\eta_\psi = \frac{4}{15}e^2, \;\; z = 2 - \eta_\psi$$

Easy route to a NFL?

Abrikosov's NFL scenario

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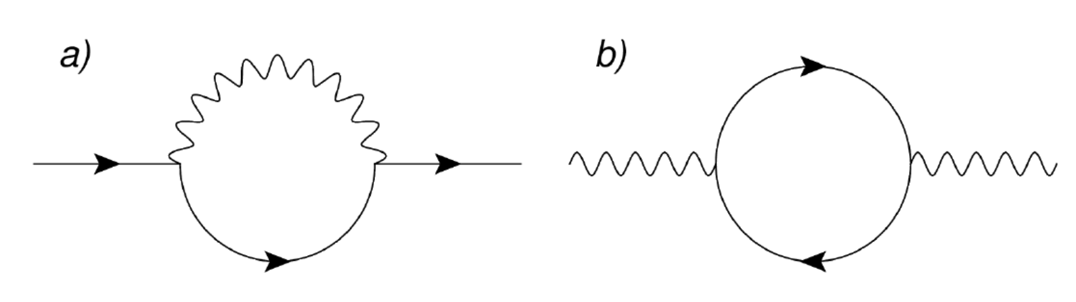

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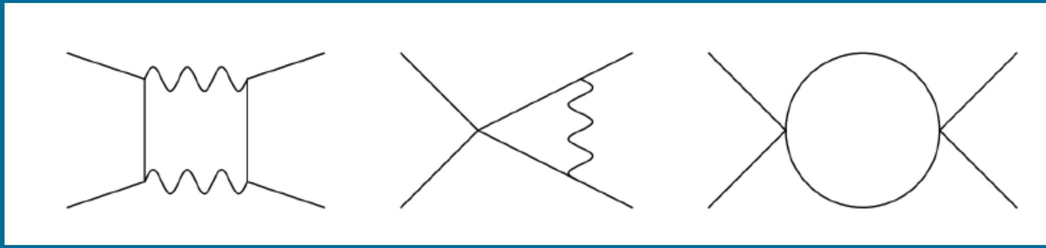
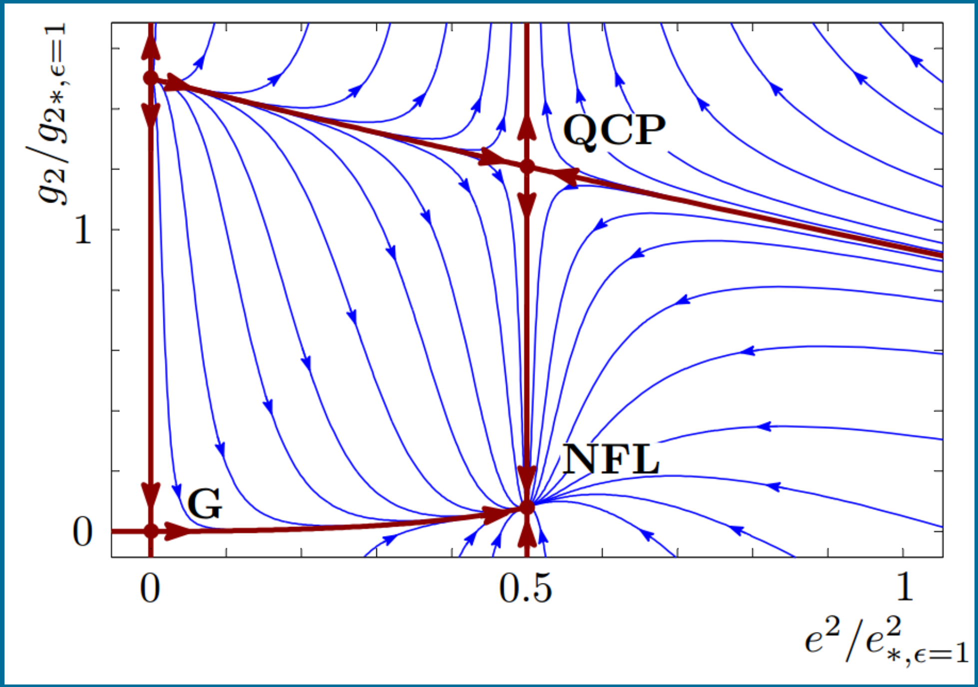
Easy route to a NFL?

