Renormalization flow of relativistic fermions

(2<d<4)

Holger Gies

Helmholtz Institute Jena & TPI, Friedrich-Schiller-Universität Jena

& FRG @ Jena

Functional Renormalization – from quantum gravity and dark energy to ultracold atoms and condensed matter
Heidelberg, March 7-10 2017

FRG

ERG

Exact RG

Exact RG

from first principles

“includes irrelevant operators”

but often only approximation solutions

NPRG

fun RG

ERG

European RG

FRG

FRG: a prediction!

(WETTERICH'93)

Physics Letters B 301 (1993) 90–94
North-Holland

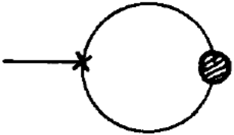
PHYSICS LETTERS B

Exact evolution equation for the effective potential

Christof Wetterich
Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, W-6900 Heidelberg, FRG

$$\frac{\partial}{\partial t}\Gamma_k[\varphi]=\tfrac{1}{2}\,\mathrm{Tr}\left(\left(\Gamma_k^{(2)}[\varphi]+R_k\right)^{-1}\frac{\partial}{\partial t}R_k\right). \qquad (3)$$

$$\frac{\partial}{\partial t}\Gamma_k = \frac{1}{2}$$



FRG: a prediction!

(WETTERICH'93)

Physics Letters B 301 (1993) 90–94
North-Holland

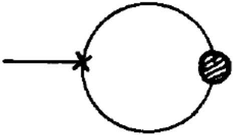
PHYSICS LETTERS B

Exact evolution equation for the effective potential

Christof Wetterich
Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, W-6900 Heidelberg, FRG

$$\frac{\partial}{\partial t}\Gamma_k[\varphi] = \tfrac{1}{2}\,\mathrm{Tr}\left(\left(\Gamma_k^{(2)}[\varphi] + R_k\right)^{-1}\frac{\partial}{\partial t}R_k\right). \qquad (3)$$

$$\frac{\partial}{\partial t}\Gamma_k = \frac{1}{2}$$



FRG: a prediction!

(WETTERICH'93)

the effective potential

, *Philosophenweg 16, W-6900 Heidelberg, FRG*

ved 17 December 1992

. . . written in FRG Land

FRG: a prediction!

(WETTERICH'93)

the effective potential

, *Philosophenweg 16, W-6900 Heidelberg, FRG*

ved 17 December 1992

. . . written in FRG Land

. . . used only by IOC and FIFA

FRG: a prediction!

(WETTERICH'93)

the effective potential

, *Philosophenweg 16, W-6900 Heidelberg, FRG*

ved 17 December 1992

. . . written in FRG Land

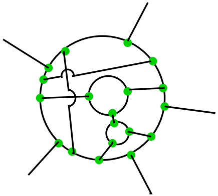
. . . use discouraged by authorities . . . considered to be a derogatory communist term

From quantum gravity . . . to . . . condensed matter

▷ low dimensional relativistic fermions & quantum gravity

▷ (perturbative) QFT:

$$\delta(\gamma) = d - \sum_i n_{E_i}[\phi_i] + \sum_\alpha n_{V_\alpha} \delta(V_\alpha)$$



$\Longrightarrow$ RG critical dimension:

$$D_{\text{RG, cr}} = \begin{cases} 4 & \text{(gauge + matter, Yukawa/Higgs)} \\ 2 & \text{(gravity, pure fermionic matter)} \end{cases}$$

▷ many similarities:

pert. nonrenormalizable, BUT: nonperturbatively renormalizable

"Asymptotic safety"

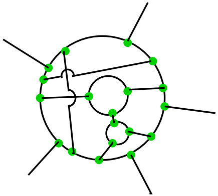
quantum phase transition

From quantum gravity . . . to . . . condensed matter

▷ low dimensional relativistic fermions & quantum gravity

▷ (perturbative) QFT:

$$\delta(\gamma) = d - \sum_i n_{E_i}[\phi_i] + \sum_\alpha n_{V_\alpha}\delta(V_\alpha)$$



$\Longrightarrow$ RG critical dimension:

$$D_{\text{RG, cr}} = \begin{cases} 4 & \text{(gauge + matter, Yukawa/Higgs)} \\ 2 & \text{(gravity, pure fermionic matter)} \end{cases}$$

▷ many similarities:

pert. nonrenormalizable, BUT: nonperturbatively renormalizable

"Asymptotic safety"

quantum phase transition

. . . no experimental evidence so far . . .

Chirality & Dirac Fermions

▷ d=3:

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}, \qquad \gamma_{\mu=1,2,3} \sim \sigma_{i=1,2,3} \quad \text{(irreducible)}$$

$\Longrightarrow$ no γ_5 ("no chirality")

▷ Dirac fermions in irreducible representation:

$$\chi, \bar{\chi} \qquad \text{2-component}$$

Chirality & Dirac Fermions

▷ d=3:

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}, \qquad \text{(reducible, 4-comp. spinors: } \psi, \bar{\psi}\text{)}$$

$$\Longrightarrow \quad \gamma_5, \qquad P_{L/R} = \frac{1}{2}(1 \pm \gamma_5)$$

$$\begin{array}{ll} \& & \gamma_4 \\ \& & \gamma_{45} = i\gamma_4\gamma_5 \qquad\qquad P^{45}_{L/R} = \tfrac{1}{2}(1 \pm \gamma_{45}) \end{array}$$

$$\mathcal{L}_{\text{kin}} = \bar{\psi}^a i\partial\!\!\!/ \psi^a \qquad = \bar{\psi}^a_L i\partial\!\!\!/ \psi^a_L + \bar{\psi}^a_R i\partial\!\!\!/ \psi^a_R = \dots$$

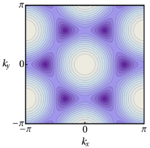
▷ max. chiral symmetry group: $U(2N_f)$

chiral symmetry (reducible) $\simeq$ flavor symmetry (irreducible)

Why 3d chiral fermions?

