Asymptotic Safety
in the ADM formalism of gravity

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J. Biemans, A. Platania and F. Saueressig, arXiv:1609.04813

J. Biemans, A. Platania and F. Saueressig, arXiv:1702.06539

W. Houthoff, A. Kurov and F. Saueressig, in preparation

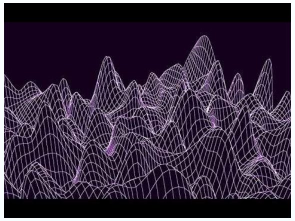
FRG2017

Heidelberg, March 10th, 2017

What’s the name of the game?

Goal: a consistent quantum field theory describing gravity on all scales

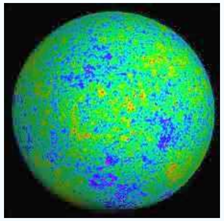
Planck scale 10^{-35} m

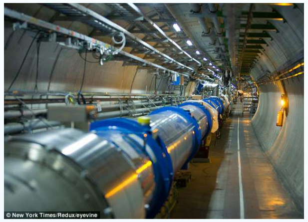


10^{-29} m

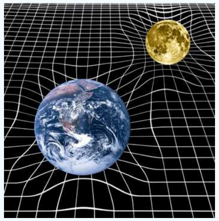
CMB scale

10^{-19} m

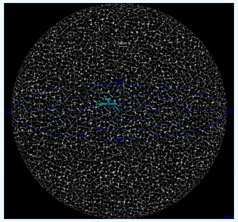




Classical General relativity 10^{12} m



Cosmic scales 10^{26} m



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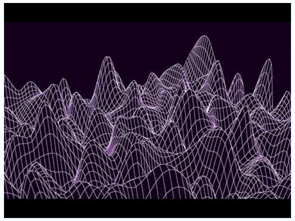

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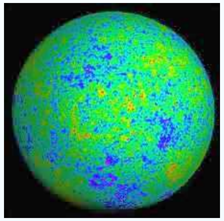

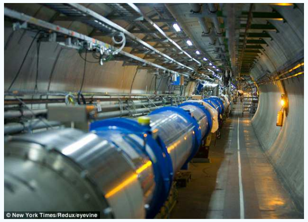

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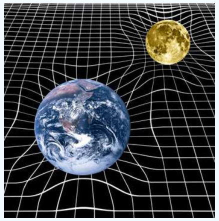

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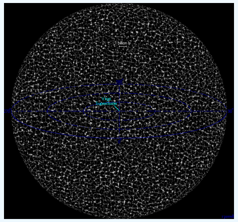

Asymptotic Safety as a successful quantum theoy (for nature)

a) non-Gaussian fixed point (NGFP)

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- tests of general relativity (cosmological signatures, ...)
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d) structural demands:

- background independence, unitarity, . . .
- resolution of singularities: (black holes, Landau poles, . . .)

The Asymptotic Safety sloganizer

Gravity rules!

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Gravity rules!

Matter matters!

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Gravity rules!

Matter matters!

$\boxed{\text{Time matters!}}$

Perturbative quantization of General Relativity

Dynamics of General Relativity governed by Einstein-Hilbert action

$$S^{\rm EH} = \frac{1}{16\pi G_N} \int d^4x \sqrt{g} \left[- R + 2\Lambda \right]$$

- Newton's constant G_N has negative mass-dimension

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Wilsonian picture of perturbative renormalization:

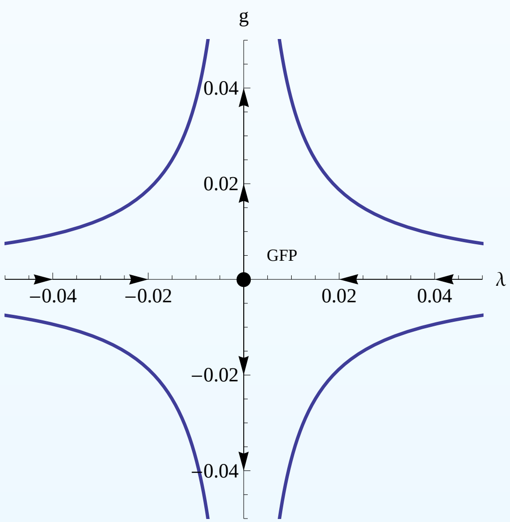
$\Rightarrow$ dimensionless coupling constant attracted to GFP (free theory) in UV

- introduce dimensionless coupling constants

$$g_k = k^2 G_N \,, \quad \lambda_k \equiv \Lambda k^{-2}$$

- GFP: flow governed by mass-dimension:

$$k\partial_k g_k = 2g + \mathcal{O}(g^2)$$
$$k\partial_k \lambda_k = -2\lambda + \mathcal{O}(g)$$



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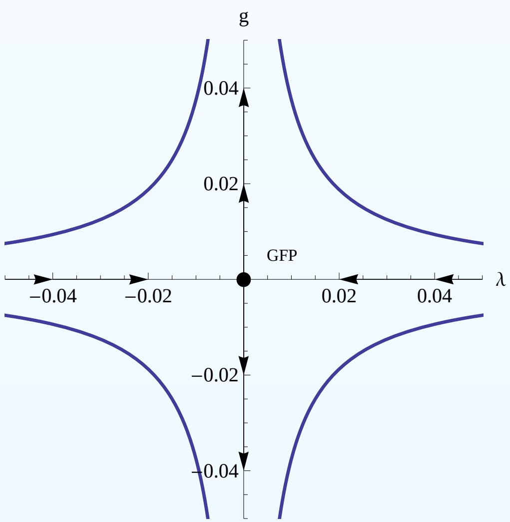

General Relativity is not asymptotically free

Quantizing general relativity

[M. H. Goroff and A. Sagnotti, Phys. Lett. B160 (1985) 81]
[A. E. M. van de Ven, Nucl. Phys. B378 (1992) 309]

quantizing the Einstein-Hilbert action

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{g}\,R$$

has a two-loop divergence $\Longrightarrow$ hallmark of perturbative non-renormalizability

$$S^{\text{div}} = \frac{1}{\epsilon}\,\frac{209}{2880}\,\frac{1}{(16\pi^2)^2}\int d^4x\sqrt{g}\,C_{\mu\nu}{}^{\rho\sigma}\,C_{\rho\sigma}{}^{\alpha\beta}\,C_{\alpha\beta}{}^{\mu\nu}\,.$$

