Exact results for (un)safe QFT

Francesco Sannino

In collaboration with:

Litim,	1406.2337
Intriligator	1508.07411
Bajc	1610.09681
Pelaggi, Strumia, Vigiani	1701.01453

D·IAS
CERN
CP³ Origins
Cosmology & Particle Physics


en·light·en

Standard Model

Fields:

Gauge fields + fermions + scalars

Interactions:

Gauge: SU(3) x SU(2) x U(1) at EW scale

Yukawa: Fermion masses/Flavour

Culprit: Higgs

Scalar self-interaction

Gauge - Yukawa theories

$$L = -\frac{1}{2}F^2 + i\overline{Q}\gamma_\mu D^\mu Q + y(\overline{Q}_L H Q_R + \text{h.c.})$$

Yukawa

Gauge

$$\mathrm{Tr}\left[\mathrm{DH}^\dagger \mathrm{DH}\right] - \lambda_\mathrm{u} \mathrm{Tr}\left[(\mathrm{H}^\dagger \mathrm{H})^2\right] - \lambda_\mathrm{v} \mathrm{Tr}\left[(\mathrm{H}^\dagger \mathrm{H})\right]^2$$

Scalar selfinteractions

4D: standard model, dark matter, …

Lower D: condensed matter, phase transitions, graphene

4D plus: extra dimensions, string theory, …

Universal description of physical phenomena

Standard Model (blind spots)

$$L = -\frac{1}{2}F^2 + i\overline{Q}\gamma_\mu D^\mu Q + y(\overline{Q}_L H Q_R + \text{h.c.})$$

Yukawa

Gauge

$$\mathrm{Tr}\left[\mathrm{DH}^\dagger \mathrm{DH}\right] - \lambda_\mathrm{u}\mathrm{Tr}\left[(\mathrm{H}^\dagger\mathrm{H})^2\right] - \lambda_\mathrm{v}\mathrm{Tr}\left[(\mathrm{H}^\dagger\mathrm{H})\right]^2$$

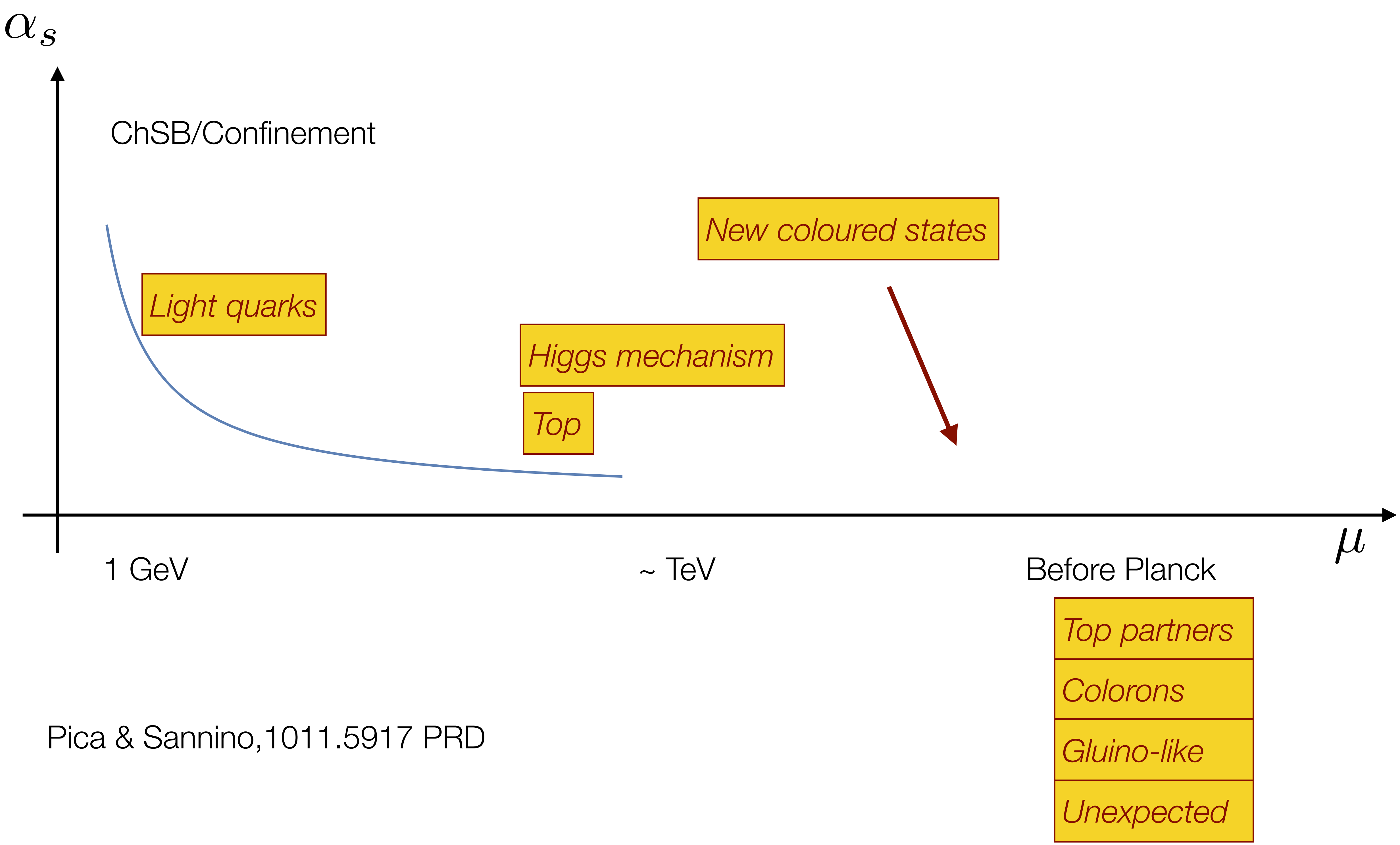
Scalar selfinteractions

- Gauge structure is established
- Yukawa structure partially constrained
- Higgs self-coupling is not directly constrained
- Unsafe field theory

But it does work well, so far!

Can QCD be safe?

