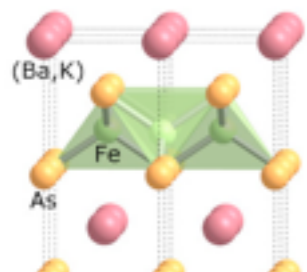


# History, Present, Future: Functional RG for unconventional superconductors

Ronny Thomale

Julius-Maximilians Universität Würzburg



Deutsche  
Forschungsgemeinschaft

DFG

SPP 1458



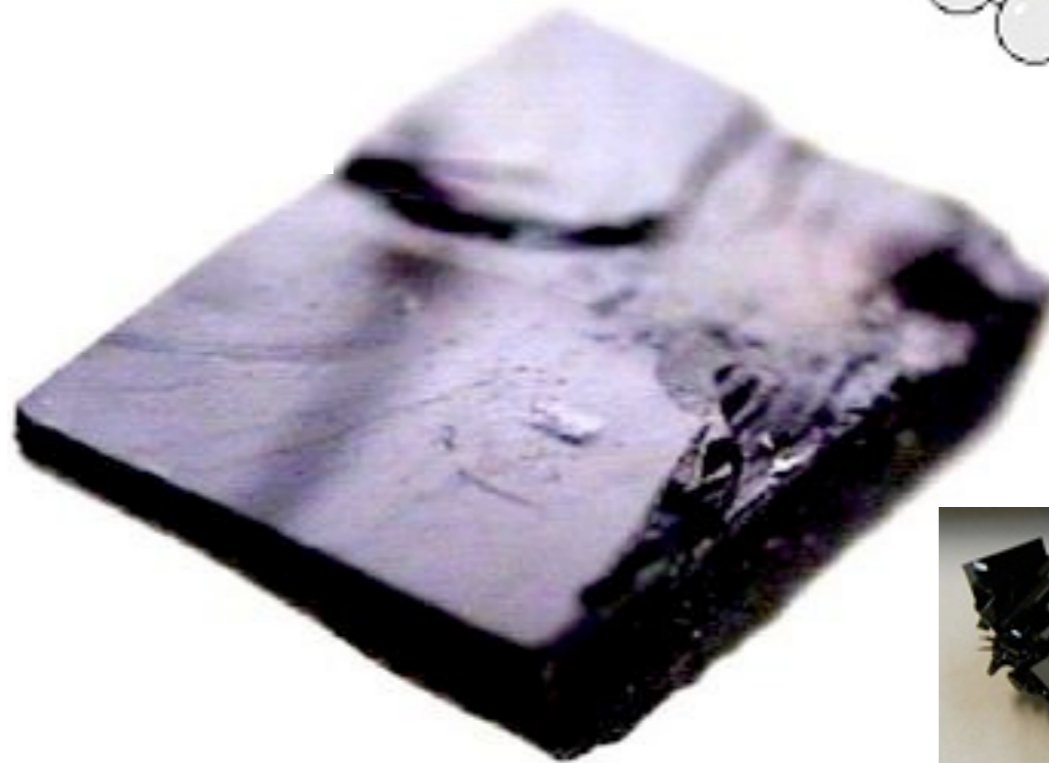
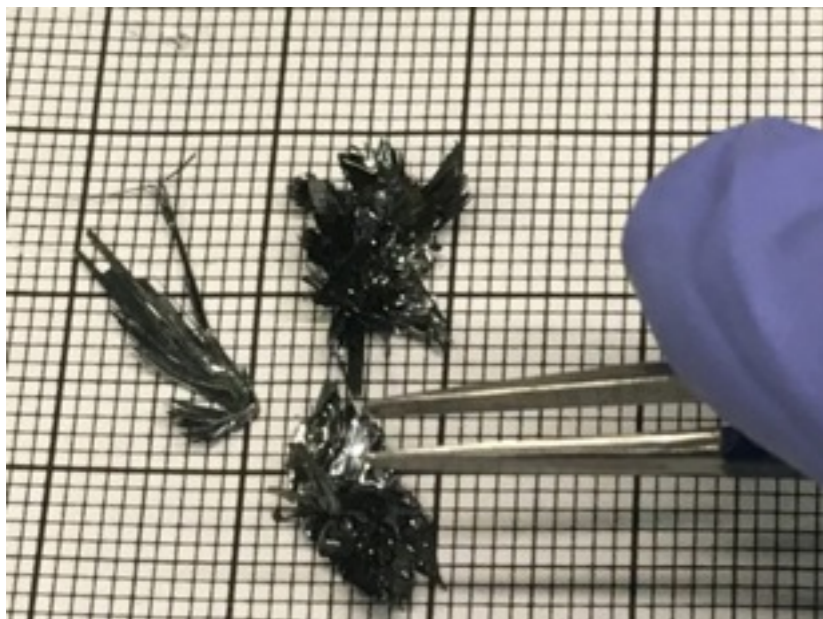
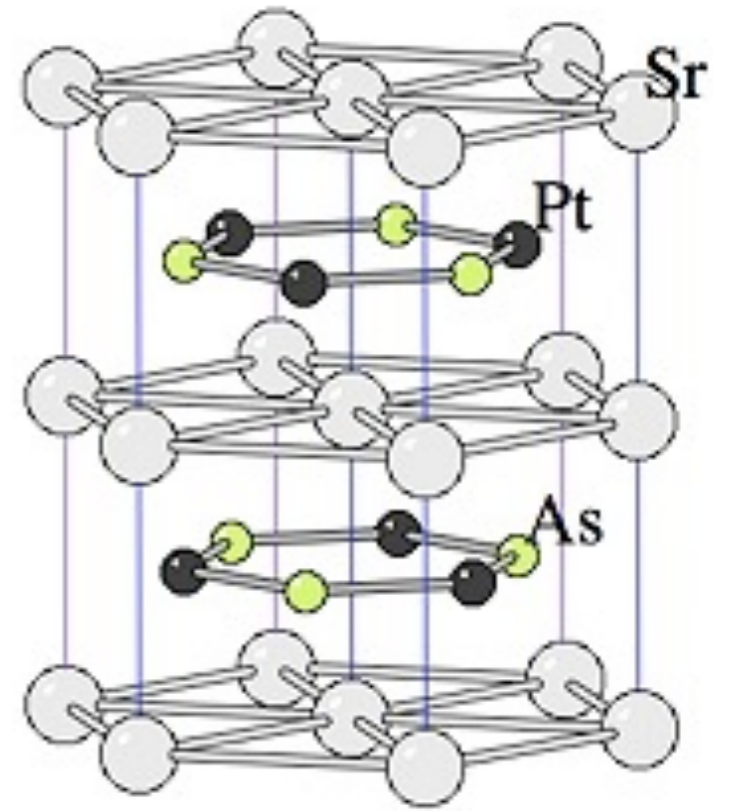
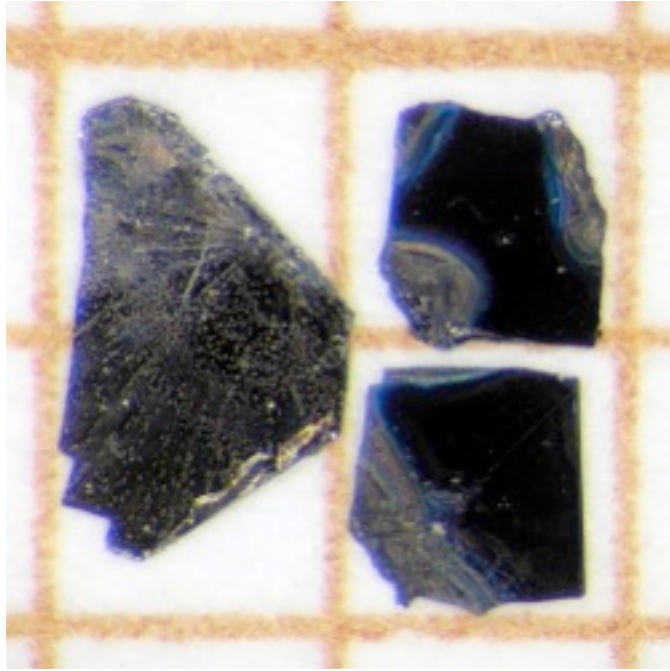
TOPOLECTRICS



SFB 1170

“Functional Renormalization - from quantum gravity and dark energy to ultracold atoms and condensed matter”

# Unconventional superconductors



# Outline

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## History

- Timeline of materials studied through FRG
- Recent methodological step: multi-band/orbital superconductivity

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- Sodium cobaltates: chiral d-wave?

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- Timeline of materials studied through FRG
- Recent methodological step: multi-band/orbital superconductivity

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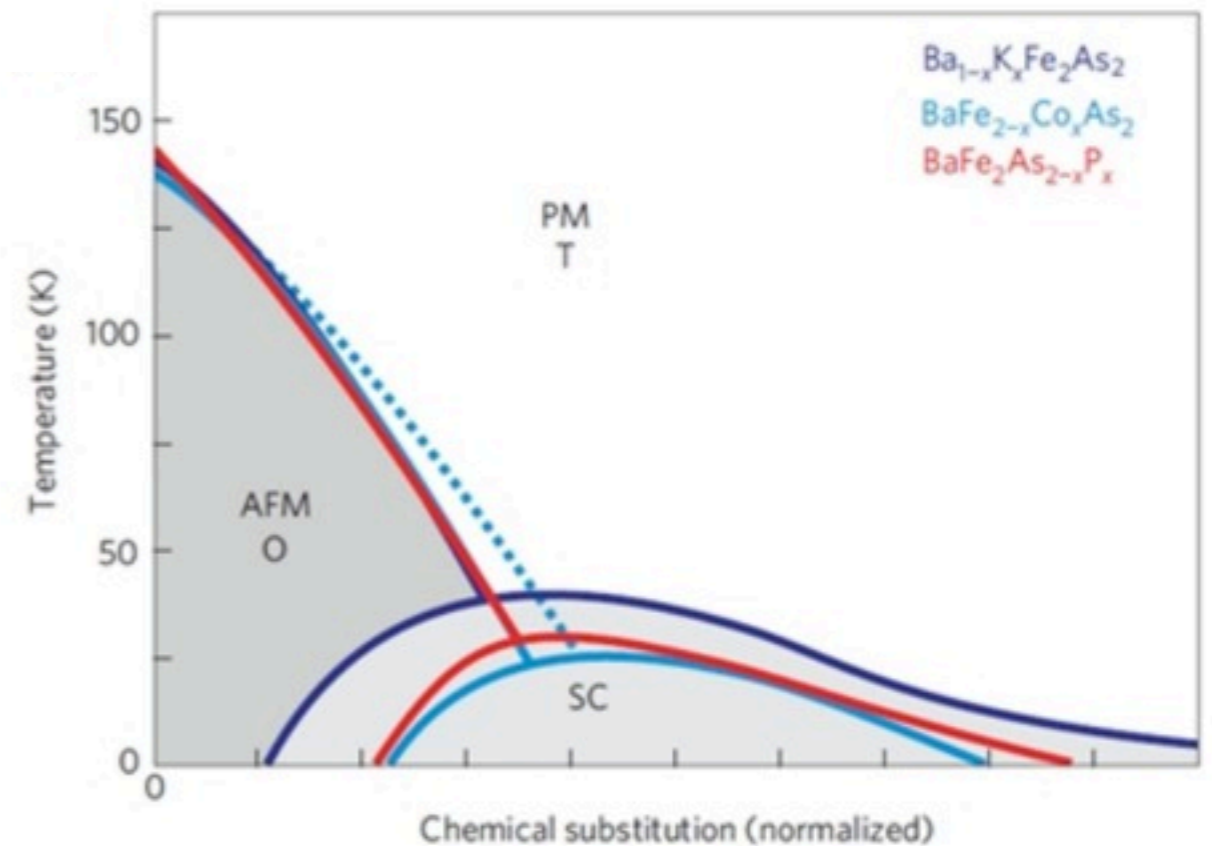
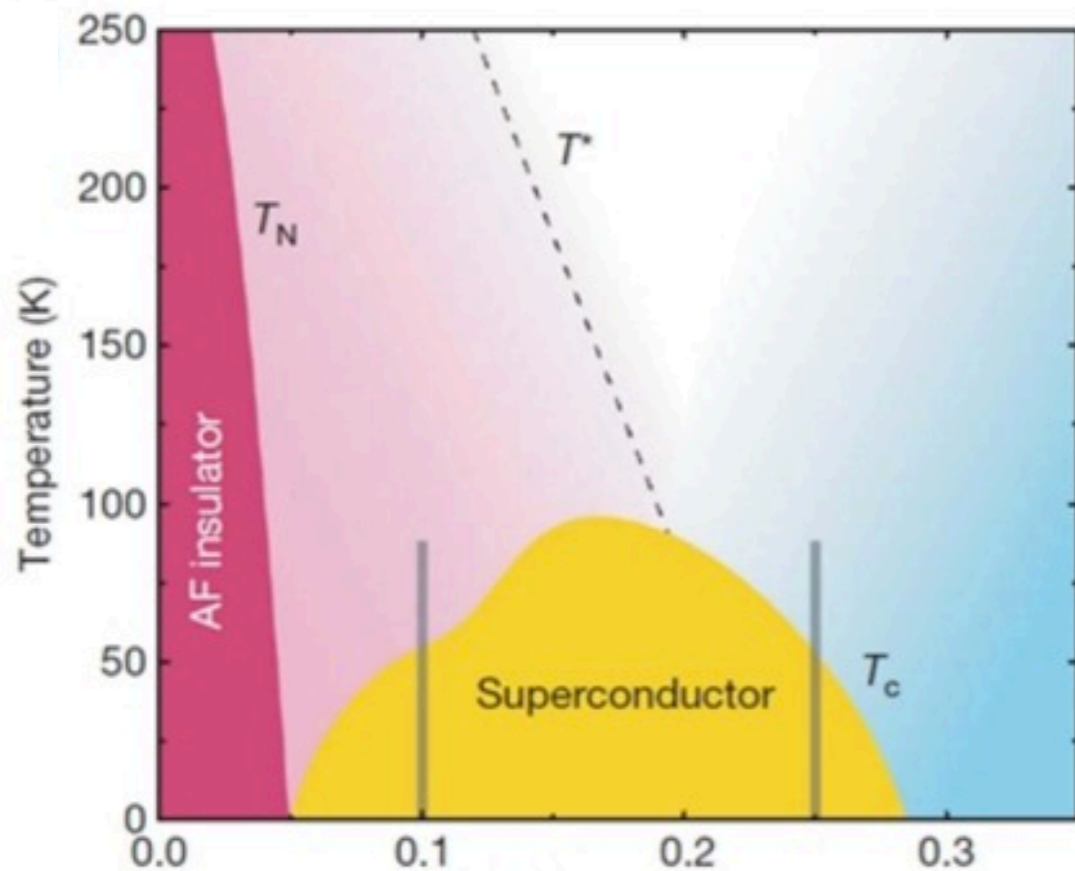
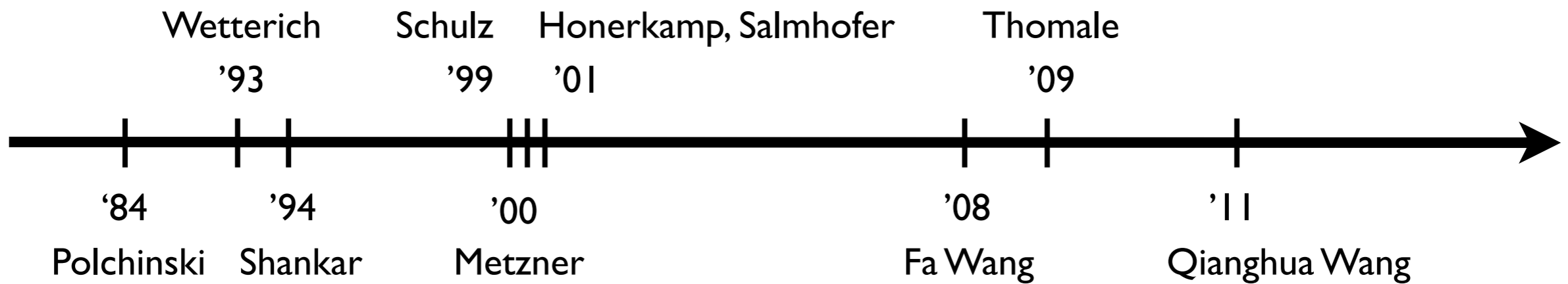
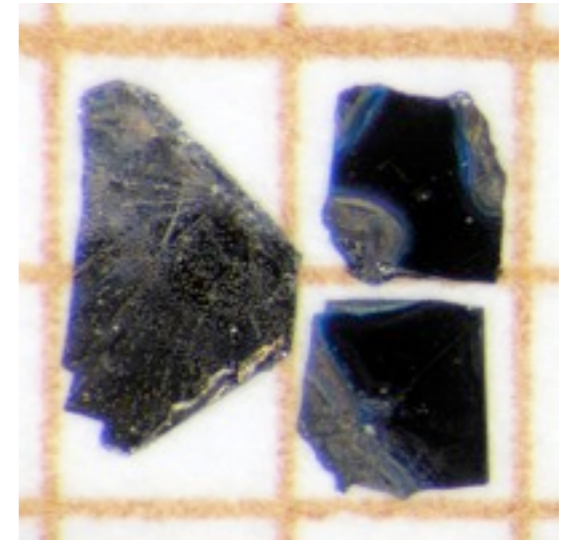
- Pnictides: extended d-wave vs. extended s-wave
- Sodium cobaltates: chiral d-wave?