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the Goroff-Sagnotti challenge mastered:

[H. Gies, S. Lippholdt, B. Knorr, F.S., Phys. Rev. Lett. 116 (2016) 211302]

$$\Gamma_k = \frac{1}{16\pi G_k}\int d^4x\sqrt{g}\,(-R+2\Lambda_k) + \bar{\sigma}_k\int d^4x\sqrt{g}\,C_{\mu\nu}{}^{\rho\sigma}\,C_{\rho\sigma}{}^{\alpha\beta}\,C_{\alpha\beta}{}^{\mu\nu} + \ldots$$

possess a NGFP with two relevant directions:

$$\text{NGFP}^{\mathrm{GS}}: \qquad \lambda_* = 0.193\,, \quad g_* = 0.707\,, \qquad \sigma_* = -0.305$$

$$\theta_{1,2} = 1.475 \pm 3.043i\,, \qquad \theta_3 = -79.39\,.$$

renormalization group flows

in the presence of a foliation

Technical routes towards asymptotic safety

starting point: path integral over metrics

$$Z = \int \mathcal{D}\hat{g}\, e^{-S[\hat{g}]}$$

- FRGE

$$\partial_t \Gamma_k[h_{\mu\nu};\bar{g}_{\mu\nu}] = \tfrac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} \partial_t \mathcal{R}_k\right]$$

  - linear split: $\hat{g}_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
  - exponential split: $\hat{g}_{\mu\nu} = \bar{g}_{\mu\rho}\,\left[e^{h}\right]^{\rho}{}_{\nu}$

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- Euclidean Dynamical Triangulations (EDT)

$$Z = \sum_{\text{triangulations}} (\det \hat{g})^\beta \; e^{-S^{\mathrm{EH}}[\hat{g}]}$$

- Causal Dynamical Triangulations (CDT)

$$Z = \sum_{\text{causal triangulations}} e^{-S^{\mathrm{EH}}[\hat{g}]}$$

A fundamental puzzle

FRGE:

- existence of NGFP well-established
- extension to gravity-matter models possible

Causal Dynamical Triangulations (CDT):

- phase diagram contains 2nd order phase transition
  $\Longrightarrow$ possible continuum limit

Euclidean Dynamical Triangulations (EDT):

- phase transition is 1st order
  $\Longrightarrow$ existence of continuum limit questionable

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Conclusions:

- measure in partition function can manifestly change physics
- foliation structure underlying CDT is important

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Conclusions:

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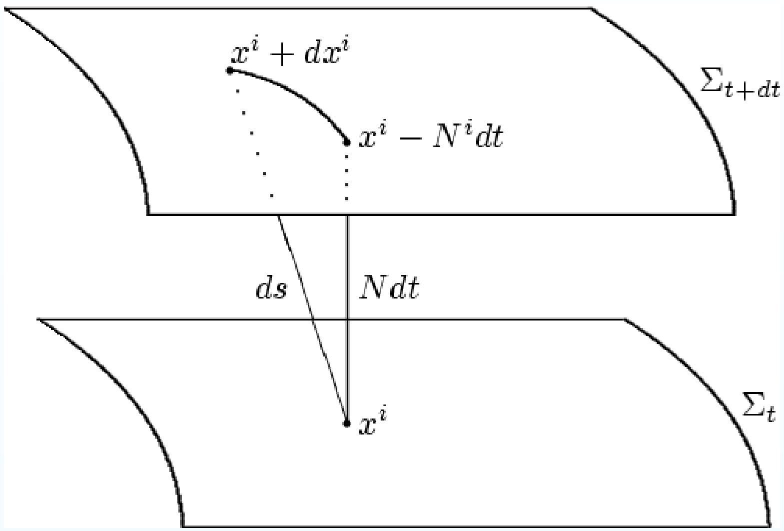
$\boxed{\text{construct a FRGE tailored to CDT}}$

renormalization group flows

in the ADM-formalism

Introducing time through the ADM formalism

Preferred “time”-direction via foliation of space-time



- foliation structure $\mathcal{M}^{d+1} = \mathbb{R} \times \mathcal{M}^d$ with $y^\mu \mapsto (t, x^a)$:

$$ds^2 = N^2 dt^2 + \sigma_{ij} \left(dx^i + N^i dt\right) \left(dx^j + N^j dt\right)$$

- fundamental fields: $g_{\mu\nu} \mapsto (N, N_i, \sigma_{ij})$

$$g_{\mu\nu} = \begin{pmatrix} N^2 + N_i N^i & N_j \\ N_i & \sigma_{ij} \end{pmatrix}$$

FRGE for the ADM-formalism

- fundamental fields: $\{N(t,x), N_i(t,x), \sigma_{ij}(t,x)\}$

$$N = \bar{N} + \hat{N}\,, \qquad N_i = \bar{N}_i + \hat{N}_i\,, \qquad \sigma_{ij} = \bar{\sigma}_{ij} + \hat{\sigma}_{ij}$$

- symmetry: general coordinate invariance inherited from $\gamma_{\mu\nu}$:

$$\delta\gamma_{\mu\nu} = \mathcal{L}_v(\gamma_{\mu\nu})\,, \qquad v^\alpha = (f(t,x)\,,\,\zeta^a(t,x))$$

induces

$$\delta N = f\partial_t N + \zeta^k\partial_k N + N\partial_t f - NN^i\partial_i f\,,$$

$$\delta N_i = N_i\partial_t f + N_k N^k\partial_i f + \sigma_{ki}\partial_t\zeta^k + N_k\partial_i\zeta^k + f\partial_t\tilde{N}_i + \zeta^k\partial_k N_i + N^2\partial_i f$$

$$\delta\sigma_{ij} = f\partial_t\sigma_{ij} + \zeta^k\partial_k\sigma_{ij} + N_j\partial_i f + N_i\partial_j f + \sigma_{jk}\partial_i\zeta^k + \sigma_{ik}\partial_j\zeta^k$$