No! (Herbut, Janssen)

Abrikosov's NFL scenario

long-range Coulomb repulsion
generates short-range interactions,
even if initially absent





Herbut, Janssen
PRL 113, 106401 (2014)

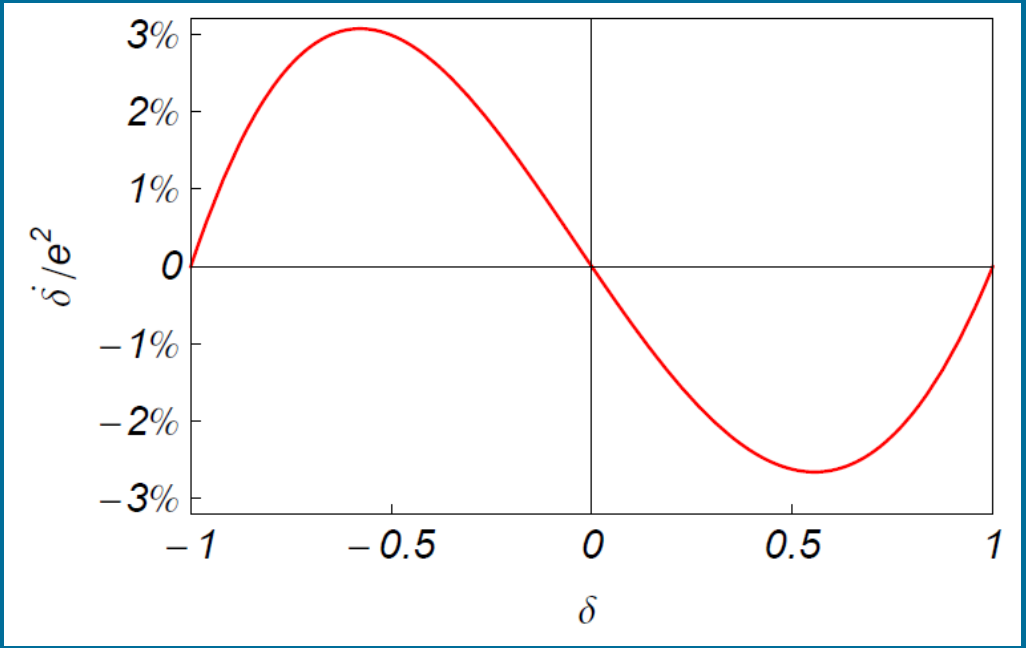
Critical dimension for survival of Abrikosov's NFL: d=3.25

Role of anisotropy δ?