Sannino, 1511.09022



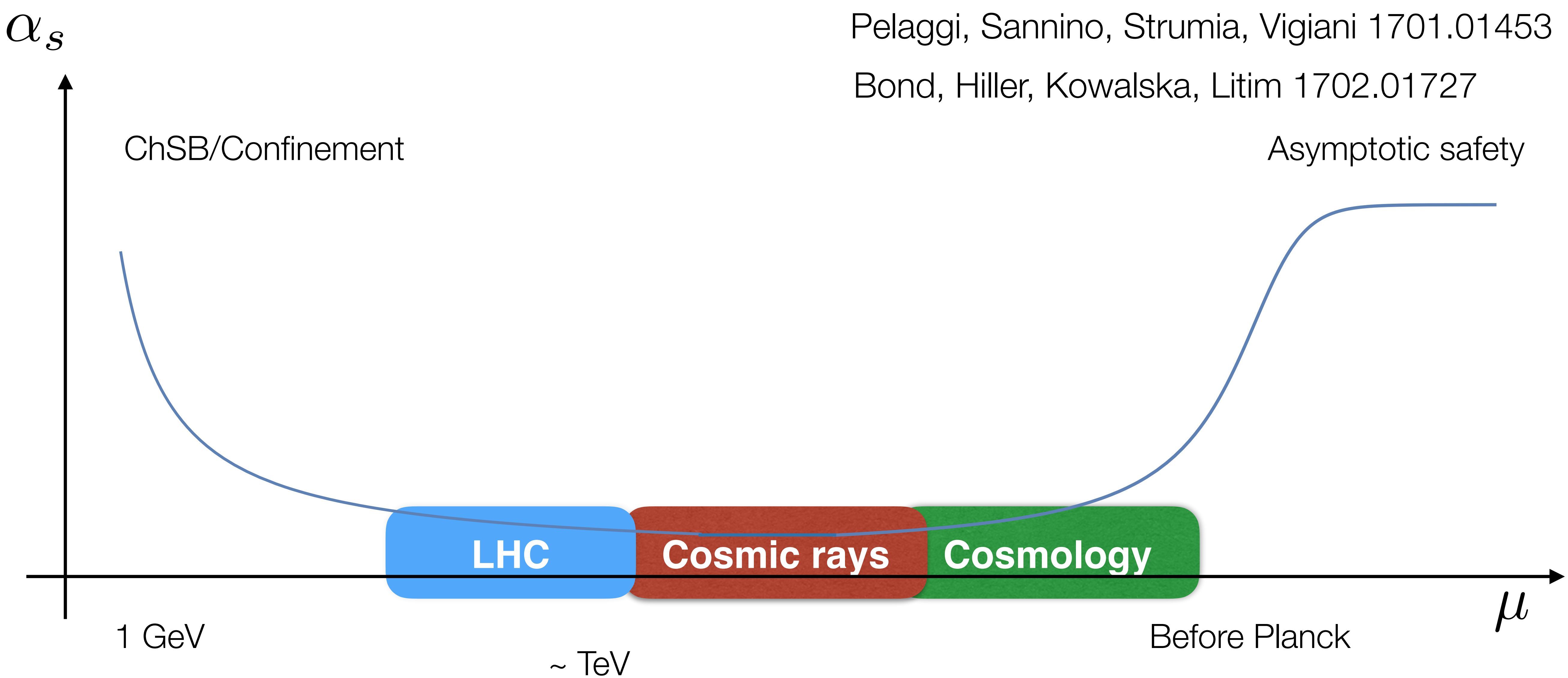
Pica & Sannino,1011.5917 PRD

Is the safe QCD scenario testable?

Sannino, 1511.09022

Pelaggi, Sannino, Strumia, Vigiani 1701.01453

Bond, Hiller, Kowalska, Litim 1702.01727

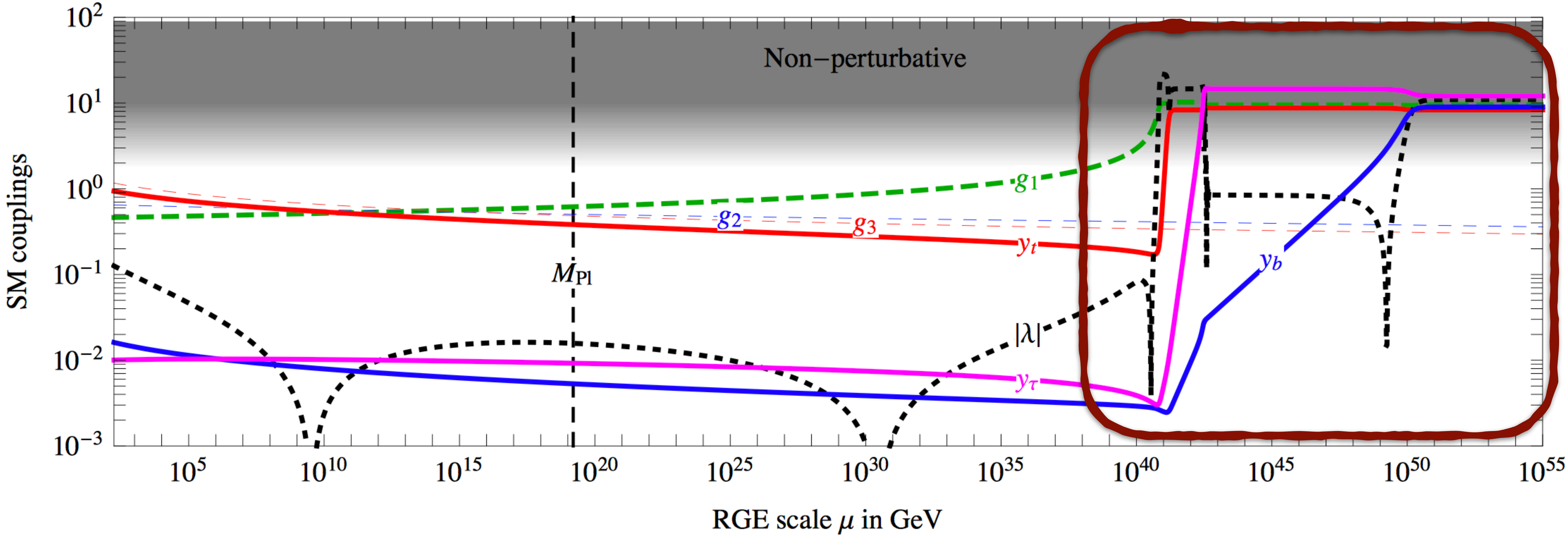


Asymptotic freedom is not a must for UV complete theories

Model independent tests of new coloured states at the LHC

Becciolini, Gillioz, Nardecchia, Sannino, Spannowsky 1403.7411, PRD

Is the Standard Model safe?



SM RGE at 3 loops in $g_{1,2,3}$, y_t, λ and at 2 loops in $y_{b,\tau}$

Pelaggi, Sannino, Strumia, Vigiani 1701.01453

Do theory like these exist?

Precise and/or nonperturbative exact results for UV interacting fixed points

Exact 4D Interacting UV Fixed Point

Antipin, Gillioz, Mølgaard, Sannino 1303.1525 PRD

Litim and Sannino, 1406.2337, JHEP

Litim, Mojaza, Sannino, 1501.03061, JHEP

$$L = -F^2 + i\overline{Q}\gamma \cdot DQ + y(\overline{Q}_L H Q_R + \mathrm{h.c.}) +$$

$$\mathrm{Tr}\left[\partial \mathrm{H}^\dagger \partial \mathrm{H}\right] - \mathrm{u}\mathrm{Tr}\left[(\mathrm{H}^\dagger \mathrm{H})^2\right] - \mathrm{v}\mathrm{Tr}\left[(\mathrm{H}^\dagger \mathrm{H})\right]^2$$

Fields	$[SU(N_c)]$	$SU_L(N_f)$	$SU_R(N_f)$	$U_V(1)$
G_μ	Adj	1	1	0
Q_L	$\square$	$\overline{\square}$	1	1
Q_R^c	$\overline{\square}$	1	$\square$	-1
H	1	$\square$	$\overline{\square}$	0