## Future

- SrPtAs: Weyl superconductors
- LAO/STO heterostructures

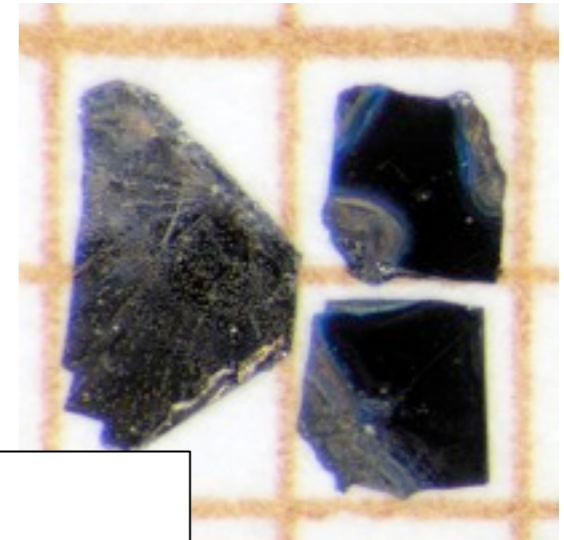
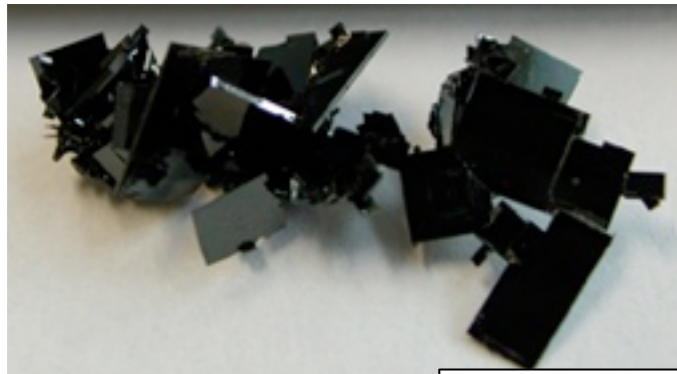
# History

# Timeline





# Timeline

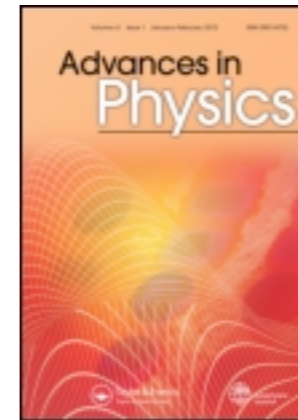


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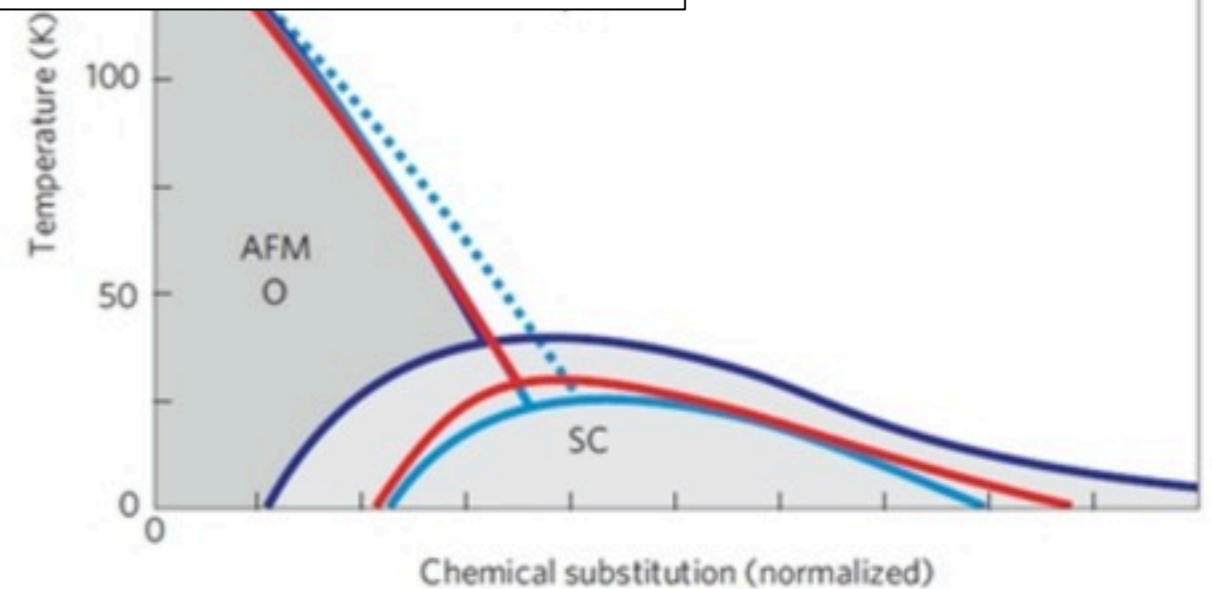
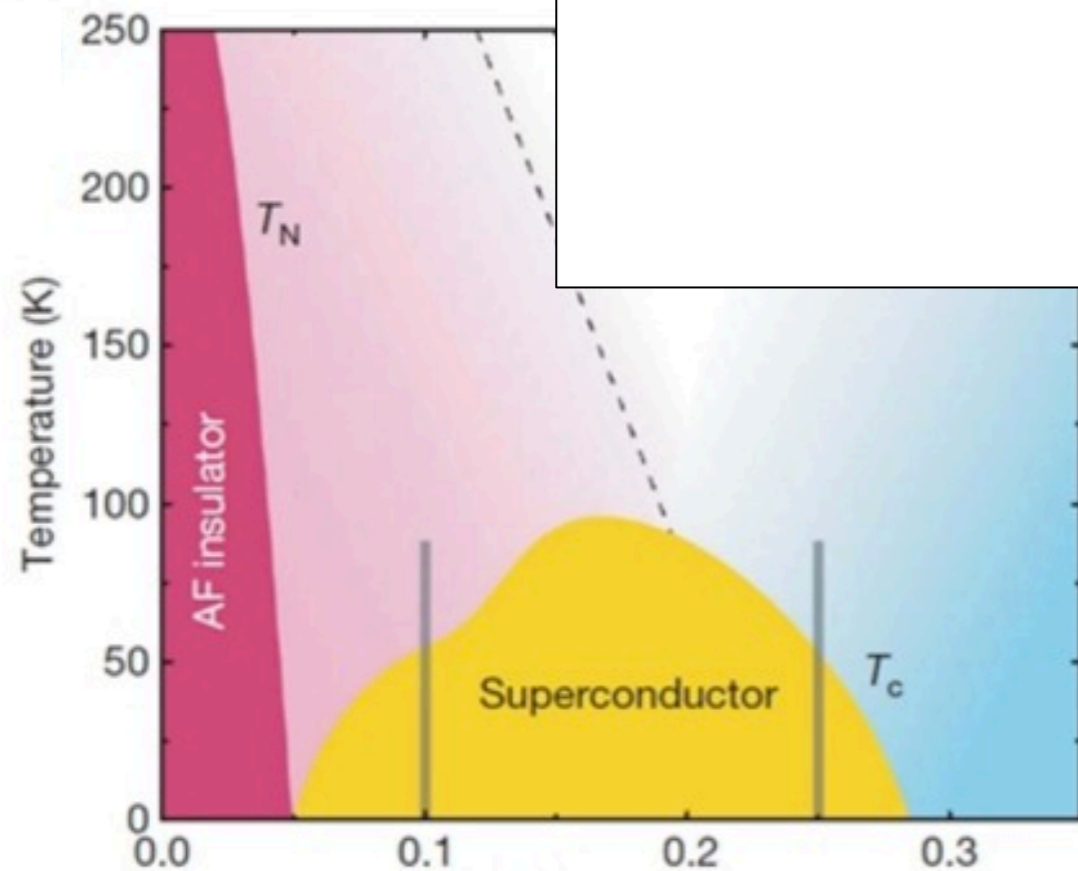


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Polchinski Shar

Platt, Hanke, Thomale  
Advances in Physics  
Vol. 62, p. 453-562, 2014



hua Wang



# Harmonic composition of SC form factors

The favored real space pairing function  $\Delta_{i,j} \sim \langle c_i^\dagger c_j^\dagger \rangle$

transforms under **irreducible point group representations**

$C_{4v}$	$E$	$C_2$	$2C_4$	$2\sigma_v$	$2\sigma_d$
$A_1$	1	1	1	1	1
$A_2$	1	1	1	-1	-1
$B_1$	1	1	-1	1	-1
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$A_2$	1	1	1	-1	-1	
$B_1$	1	1	-1	1	-1	$d_{x^2-y^2}$
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The point group representation **does not fix** the form of the superconducting gap function, even without further symmetry breaking

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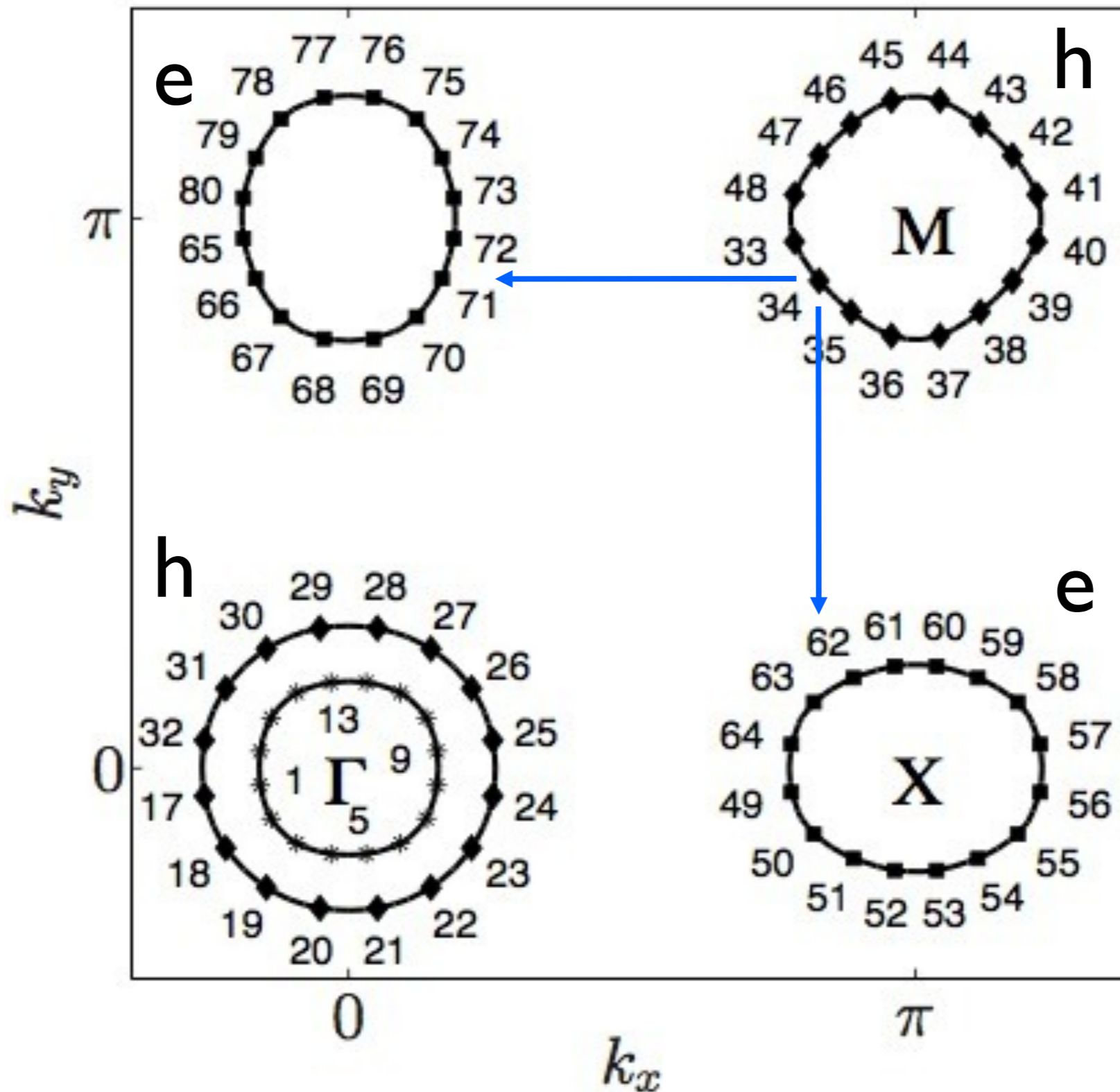
$C_{4v}$	$E$	$C_2$	$2C_4$	$2\sigma_v$	$2\sigma_d$	
$A_1$	1	1	1	1	1	<b>s</b>
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The point group representation **does not fix** the form of the superconducting gap function, even without further symmetry breaking

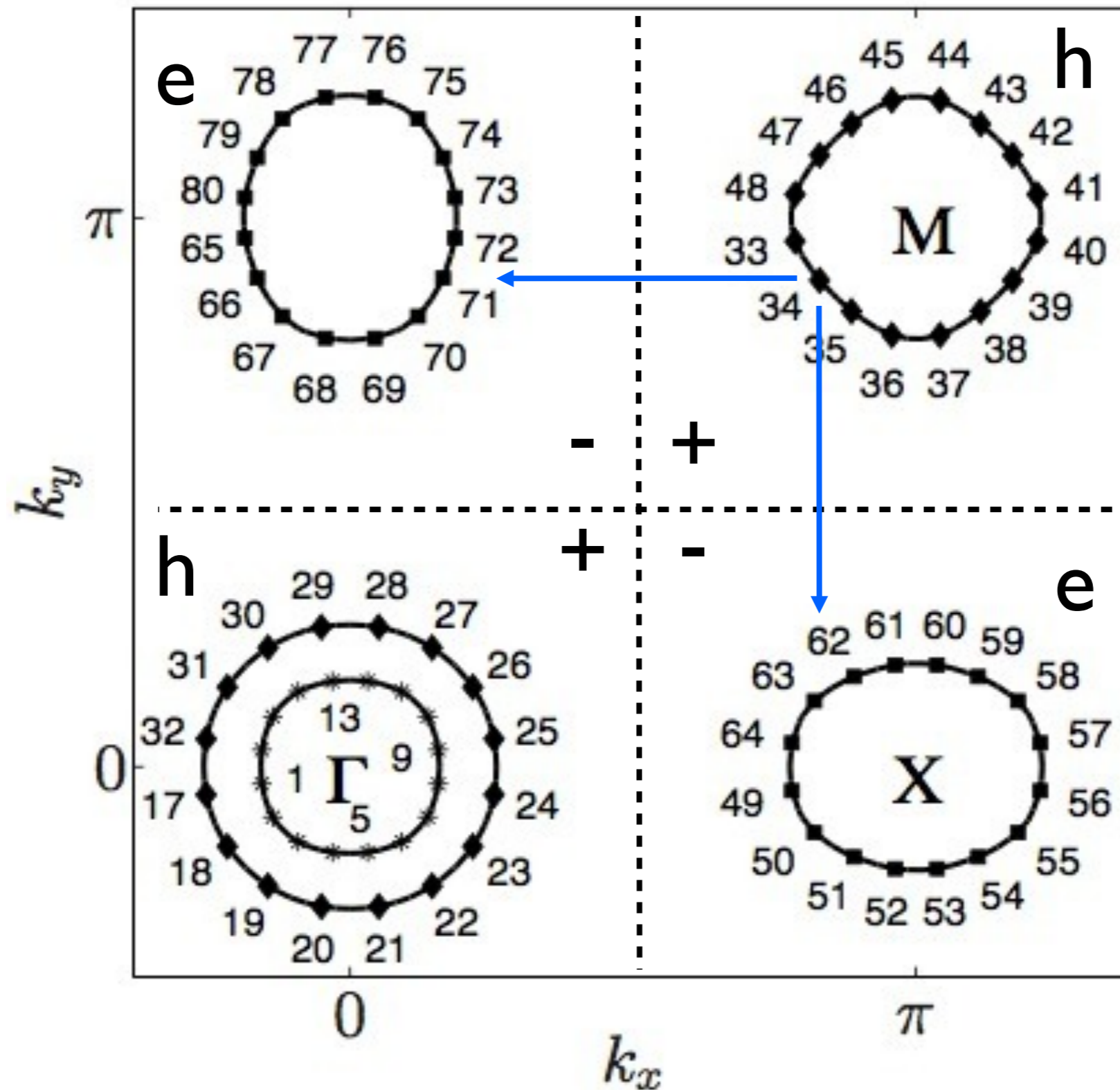
$$\Delta_s(\mathbf{k}) = 1 + \alpha(\cos k_x + \cos k_y)$$

$$\Delta_{d_{x^2-y^2}}(\mathbf{k}) = \cos k_x - \cos k_y + \alpha(\cos 2k_x - \cos 2k_y)$$