▷ Goal: understanding QPTs with

order parameter $\phi \leftrightarrow \psi, \bar{\psi}$ gapless fermions



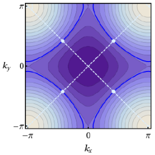
...beyond the ϕ^4 paradigm

(HERBUT'06)

▷ relativistic fermions from electrons on

- honeycomb lattice
- π-flux square lattice

$\Longrightarrow$ robust against weak interactions



Hubbard model on honeycomb lattice (VOJTA ET AL.'00)

Nodal d-wave superconductors (SACHDEV'10)

▷ for increasing coupling (Hubbard U or NN repulsion V):

phase transition: semi-metal $\rightarrow$ (Mott) insulator

$\Longrightarrow$ long-range order: AF, CDW, QAHS

Gross-Neveu model

a.k.a. "chiral Ising"

▷ classical action, e.g., in d=3: $a = 1, \ldots, N_f$

$$S = \int d^3x \left[\bar{\psi}^a i\partial\!\!\!/ \psi^a + \frac{1}{2N_f} \bar{g} (\bar{\psi}^a \psi^a)^2 \right], \qquad [\bar{g}] = -1$$

▷ symmetries of reducible model:

- discrete "chiral" symmetry:

$$\mathbb{Z}_2^5 : \qquad \psi^a \to \gamma_5 \psi^a, \qquad \bar{\psi}^a \to -\bar{\psi}^a \gamma_5$$

- flavor symmetry:

$$P_{L/R}^{45} = \frac{1}{2}(1 \pm \gamma_{45}) : \qquad U(N_f)_L \times U(N_f)_R$$

Gross-Neveu model

a.k.a. "chiral Ising"

▷ classical action, e.g., in d=3: $a = 1, \dots, 2N_\mathrm{f}$

$$S = \int d^3x \left[\bar{\chi}^a \mathrm{i}\partial\!\!\!/ \chi^a + \frac{1}{2N_\mathrm{f}} \bar{g} (\bar{\chi}^a \chi^a)^2 \right], \qquad [\bar{g}] = -1$$

▷ symmetries of irreducible model:

- parity symmetry:

$$\mathbb{Z}_2^P : \qquad \chi^a(x) \to \chi^a(-x), \qquad \bar{\chi}^a(x) \to -\bar{\chi}^a(-x)$$

- flavor symmetry:

$$U(2N_\mathrm{f})$$

▷ irreducible model in reducible notation ($2N_\mathrm{f} \in \mathbb{N}$):

$$(\bar{\chi}^a \chi^a)^2 \sim (\bar{\psi}\gamma_{45}\psi)^2$$

Gross-Neveu model

a.k.a. "chiral Ising"

▷ classical action, e.g., in d=3: $a = 1, \ldots, N_\mathrm{f}$

$$S = \int d^3x \left[\bar{\psi}^a \mathrm{i}\partial\!\!\!/\, \psi^a + \frac{1}{2N_\mathrm{f}} \bar{g} (\bar{\psi}^a \psi^a)^2 \right], \qquad [\bar{g}] = -1$$

▷ discrete "chiral" symmetry:

$$\mathbb{Z}_2^5: \qquad \psi^a \to \gamma_5 \psi^a, \qquad \bar{\psi}^a \to -\bar{\psi}^a \gamma_5$$

Gross-Neveu model

a.k.a. "chiral Ising"

▷ classical action, e.g., in d=3: $a = 1, \ldots, N_\mathrm{f}$

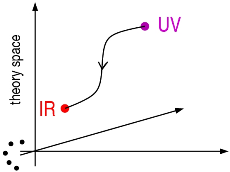
$$S = \int d^3x \left[\bar{\psi}^a \mathrm{i}\partial\!\!\!/\, \psi^a + \frac{1}{2N_\mathrm{f}} \bar{g} (\bar{\psi}^a \psi^a)^2 \right], \qquad [\bar{g}] = -1$$

▷ discrete "chiral" symmetry:

$$\mathbb{Z}_2^5: \qquad \psi^a \to \gamma_5 \psi^a, \qquad \bar{\psi}^a \to -\bar{\psi}^a \gamma_5$$

▷ *Recette: On prend . . .* (WETTERICH'93)

$$\partial_t \Gamma_k = \frac{1}{2} \,\mathrm{Tr}\, \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1}$$



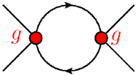
Gross-Neveu model

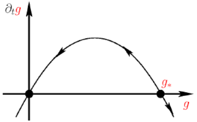
Simplest approximation: “pointlike” vertices:

$$\Gamma_k = \int d^3x \left[\bar{\psi}^a \mathrm{i}\partial\!\!\!/\, \psi^a + \frac{1}{2N_\mathrm{f}}\, \bar{g}_k (\bar{\psi}^a \psi^a)^2 \right]$$

▷ RG flow of dim’less coupling $g = k^{d-2}\bar{g}_k$:

$$\partial_t g \;\propto\; (d-2)g \;-\; \left(\frac{4N_\mathrm{f}-2}{N_\mathrm{f}}\right)\tilde{\partial}_t$$





▷ UV fixed point: g_*

▷ IR divergence in scalar channel for $g_\Lambda > g_*$ indication for χSB

▷ critical exponent $\Theta = 1/\nu = 1$ (in $d = 3$)