Veneziano Limit

Litim and Sannino, 1406.2337, JHEP

Litim, Mojaza, Sannino, 1501.03061, JHEP

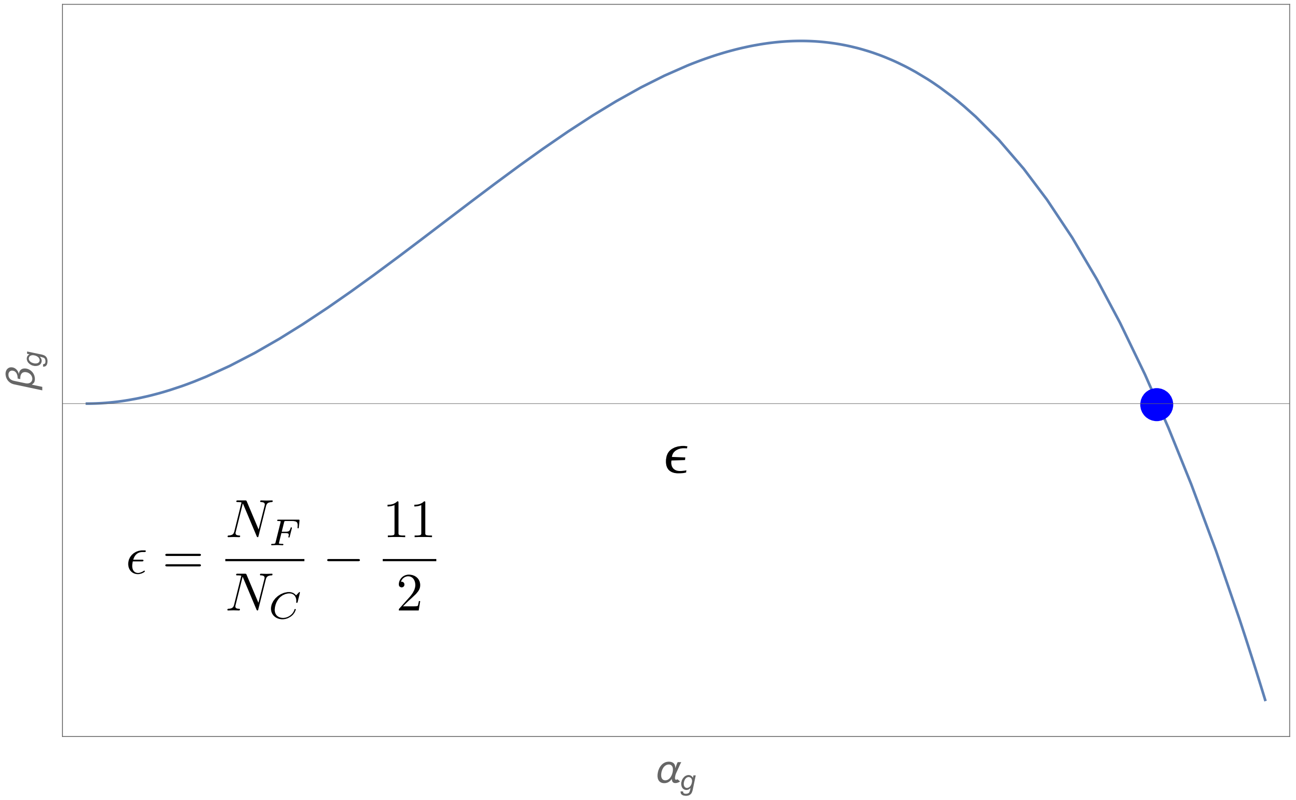
- Normalised couplings

$$\alpha_g = \frac{g^2\, N_C}{(4\pi)^2}\,, \quad \alpha_y = \frac{y^2\, N_C}{(4\pi)^2}\,, \quad \alpha_h = \frac{u\, N_F}{(4\pi)^2}\,, \quad \alpha_v = \frac{v\, N_F^2}{(4\pi)^2}$$

$$\frac{v}{u} = \frac{\alpha_v}{\alpha_h N_F}$$

At large N

$$\frac{N_F}{N_C} \in \Re^+$$



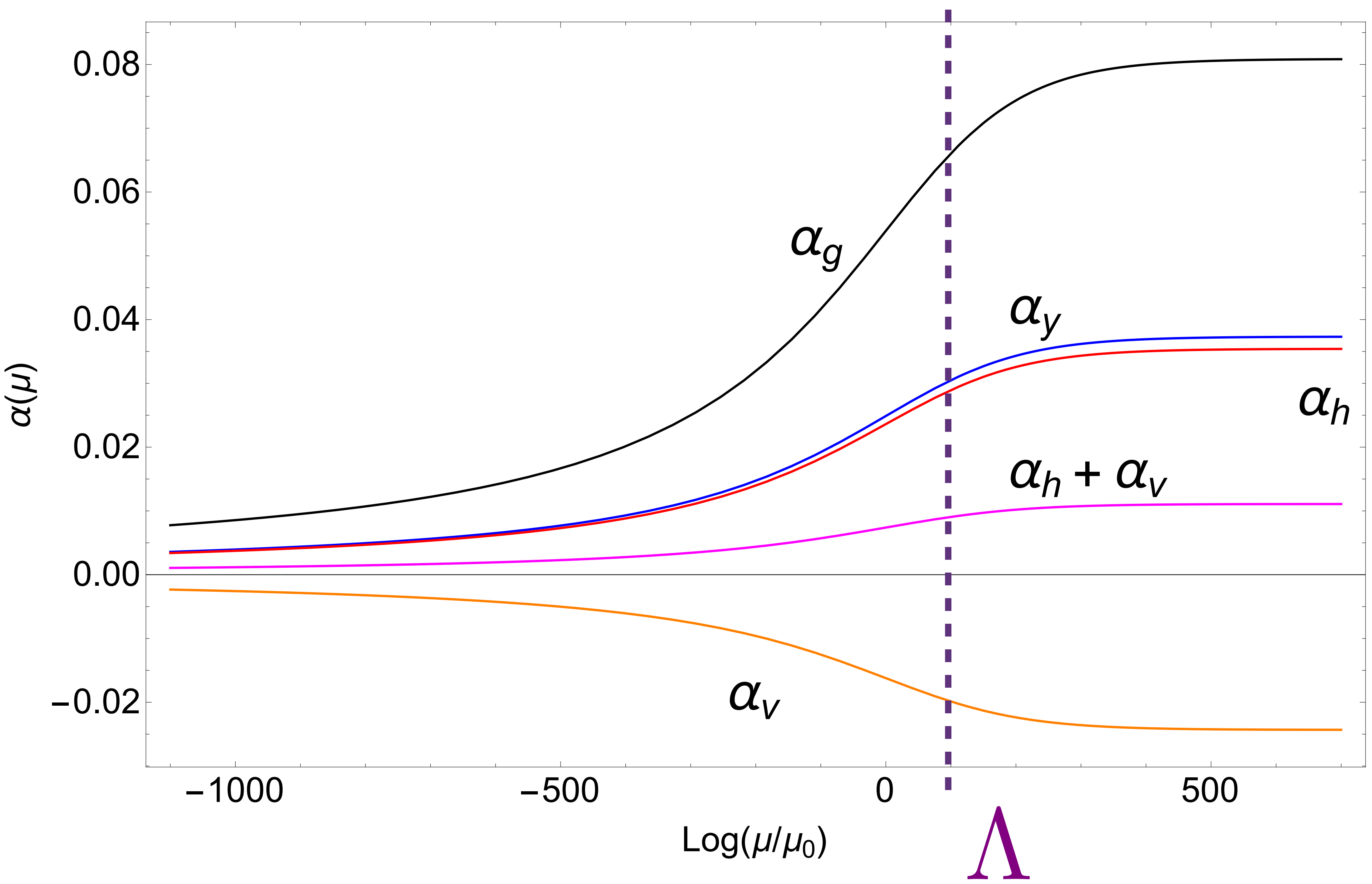
Impossible in Gauge Theories with Fermions alone

Caswell, PRL 1974

Complete asymptotic safety

Litim and Sannino, 1406.2337, JHEP

Gauge + fermion + scalars theories can be fund. at any energy scale



Scalars are needed to make the theory fundamental

Violation of the thermal d.o.f. count

Thermal d.o.f. conjecture

Appelquist, Cohen, Schmaltz, th/9901109 PRD

Corrected SU(2) GB count in Sannino 0902.3494 PRD

$$f(T) = -\frac{\mathcal{F}(T)}{T^4}\frac{90}{\pi^2} = \frac{p(T)}{T^4}\frac{90}{\pi^2}$$

$$f_{IR} \leq f_{UV}$$

Thermal d.o.f. is violated

$$f_{IR} \geq f_{UV}$$

Rischke & Sannino 1505.07828, PRD

Although the thermal d.o.f. count is violated the a-theorem works!