Non-linearity of ADM-decomposition: symmetry realized non-linearly

$\Longrightarrow$ background Diff($\mathcal{M}$)-symmetry broken to foliation preserving diffeos

$$k\partial_k\Gamma_k[h_{\mu\nu};\bar{g}_{\mu\nu}] = \tfrac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k\mathcal{R}_k\right]$$

renormalization group flows on

cosmological backgrounds

Off-shell flows in the ADM formalism

Einstein-Hilbert action in ADM variables

$$\Gamma_k^{\rm grav} = \frac{1}{16\pi G_k} \int dt d^d x N \sqrt{\sigma} \left[K_{ij} K^{ij} - K^2 - R + 2\Lambda_k \right]$$

background: flat Friedmann-Robertson-Walker spacetime

$$\bar{N} = 1 \, , \qquad \bar{N}_i = 0 \, , \qquad \bar{\sigma}_{ij} = a^2(t) \delta_{ij} \, .$$

fluctuations: adapted to cosmological perturbation theory

$$\begin{aligned} \hat{N}_i =& u_i + \partial_i \, \tfrac{1}{\sqrt{\Delta}} \, B \, , \qquad \partial^i \, u_i = 0 \\ \hat{\sigma}_{ij} =& h_{ij} - \left(\bar{\sigma}_{ij} + \partial_i \partial_j \, \tfrac{1}{\Delta} \right) \psi + \partial_i \partial_j \, \tfrac{1}{\Delta} \, E + \partial_i \, \tfrac{1}{\sqrt{\Delta}} v_j + \partial_j \, \tfrac{1}{\sqrt{\Delta}} \, v_i \, . \end{aligned}$$

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unique gauge-fixing of diffeomorphisms

$$\begin{aligned} F =& \partial_t \hat{N} + \partial^i \hat{N}_i - \tfrac{1}{2} \partial_t \hat{\sigma} + \tfrac{2(d-1)}{d} \bar{K} \hat{N} \,, \\ F_i =& \partial_t \hat{N}_i - \partial_i \hat{N} - \tfrac{1}{2} \partial_i \hat{\sigma} + \partial^j \hat{\sigma}_{ji} + (d-2) \bar{K}_{ij} \hat{N}^j \,, \end{aligned}$$

Gauge-fixing in the ADM formalism (flat space)

$$\delta^2\Gamma_k^{\mathrm{grav}}$$

Index	matrix element of $32\pi G_k\,\delta^2\Gamma_k^{\mathrm{grav}}$
$h\,h$	$-\partial_t^2+\Delta-2\Lambda_k$
$v\,v$	$2\big[-\partial_t^2-2\Lambda_k\big]$
$E\,E$	$-\Lambda_k$
$\psi\,\psi$	$-(d-1)\big[(d-2)\,(-\partial_t^2+\Delta)-(d-3)\,\Lambda_k\big]$
$\psi\,E$	$-\frac{1}{2}(d-1)\big[-\partial_t^2-2\Lambda_k\big]$
$\hat{N}\,\hat{N}$	0
$B\,B$	0
$u\,u$	$2\,\Delta$
$u\,v$	$-2\,\partial_t\sqrt{\Delta}$
$B\,\psi$	$2\,(d-1)\sqrt{\Delta}\,\partial_t$
$\hat{N}\,\psi$	$2\,(d-1)\big[\Delta-\Lambda_k\big]$
$\hat{N}\,E$	$-2\,\Lambda_k$

Gauge-fixing in the ADM formalism (flat space)

$$\delta^2\Gamma_k^{\mathrm{grav}} + S^{\mathrm{gf}}$$

Index	matrix element of $32\pi G_k\,\left(\delta^2\Gamma_k^{\mathrm{grav}} + S^{\mathrm{gf}}\right)$
$h\,h$	$-\partial_t^2 + \Delta - 2\Lambda_k$
$v\,v$	$2\big[-\partial_t^2 + \Delta - 2\Lambda_k\big]$
$E\,E$	$\frac{1}{2}\left[-\partial_t^2 + \Delta - \Lambda_k\right]$
$\psi\,\psi$	$\frac{(d-1)(d-3)}{4}\big[-\partial_t^2 + \Delta - 2\Lambda_k\big]$
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well-defined Hessian $\Gamma_k^{(2)}$ with relativistic propagators

beta functions for the gravity-matter system

$$\beta_g = (2+\eta)\, g\,,$$

$$\beta_\lambda = (\eta - 2)\lambda + \frac{g}{4\pi}\left[\left(3 + \tfrac{4}{1-2\lambda} + \tfrac{6-10\lambda}{B_{\rm det}(\lambda)}\right)\left(1 - \tfrac{\eta}{6}\right) - 8\right]$$

anomalous dimension of Newton's constant:

$$\eta = \frac{16\pi g\, B_1(\lambda)}{(4\pi)^2 + 16\pi g\, B_2(\lambda)}$$

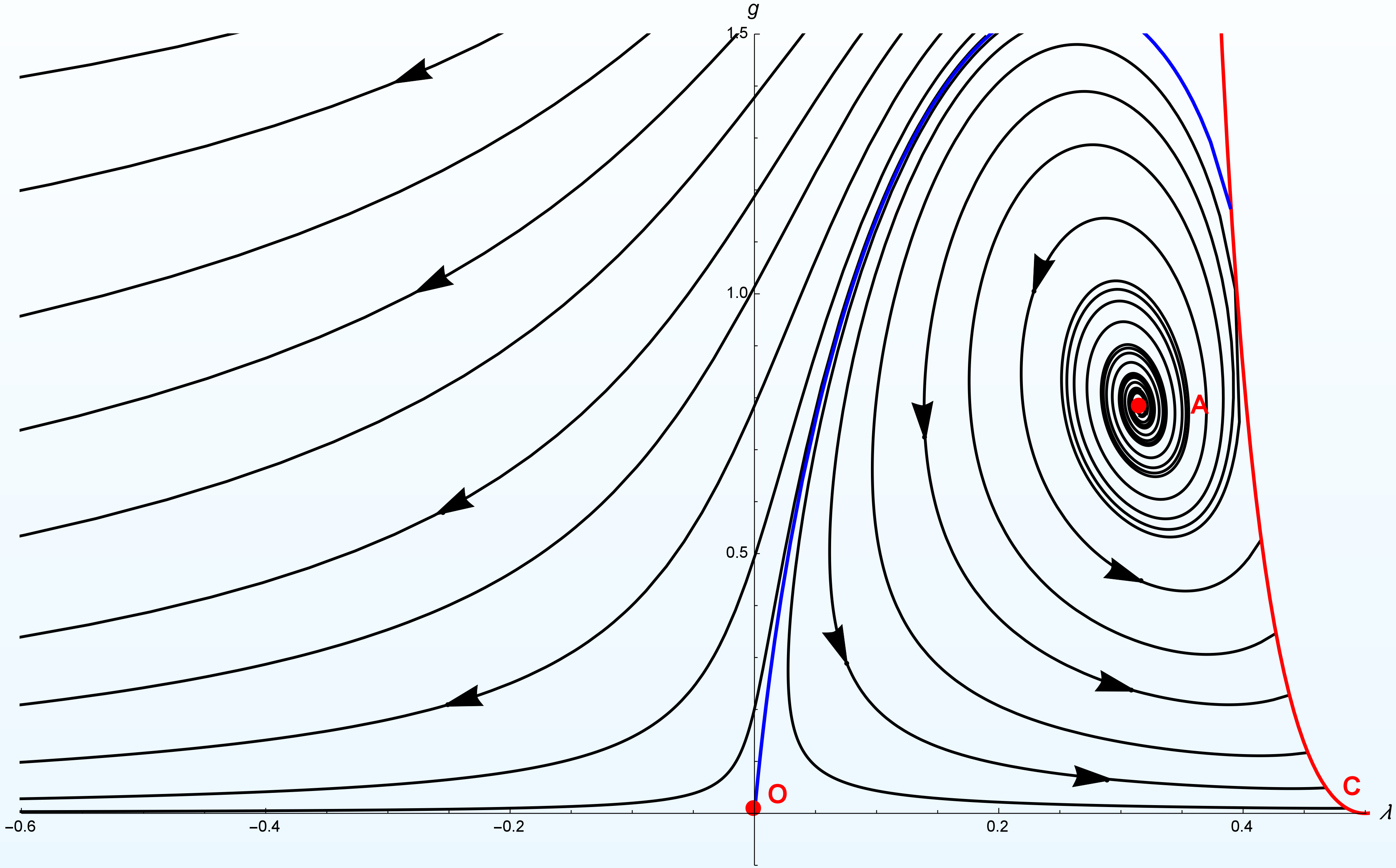
and

$$B_1(\lambda) \equiv -\tfrac{41}{9} + \tfrac{11}{6\,(1-2\lambda)} - \tfrac{5}{6\,(1-2\lambda)^2} + \tfrac{3-5\lambda}{3\, B_{\rm det}(\lambda)} + \tfrac{7-124\lambda+196\lambda^2}{24\, B_{\rm det}(\lambda)^2}$$

$$B_2(\lambda) = \tfrac{1}{36} + \tfrac{11}{24\,(1-2\lambda)} - \tfrac{5}{36\,(1-2\lambda)^2} + \tfrac{3-5\lambda}{12\, B_{\rm det}(\lambda)} + \tfrac{7-124\lambda+196\lambda^2}{144\, B_{\rm det}(\lambda)^2}$$

Einstein-Hilbert-truncation on cosmological background

J. Biemans, A. Platania and F. Saueressig, arXiv:1609.04813



adding minimally coupled matter fields

Minimally coupled matter fields in ADM

Supplement Einstein-Hilbert action by

- N_S minimally coupled scalar fields

$$S^{\text{scalar}} = \frac{1}{2} \sum_{i=1}^{N_S} \int dt d^d x N \sqrt{\sigma} \left[\phi^i \, \Delta_0 \, \phi^i \right]$$

- N_V abelian gauge fields (gauge-fixed)

$$S^{\text{vector}} = \frac{1}{4} \sum_{i=1}^{N_V} \int dt d^d x N \sqrt{\sigma} \left[g^{\mu\nu} g^{\alpha\beta} F^i_{\mu\alpha} F^i_{\nu\beta} \right] + \frac{1}{2\xi} \sum_{i=1}^{N_V} \int dt d^d x \bar{N} \sqrt{\bar{\sigma}} \left[\bar{g}^{\mu\nu} \bar{D}_\mu A^i_\nu \right]^2 + \sum_{i=1}^{N_V} \int dt d^d x \bar{N} \sqrt{\bar{\sigma}} \left[\bar{C}^i \, \Delta_0 \, C^i \right]$$

- N_D Dirac spinors

$$S^{\text{fermion}} = i \sum_{i=1}^{N_D} \int dt d^d x N \sqrt{\sigma} \left[\bar{\psi}^i \, \nabla\!\!\!/ \, \psi^i \right] .$$

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and

$$B_1(\lambda) \equiv -\tfrac{41}{9} + \tfrac{11}{6\,(1-2\lambda)} - \tfrac{5}{6\,(1-2\lambda)^2} + \tfrac{3-5\lambda}{3\,B_{\rm det}(\lambda)} + \tfrac{7-124\lambda+196\lambda^2}{24\,B_{\rm det}(\lambda)^2} + \tfrac{1}{6}d_g$$