# Extended s-wave in the pnictides



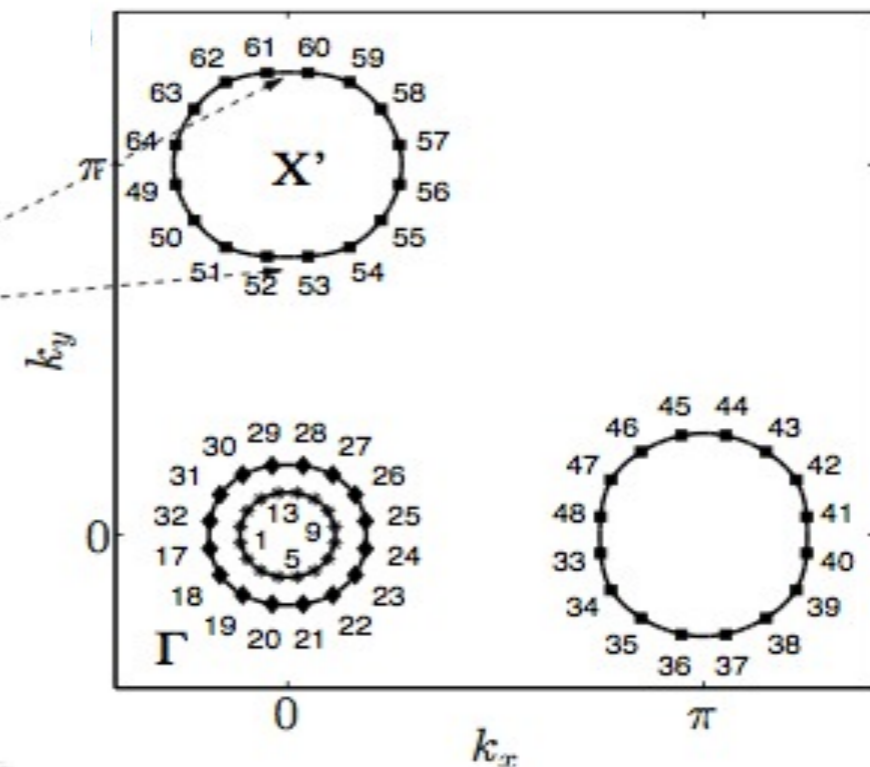
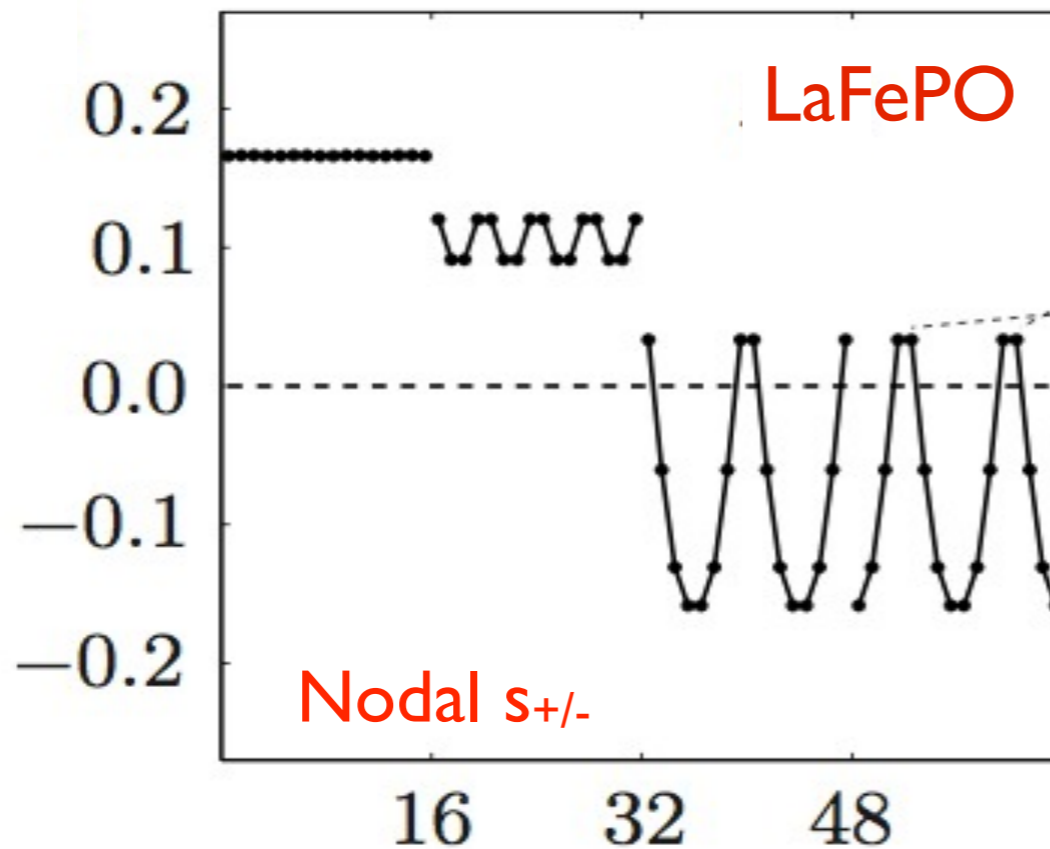
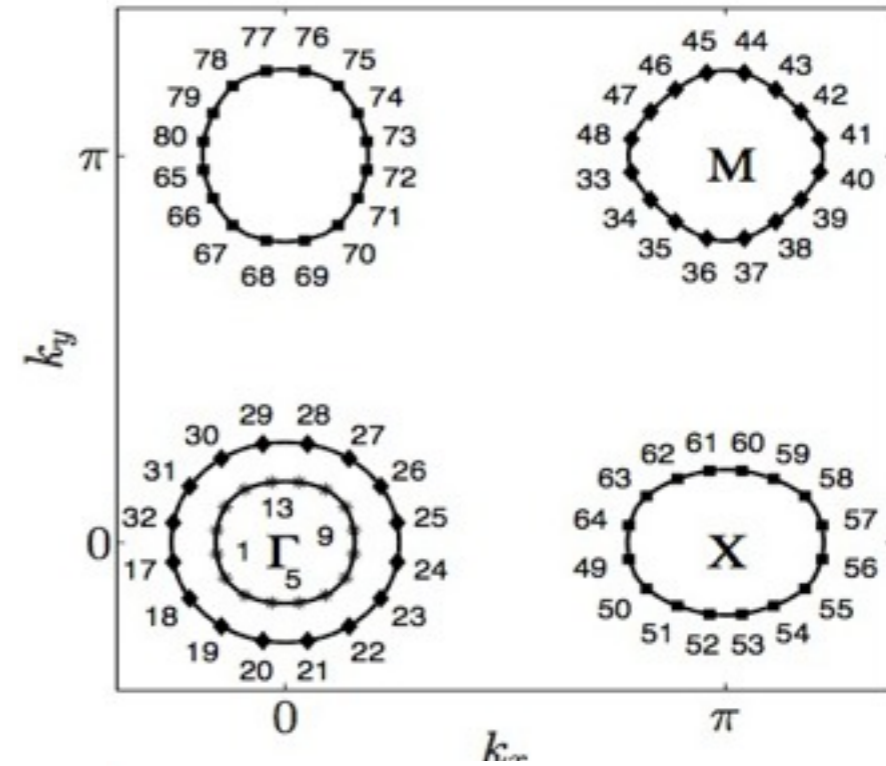
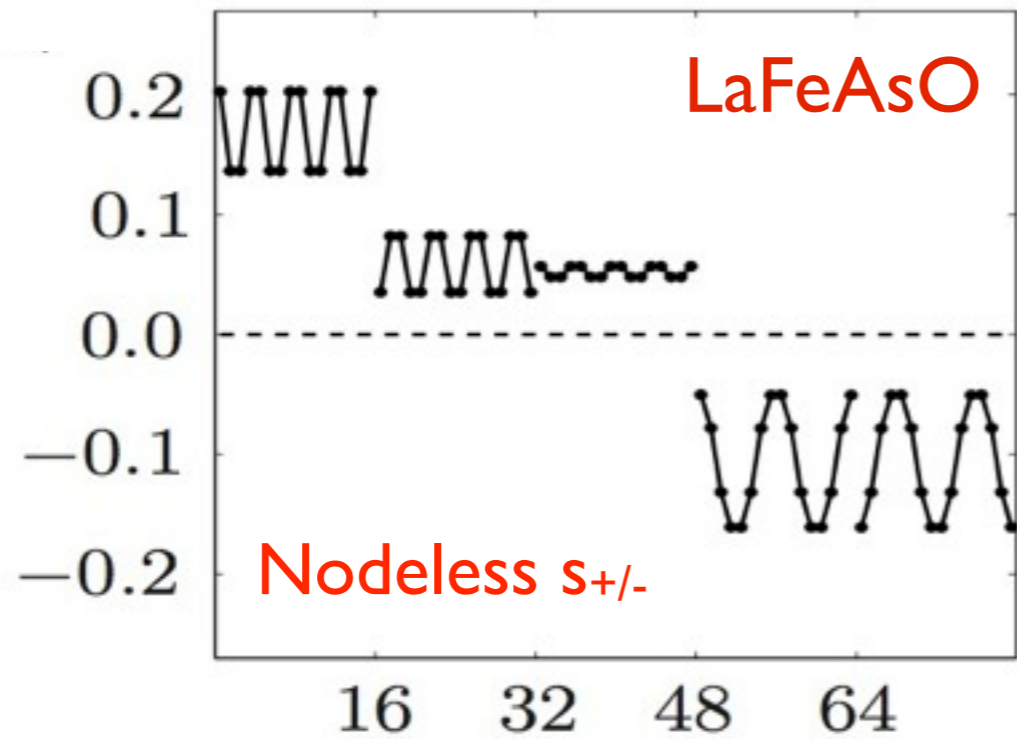
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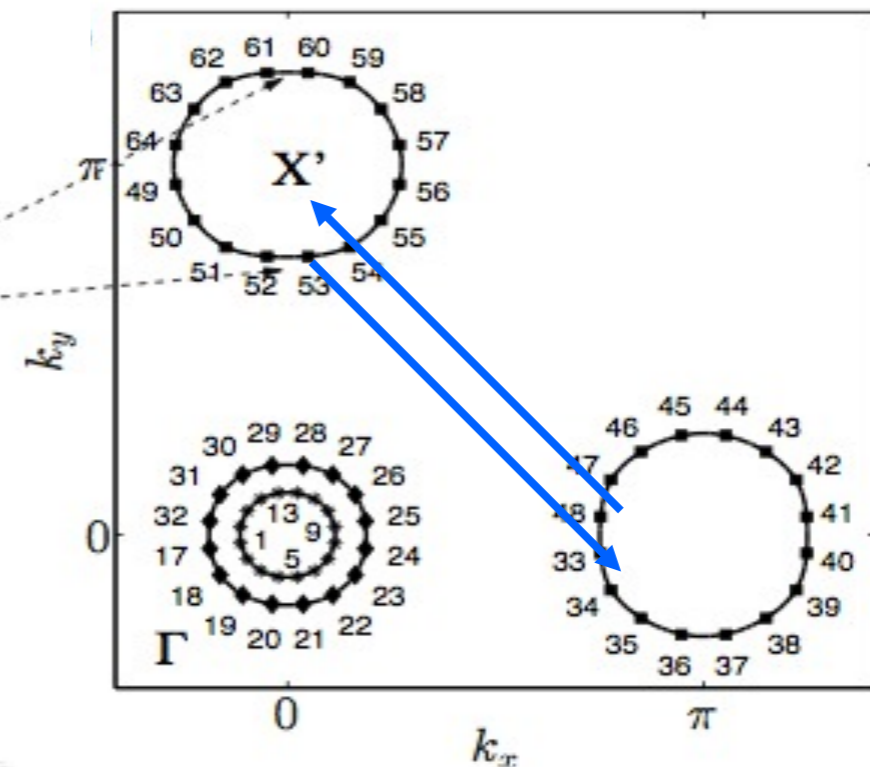
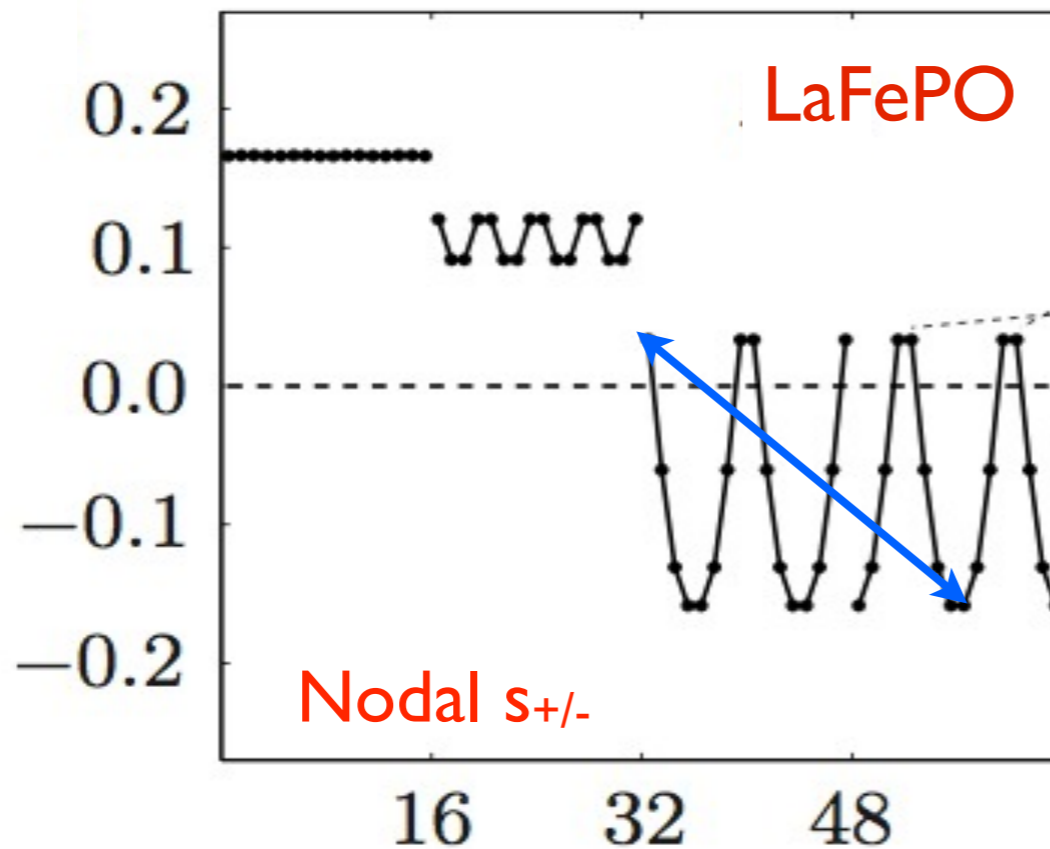
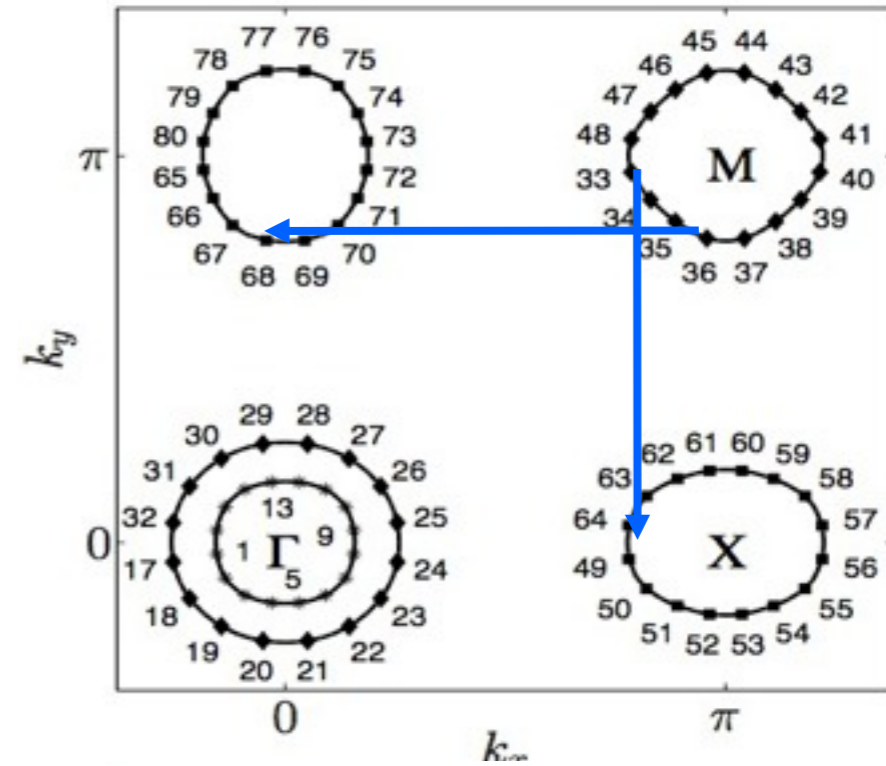
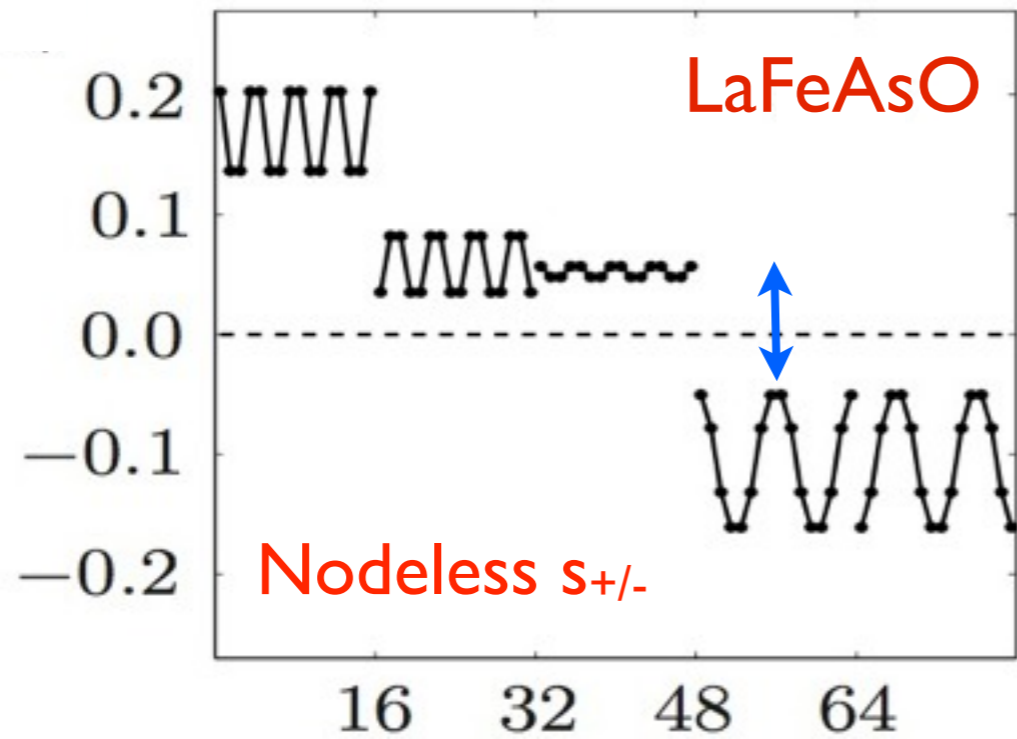
# LaFeAsO vs. LaFePO: s-wave anisotropy

Thomale, Platt, Hanke, Bernevig, PRL 106, 187003 (2011)



# LaFeAsO vs. LaFePO: s-wave anisotropy

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# Prediction of extended d-wave in pnictides

RAPID COMMUNICATIONS

PHYSICAL REVIEW B **80**, 180505(R) (2009)