Anisotropic non-Fermi-liquid

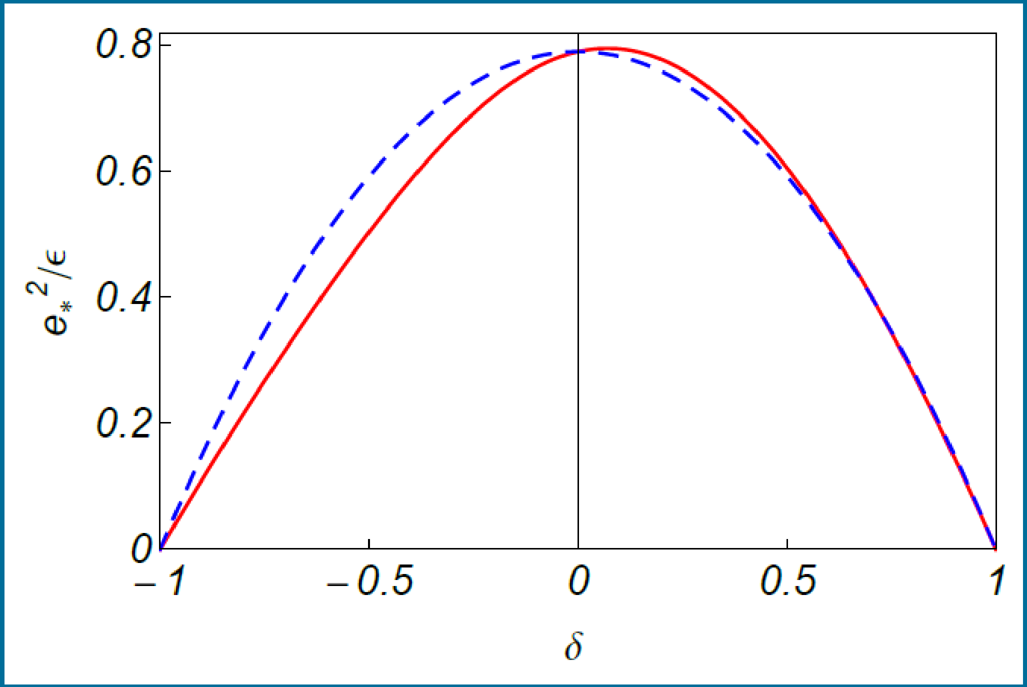
Flow of the anisotropy

$$\dot{\delta} = -\frac{2}{15}(1-\delta^2)\Big[f_{1\mathrm{e}}(\delta) - f_{1\mathrm{t}}(\delta)\Big]e^2$$



Anisotropy constant for all practical purposes

Anisotropic non-Fermi-liquid



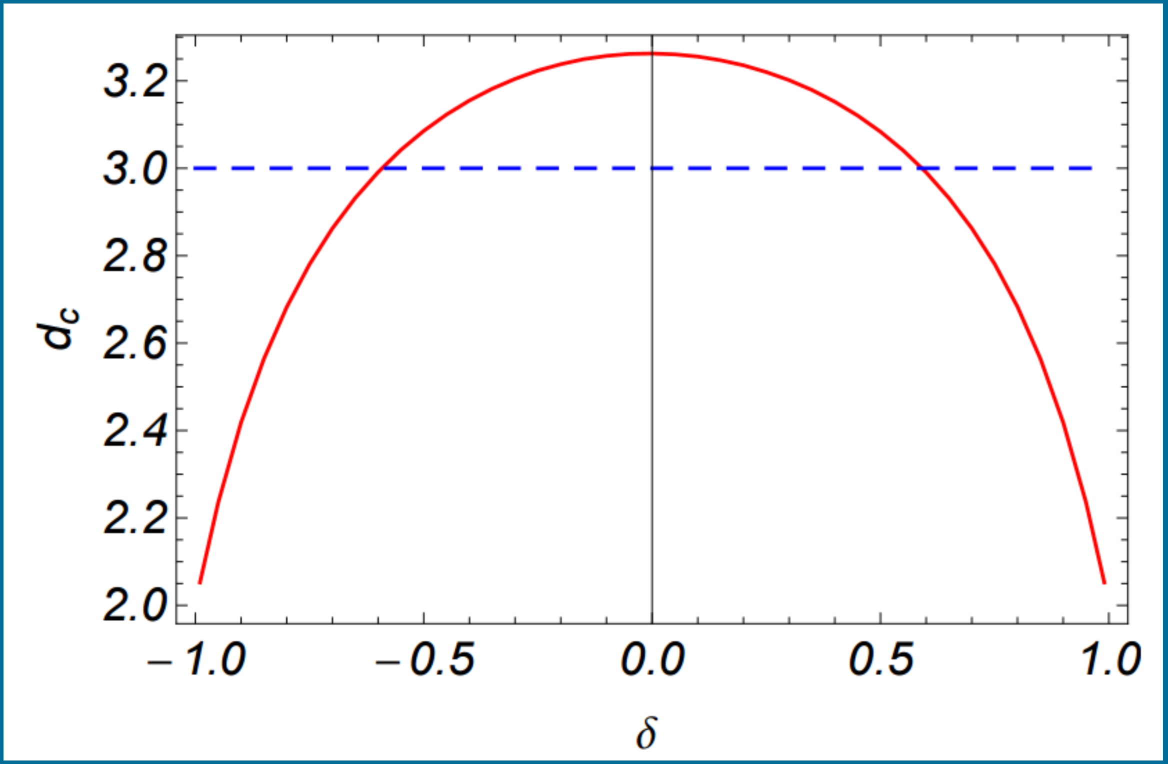
$$e_\star^2 \simeq \frac{15}{19}(1-\delta^2)\varepsilon$$

- Abrikosov fixed point and NFL scaling for each δ
- Fixed point weakly coupled for strong anisotropy

Anisotropic non-Fermi-liquid

- Fixed point collision scenario also with anisotropy
- Critical dimension lowered due to $e_\star^2 \simeq \frac{15}{19}(1-\delta^2)\varepsilon \to 0$