Gauged Higgs UV Fixed Point

Pelaggi, Sannino, Strumia, Vigiani, 1701.01453

Fields	Gauge symmetries		Global symmetries	
	Spin	$SU(N_c)$	$U(N_F)_L$	$U(N_F)_R$
ψ	$1/2$	$\square$	$\bar{\square}$	1
$\bar{\psi}$	$1/2$	$\bar{\square}$	1	$\square$
S	0	1	$\square$	$\bar{\square}$
H	0	$\square$	1	1
N	$1/2$	1	1	$\bar{\square}$
N'	$1/2$	1	$\square$	1

$$V = \lambda_{S1}(\mathrm{Tr}S^\dagger S)^2 + \lambda_{S2}\,\mathrm{Tr}(S^\dagger S S^\dagger S) + \lambda_H (H^\dagger H)^2 + \lambda_{HS}(H^\dagger H)\,\mathrm{Tr}(SS^\dagger)$$

$$\mathscr{L}_Y = y\, S_{ij}\psi_i\bar{\psi}_j + y'\, S^*_{ij} N_i N'_j + \boxed{\tilde{y}\, H\bar{\psi}_i N_i + \tilde{y}'\, H^*\psi_i N'_i + \mathrm{h.c.}}$$

Controllably safe in all couplings

Supersymmetric (un)safety

Intriligator and Sannino, 1508.07413, JHEP

Bajc and Sannino, 1610.09681, JHEP

Exact results beyond perturbation theory

Unitarity constraints

- Operators belong to unitary representations of the superconf. group
- Dimensions have different lower bounds
- Gauge invariant spin zero operators

$$D(\mathcal{O}) \geq 1, \qquad D(\mathcal{O}) = 1 \leftrightarrow P_\mu P^\mu(\mathcal{O}) = 0,$$

- Chiral primary operators have dim. D and $U(1)_R$ charge R

$$D(\mathcal{O}) = \frac{3}{2} R(\mathcal{O})$$

$$D(Q_i) \equiv 1 + \frac{1}{2}\gamma_i(g) = \frac{3}{2} R(Q_i) \equiv \frac{3}{2} R_i$$

Central charges

- Positivity of coefficients related to the stress-energy trace anomaly
- ‘a(R)’ Conformal anomaly of SCFT = $U(1)_R$ ’t Hooft anomalies
  [proportional to the square of the dual of the Rieman Curvature]

$$a(R) = 3\mathrm{Tr}U(1)_R^3 - \mathrm{Tr}U(1)_R$$

- ‘c(R)’
  [proportional to the square of the Weyl tensor]

$$c(R) = 9\mathrm{Tr}U(1)_R^3 - 5\mathrm{Tr}U(1)_R$$

- ‘b(R)’
  [proportional to the square of the flavor symmetry field strength]

$$b(R) = \mathrm{Tr}U(1)_R F^2$$

a-theorem

For any super CFT besides positivity we also have, following Cardy

$$\Delta a \equiv a_{UV} - a_{IR} > 0$$

$$\Delta a = a_{\mathrm{UV}} - a_{\mathrm{IR}} = \pm\frac{1}{9}\sum_i |r_i| \left[(3R_i - 2)^2(3R_i - 5)\right] > 0$$

r_i = dim. of matter rep.

+(-) for asymptotic safety (freedom)

Stronger constraint for asymp. safety, since at least one large R > 5/3

SQCD with H

Fields	$[SU(N_c)]$	$SU_L(N_f)$	$SU_R(N_f)$	$U_V(1)$	$U(1)_R$
W_α	Adj	1	1	0	1
Q	$\square$	$\overline{\square}$	1	1	$1-\frac{N_c}{N_f}$
$\widetilde{Q}$	$\overline{\square}$	1	$\square$	-1	$1-\frac{N_c}{N_f}$
H	1	$\square$	$\overline{\square}$	0	$2\frac{N_c}{N_f}$

AF is lost

$$N_f > 3N_c$$

$$W = y\,\mathrm{Tr}\,\mathrm{QH}\widetilde{\mathrm{Q}}$$

$$\beta(\alpha_g) \approx -2\alpha_g^2\left[3-\frac{N_f}{N_c}+\left(6-4\frac{N_f}{N_c}\right)\alpha_g+2\frac{N_f^2}{N_c^2}\alpha_y+\mathcal{O}(\alpha^2)\right]$$

$$\beta(\alpha_y) \approx 2\alpha_y\left[\left(2\frac{N_f}{N_c}+1\right)\alpha_y-2\alpha_g+\mathcal{O}(\alpha^2)\right]$$

No perturbative UV fixed point

$$\beta(\alpha_g) \approx 2\alpha_g^2\left[\epsilon+\frac{6}{7}\alpha_g\right]$$

SQCD with H

Assume a nonperturbative fixed point, however

$$D(H) = \frac{3}{2}R(H) = 3\frac{N_c}{N_f} < 1 \quad \text{for} \quad N_f > 3N_c$$

Violates the unitarity bound

$$D(\mathcal{O}) \geq 1$$

Potential loophole: H is free and decouples at the fixed point

Check if SQCD without H has an UV fixed point

SQCD

Unitarity bound is not sufficient

$$\mathcal{M} = Q\tilde{Q} \qquad D_{SCFT}(\mathcal{M}) = \frac{3}{2}R_{SCFT}(\mathcal{M}) = 3\frac{N_f - N_c}{N_f}$$

$$\mathcal{B} = Q^{N_c} \qquad D_{SCFT}(\mathcal{B}) = D_{SCFT}(\widetilde{\mathcal{B}}) = \frac{3}{2}R_{SCFT}(\mathcal{B}) = \frac{3}{2}N_c\frac{N_f - N_c}{N_f}$$

Can be ruled out via a-theorem $\quad a(R) = 3\mathrm{Tr}U(1)_R^3 - \mathrm{Tr}U(1)_R.$

$$a_{\mathrm{UV-safe}} - a_{\mathrm{IR-safe}} < 0$$

Non-abelian SQED with(out) H cannot be asymptotically safe

*Generalisation to several susy theories using a-maximisation**

Super safe GUTs

Bajc and Sannino, 1610.09681, JHEP

Exact results

Gaining R parity… but

R-symmetry from SO(10) Cartan subalgebra generator B-L

$$R = (-1)^{3(B-L)+2S} = M(-1)^{2S} \quad \text{with} \quad M = (-1)^{3(B-L)}$$

M = matter parity

Elegant breaking of SO(10) preserving R-parity:

Introduce 126 + 126* Higgs in SO(10)

126(126*) SM and SU(5) singlet has B-L=-2(2) preserving R-parity

asymp. freedom is lost

$$W_{Yukawa} = 16_a \left(Y_{10}^{ab} 10 + Y_{126}^{ab} \overline{126} + Y_{120}^{ab} 120 \right) 16_b$$

a, b run over generations

To fully break SO(10) to SM add 210 of SO(10)

In summary: 3 x 16 + 126 + 126* + 10 + 210 contributes

$$\beta_{1-loop} = -109$$

Asymptotic freedom is badly lost!