## Functional renormalization-group study of the doping dependence of pairing symmetry in the iron pnictide superconductors

Ronny Thomale,<sup>1</sup> Christian Platt,<sup>2</sup> Jiangping Hu,<sup>3</sup> Carsten Honerkamp,<sup>2</sup> and B. Andrei Bernevig<sup>4</sup>

<sup>1</sup>*Institut für Theorie der Kondensierten Materie, Universität Karlsruhe, D 76128 Karlsruhe, Germany*

<sup>2</sup>*Theoretical Physics, University of Würzburg, D-97074 Würzburg, Germany*

<sup>3</sup>*Department of Physics, Purdue University, West Lafayette, Indiana 47907, USA*

<sup>4</sup>*Department of Physics, Princeton University, Princeton, New Jersey 08544, USA*

(Received 19 October 2009; published 13 November 2009)

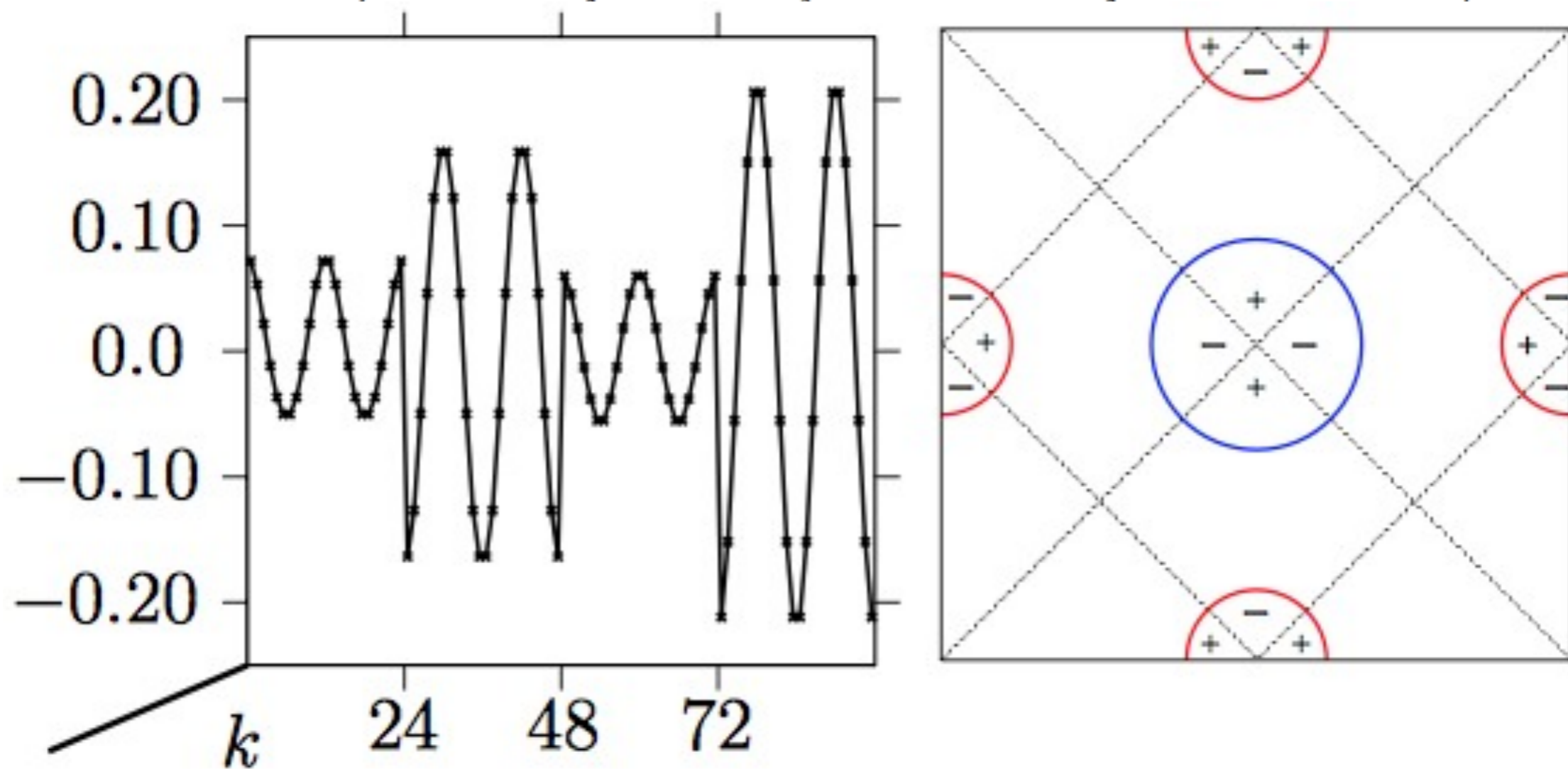
We use the functional renormalization group to analyze the phase diagram of a four-band model for the iron pnictides subject to band interactions with certain  $A_{1g}$  momentum dependence. We determine the parameter regimes where an extended  $s$ -wave pairing instability with and without nodes emerges. For electron doping, the parameter regime in which a nodal gap appears is in correspondence to recent predictions [A. Chubukov *et al.*, arXiv:0903.5547 (unpublished)], however, at very low  $T_c$ . Upon hole doping, the  $s$ -wave gap never becomes nodal: above a critical strength of the intraband repulsion, the system favors an exotic extended  $d$ -wave instability on the enlarged hole pockets. At half filling, we find that a strong momentum dependence of

# Prediction of extended d-wave in pnictides

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PHYSICAL REVIEW B **80**, 180505(R) (2009)

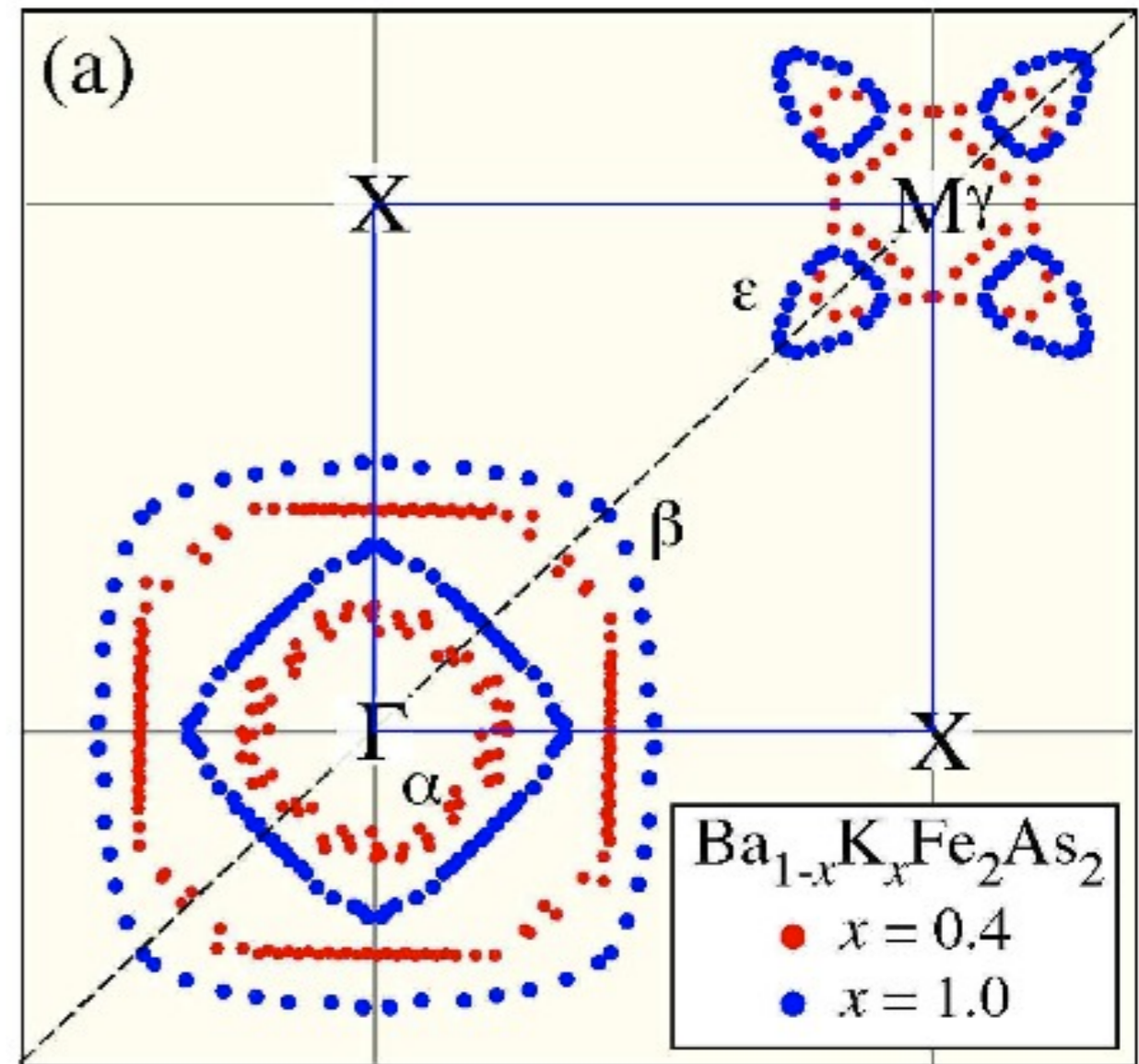
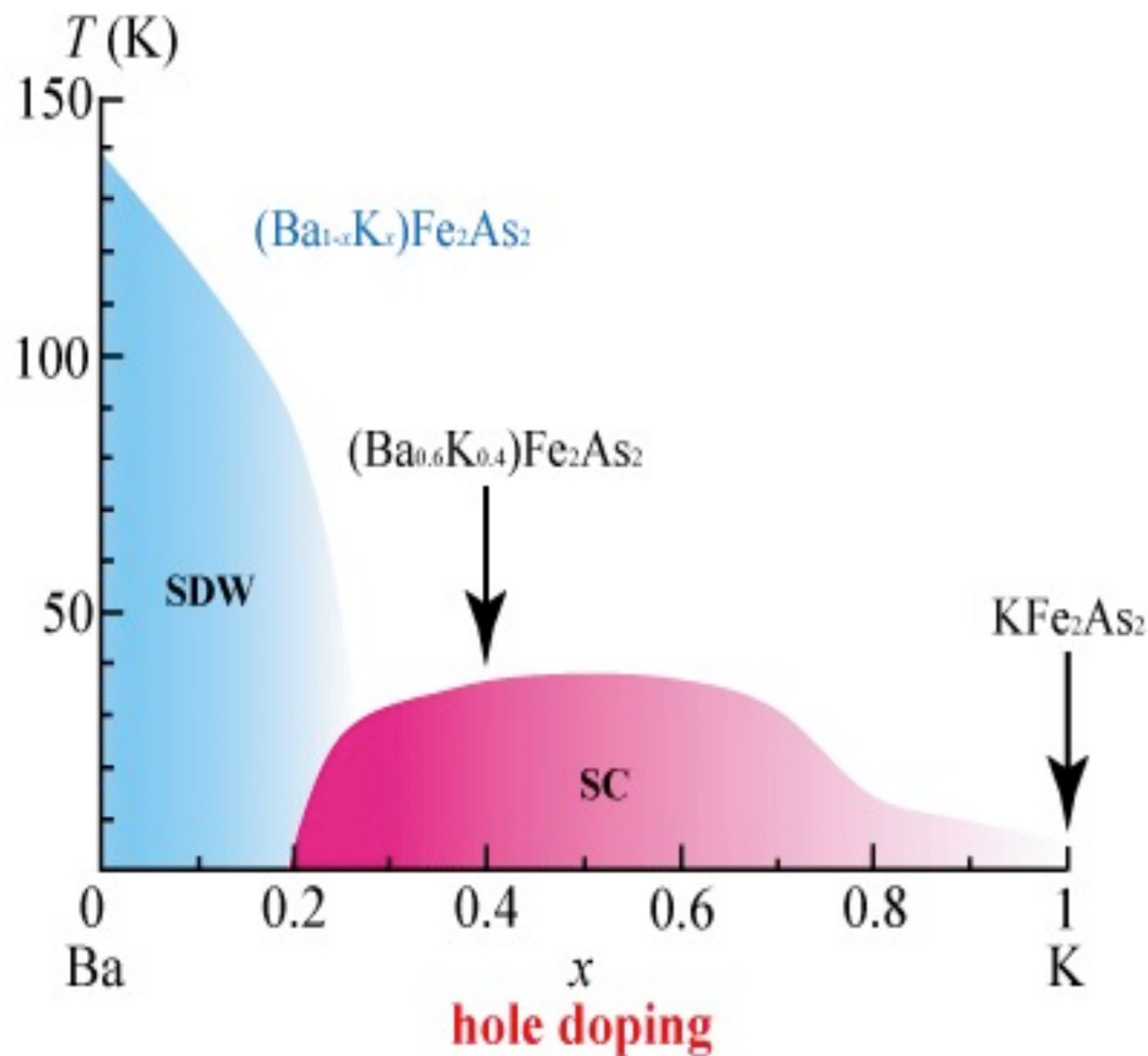
Functional renormalization-group study of the doping dependence of pairing symmetry in the iron pnictide superconductors



**Present**

# D-wave in hole doped iron pnictides?

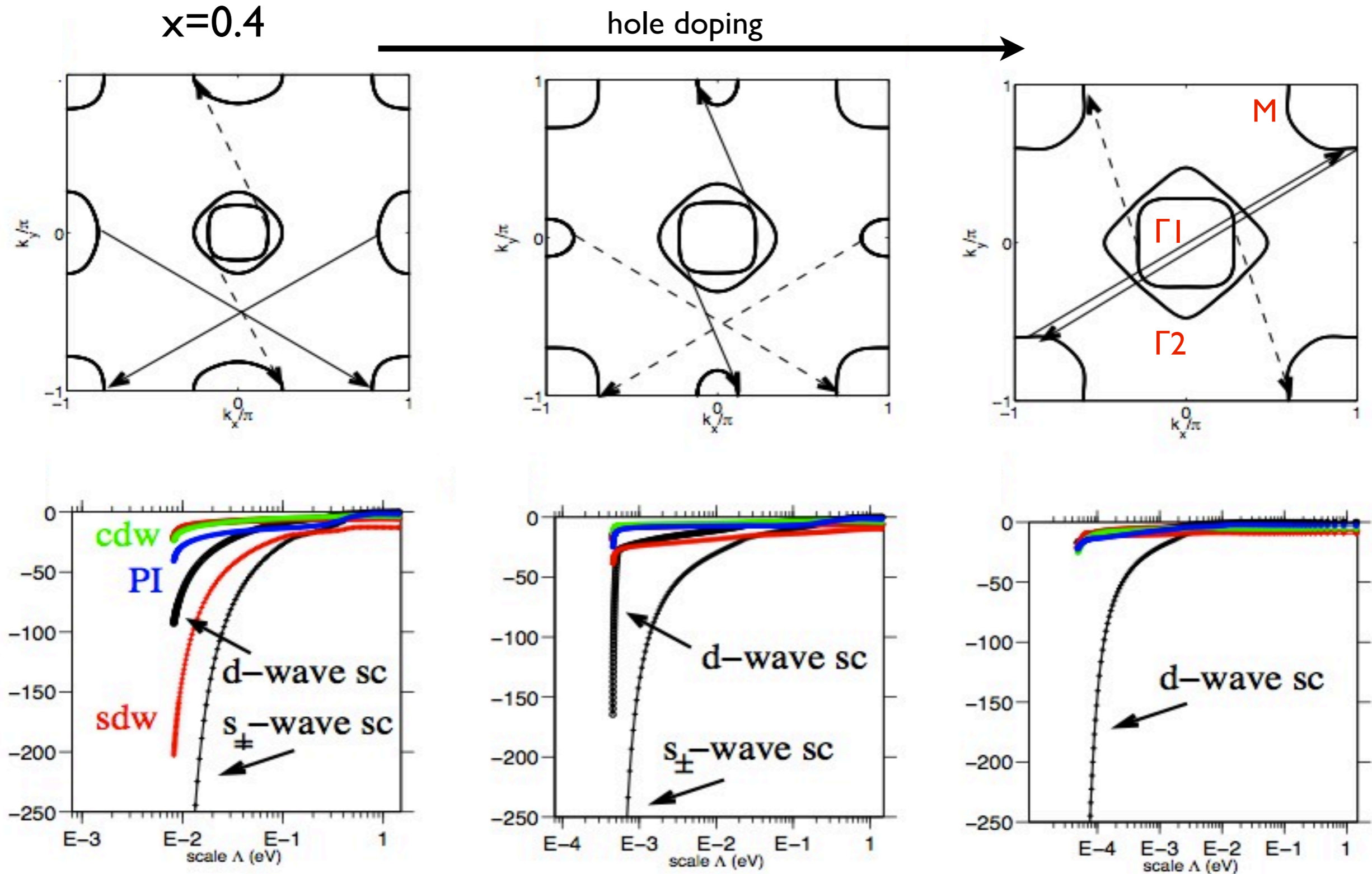
- moderate hole doping: electron **and** hole pockets
- strong hole doping: **only** hole pockets present



Ding group (China)