NFL from anisotropy →



Short-range interactions

Generic short-range interaction

$$L_{\text{int}} \sim g(\psi^\dagger M\psi)^2$$

- Construct orthogonal basis of Hermitean matrices M (16 elements)
- Classify them via tensor rank under SO(3)

Short-range interactions

rank n under SO(3)

$$T_{i_1 \ldots i_n} \to R_{i_1 j_1} \cdots R_{i_n j_n} T_{j_1 \ldots j_n}$$

reduce rank by 2 $\delta_{i_1 i_2} T_{i_1 \ldots i_n}$

reduce rank by 1 $\varepsilon_{i_1 i_2 j} T_{i_1 \ldots i_n}$

Irreducible tensors = symmetric traceless tensors

Short-range interactions

Idea: start from products $J_i \cdots J_j$ (operator valued tensors)

$$J_x = \begin{pmatrix} 0 & \frac{\sqrt{3}}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{\sqrt{3}}{2} \\ 0 & 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} \quad J_y = \begin{pmatrix} 0 & -\mathrm{i}\frac{\sqrt{3}}{2} & 0 & 0 \\ \mathrm{i}\frac{\sqrt{3}}{2} & 0 & -\mathrm{i} & 0 \\ 0 & \mathrm{i} & 0 & -\mathrm{i}\frac{\sqrt{3}}{2} \\ 0 & 0 & \mathrm{i}\frac{\sqrt{3}}{2} & 0 \end{pmatrix} \quad J_z = \begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix}$$

$$S_{ij} = J_i J_j + J_j J_i - \frac{5}{2}\delta_{ij} 1_4$$

$$B_{ijk} = J_i J_j J_k + \text{permutations of } ijk - \frac{41}{10}\left(\delta_{ij} J_k + \delta_{ik} J_j + \delta_{jk} J_i\right)$$

Short-range interactions

$$
\begin{aligned}
K_{ijkl} &= J_i J_j J_k J_l + \text{permutations of } ijkl \\
&\quad - 5\Big(\delta_{ij} S_{kl} + \delta_{ik} S_{jl} + \delta_{il} S_{jk} + \delta_{jk} S_{il} + \delta_{jl} S_{ik} + \delta_{lk} S_{ij}\Big) \\
&\quad - \frac{41}{2}\Big(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\Big) 1_4
\end{aligned}
$$

Cayley-Hamilton theorem:
Matrix A is zero of its characteristic polynomial $\chi(A) = 0$

$$
0 = \left(\vec{h}\cdot\vec{J} - \frac{3}{2}\right)\left(\vec{h}\cdot\vec{J} - \frac{1}{2}\right)\left(\vec{h}\cdot\vec{J} + \frac{1}{2}\right)\left(\vec{h}\cdot\vec{J} + \frac{3}{2}\right)
$$

$$
\Longrightarrow 0 = \int \mathrm{d}^3 h \,\Big(\dots\Big)\, h_i h_j h_k h_l = K_{ijkl}
$$

Short-range interactions

four-fermion terms with rotation symmetry $\delta = 0$

$$L_{\text{int}} = g_1(\psi^\dagger\psi)^2 + g_J(\psi^\dagger \mathcal{J}_i\psi)^2 + g_2(\psi^\dagger\gamma_a\psi)^2 + g_W(\psi^\dagger W_\mu\psi)^2$$

1 rank-0-tensor: 1 component, density

$\mathcal{J}_i$ rank-1-tensor: 3 components, magnetic order

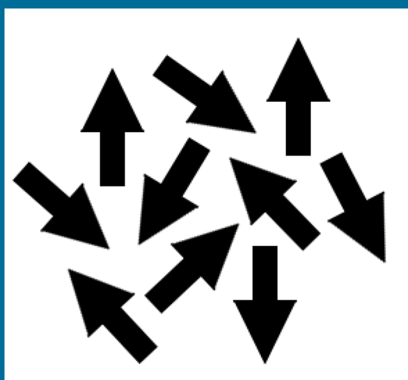
γ_a rank-2-tensor: 5 components, nematic order