$\Longrightarrow$ asymptotically safe proven to all orders in $1/N_\mathrm{f}$ expansion

(Gawedzki, Kupiainen’85; Rosenstein, Warr, Park’89; de Calan et al.’91)

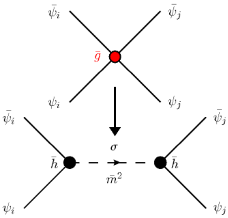
Partial Bosonization

▷ mapping to Yukawa model: (STRATONOVICH'58, HUBBARD'59)

$$S = \int d^3x \left[\bar{\psi}^a \mathrm{i}\partial\!\!\!/ \, \psi^a + \frac{1}{2N_\mathrm{f}} \bar{g} (\bar{\psi}^a \psi^a)^2 \right]$$

$$\downarrow$$

$$S_{FB} = \int d^3x \left[\bar{\psi}^a (\mathrm{i}\partial\!\!\!/ + \mathrm{i}\bar{h}\sigma) \, \psi^a + \frac{N_\mathrm{f}}{2} \bar{m}^2 \sigma^2 \right]$$



Pros:

+ RG flow into χSB regime
+ access to long-range observables

Cons:

- use in FRG trunc's: assumes dominance of bosonized channel
- can be affected by "Fierz ambiguity"

Cons less relevant for GN case

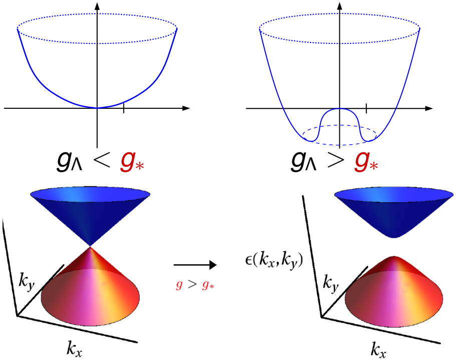
RG flow of Gross Neveu model

(Rosa, Vitale, Wetterich'01; Hofling, Nowak, Wetterich'02; Braun, HG, Scherer'10)

▷ NLO derivative expansion:

$$\Gamma_k = \int \left[Z_\psi \bar{\psi}^a (\mathrm{i}\partial\!\!\!/ + \mathrm{i}\bar{h}\sigma)\, \psi^a + \frac{1}{2} Z_\sigma (\partial_\mu \sigma)^2 + U(\sigma) \right]$$

▷ quantum phase transition



Exact large-N_{f} fixed-point solution

▷ anomalous dimensions: (BRAUN,HG,SCHERER'10)

$$\eta_\psi = 0, \qquad \eta_\sigma = 1$$

▷ large-N_{f} fixed point effective potential for $2 < d < 4$:

$$u_*(\rho) = -\frac{2d-8}{3d-4}\rho\, {}_2F_1\left(1-\frac{d}{2}, 1; 2-\frac{d}{2}; \frac{(d-4)(d-2)}{6d-8}\frac{d}{d_\gamma v_d}\rho\right), \rho = \frac{\sigma^2}{2}$$

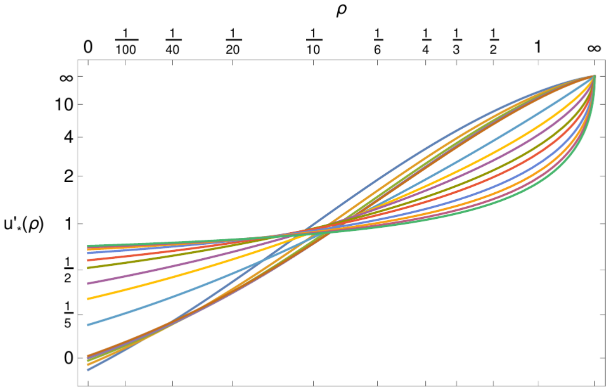
▷ exact critical exponents:

$$\Theta = 1, -1, -1, -3, -5, -7, \dots$$

$\Longrightarrow$ critical surface: dim $\mathcal{S} = 1$ physical parameter

Global effective potential and finite N_f

▷ FP solver with pseudo-spectral methods (BORCHARDT, KNORR'15)

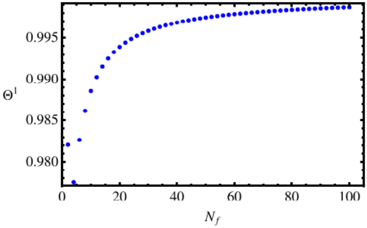


$3d$ Gross-Neveu universality class, (arbitrary N_f)

(Braun,HG,Scherer'10)

correlation exponent:

$$\nu = \frac{1}{\Theta^1}$$



N_f	Θ^1	Θ^2	Θ^3	Θ^4	Θ^5	Θ^6
2	0.9821	−0.8722	−1.0916	−3.5135	−6.0514	−8.5820
4	0.9775	−0.9240	−1.1010	−3.3910	−5.7739	−8.2429
12	0.9903	−0.9735	−1.0506	−3.1810	−5.3665	−7.6004
50	0.9975	−0.9936	−1.0143	−3.0510	−5.1062	−7.1789
100	0.9987	−0.9968	−1.0073	−3.0263	−5.0550	−7.0934
∞	1	−1	−1	−3	−5	−7

▷ leading-order derivative expansion

identical results for irreducible model (Rosa,Vitale,Wetterich'01; Hofling,Nowak,Wetterich'02)

FRG goes quantitative

▷ Derivative expansion:

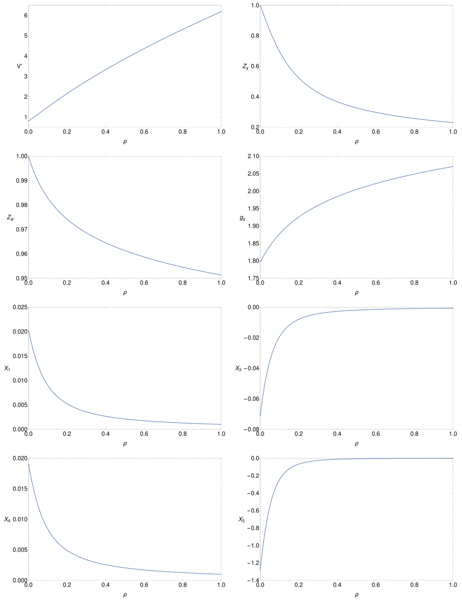
$$
\begin{aligned}
\Gamma_k \quad = \quad & \int \Big[\frac{1}{2} Z_\psi(\rho)(\bar\psi \partial\!\!\!/ \psi - (\partial_\mu \bar\psi)\gamma_\mu \psi) + h(\rho)\bar\psi\psi + \frac{1}{2} Z_\sigma(\rho)(\partial_\mu \sigma)^2 \\
& - U(\sigma) + iJ_\psi(\rho)(\partial_\mu \rho)\bar\psi\gamma_\mu\psi + X_1(\rho)\sigma(\partial_\mu\bar\psi)(\partial_\mu\psi) \\
& + \frac{i}{2} X_2(\rho)(\partial_\mu\sigma)[\bar\psi\partial\!\!\!/\psi - (\partial_\mu\bar\psi)\gamma_\mu\psi] + X_3(\rho)(\partial^2\sigma)\bar\psi\psi \\
& + \frac{1}{2} X_4(\rho)(\partial_\mu\sigma)[\bar\psi\Sigma_{\mu\nu}\partial_\nu\psi - (\partial_\nu\bar\psi)\Sigma_{\mu\nu}\psi] \\
& + \frac{1}{2}[X_5(\rho) + 2X_3'(\rho)](\partial_\mu\sigma)^2\sigma\bar\psi\psi \Big]
\end{aligned}
$$

- FRG LO: $U(\rho)$, h, Z_ψ, Z_σ (BRAUN,HG,SCHERER'10)
- FRG LO': $U(\rho)$, $h(\rho), Z_\psi, Z_\sigma$ (VACCA,ZAMBELLI'15)
- FRG NLO (KNORR'16)

(+regulator optimization, + pseudospectral solver + XACT)

FRG goes quantitative

(KNORR'16)



FRG goes quantitative

▷ critical exponents $N_f = 2$:

	FRG LO iGN (HNW'02)	FRG LO rGN (BGS'10)	FRG LO+ps rGN (BK'15)	FRG LO' rGN (VZ'15)	FRG NLO rGN (K'16)
ν	1.018	1.018	1.018	1.004	1.006(2)
η_σ	0.756	0.760	0.760	0.789	0.7765
η_ψ	0.032	0.032	0.032	0.031	0.0276

(HOFLING,NOWAK,WETTERICH'02; BRAUN,HG,SCHERER'10; BORCHARDT,KNORR'15; VACCA,ZAMBELLI'15; KNORR'16)

$\Longrightarrow$ satisfactory apparent convergence

FRG performs rather well already at LO

FRG goes quantitative

▷ critical exponents $N_f = 2$: method comparison

	FRG NLO (K'16)	MC (KLLP'94)	$1/N_f$ (G'94;HJ'14)	$2+\epsilon$ 3rd (G'90'91;LR'91)	$2+\epsilon$ 4th +res. (GLS'16)	$4-\epsilon$ 2nd (RYK'93)	2-sided Padé (FGKT'16)
ν	1.006(2)	1.00(4)	1.04	1.309	1.074	0.948	1.055
η_σ	0.7765	0.754(8)	0.776	0.602	0.745	0.695	0.739
η_ψ	0.0276	–	0.044	0.081	0.082	0.065	0.041

(Knorr'16) (Karkkainen,Lacaze,Lacock,Petersson'94) (Gracey'94; Herbut,Janssen'14) (Gracey'90'91; Luperini,Rossi'91) (Gracey,Luth,Schroder'16) (Rosenstein,Yu,Kovner'93) (Fei,Giombi,Klebanov,Tarnopolsky'16)

(Poster: B. Ihrig)

$\Longrightarrow$ acceptable overall agreement

with minor exceptions

FRG goes quantitative

▷ critical exponents $N_{\mathrm{f}} = 1$: method comparison

FRG goes quantitative

▷ critical exponents $N_f = 1$: method comparison

	FRG NLO (K'16)	MC CT-INT (WCT'14)	MC CT-INT f.T. (HW'16)	MC MQMC (LJY'15)	$1/N_f$ (G'94;HJ'14)	$4-\epsilon$ 2nd (RYK'93)	2-sided Padé (FGKT'16)
ν	0.930(4)	0.80(3)	0.74(4)	0.77(3)	0.735	0.862	1.174
η_σ	0.5506	0.302(7)	0.275(25)	0.45(2)	0.635	0.502	0.506
η_ψ	0.0645	–	–	–	0.105	0.110	0.096

(Knorr'16) (Wang,Corboz,Troyer'14) (Hesselmann,Wessel'16) (Li,Jiang,Yao'15) (Gracey'94; Herbut,Janssen'14)

(Rosenstein,Yu,Kovner'93) (Fei,Giombi,Klebanov,Tarnopolsky'16)