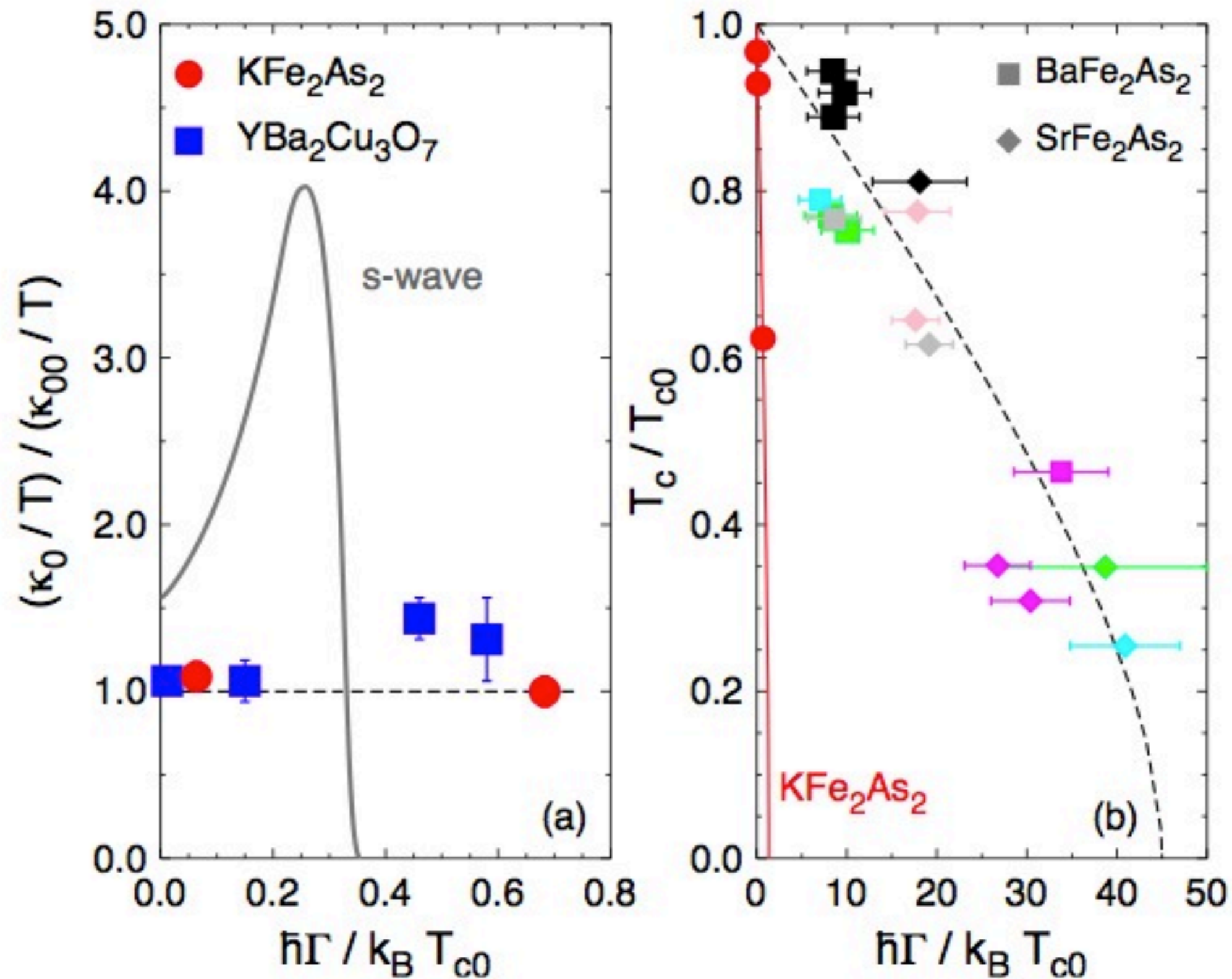
# FRG: s to d-wave transition in K-doped Ba-122

Thomale, Platt, Hanke, Hu, Bernevig, PRL 107, 117001 (2011).



# Evidence for d-wave: thermal transport

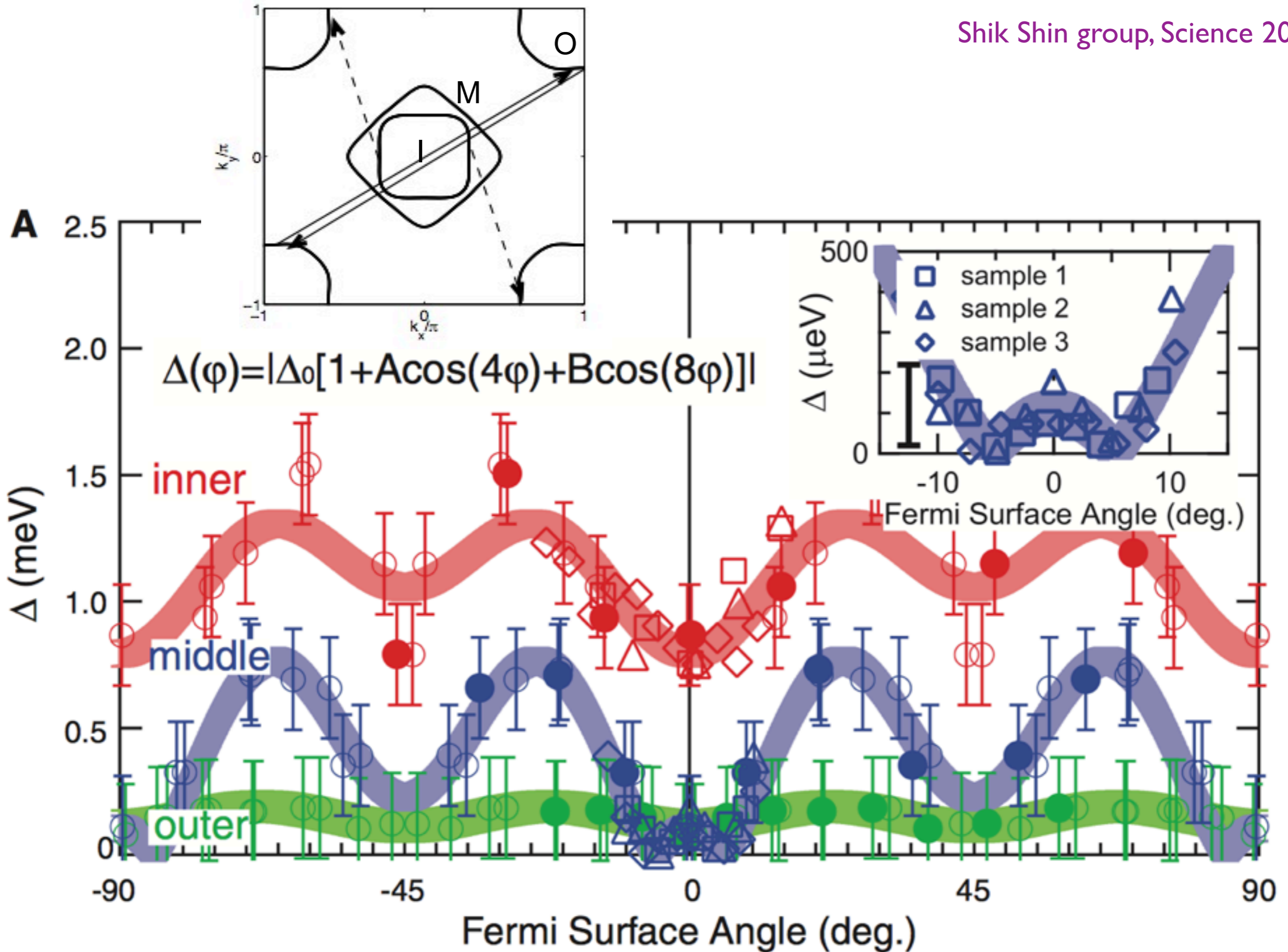
Taillefer group, Reid et al. PRL 2012





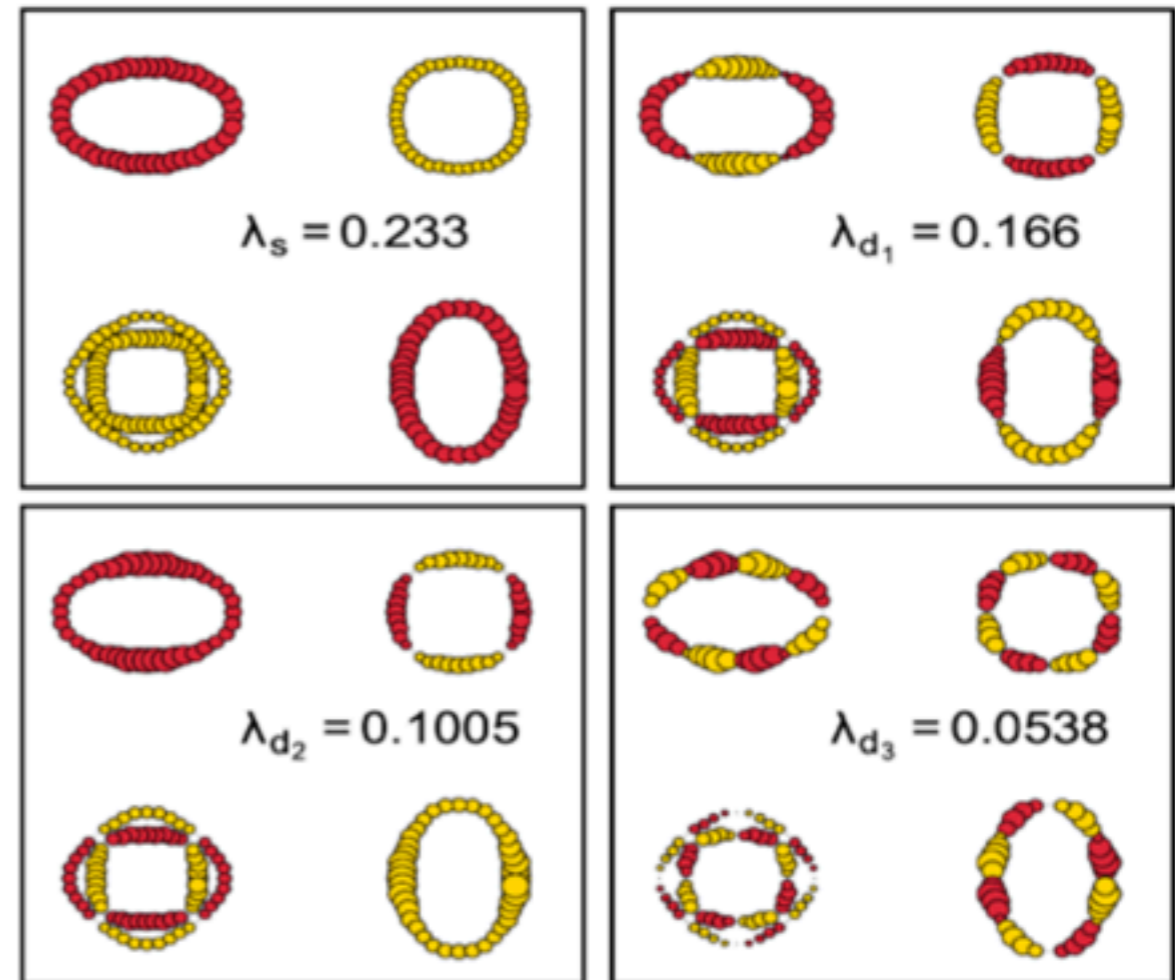
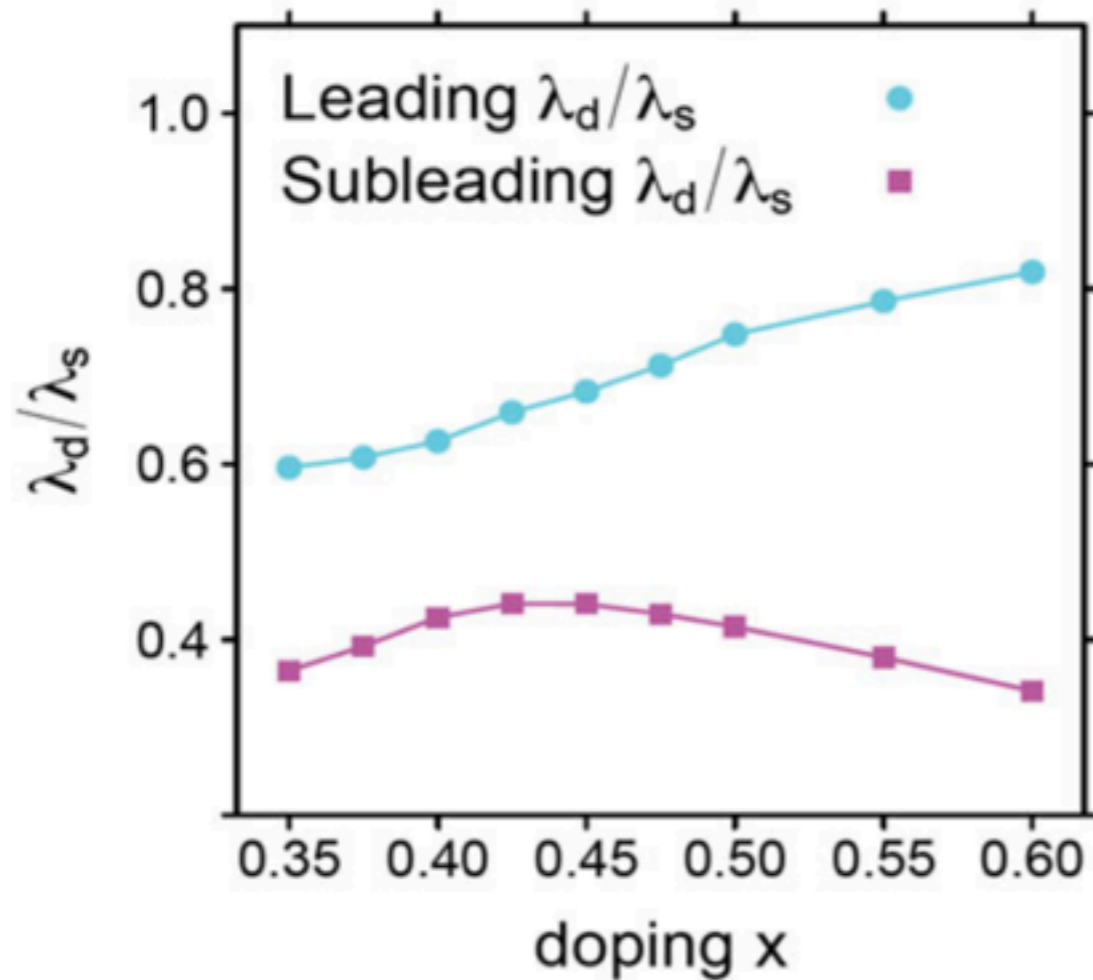
# Evidence against d-wave: Laser ARPES

Shik Shin group, Science 2012



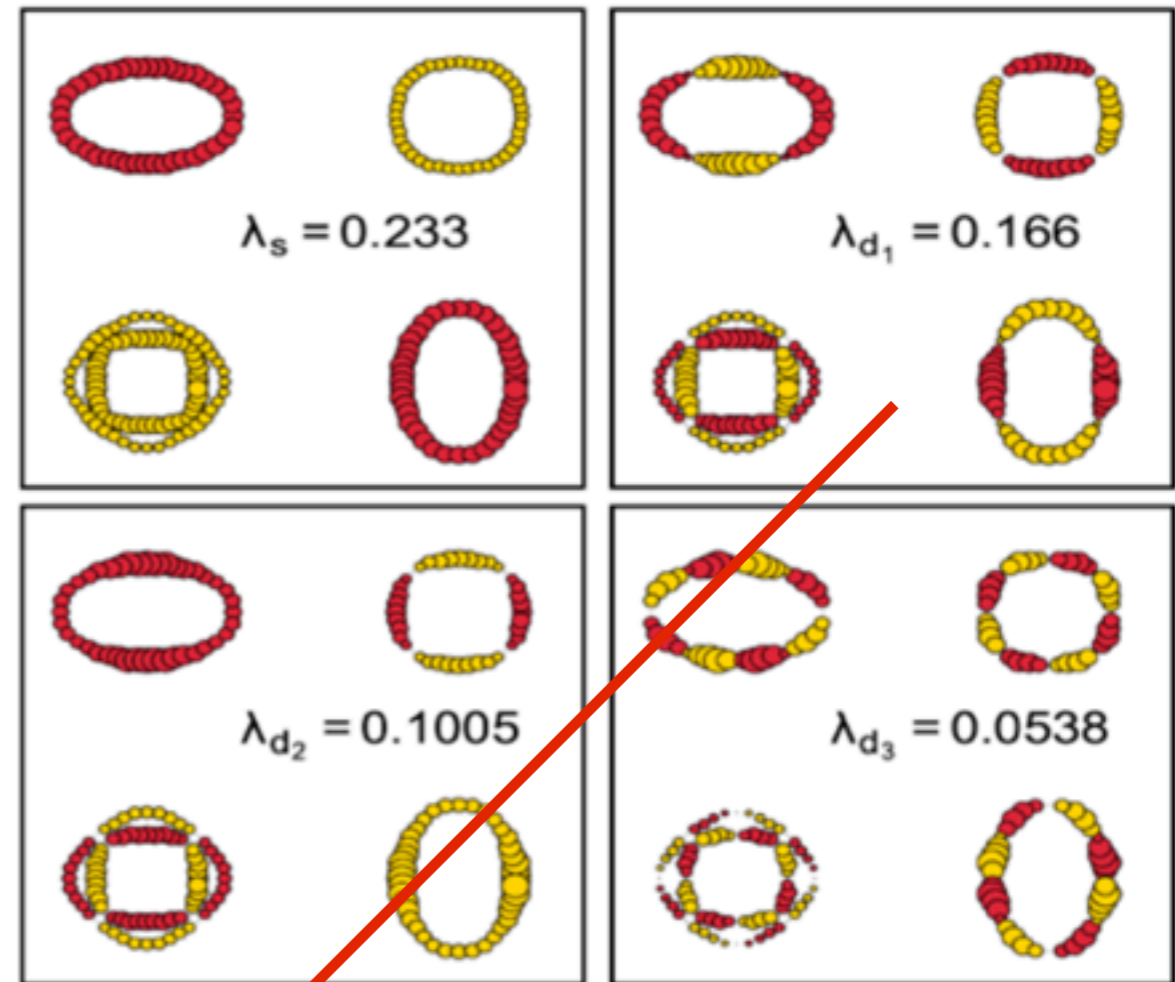
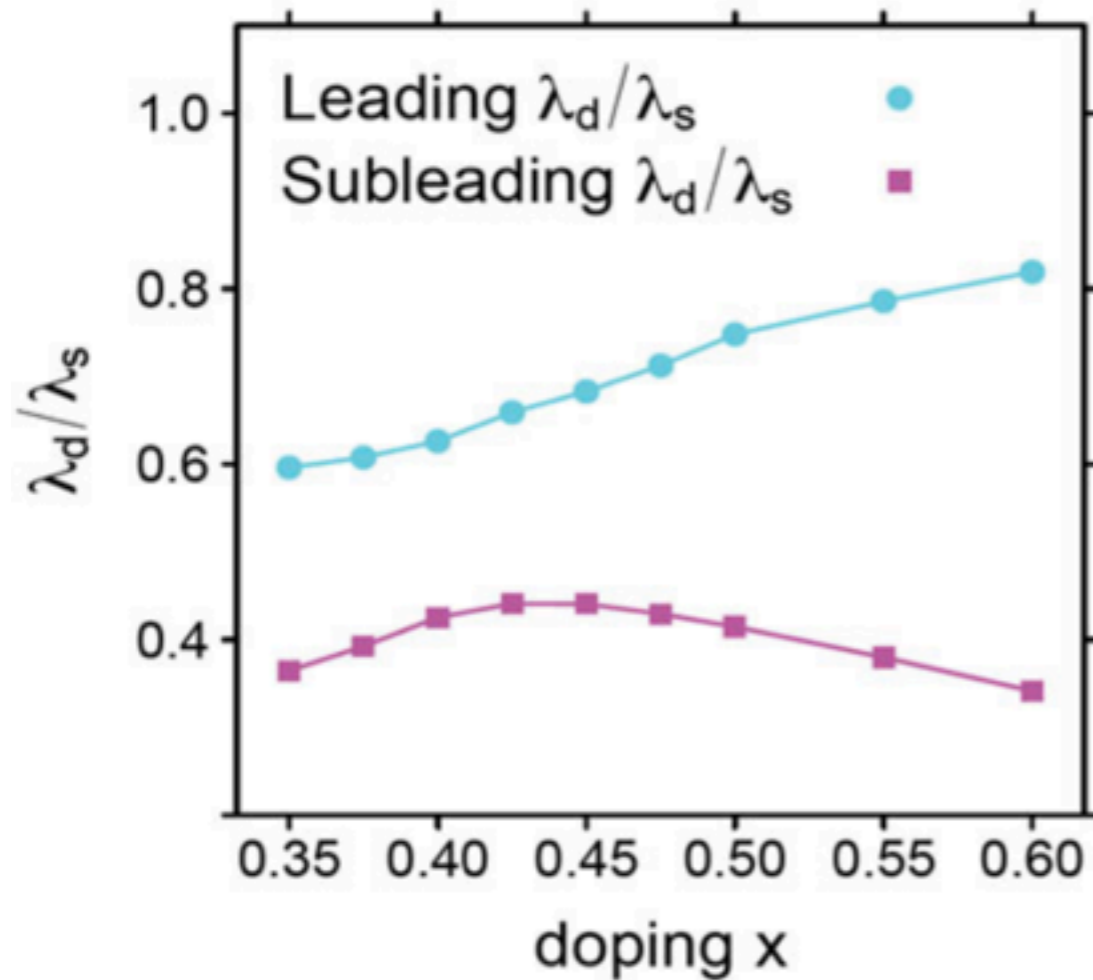
# Raman: extended d-wave dominant subleading order