W_μ rank-3-tensor: 7 components, nemagnetic order

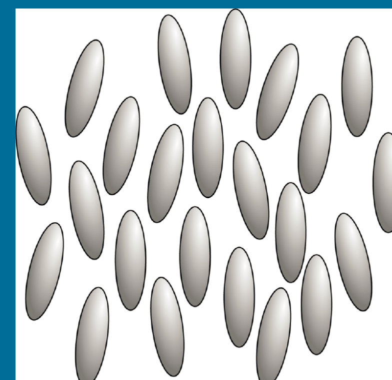
2 independent couplings after Fierz

Tensor orders

think of coarse-grained microscopic orders



Magnetic order
- rank 1 under SO(3)
- breaks TRS



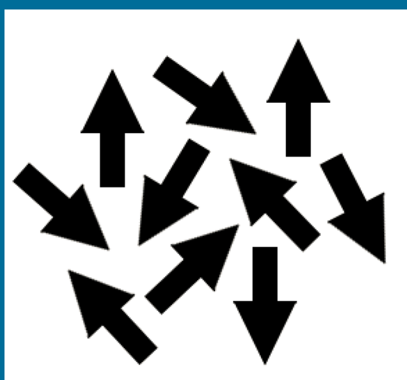
Nematic order
- rank 2 under SO(3)
- preserves TRS

$$m_i(\vec{x}) = \frac{1}{V}\sum_V \mu_i$$

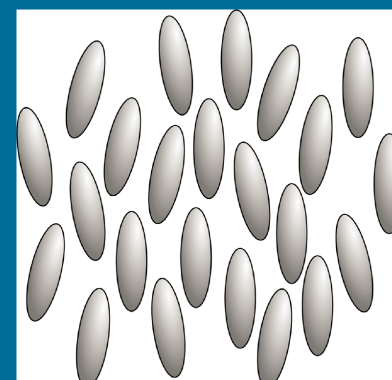
$$\langle u_i \rangle = 0,\ \ S_{ij}(\vec{x}) = \frac{1}{V}\sum_V \left(u_i u_j - \frac{1}{3}\delta_{ij} \right)$$

Tensor orders

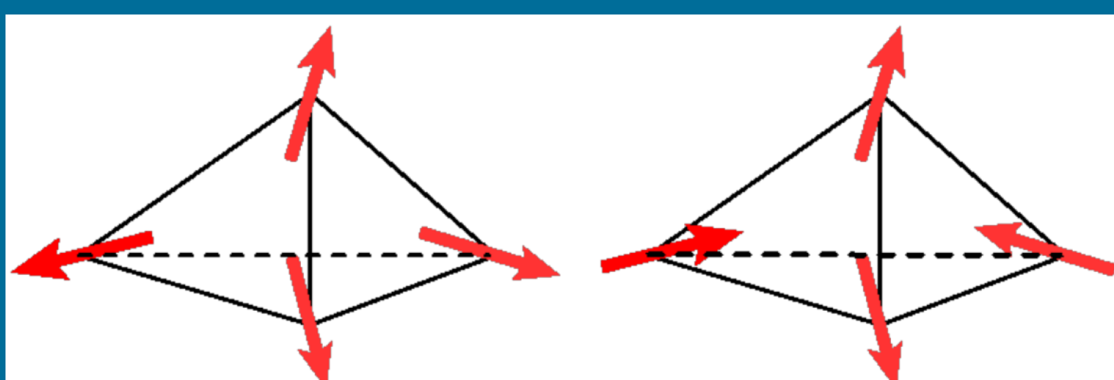
think of coarse-grained microscopic orders