FRG goes quantitative

▷ critical exponents $N_f = 1$: method comparison

	FRG NLO (K'16)	MC CT-INT (WCT'14)	MC CT-INT f.T. (HW'16)	MC MQMC (LJY'15)	$1/N_f$ (G'94;HJ'14)	$4-\epsilon$ 2nd (RYK'93)	2-sided Padé (FGKT'16)
ν	0.930(4)	0.80(3)	0.74(4)	0.77(3)	0.735	0.862	1.174
η_σ	0.5506	0.302(7)	0.275(25)	0.45(2)	0.635	0.502	0.506
η_ψ	0.0645	–	–	–	0.105	0.110	0.096

(Knorr'16) (Wang,Corboz,Troyer'14) (Hesselmann,Wessel'16) (Li,Jiang,Yao'15) (Gracey'94; Herbut,Janssen'14)

(Rosenstein,Yu,Kovner'93) (Fei,Giombi,Klebanov,Tarnopolsky'16)

$\Longrightarrow$ overall confusion

$\Longrightarrow$ no MC data within lattice field theory so far

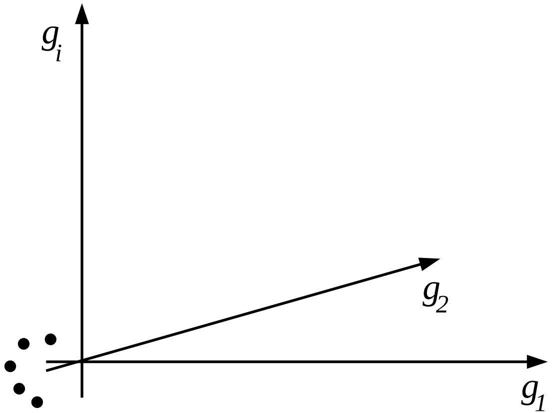
new sign-problem free algorithm with SLAC fermions

(Schmidt,Wellegehausen,Wipf in prep.)

stay tuned!

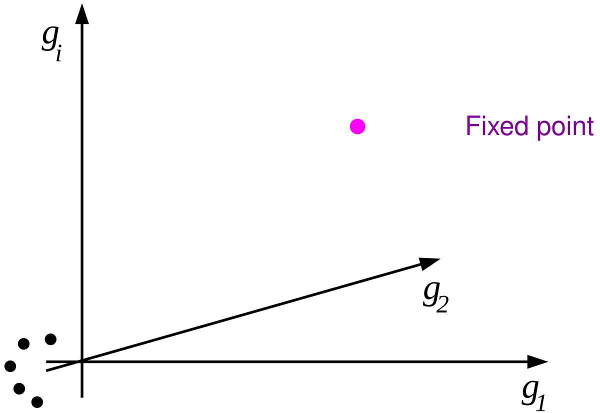
Emergent supersymmetry

- ▷ $d = 2 + 1$ lattice model $\sim 2\times$ Wess-Zumino (LEE'08)
- ▷ for "$N_\mathrm{f} = 1/4$": field content of GN compatible with supersymmetry
  $\Longrightarrow$ emergent susy? (BASHIROV'13; GROVER,SHENG,VISHWANATH'14)
  (SHIMADA,HIKAMI'15; ILIESIU ET AL.'16)
- ▷ RG flow in theory space



Emergent supersymmetry

- ▷ $d = 2 + 1$ lattice model $\sim 2\times$ Wess-Zumino (LEE'08)
- ▷ for "$N_\mathrm{f} = 1/4$": field content of GN compatible with supersymmetry
$\Longrightarrow$ emergent susy? (BASHIROV'13; GROVER,SHENG,VISHWANATH'14)
(SHIMADA,HIKAMI'15; ILIESIU ET AL.'16)
- ▷ RG flow in theory space



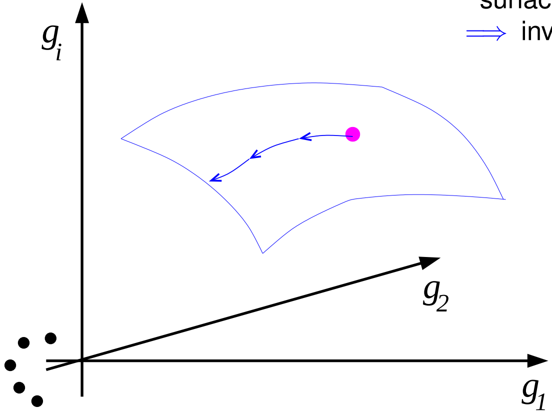
Emergent supersymmetry

▷ $d = 2 + 1$ lattice model $\sim 2\times$ Wess-Zumino (Lee'08)

▷ for "$N_f = 1/4$": field content of GN compatible with supersymmetry
$\Longrightarrow$ emergent susy? (Bashirov'13; Grover,Sheng,Vishwanath'14)

(Shimada,Hikami'15; Iliesiu et al.'16)

▷ RG flow in theory space



surface of higher symmetry
$\Longrightarrow$ invariant hyperplane

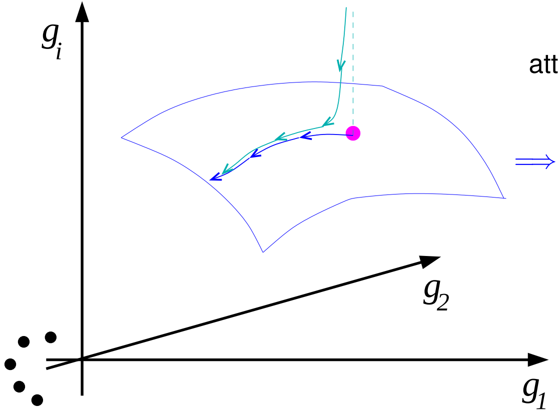
Emergent supersymmetry

▷ $d = 2 + 1$ lattice model $\sim 2\times$ Wess-Zumino (LEE'08)