Maiti et al., PRL 117, 257001 (2016); Böhm et al., in preparation.



# Raman: extended d-wave dominant subleading order

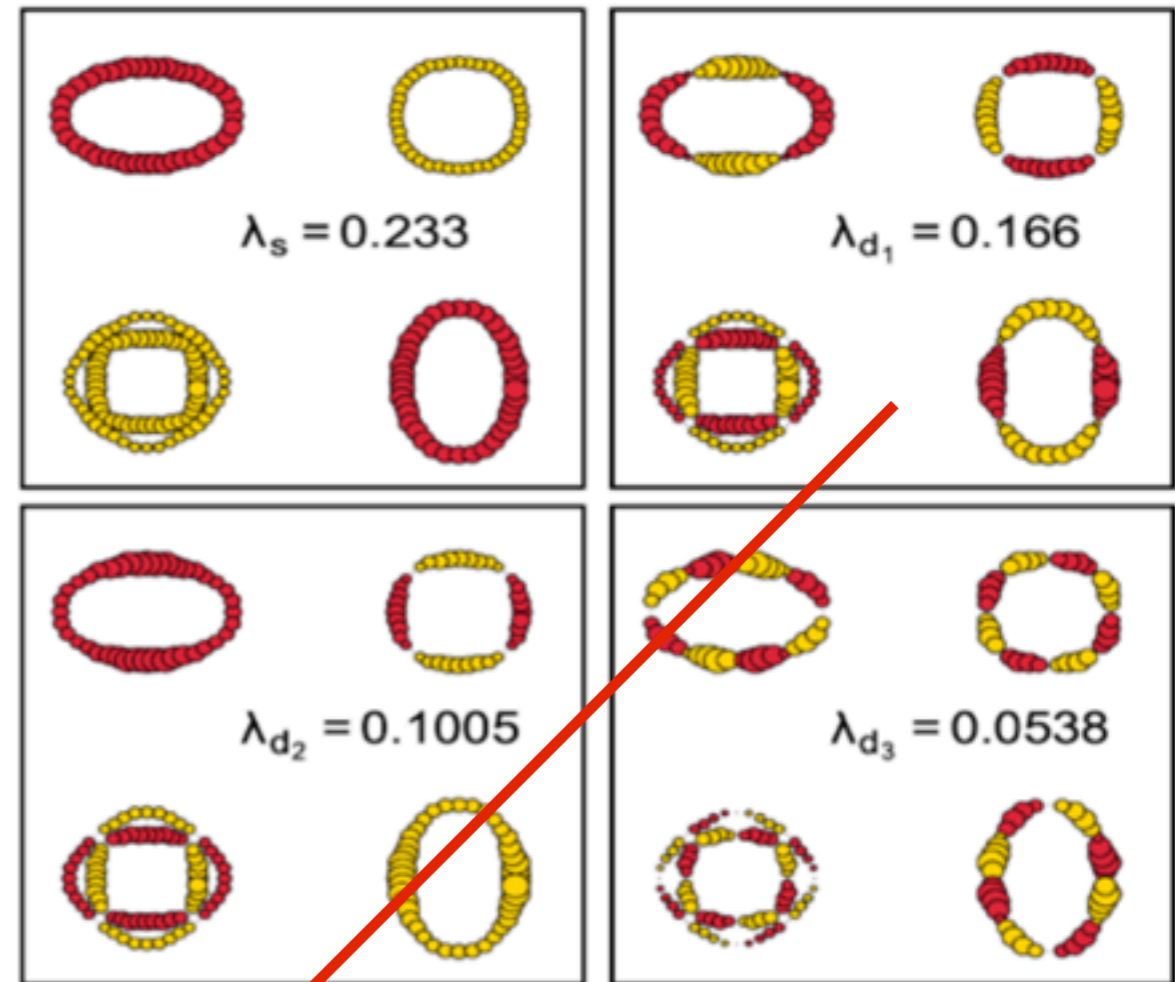
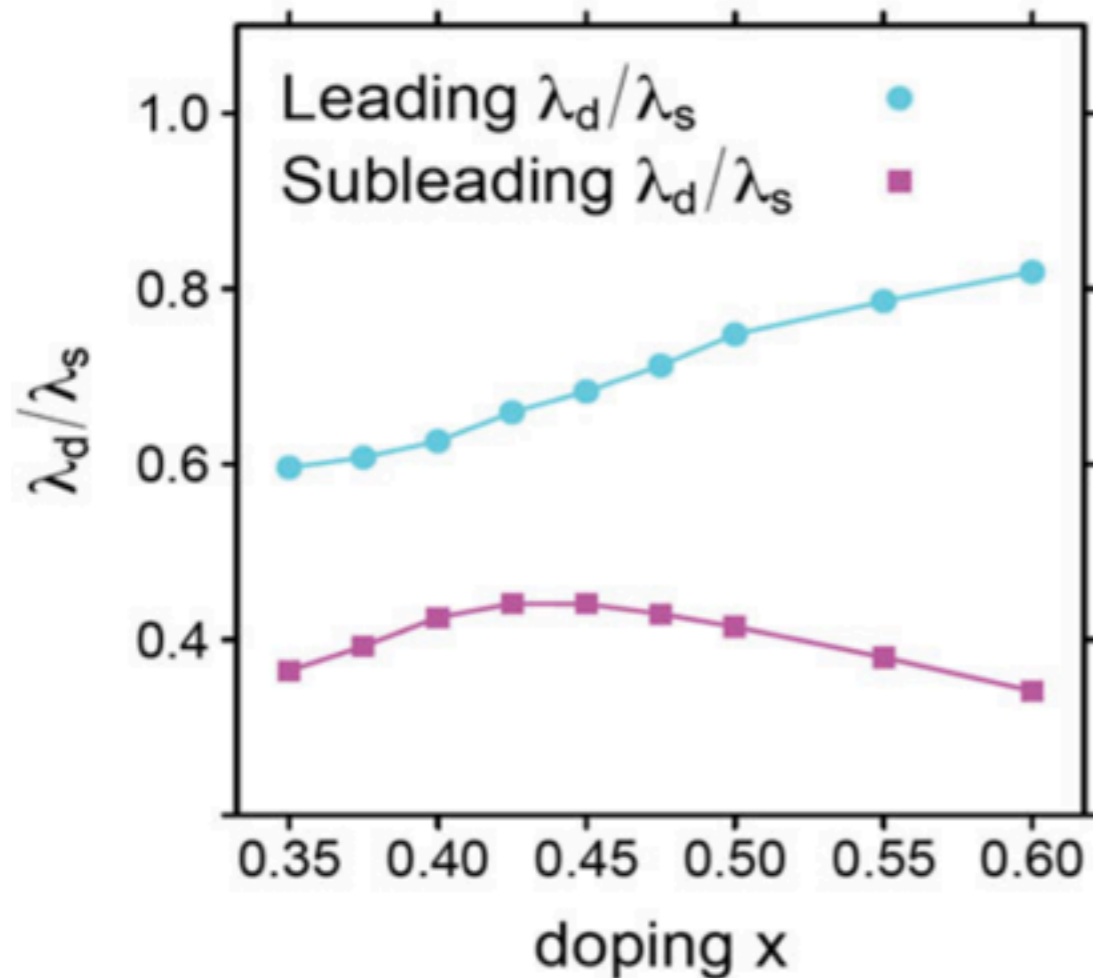
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Extended d-wave!

# Raman: extended d-wave dominant subleading order

Maiti et al., PRL 117, 257001 (2016); Böhm et al., in preparation.



Extended d-wave!

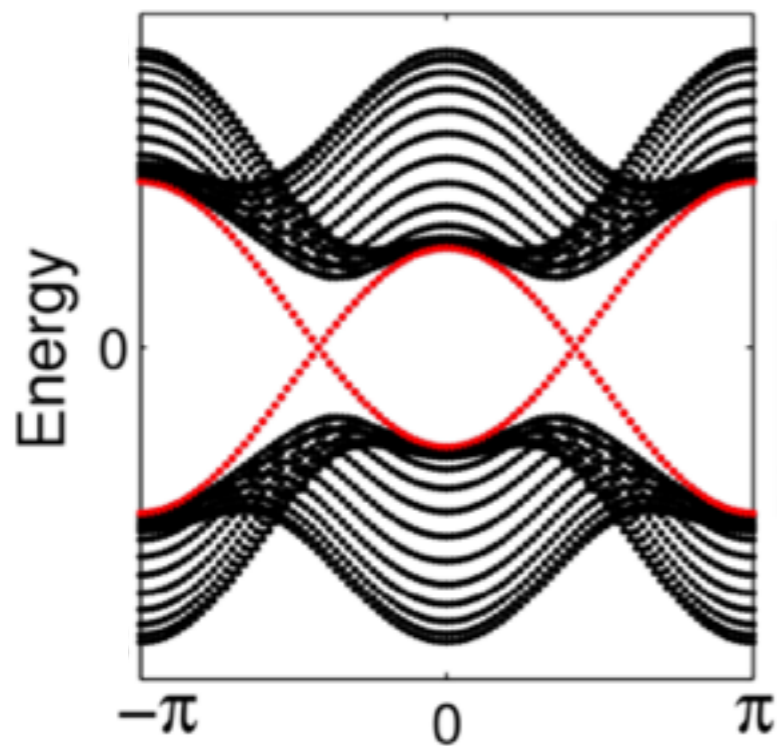
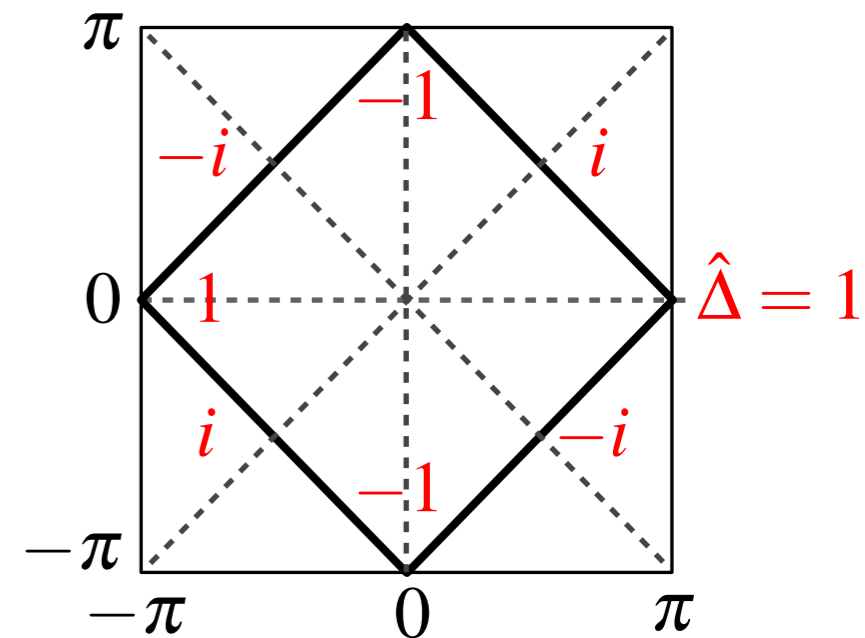
To be submitted: Raman scattering **confirms** the prediction of extended d-wave as the dominant subleading d-wave instability in K-doped Ba-122.

# Chiral d-wave superconductivity

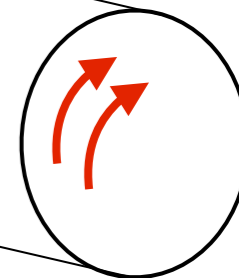
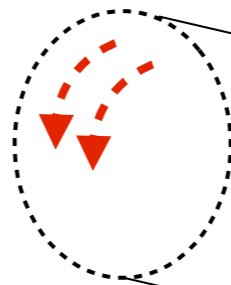
$$H_{d+id} = \sum_{\mathbf{k}} (c_{\mathbf{k}\uparrow}^\dagger, c_{-\mathbf{k}\downarrow}) \begin{pmatrix} \xi_{\mathbf{k}} & \Delta_{\mathbf{k}} \\ \Delta_{\mathbf{k}}^* & -\xi_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c_{-\mathbf{k}\downarrow}^\dagger \end{pmatrix}$$

$$\Delta_{\mathbf{k}} = \cos(k_x) - \cos(k_y) + i \sin(k_x) \sin(k_y) = |\Delta_{\mathbf{k}}| e^{i\varphi_{\mathbf{k}}}$$

$$\xi_{\mathbf{k}} = \cos(k_x) + \cos(k_y)$$



$$C_{d\pm id} = \frac{1}{2\pi} \oint \nabla_{\mathbf{k}} \varphi_{\mathbf{k}} d\mathbf{k} = \pm 2$$