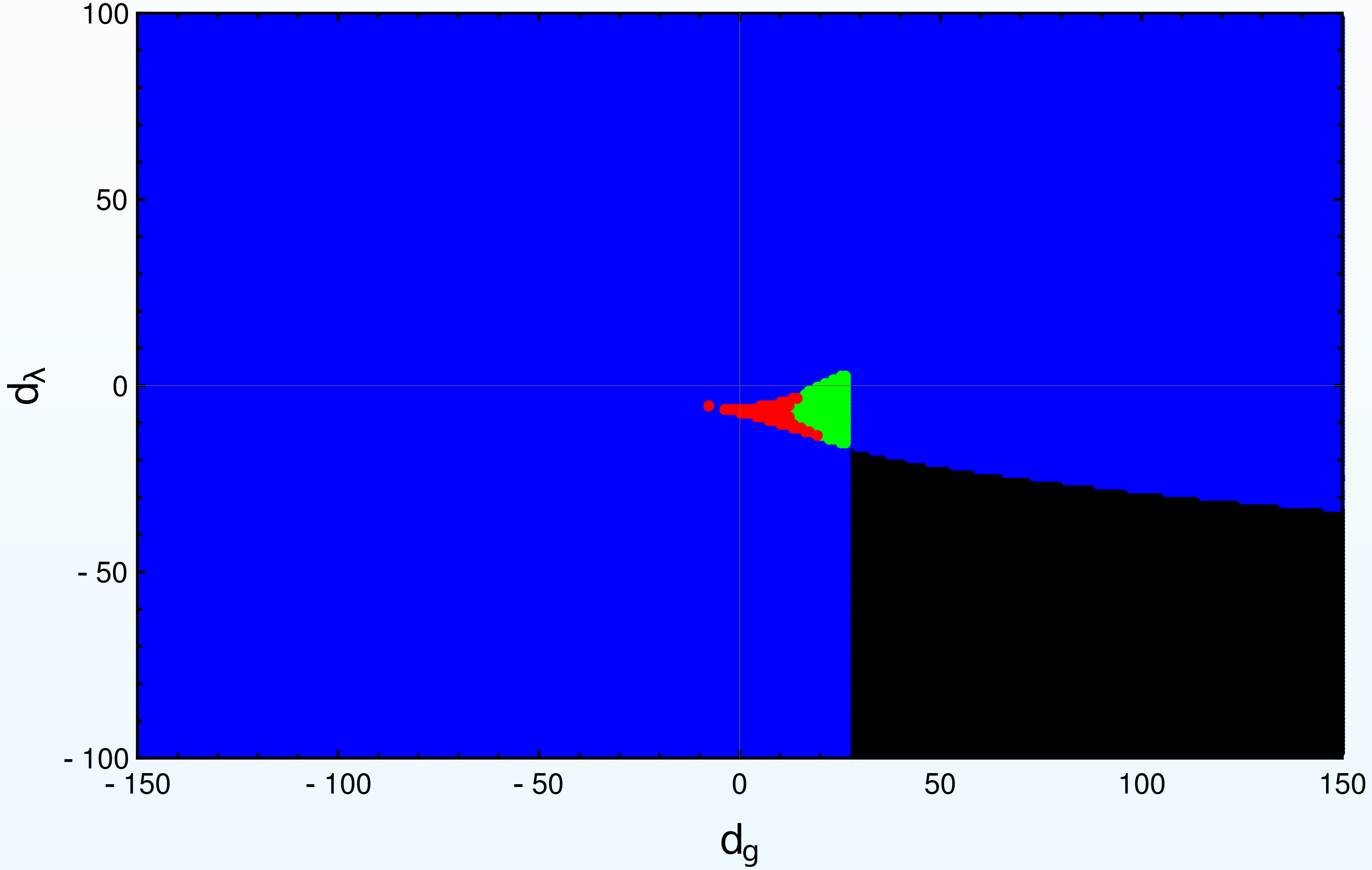
$$B_2(\lambda) = \tfrac{1}{36} + \tfrac{11}{24\,(1-2\lambda)} - \tfrac{5}{36\,(1-2\lambda)^2} + \tfrac{3-5\lambda}{12\,B_{\rm det}(\lambda)} + \tfrac{7-124\lambda+196\lambda^2}{144\,B_{\rm det}(\lambda)^2}$$

Type I regulator:

$$d_g = N_S - N_V - N_D\,, \quad d_\lambda = N_S + 2N_V - 4N_D$$

Gravity-matter fixed points are quite common

$$d_g = N_S - N_V - N_D\,, \quad d_\lambda = N_S + 2N_V - 4N_D$$



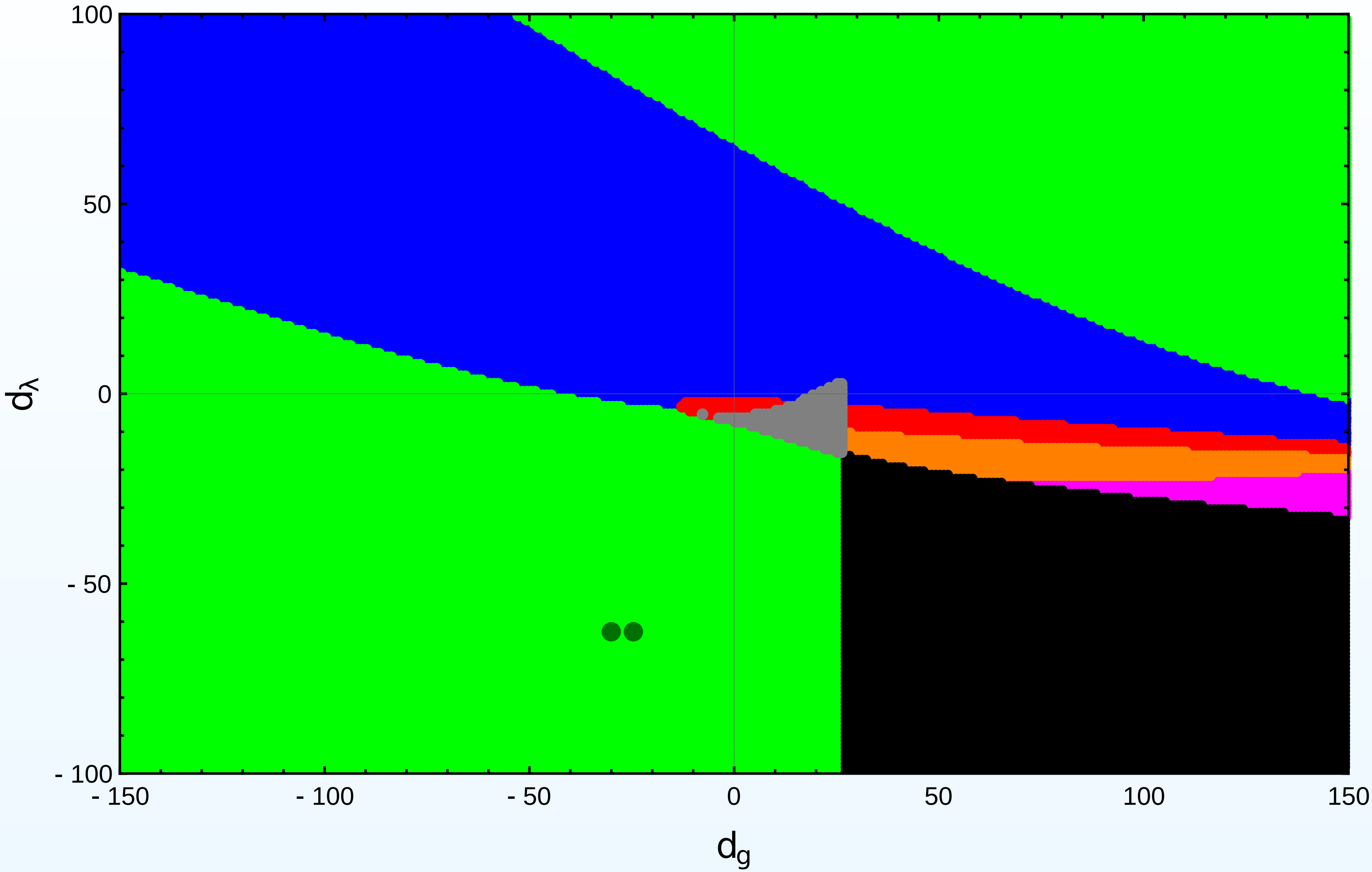
blue: 1 NGFP

red: 3 NGFP

black: 0 NGFPs

green: 2 NGFPs

most of them are UV fixed points



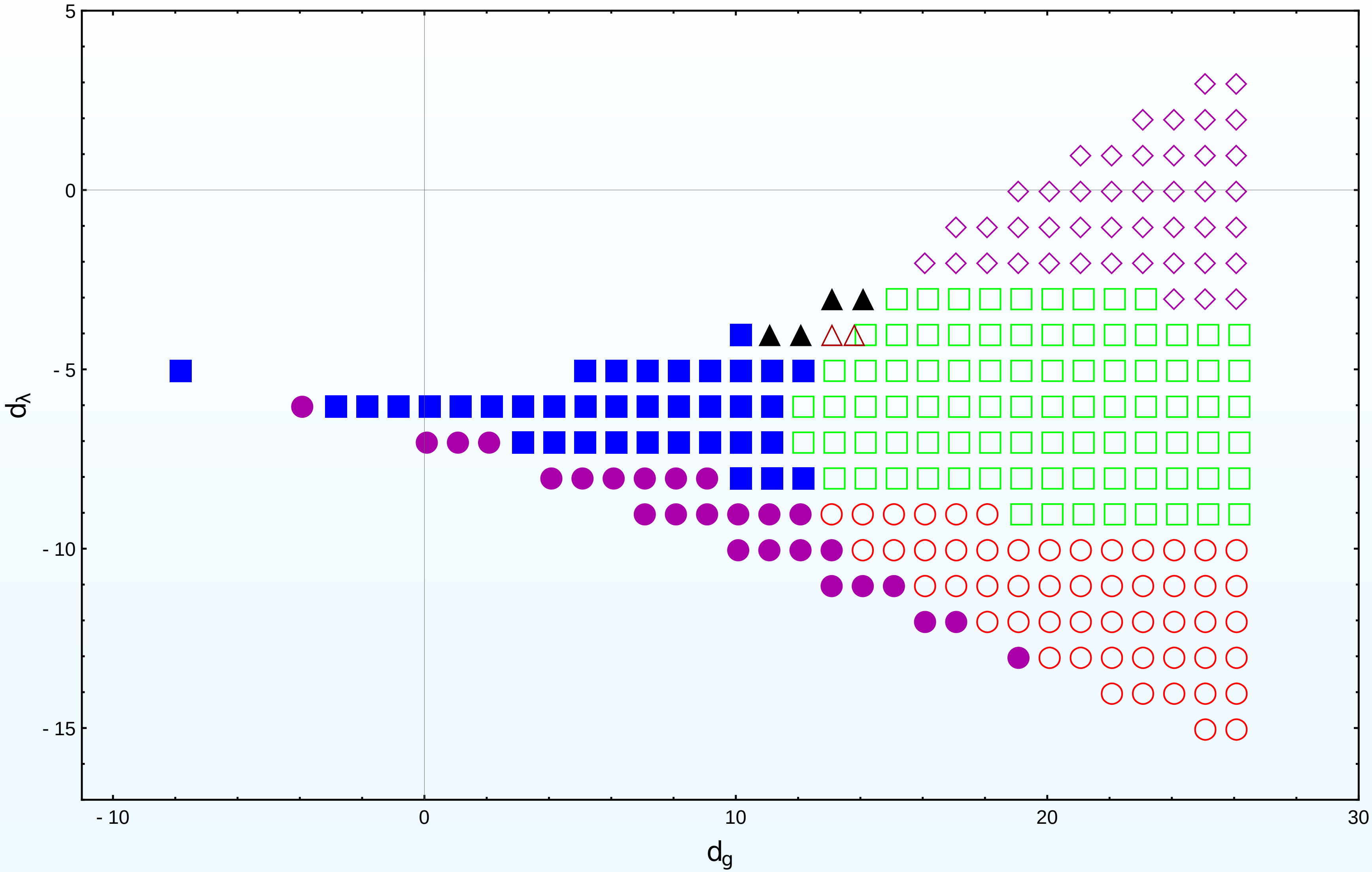
green: UV-FP: real exponents

blue: UV-FP: complex exponents

black: 0 NGFPs

rest: saddle/IR fixed points

multi-fixed point systems exist



filled symbols: 3 fixed point systems

open symbols: 2 fixed point systems

most matter sectors are in a good region!

model	N_S	N_D	N_V	g_*	λ_*	θ_1	θ_2
pure gravity	0	0	0	0.78	$+0.32$	$0.50 \pm 5.38\,i$	
Standard Model (SM)	4	$\frac{45}{2}$	12	0.75	-0.93	3.871	2.057
SM, dark matter (dm)	5	$\frac{45}{2}$	12	0.76	-0.94	3.869	2.058
SM, $3\,\nu$	4	24	12	0.72	-0.99	3.884	2.057
SM, $3\,\nu$, dm, axion	6	24	12	0.75	-1.00	3.882	2.059
MSSM	49	$\frac{61}{2}$	12	2.26	-2.30	3.911	2.154
SU(5) GUT	124	24	24	0.17	$+0.41$	25.26	6.008
SO(10) GUT	97	24	45	0.15	$+0.40$	19.20	6.010

renormalization group flows on

CDT backgrounds $S^1 \times S^d$

Off-shell flows in the ADM formalism

Einstein-Hilbert action in ADM variables

$$\Gamma_k^{\rm grav} = \frac{1}{16\pi G_k} \int dt d^d x N \sqrt{\sigma} \left[K_{ij} K^{ij} - K^2 - R + 2\Lambda_k \right]$$

background: $S^1 \times S^d$

$$\bar{N} = 1 \,, \qquad \bar{N}_i = 0 \,, \qquad \bar{\sigma}_{ij} = \bar{\sigma}_{ij}(r)|_{S^d} \;.$$