Exact results

Minimal SO(10) without super potential

3 x 16 + 126 + 126 + 10 + 210* ***is unsafe****.*

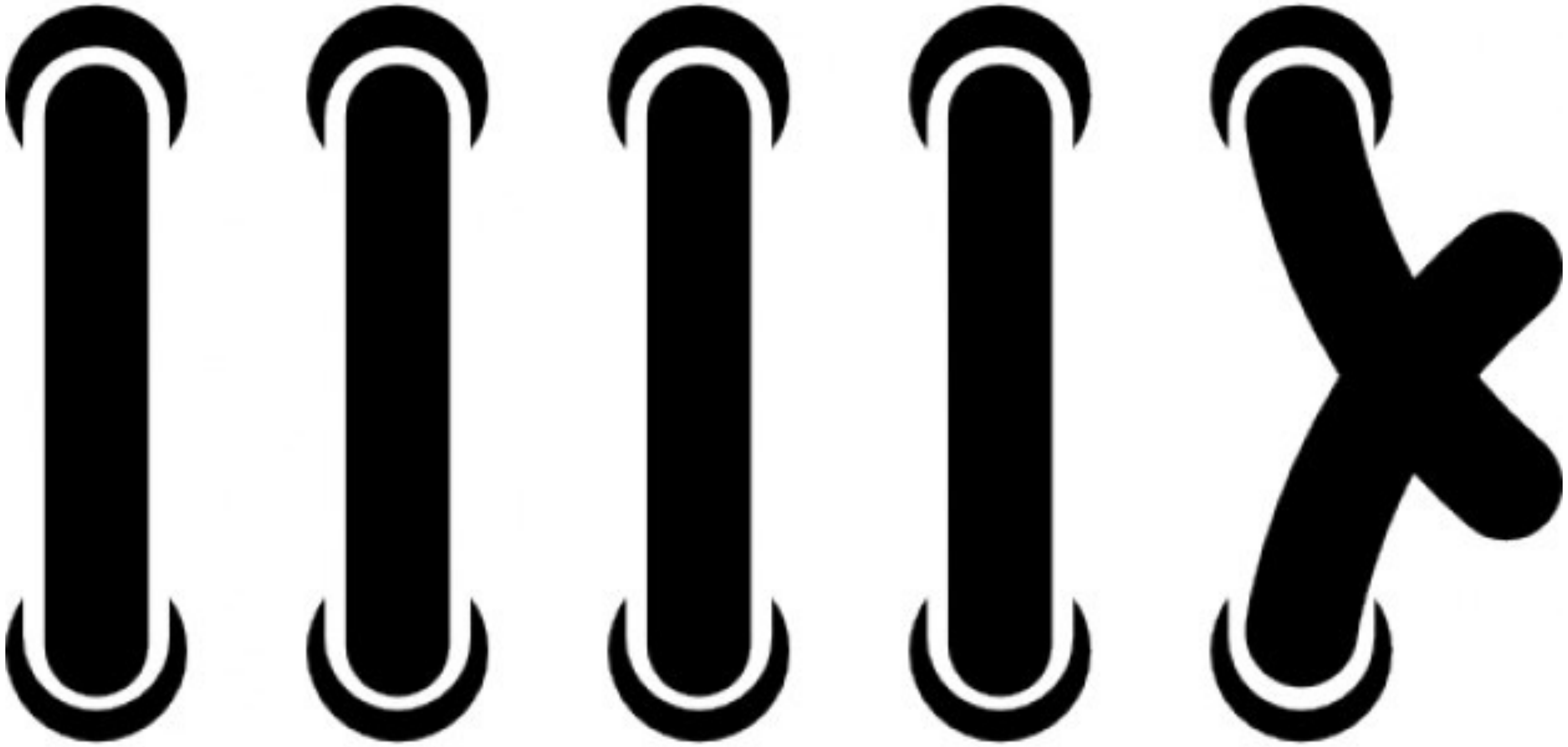
Exotic examples exist requiring thousands of generations!

Minimal SO(10) with general 3-linear super potential

$$W = y_1\, 210^3 + y_2\, 210\, 126\, \overline{126} + y_3\, 210\, 126\, 10 + y_4\, 210\, \overline{126}\, 10 + \sum_{a,b=1,2,3} 16_a\, 16_b\, \left(y_{5,ab}\, 10 + y_{6,ab}\, \overline{126}\right)$$

- *All trilinear present then: R=2/3 for all fields and no NSVZ UV fixed point*
- *Eliminate one 16 from super potential passes the constraints*

Super GUTs with R-charge are challenging!



Higgs as shoelace

Outlook

Extend to other (chiral) gauge theories/space-time dim

[Ebensen, Ryttov, Sannino,1512.04402 PRD, Codello, Langaeble, Litim, Sannino, JHEP 1603.03462, Mølgaard and Sannino 1610.03130]

N=1 Susy GUTs with R-parity are unlikely

Go beyond P.T. [Lattice, dualities, holography, truncations]

New ways to unify flavour?

Models of DM and/or Inflation

Challenging QCD asymptotic freedom

Is there a 4D alternative to asymptotically safe gravity ?