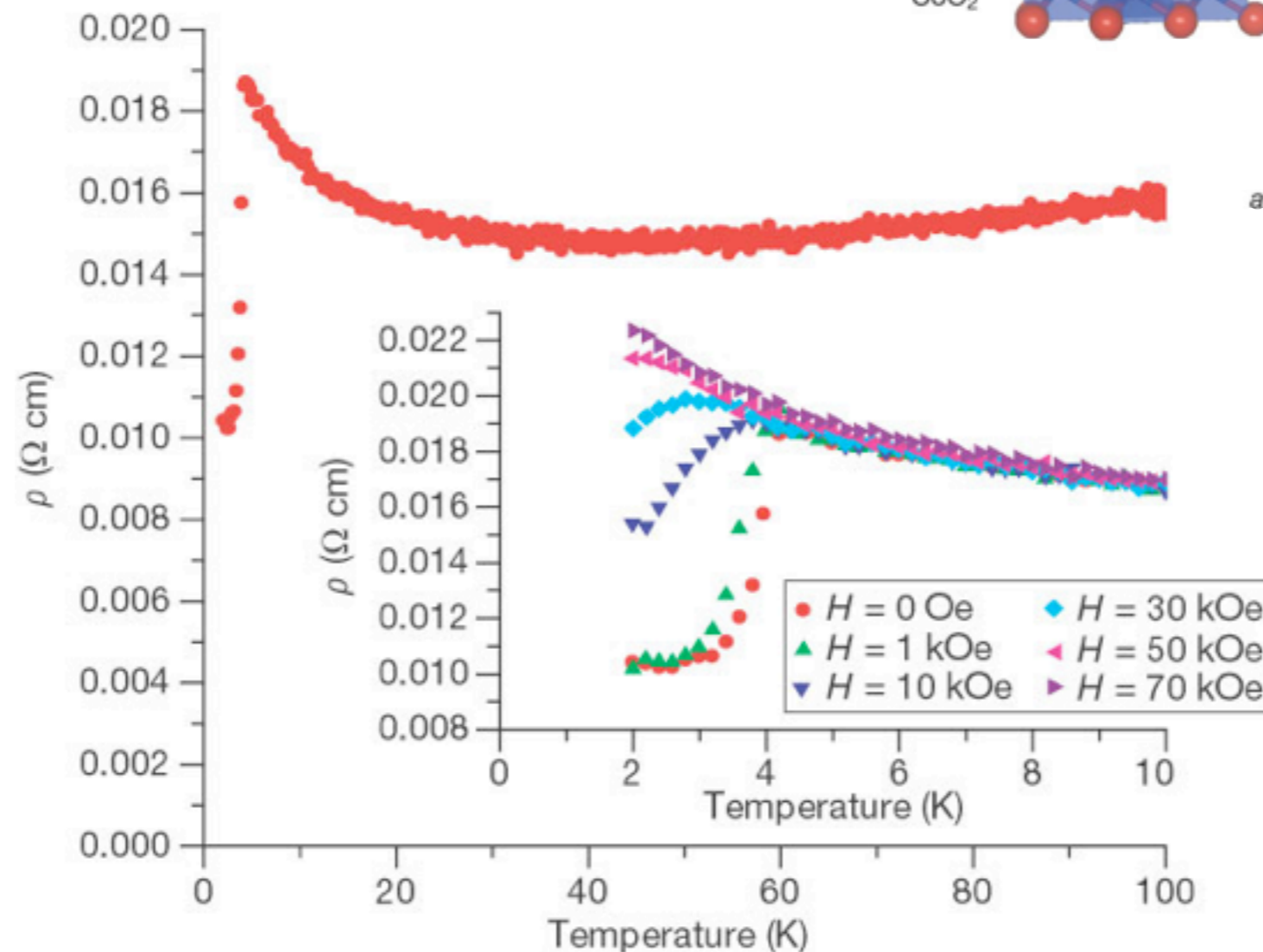
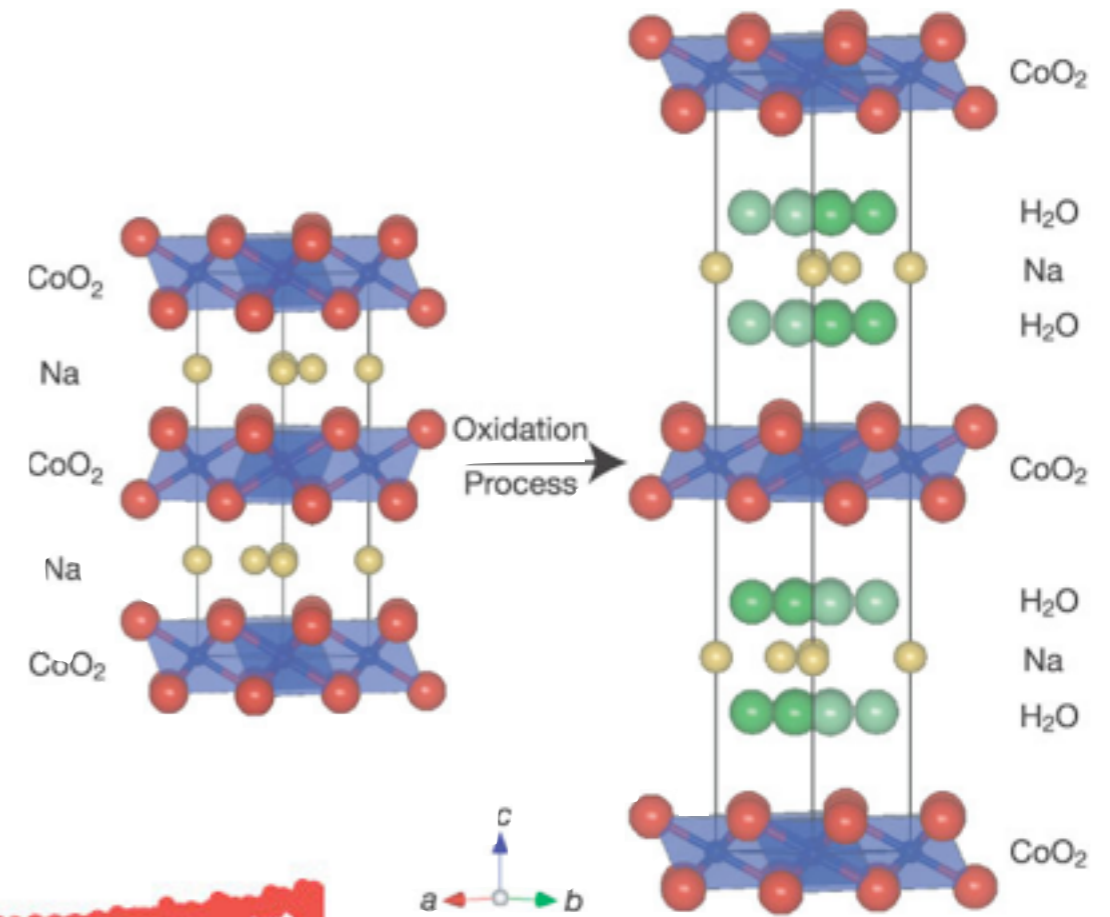
# Superconductivity in two-dimensional $\text{CoO}_2$ layers

Kazunori Takada<sup>\*‡</sup>, Hiroya Sakurai<sup>†</sup>, Eiji Takayama-Muromachi<sup>†</sup>,  
Fujio Izumi<sup>\*</sup>, Ruben A. Dilanian<sup>\*</sup> & Takayoshi Sasaki<sup>\*‡</sup>

<sup>\*</sup> Advanced Materials Laboratory, National Institute for Materials Science,  
Tsukuba, Ibaraki 305-0044, Japan

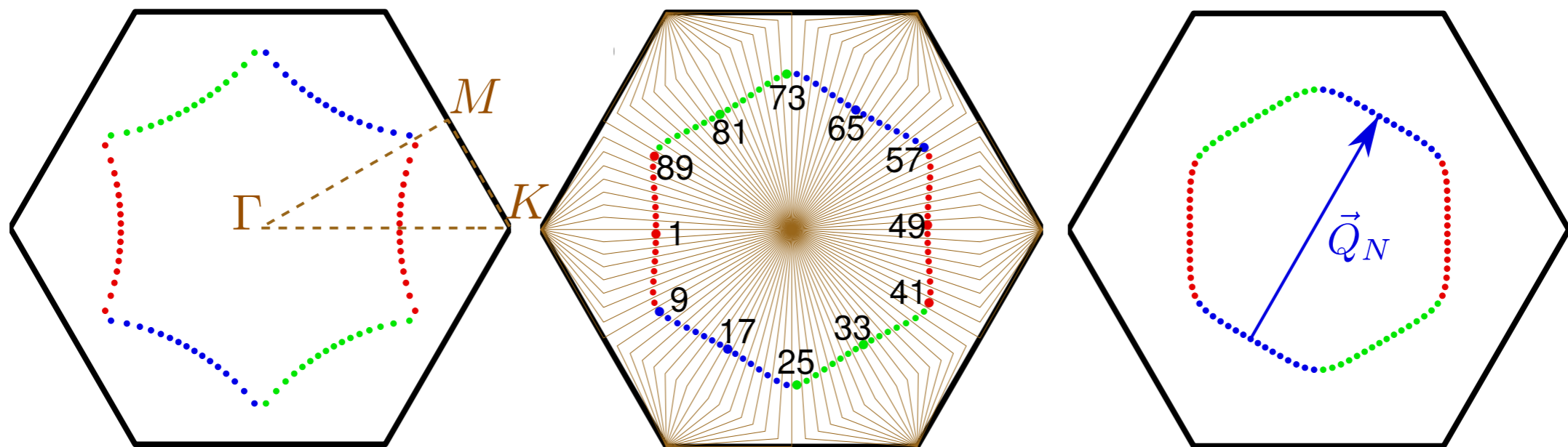
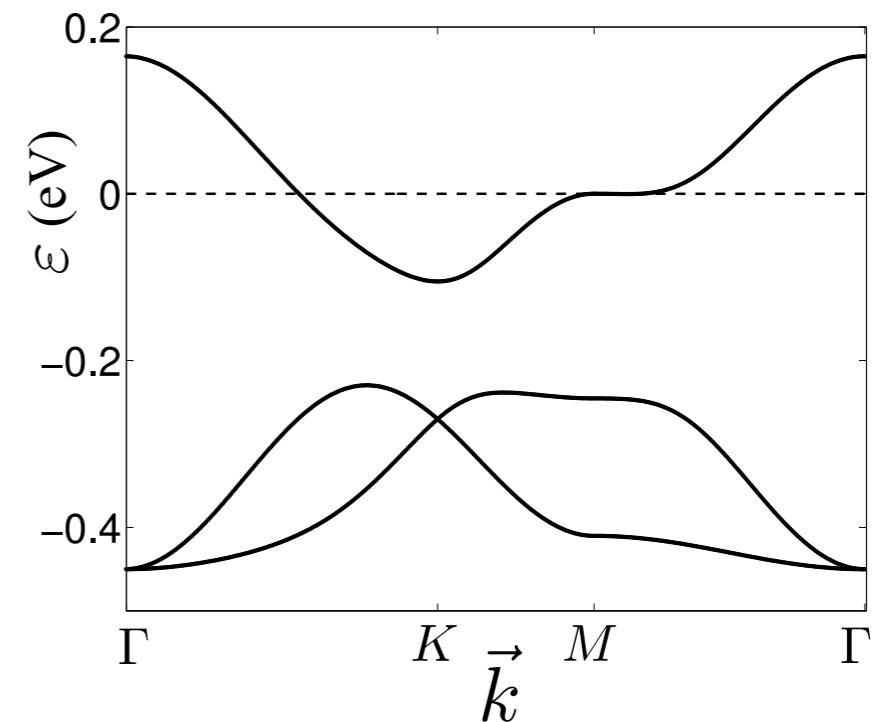
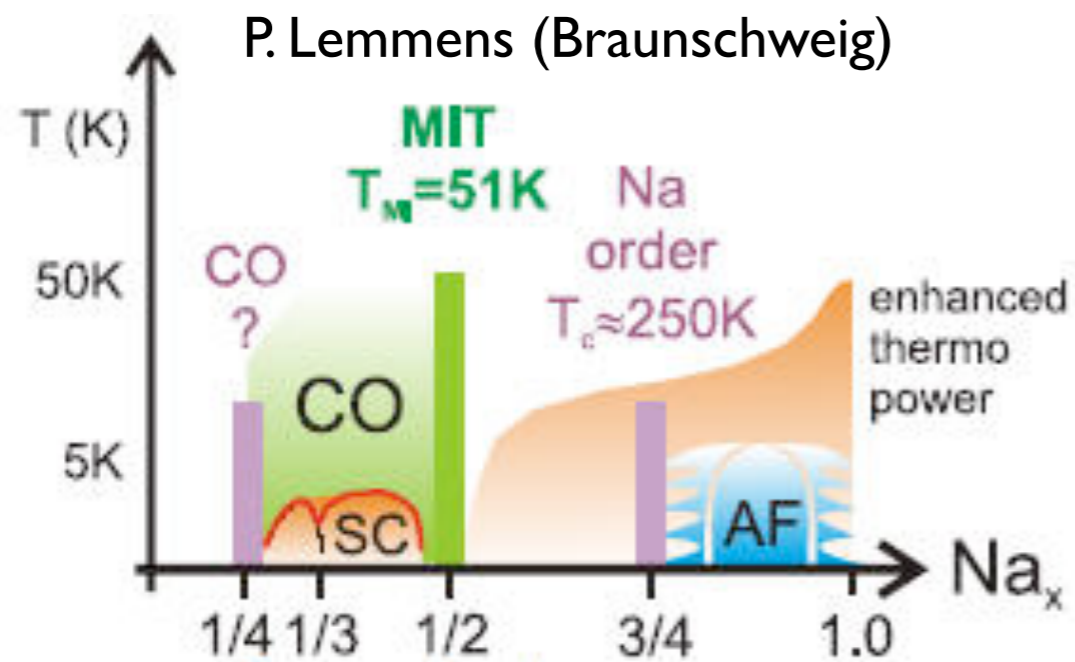
<sup>†</sup> Superconducting Materials Center, National Institute for Materials Science,  
Tsukuba, Ibaraki 305-0044, Japan

<sup>‡</sup> CREST, Japan Science and Technology Corporation



# Cobaltate phase diagram

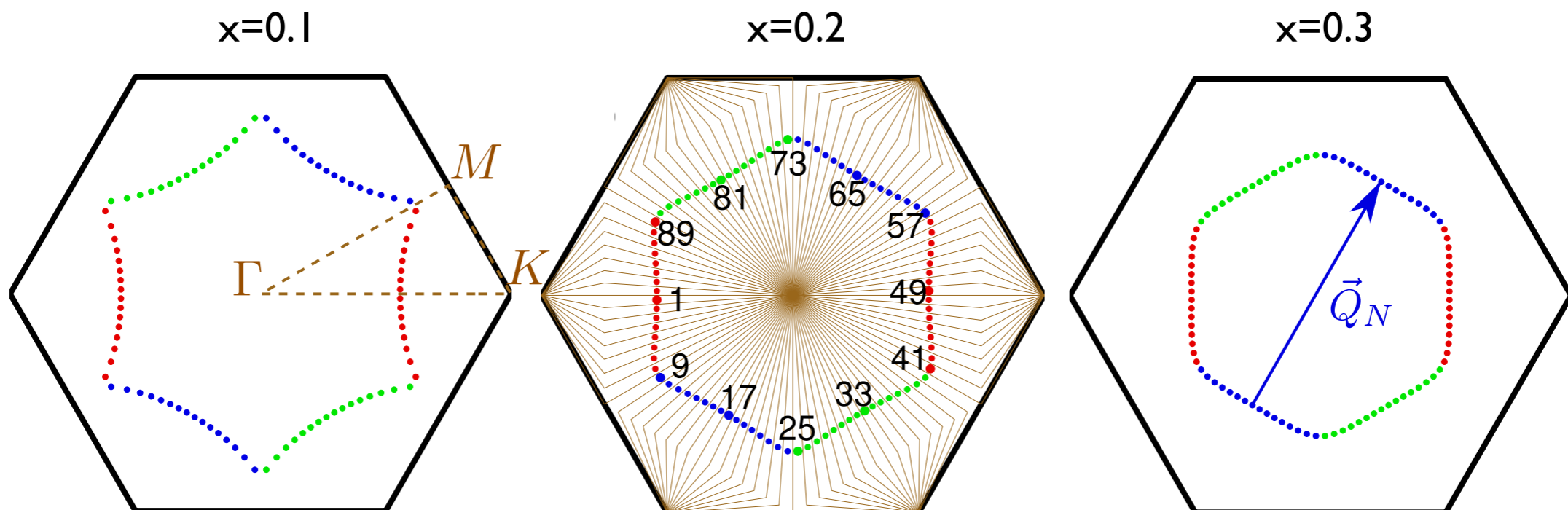
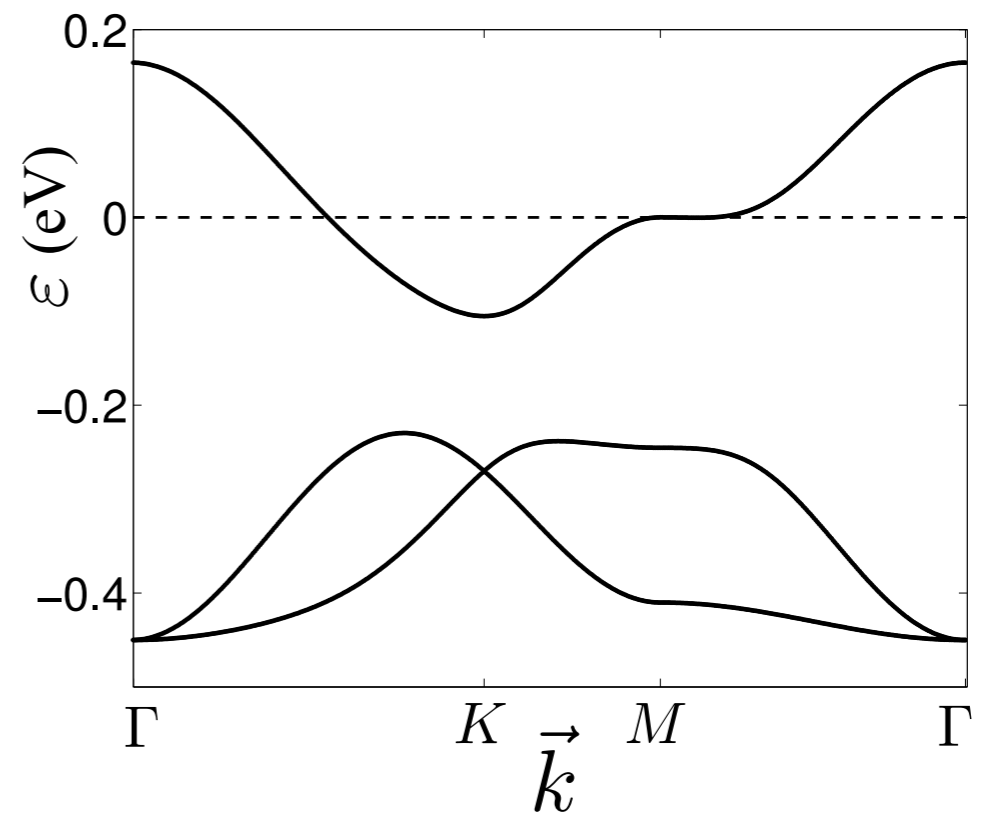
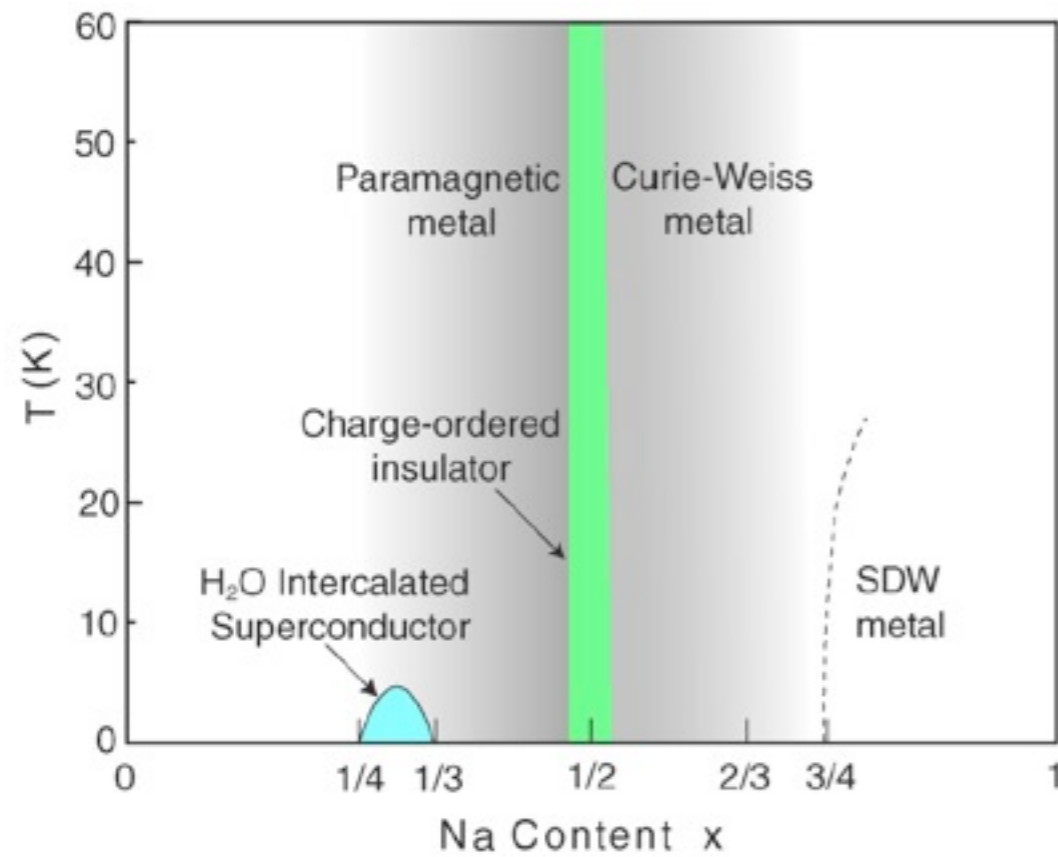
The phase diagram hosts a plethora of phases interpolating between the **parent insulating compound** ( $x=0$ ) and **Na-doped insulating limit** ( $x=1$ )