Magnetic order
- rank 1 under SO(3)
- breaks TRS



Nematic order
- rank 2 under SO(3)
- preserves TRS



All-In-All-Out   Spin Ice

Nemagnetic order
- rank 3 under SO(3)
- breaks TRS

*electrons on the pyrochlore lattice

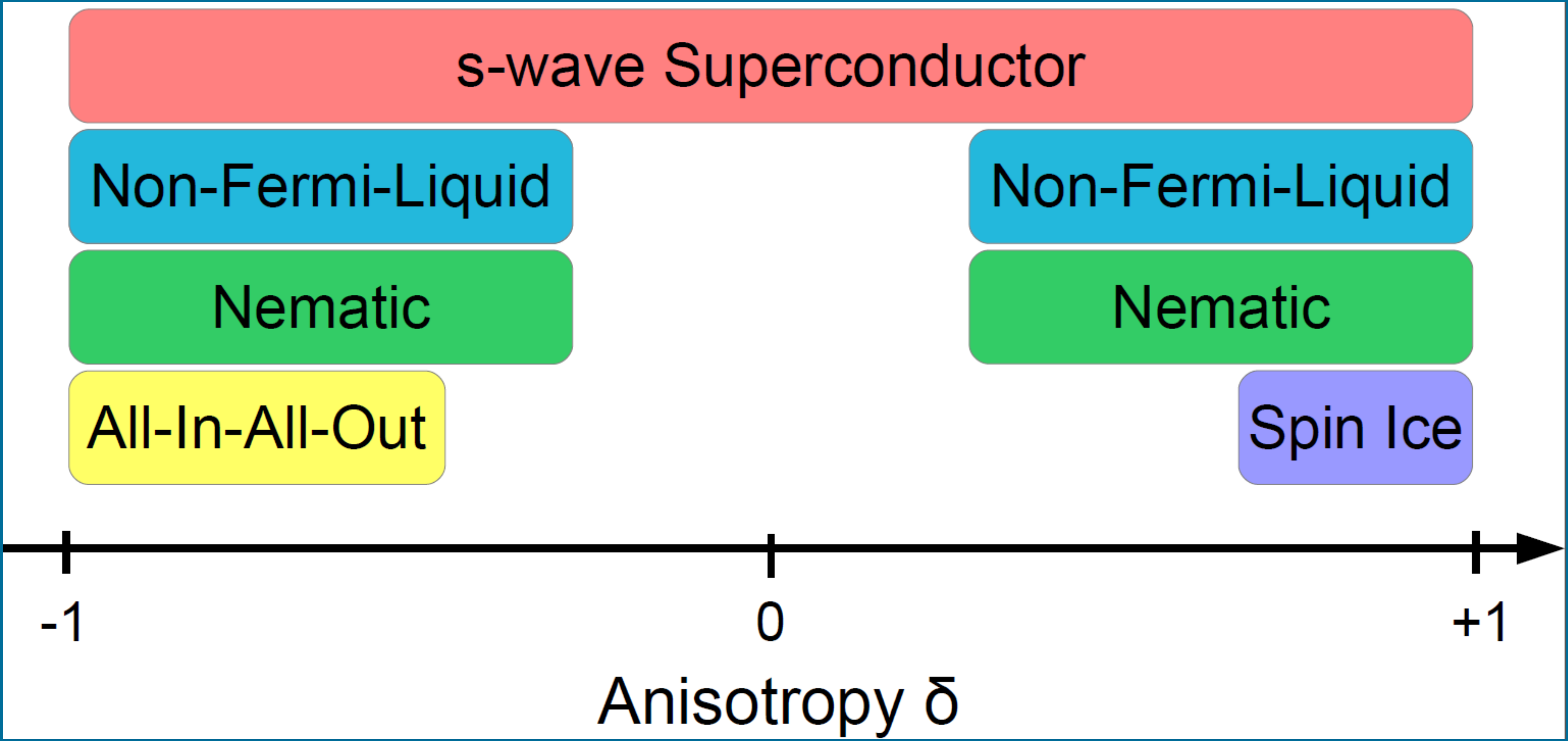
Spin Pics: Goswami, Roy, Das Sarma, PRB 95, 085120 (2017)

Tensor orders

Σ^A	order	rank	#	$\mathcal{I}$	$\mathcal{T}$	cubic case
$\mathbb{1}$	density $\rho = \langle \psi^\dagger \mathbb{1} \psi \rangle$	0	1	+	+	$\mathbb{1}$
$\mathcal{J}_i$	magnetic $m_i = \langle \psi^\dagger \mathcal{J}_i \psi \rangle$	1	3	+	−	$\vec{\mathcal{J}} = \begin{pmatrix} \mathcal{J}_1 \\ \mathcal{J}_2 \\ \mathcal{J}_3 \end{pmatrix}$
γ_a	nematic $\phi_a = \langle \psi^\dagger \gamma_a \psi \rangle$	2	5	+	+	$\vec{E} = \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix}$ $\vec{T} = \begin{pmatrix} \gamma_3 \\ \gamma_4 \\ \gamma_5 \end{pmatrix}$
W_μ	nemagnetic $\chi_\mu = \langle \psi^\dagger W_\mu \psi \rangle$	3	7	+	−	$\vec{W} = \begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix}$ $\vec{W}' = \begin{pmatrix} W_4 \\ W_5 \\ W_6 \end{pmatrix}$ W_7 (AIAO)