▷ for "$N_\mathrm{f} = 1/4$": field content of GN compatible with supersymmetry
$\Longrightarrow$ emergent susy? (BASHIROV'13; GROVER,SHENG,VISHWANATH'14)

(SHIMADA,HIKAMI'15; ILIESIU ET AL.'16)

▷ RG flow in theory space



if perturbations are attracted by hyperplane

RG irrelevant

$\Longrightarrow$ emergent symmetry towards IR

Emergent supersymmetry

▷ FRG in GN at LO' (VACCA,ZAMBELLI'15)

$$\nu \simeq 0.693, \quad \eta_\sigma \simeq 0.154, \quad \eta_\psi \simeq 0.221 \neq \eta_\sigma$$

non-susy regularization

▷ manifestly supersymmetric FRG (BERGNER,HG,SYNATSCHKE,WIPF'08)

(HG,SYNATSCHKE,WIPF'09)

▷ FRG for WZ at NNLO (HEILMANN,HELLWIG,KNORR,ANSORG,WIPF'14)

$$\nu \simeq 0.710, \quad \eta_\sigma \simeq 0.180, \quad \eta_\psi \simeq 0.180 \equiv \eta_\sigma$$

▷ superscaling relation satisfied (HG,SYNATSCHKE,WIPF'09)

$$\frac{1}{\nu_W} = \frac{1}{2}(d - \eta), \quad d \geq 2$$

holds to all orders (HEILMANN,HELLWIG,KNORR,ANSORG,WIPF'14)

Emergent supersymmetry

▷ "$N_f = 1/4$"-GN-Yukawa model in susy + rest notation:

(HELLWIG,WIPF,ZANUSSO IN PREP.)

$$\Gamma_k = \int \Big[\frac{1}{2}(Z+Z_\psi)\bar{\psi} i\partial\!\!\!/ \psi - \frac{1}{2}(Z+Z_\sigma)(\partial_\mu \sigma)^2 - \frac{1}{2} Z F^2 + F W'(\phi) - \frac{1}{4} W''(\phi)\bar{\psi}\psi + V_0 + h(\phi)\bar{\psi}\psi\Big]$$

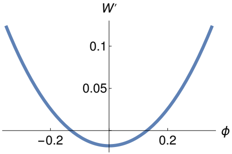
▷ control of higher-order operators by "dynamical supersymmetrization"

$$F \to F_k[\phi, \bar{\psi}\psi, F]$$

cf. dynamical hadronization

(HG,WETTERICH'01; PAWLOWSKI'05)

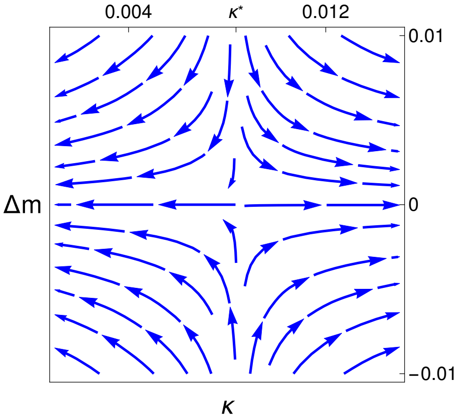
FP superpotential



(HELLWIG,WIPF,ZANUSSO IN PREP.)

Emergent supersymmetry

▷ "$N_f = 1/4$"-GN-Yukawa model: phase diagram (HELLWIG,WIPF,ZANUSSO IN PREP.)



▷ supersymmetric hyperplane: IR attractive

e.g. $\Delta m =$ fermion mass − boson mass $\to 0$

$\Longrightarrow$ emergent supersymmetry

Emergent supersymmetry

▷ "$N_f = 1/4$"-GN-Yukawa model:

	GN-FRG LO' (VZ'15)	WZ-FRG NNLO (HHKAW'14)	SUSY-FRG +GN+d.s. (HWZ'16)	$4-\epsilon$ 2nd (RYK'93)	2-sided Padé (FGKT'16)	CBS earl. est. (B'13)	CBS impr. est. (IKPPSY'15)
ν	0.693	0.710	0.722	0.710	–	–	–
η_σ	0.154	0.180	0.167	0.184	0.180	0.13	0.164
η_ψ	0.221	0.180	0.167	0.184	0.180	0.13	0.164
θ_2	- 0.796	-0.715	-0.765	–	–	–	–

(Vacca,Zambelli'15)(Heilmann,Hellwig,Knorr,Ansorg,Wipf'14) (Hellwig,Wipf,Zanusso in prep.)