Backup slides

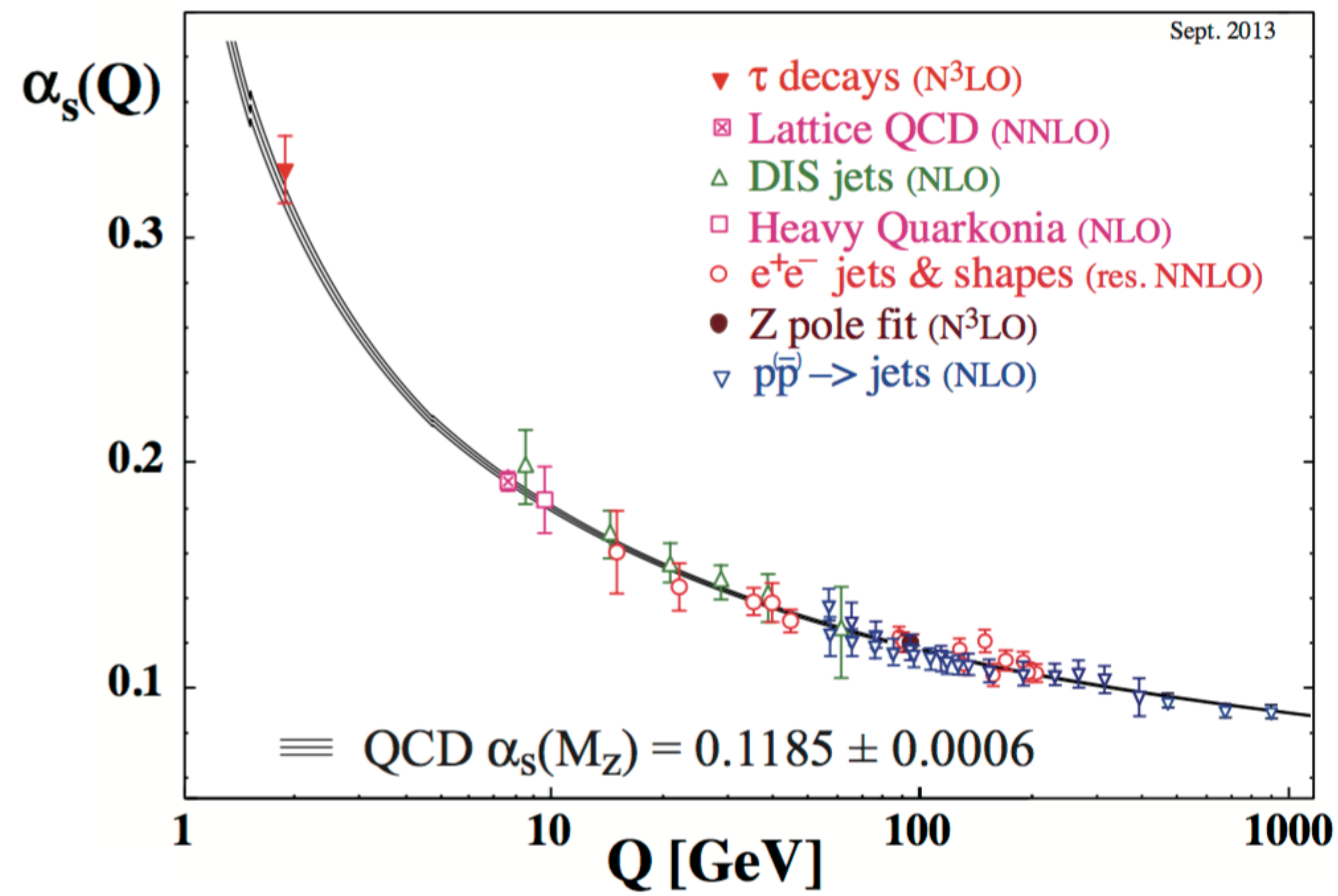
Phenomenological Applications

Safe QCD

QCD

QCD is not IR conformal because

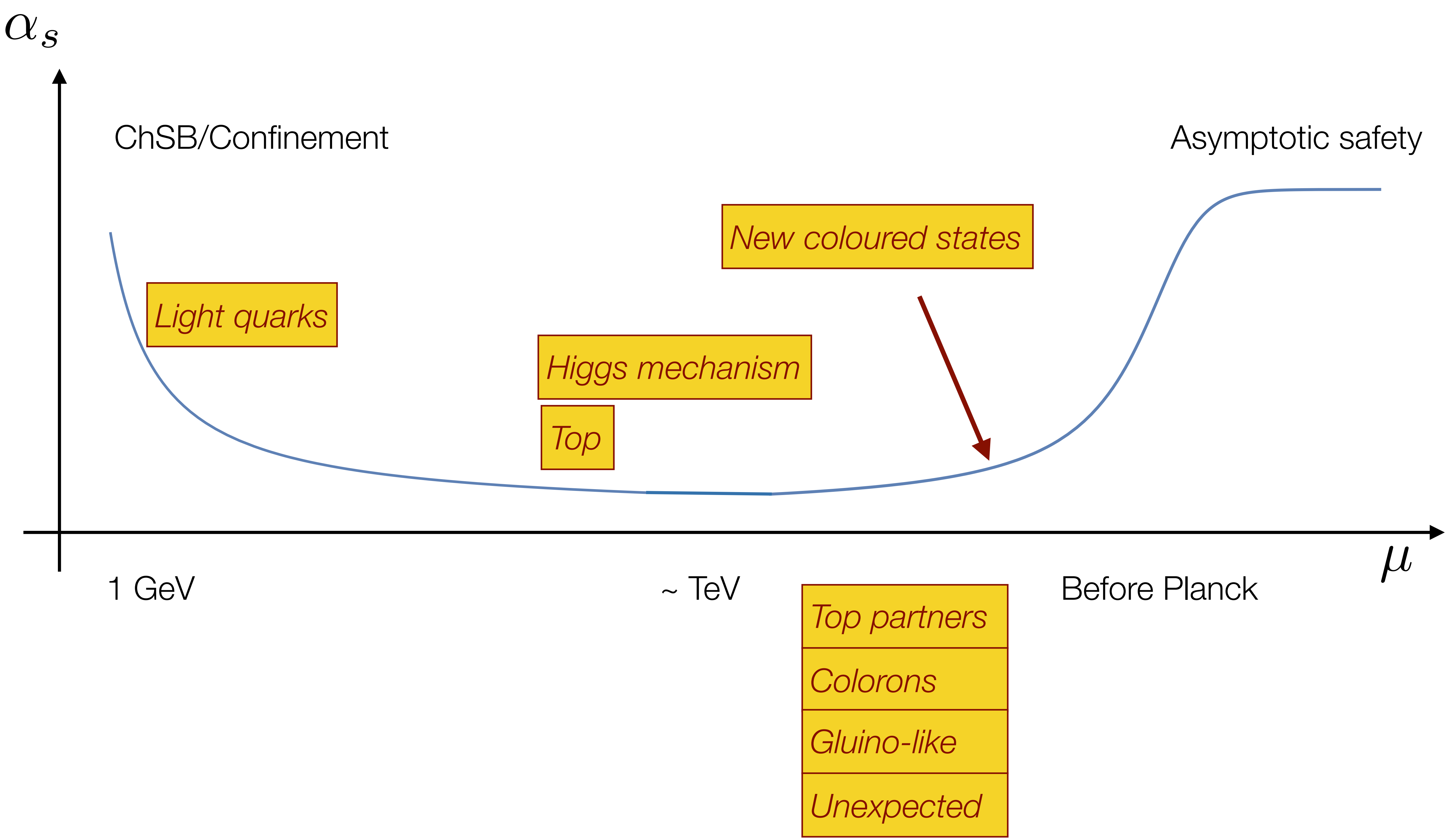
- Hadronic spectrum/dyn. mass
- Pions <-> Spont. ChSB



Asymptotic freedom verified < TeV

If above TeV asymptotic freedom is lost, then what?