Nemagnetic order

RG fixed points - possible 2nd order quantum phase transitions



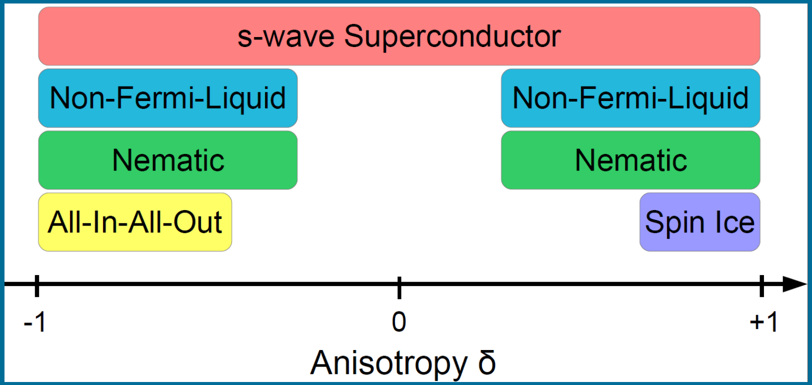
IB, Herbut, PRB 95, 075149 (2017)

Nemagnetic order

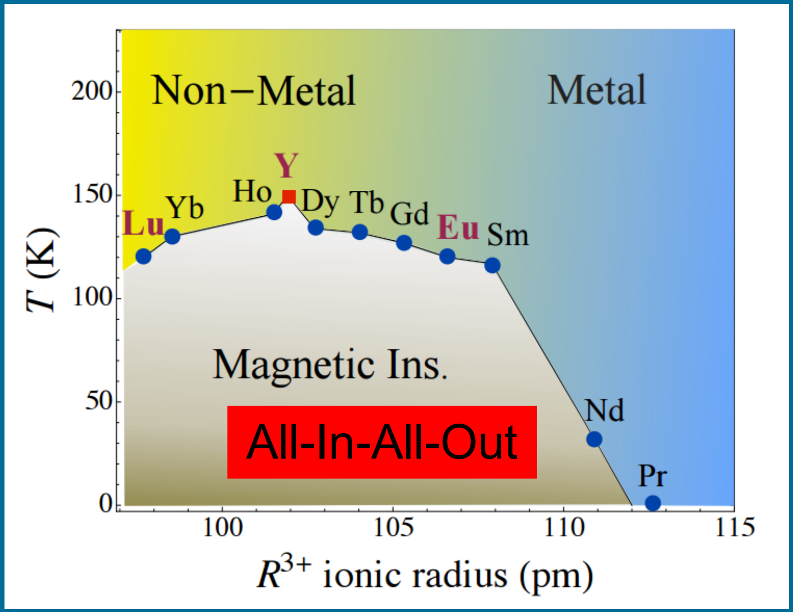
All-In-All-Out

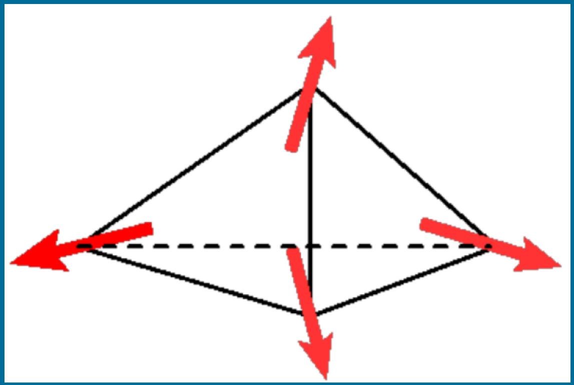
$$\chi = \langle \psi^\dagger W_7 \psi \rangle$$

$$\propto \langle \psi^\dagger (J_x J_y J_z + J_z J_y J_x) \psi \rangle$$



Savary, Moon, Balents
Goswami, Roy, Das Sarma

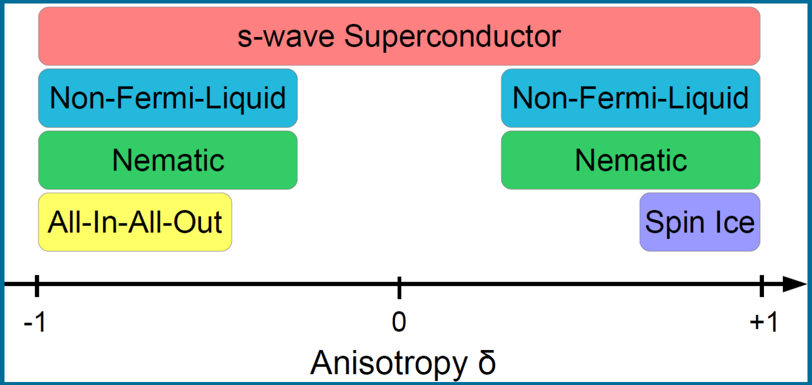




Pyrochlore Iridates: δ < 0

IB, Herbut, PRB 95, 075149 (2017)