(Rosenstein,Yu,Kovner'93) (Fei,Giombi,Klebanov,Tarnopolsky'16) (Bashkirov'13)

(Iliesu,Kos,Poland,Dufu,Simmons-Duffin,Yacoby'15)

$\Longrightarrow$ acceptable overall agreement

with minor exceptions

Models with GN symmetry: chiral $\mathrm{U}(N_\mathrm{f})_L \times \mathrm{U}(N_\mathrm{f})_R$

▷ symmetries of the reducible Gross-Neveu model (GEHRING,HG,JANSSEN'15)

$$S = \int d^3x \left[\bar{\psi}^a \mathrm{i}\partial\!\!\!/\, \psi^a + \frac{\bar{g}}{2N_\mathrm{f}} (\bar{\psi}^a \psi^a)^2 \right],$$

▷ chiral projector:

$$P^{45}_{L/R} = \frac{1}{2}(\mathbb{1} \pm \gamma_{45})$$

▷ independent chiral subsectors:

$$S = \int d^3x \left[\bar{\psi}^a_L \mathrm{i}\partial\!\!\!/\, \psi^a_L + \frac{\bar{g}}{2N_\mathrm{f}} (\bar{\psi}^a_L \psi^a_L)^2 \right] + \quad (L \leftrightarrow R)$$

Chiral $U(N_f)_L \times U(N_f)_R$ model

▷ complete pointlike Fierz basis: (GEHRING,HG,JANSSEN'15)

$$(S)^2 = (\bar{\psi}^a \psi^a)^2, \qquad (P)^2 = (\bar{\psi}^a \gamma_{45} \psi^a)^2,$$

$$(V)^2 = (\bar{\psi}^a \gamma_\mu \psi^a)^2, \qquad (T)^2 = \frac{1}{2}(\bar{\psi}^a \gamma_{\mu\nu} \psi^a)^2.$$

▷ chiral model contains (as invariant subspaces):

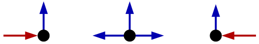
- reducible Gross-Neveu
- irreducible Gross-Neveu
- Thirring
- an some more ...

"meta-theory"

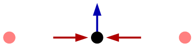
Chiral $U(N_f)_L \times U(N_f)_R$ model

- ▷ $N_f > 1$: 4 independent pointlike interactions (GEHRING,HG,JANSSEN'15)
- ▷ real fixed points: $\begin{cases} 12 & \text{for } 2 \geq N_f < 3.76 \\ 16 & \text{for } N_f \geq 3.76 \end{cases}$
- ▷ collision of 3 (!) fixed points at $N_{f,cr}^{(1)} \simeq 3.76$.

$N_f > N_{f,cr}^{(1)}$:



$N_f < N_{f,cr}^{(1)}$:



- ▷ red. and irred. Gross-Neveu: "critical FPs" for any $N_f > 1$
- ▷ Thirring: "critical FP" for $N_f > 6$ (2 rel. dir. for $N_f < 6$)

connection to MC result $N_{f,cr} \simeq 6.6$ for staggered fermions?

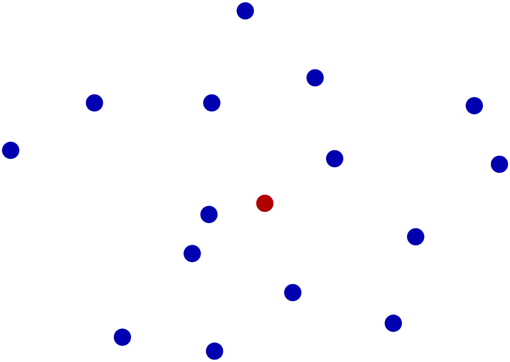
Chiral $\mathrm{U}(N_\mathrm{f})_L \times \mathrm{U}(N_\mathrm{f})_R$ model

(HG,Jaeckel,Wetterich'04; Gehring,HG,Janssen'15)

less symmetry $\Longrightarrow$ richer FP structure

Chiral $\mathrm{U}(N_\mathrm{f})_L \times \mathrm{U}(N_\mathrm{f})_R$ model

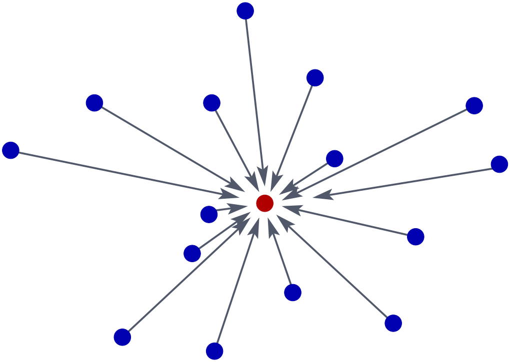
(HG,Jaeckel,Wetterich'04; Gehring,HG,Janssen'15)



for i pointlike interactions: 2^i FPs

Chiral $U(N_f)_L \times U(N_f)_R$ model

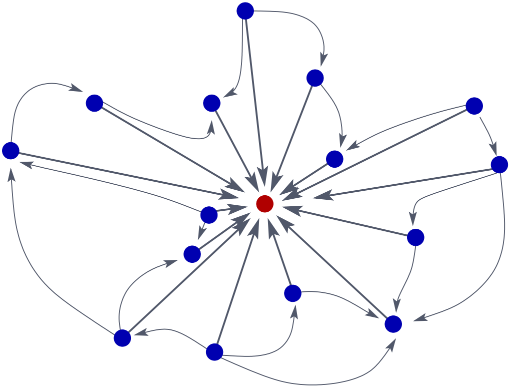
(HG,JAECKEL,WETTERICH'04; GEHRING,HG,JANSSEN'15)



each non-Gaußian FP has a critical exponent $\Theta = d - 2$

Chiral $\mathrm{U}(N_\mathrm{f})_L \times \mathrm{U}(N_\mathrm{f})_R$ model

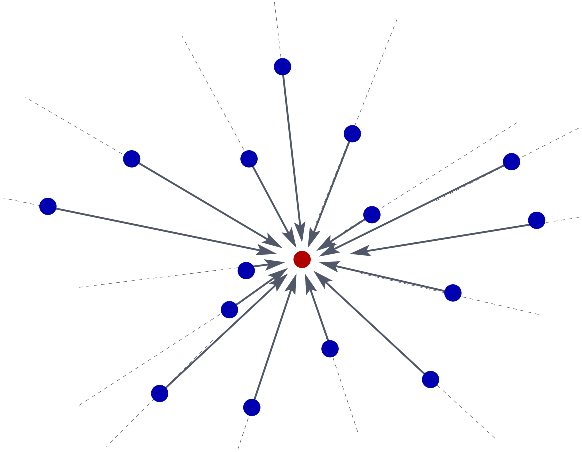
(HG, Jaeckel, Wetterich'04; Gehring, HG, Janssen'15)