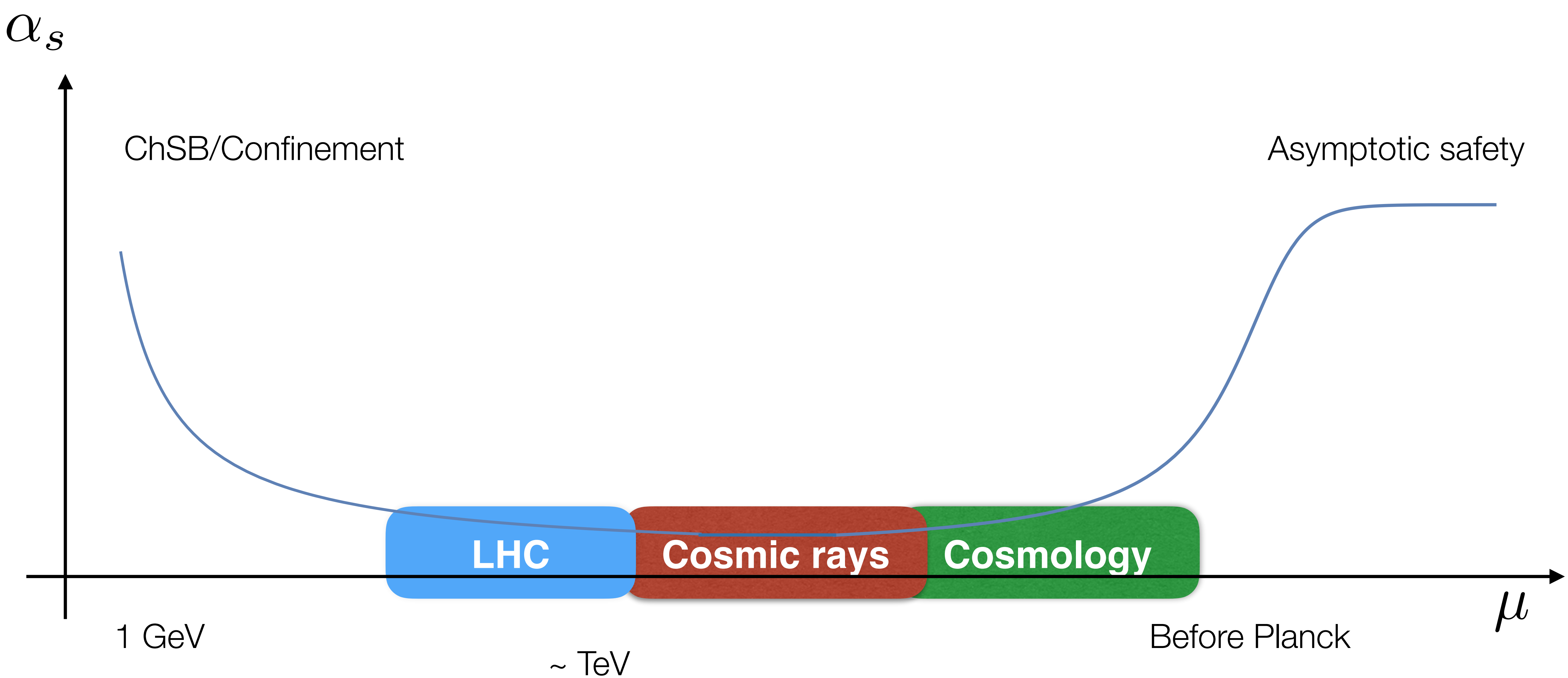
Safe QCD scenario



Sannino, 1511.09022

Is the safe QCD scenario testable?

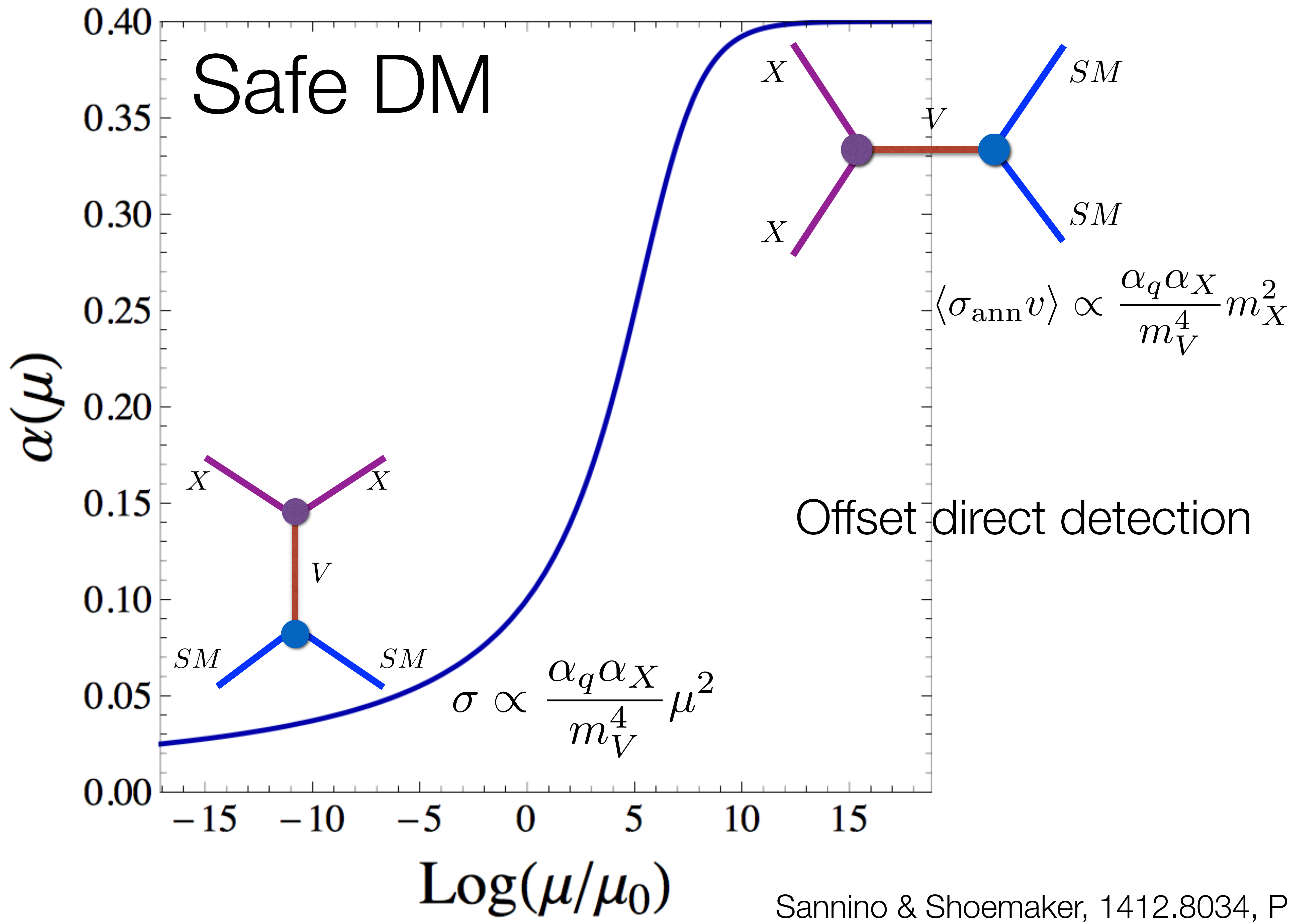
Sannino, 1511.09022



Asymptotic freedom is not a must for UV complete theories

Large Nf, QCD, Holdom 1006.2119 PLB & Pica & Sannino,1011.5917 PRD

Safe Dark Matter



Sannino & Shoemaker, 1412.8034, PRD

Anomalous dimensions

$$H_B = Z_H^{\frac{1}{2}} H$$

$$\gamma_H = -\frac{1}{2}\frac{d\ln Z_H}{d\ln\mu}$$

$$\Delta_H = 1 + \gamma_H$$

$$\gamma_H = \frac{4\epsilon}{19} + \frac{14567 - 2376\sqrt{23}}{6859}\epsilon^2 + \mathcal{O}(\epsilon^3)$$

Mass dimensions

Fermion

$$M\overline{Q}Q$$

$$\Delta_F = 3 - \gamma_F$$

$$\gamma_F = \frac{d \ln M}{d \ln \mu}$$

$$\gamma_F = \frac{4}{19}\epsilon + \frac{4048\sqrt{23} - 59711}{6859}\epsilon^2 + \mathcal{O}(\epsilon^3)$$

Mass dimensions

Scalar

$$m^2 \mathrm{Tr}\left[H^\dagger H\right]$$

$$\gamma_m = \frac{1}{2}\frac{d\ln m^2}{d\ln\mu}$$

$$\gamma_m^{(1)} = 2\alpha_y + 4\alpha_h + 2\alpha_v$$

Small perturb., hence $m^2 = 0$ at the UV-FP

UV critical surface

(Ir)relevant directions implies UV lower dim. critical

$$\alpha_i = F_i(\alpha_g) \qquad \alpha_i(\mu) = \alpha_i^* + \sum_n c_n\, V_i^n \left(\frac{\mu}{\Lambda_c}\right)^{\vartheta_n} + \text{subleading}$$

$$F_y(\alpha_g) = \quad (0.4615 + 0.6168\,\epsilon)\,\alpha_g$$
$$F_h(\alpha_g) = \quad (0.4380 + 0.5675\,\epsilon)\,\alpha_g$$
$$F_v(\alpha_g) = -(0.3009 + 0.3241\,\epsilon)\,\alpha_g$$

Near the fixed point

$$\alpha_g(\mu) = \alpha_g^* + \Big(\alpha_g(\Lambda_c) - \alpha_g^*\Big)\left(\frac{\mu}{\Lambda_c}\right)^{\vartheta_1(\epsilon)}$$

Double - trace and stability

$$\alpha^*_{v1,v2} = -\frac{1}{19}\left(2\sqrt{23} \mp \sqrt{20 + 6\sqrt{23}}\right)\,\epsilon + \mathcal{O}(\epsilon^2)$$

- Is the potential stable at FP?
- Which FP survives?