fluctuations: adapted to cosmological perturbation theory

$$\begin{aligned} \hat{N}_i &= u_i + \bar{D}_i \frac{1}{\sqrt{\Delta}} B \,, \qquad \bar{D}^i u_i = 0 \\ \hat{\sigma}_{ij} &= h_{ij} + \bar{D}_i \frac{1}{\sqrt{\Delta}} v_j + \bar{D}_j \frac{1}{\sqrt{\Delta}} v_i + \left(\bar{D}_i \bar{D}_j + \tfrac{1}{d} \bar{\sigma}_{ij} \Delta \right) \psi + \tfrac{1}{d} \bar{\sigma}_{ij} h \,. \end{aligned}$$

unique gauge-fixing of diffeomorphisms

$$S^{\rm gf} = \frac{1}{32\pi G_k} \int dt d^d y \sqrt{\bar{\sigma}} \left[F^2 + F_i \bar{\sigma}^{ij} F_j \right]$$

$$F = \partial_t \hat{N} + \bar{D}^i \hat{N}_i - \tfrac{1}{2} \partial_t \hat{\sigma} \,, \quad F_i = \partial_t \hat{N}_i - \bar{D}_i \hat{N} - \tfrac{1}{2} \bar{D}_i \hat{\sigma} + \bar{D}^j \hat{\sigma}_{ji}$$

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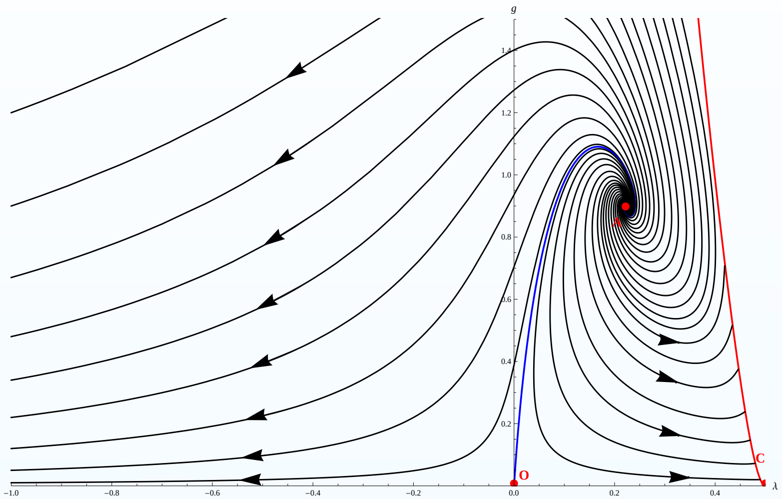
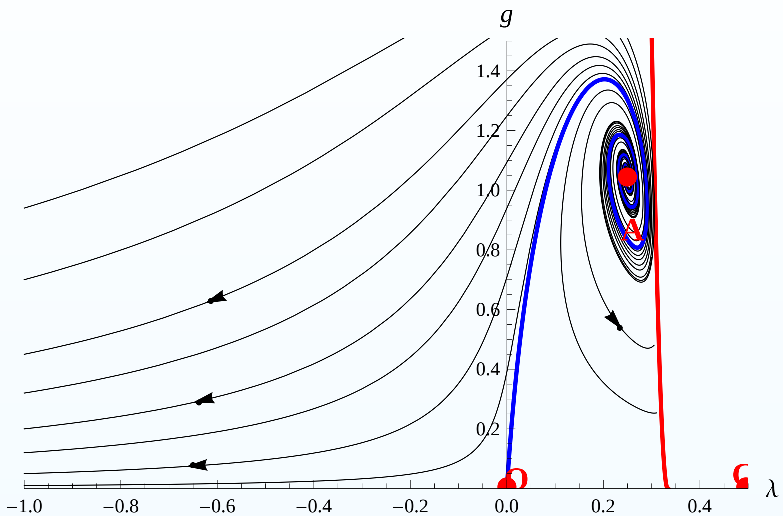
Phase diagrams on $S^1 \times S^3$