$\begin{pmatrix} i \\ k \end{pmatrix}$ FPs with k relevant directions

Chiral $U(N_f)_L \times U(N_f)_R$ model

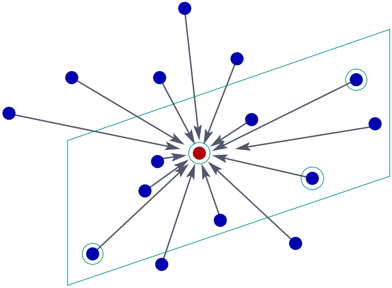
(HG, Jaeckel, Wetterich'04; Gehring, HG, Janssen'15)



all FP rays from Gaußian FP $\mathcal{O}$ are invariant subspaces

Chiral $\mathrm{U}(N_\mathrm{f})_L \times \mathrm{U}(N_\mathrm{f})_R$ model

(HG, Jaeckel, Wetterich'04; Gehring, HG, Janssen'15)



a plane containing four pairwise linearly independent FPs

$$\mathcal{O}, \mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3$$

is an invariant subspace.

$\Longrightarrow$ candidate for emergent symmetry e.g., $\mathrm{U}(2N_\mathrm{f})$ for $N_\mathrm{f} > 6$

Irreducible vs. reducible GN

▷ $N_f > 1$

(Gehring,HG,Janssen'15)

irreducible GN

$$\sim (\bar{\psi}\gamma_{45}\psi)^2$$

FP: $\mathcal{A}$

1 relevant direction

SB pattern:

$\mathbb{Z}_2$ parity

QAHS

reducible GN

$$\sim (\bar{\psi}\psi)^2$$

FP: $\mathcal{D}$

1 relevant direction

SB pattern:

$\mathbb{Z}_2$ "discrete chirality"

CDW

$\Longrightarrow$ same dimension d

symmetry of σ

\# of long-range degrees of freedom

Irreducible vs. reducible GN

▷ $N_\mathrm{f} = 1$ (GEHRING,HG,JANSSEN'15)

irreducible GN

$$\sim (\bar{\psi}\gamma_{45}\psi)^2$$

FP: $\mathcal{A}$

1 relevant direction

SB pattern:

$$\mathbb{Z}_2 \quad \text{parity}$$

QAHS

critical exponents:

$$\nu \simeq 1, \;\; \theta_2 \simeq -2$$

$\Longrightarrow$ spectator: $\mathrm{U}(2N_\mathrm{f})$

reducible GN

$$\sim (\bar{\psi}\psi)^2$$

FP: $\mathcal{D}$

2 relevant direction

FP: $\mathcal{E}$

1 relevant direction

SB pattern:

$$\mathbb{Z}_2 \quad \text{“discrete chirality”}$$

CDW

critical exponents:

$$\nu \simeq 1, \;\; \theta_2 \simeq -(3 - \sqrt{5})$$

$\Longrightarrow$ spectator: $\mathrm{U}(N_\mathrm{f})_L \times \mathrm{U}(N_\mathrm{f})_R$

Universality class conjecture

▷ universality classes are determined by (Gehring,HG,Janssen'15)

- dimension d
- symmetry of order parameter
- # long-range degrees of freedom
- & spectator symmetries

Conclusions

Low-dimensional chiral fermion systems:

- plethora of theories

  2$\times$Gross-Neveu,Thirring, NJL, chiral ...

- “perfect” quantum field theories

  non-perturbatively renormalizable, asymptotically safe

- wide variety of universality classes

  & variety of symmetry breaking patterns
  quantitative playground for FRG

- emergent (super-)symmetries

  ...general mechanism?

- specification of universality classes

  + spectator symmetries

Enjoy FRG Land!

Enjoy FRG Land!

Chiral $U(N_f)_L \times U(N_f)_R$ model

▷ e.g., $N_f = 2$ ($\sim$ graphen ?):

$$U(2)_L \times U(2)_R \simeq U_V(1) \times U_A(1) \times SU_L(2) \times SU_R(2)$$

- $U_V(1)$: charge conservation
- $U_A(1)$: translational symmetry on honeycomb lattice

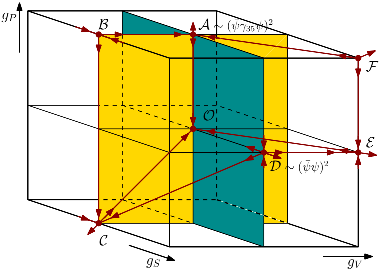
(HERBUT,JURICIC,ROY'09)

- $SU(2)_{L/R}$: independent spin rotation in the two Dirac-cone sectors (expected to be broken at strong coupling)

(JANSSEN,HERBUT'14)

Spinless chiral $U(1)_L \times U(1)_R$ model

- $N_f = 1$: 3 independent pointlike interactions (GEHRING,HG,JANSSEN'15)
- RG flow: 7 fixed points $(\mathcal{O}, \mathcal{A} - \mathcal{F})$



- 1 relevant direction at Thirring and irreducible Gross-Neveu FP

$\sim$ critical point, 2nd order QPT ?

- 2 relevant directions at reducible Gross-Neveu FP

1st order transition ?