Moduli

Classical moduli space

$$V = u\,\mathrm{Tr}\,(H^\dagger H)^2 + v\,(\mathrm{Tr}\,H^\dagger H)^2$$

Use $U(N_f) \times U(N_f)$ symmetry $\quad H_c = \mathrm{diag}(h_1, \ldots, h_{N_F})$

$$V = u \sum_{i=1}^{N_F} h_i^4 + v \left(\sum_{i=1}^{N_F} h_i^2 \right)^2 - 2\lambda\left(\sum_i h_i^2 - 1\right)$$

If V vanishes on H_c it will vanish for any multiple of it

Litim, Mojaza, Sannino 1501.03061 JHEP

Ground state conditions at any Nf

$\alpha_h > 0 \quad \text{and} \quad \alpha_h + \alpha_v \geq 0$ $\qquad H_c \propto \delta_{ij}$

$\alpha_h < 0 \quad \text{and} \quad \alpha_h + \alpha_v / N_F \geq 0$ $\qquad H_c \propto \delta_{i1}$

$V_\phi = (4\pi)^2 (\alpha_h + \alpha_v) \phi^4$ $\qquad \alpha_h^* + \alpha_{v_2}^* < 0 < \alpha_h^* + \alpha_{v_1}^*$

Stability for $\alpha_{v_1}^*$

Quantum Potential

The QP obeys an exact RG equation

$$\left(\mu_0 \frac{\partial}{\partial \mu_0} - \gamma(\alpha_j)\, \phi_c \frac{\partial}{\partial \phi_c} + \sum_i \beta_i(\alpha_j) \frac{\partial}{\partial \alpha_i}\right) V_{\rm eff}(\phi_c, \mu_0, \alpha_j) = 0$$

$$H_c = \phi_c \delta_{ij} \qquad \gamma = -\frac{1}{2} d \ln Z / d \ln \mu$$

Litim, Mojaza, Sannino 1501.03061, JHEP

Resumming logs

Dimensional analysis $$V_{\text{eff}}(\phi_c;\mu_0,\alpha_i) = \lambda_{\text{eff}}(\phi_c/\mu_0,\alpha_i)\cdot\phi_c^4$$

$$\left(\phi_c\frac{\partial}{\partial\phi_c} + 4\,\bar{\gamma}(\alpha_j) - \sum_i \bar{\beta}_i(\alpha_j)\frac{\partial}{\partial\alpha_i}\right)\lambda_{\text{eff}}(\phi_c) = 0$$

$$\lambda_{\text{eff}}(\phi_c) = \lambda(\phi_c)\,\exp\left(-4\int_{\mu_0}^{\phi_c}\frac{d\mu}{\mu}\,\bar{\gamma}(\mu)\right)$$

$$\bar{\gamma}(\alpha_i) = \frac{\gamma(\alpha_i)}{1+\gamma(\alpha_i)}$$

The Potential

$$V_{\text{cl}}(\phi_c) = \lambda_* \, \phi_c^4$$

$$\lambda_* = \epsilon \, \frac{16\pi^2}{19} (\sqrt{20 + 6\sqrt{23}} - \sqrt{23} - 1)$$

$$V_{\text{eff}}(\phi_c) = \frac{V_{\text{cl}}(\phi_c)}{1 + W(\phi_c)} \left(\frac{W(\phi_c)}{W(\mu_0)} \right)^{-4D/B}$$

$$-4D/B = 18/(13 \cdot \epsilon) > 0$$

Lambert Function

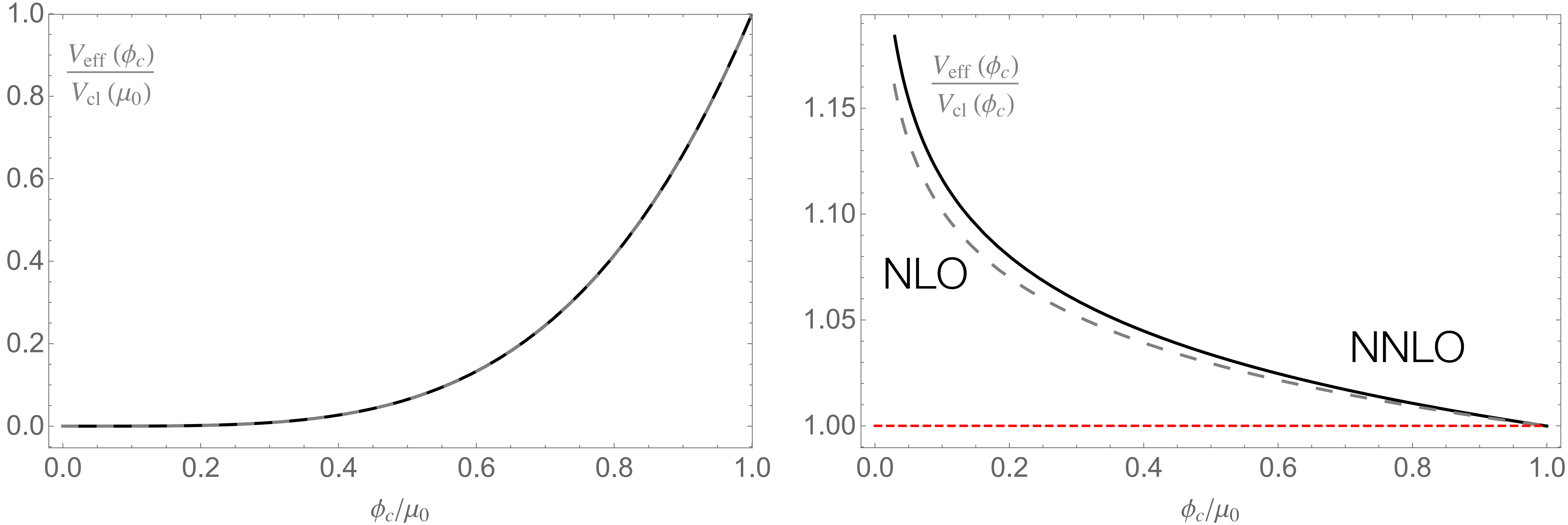
$$z = W \exp W$$

$$z = \left(\frac{\mu_0}{\mu} \right)^{-B \cdot \alpha_*} \left(\frac{\alpha_*}{\alpha_0} - 1 \right) \exp \left(\frac{\alpha_*}{\alpha_0} - 1 \right)$$

$$\alpha(\mu) = \frac{\alpha_*}{1 + W(\mu)}$$

Effective gauge coupling

Visualisation



QFT is controllably defined to arbitrary short scales

Gauge - Yukawa theories/Gradient Flow

omega is an exact form

Osborn 89 & 91, Jack & Osborn 90

$$\frac{\partial \tilde{a}}{\partial g_i} = \left(-\chi^{ij} + \frac{\partial w^i}{\partial g_j} - \frac{\partial w^j}{\partial g_i}\right)\beta_j \quad \Rightarrow \quad \frac{\partial \tilde{a}}{\partial g_i} = -\beta^i \,, \quad \beta^i \equiv \chi^{ij}\beta_j$$

Gradient flow fundamental relation

$$\frac{\partial \beta^j}{\partial g_i} = \frac{\partial \beta^i}{\partial g_j} \,,$$

Relations among the modified β of different couplings

Precise prescription for expanding beta functions in perturb. theory

Antipin, Gillioz, Mølgaard, Sannino 13

Jack and Poole 15