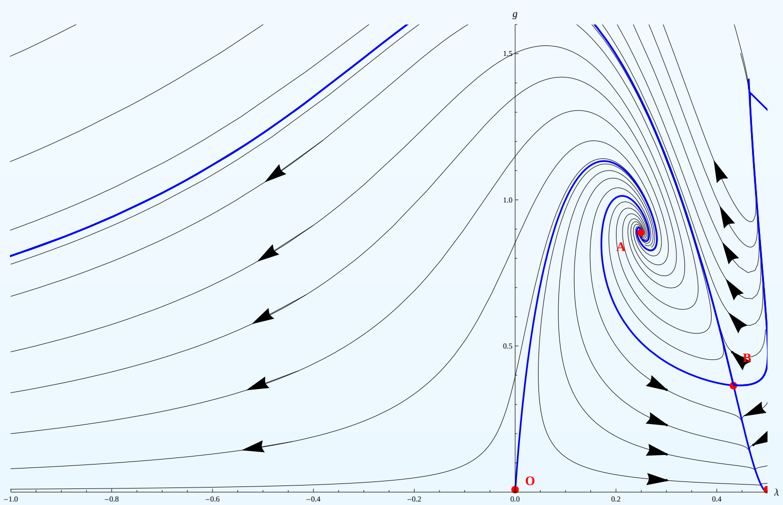
Type I, linear



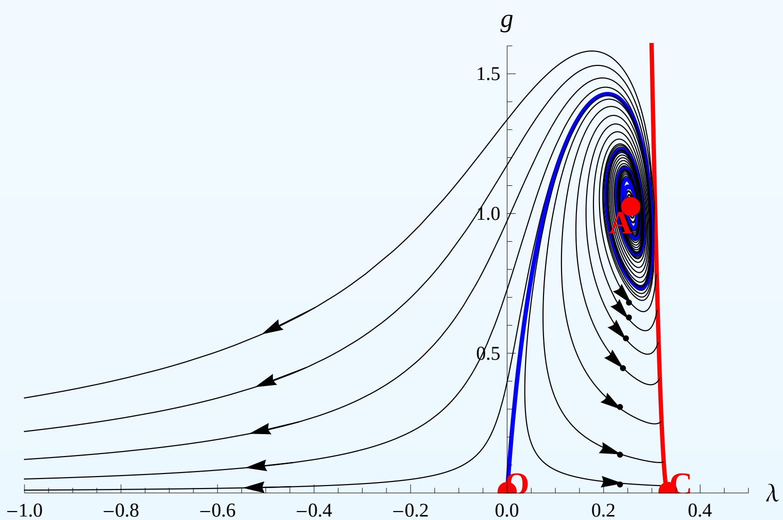
Type I, exponential



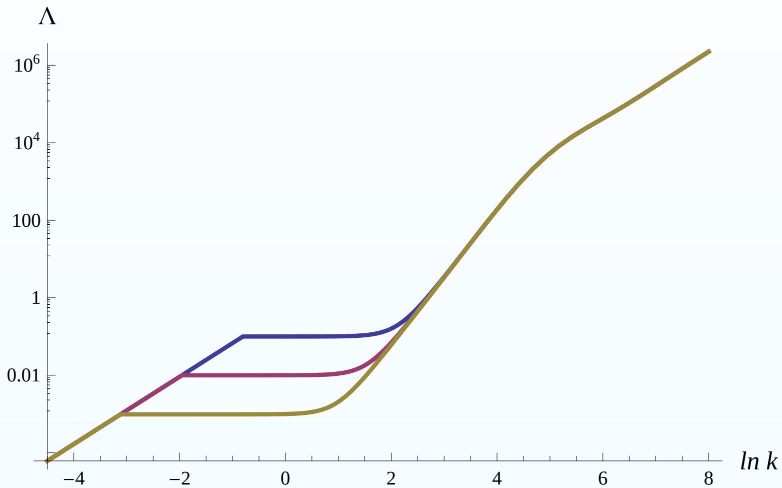
Type II, linear

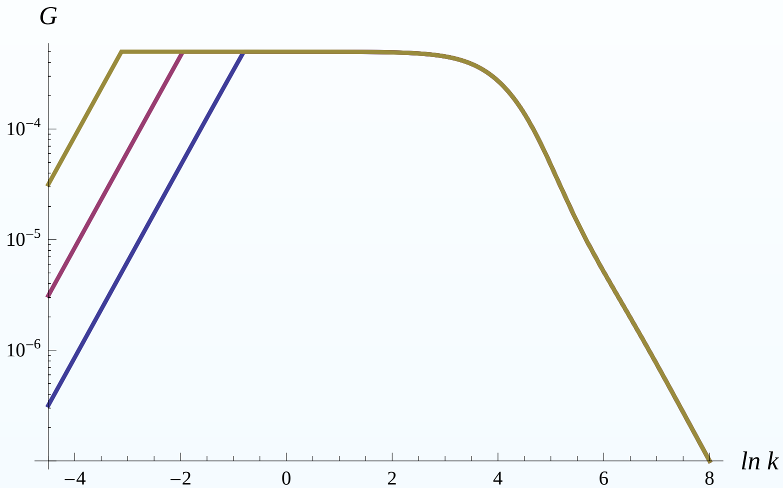


Type II, exponential



IR completion of the flow: Type II - linear





flow approaches quasi-fixed point C:

- dimensionful Newton's constant: $\lim_{k\to 0} G_k = 0$
- dimensionful cosmological constant: $\lim_{k\to 0} \Lambda_k = 0$

!!! transition to IR-phase at Hubble-horizon scales !!!

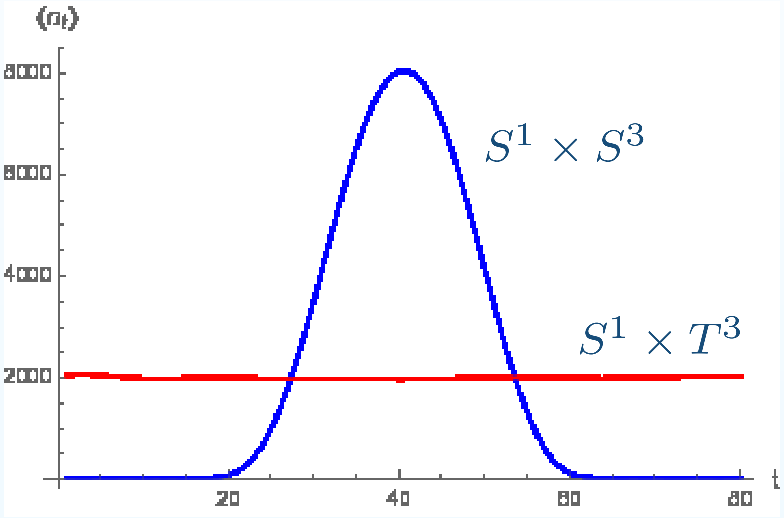
outlook

CDT correlation functions

[J. Ambjørn, et. al., Phys. Rev. D **94** (2016) 044010]

Best measured correlation functions build from spatial volumes V_3:

- expectation value of volume profiles: $\langle\, V_3(t)\, \rangle$
- correlators for volume fluctuations: $\langle\, (\delta V_3(t))\, (\delta V_3(t'))\, \rangle$



now within reach of the FRGE based on the ADM-formalism

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[J. Ambjørn, et. al., Phys. Rev. D **94** (2016) 044010]

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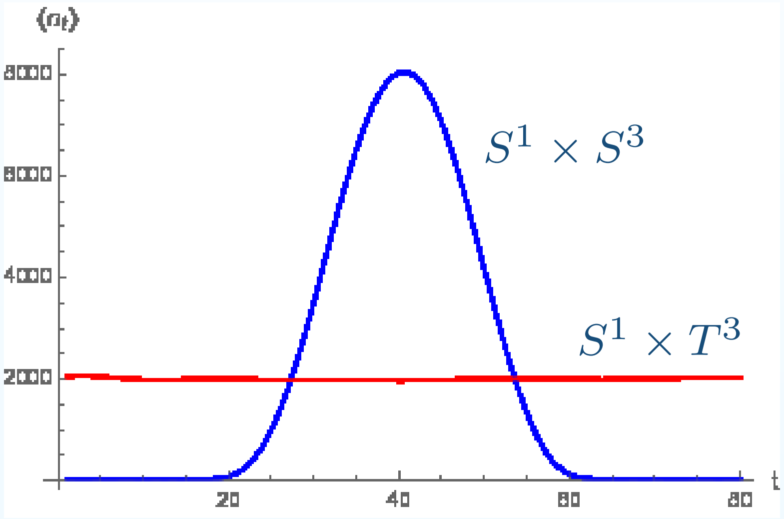

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Thank you!