Nemagnetic order



Order with index i

$$\chi_i = \langle \psi^\dagger V_i \psi \rangle$$
$$= \langle \psi^\dagger (\alpha \mathcal{J}_i + \beta W_i) \psi \rangle, \ \alpha^2 + \beta^2 = 1$$

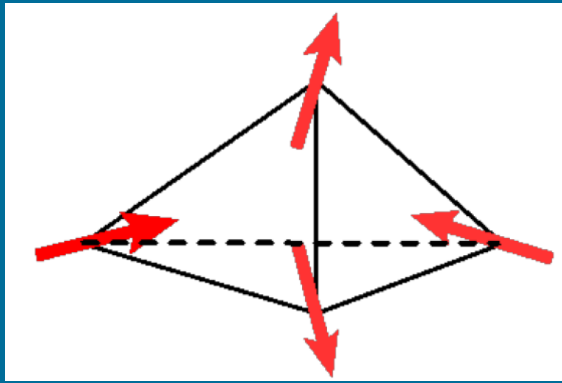
Instability analysis *selects* Spin ice (2-In-2-Out)

$$V_i = \frac{1}{\sqrt{5}} (\mathcal{J}_i + 2W_i) \propto -7J_i + 4J_i^3$$

Goswami, Roy, Das Sarma

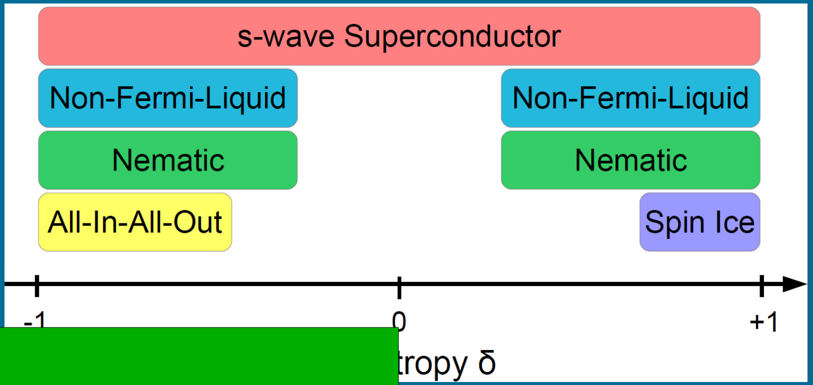
$$\{V_i, V_j\} = 2\delta_{ij}$$

Isobe, Fu



IB, Herbut, PRB 95, 075149 (2017)

Nemagnetic order



Order with index i

$$\chi_i = \langle \psi^\dagger V_i \psi \rangle$$

$$= \langle \psi^\dagger (a \quad 1$$

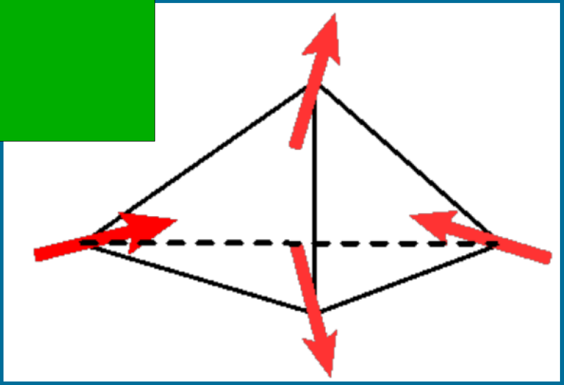
Thanks

Instability ana)

$$V_i = \frac{1}{\sqrt{5}}(\mathcal{J}_i$$

Goswami, Roy, Das Sarma

$$\{V_i, V_j\} = 2\delta_{ij}$$ Isobe, Fu



IB, Herbut, PRB 95, 075149 (2017)