# Cobaltate phase diagram

Ong group, PRB 2004

Aligia et al., PRL 2008

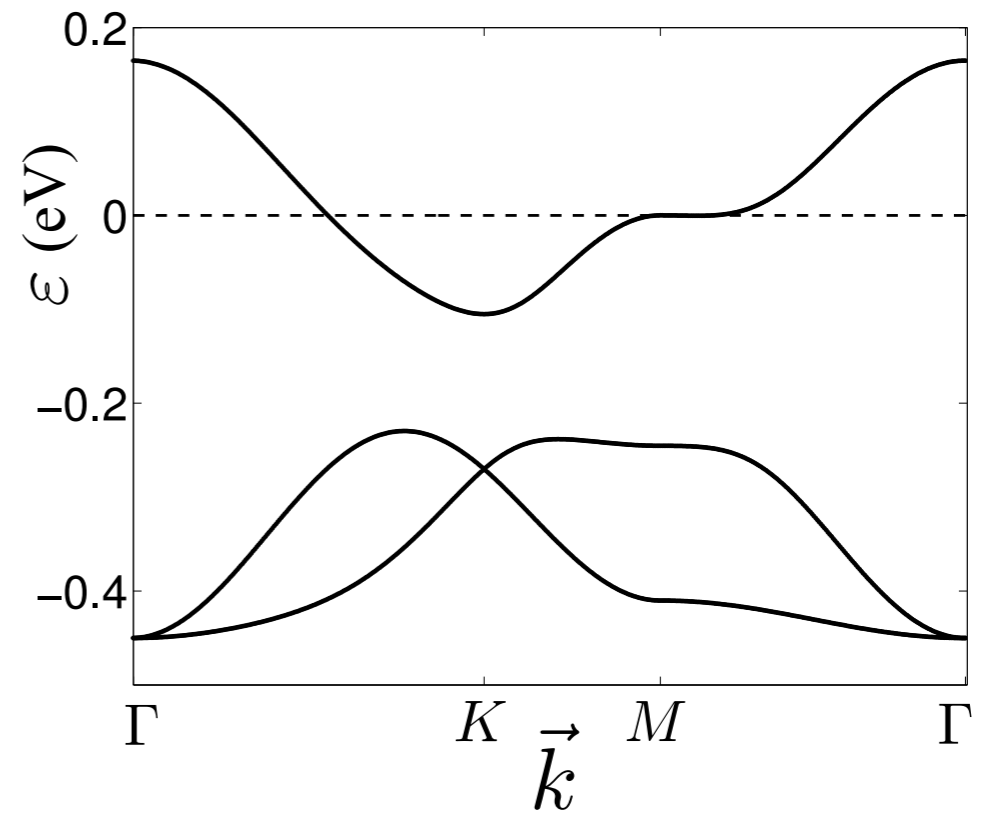
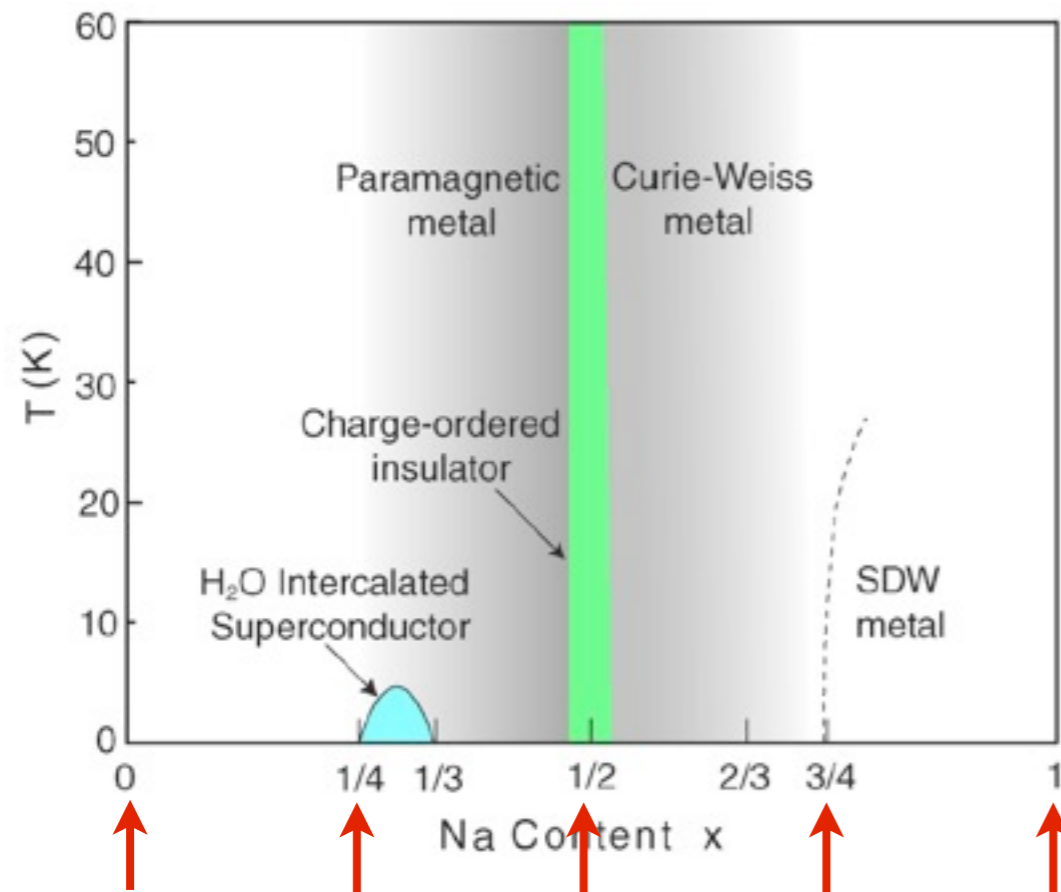




# Cobaltate phase diagram

Ong group, PRB 2004

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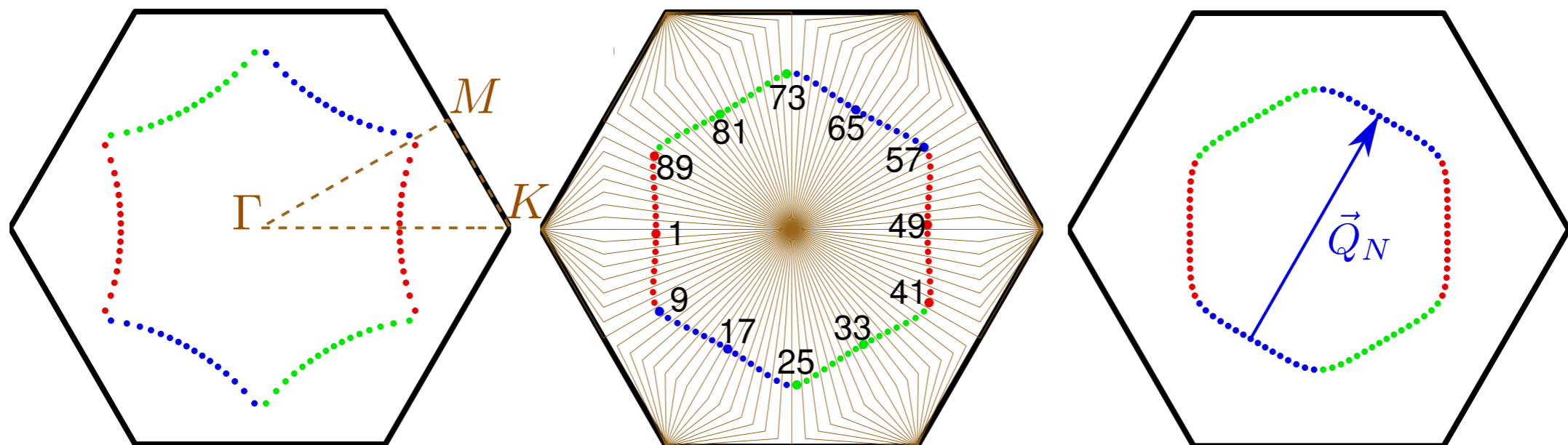


Insulators

$x=0.1$

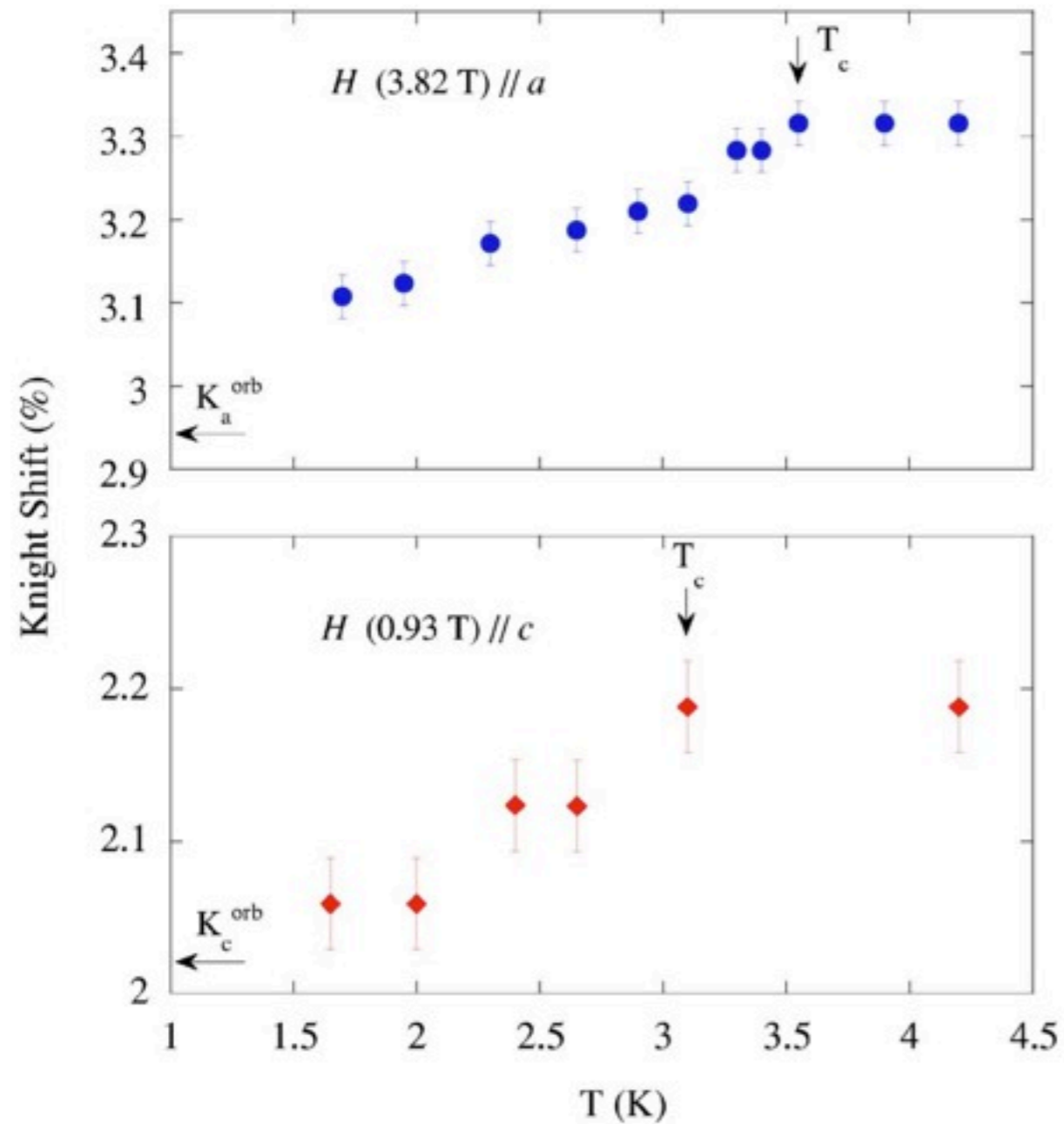
$x=0.2$

$x=0.3$

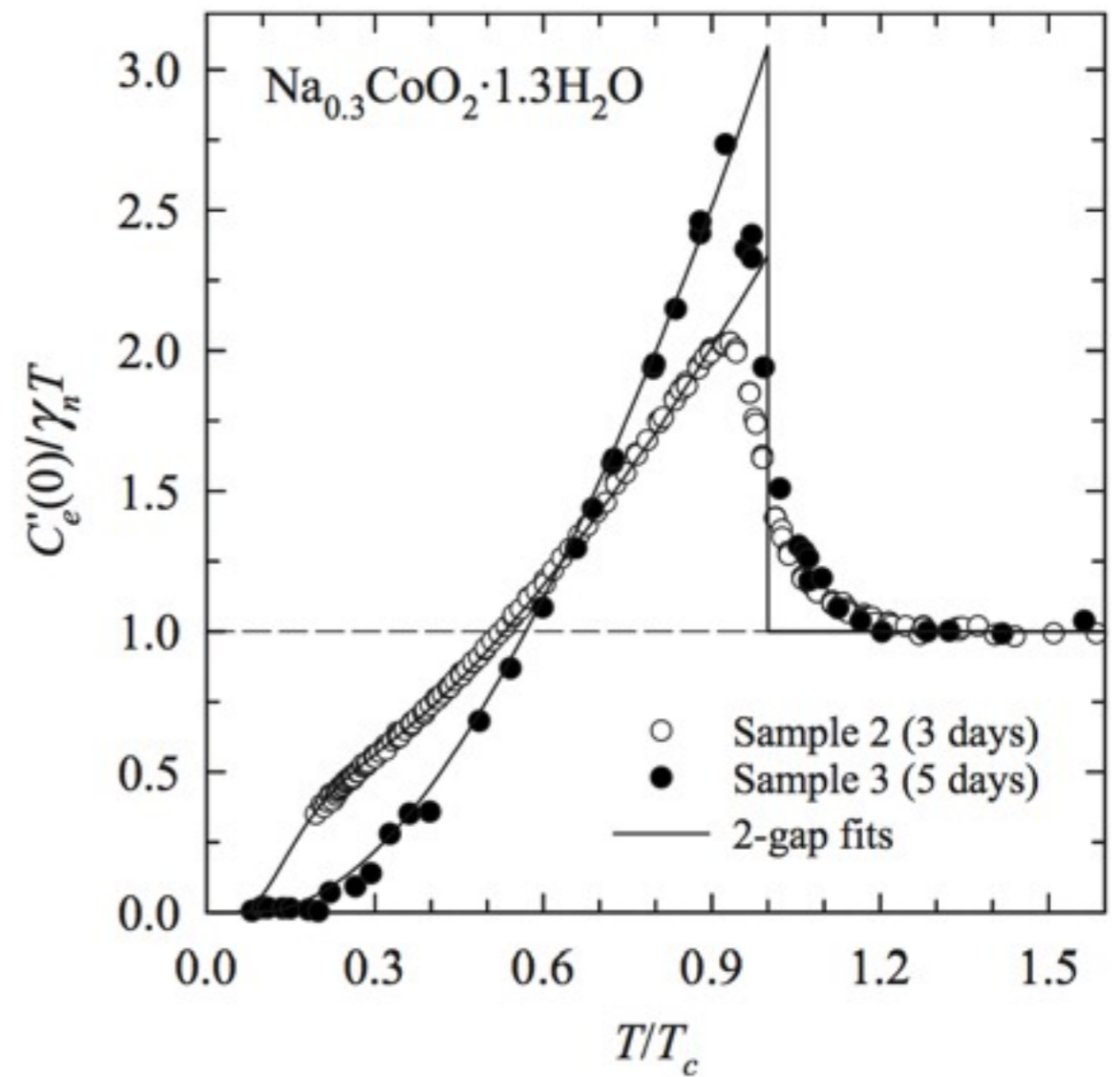


# Evidence at $x=0.3$ : Singlet, close-to-nodal superconductor

Knight shift: Lin group, PRB 2006



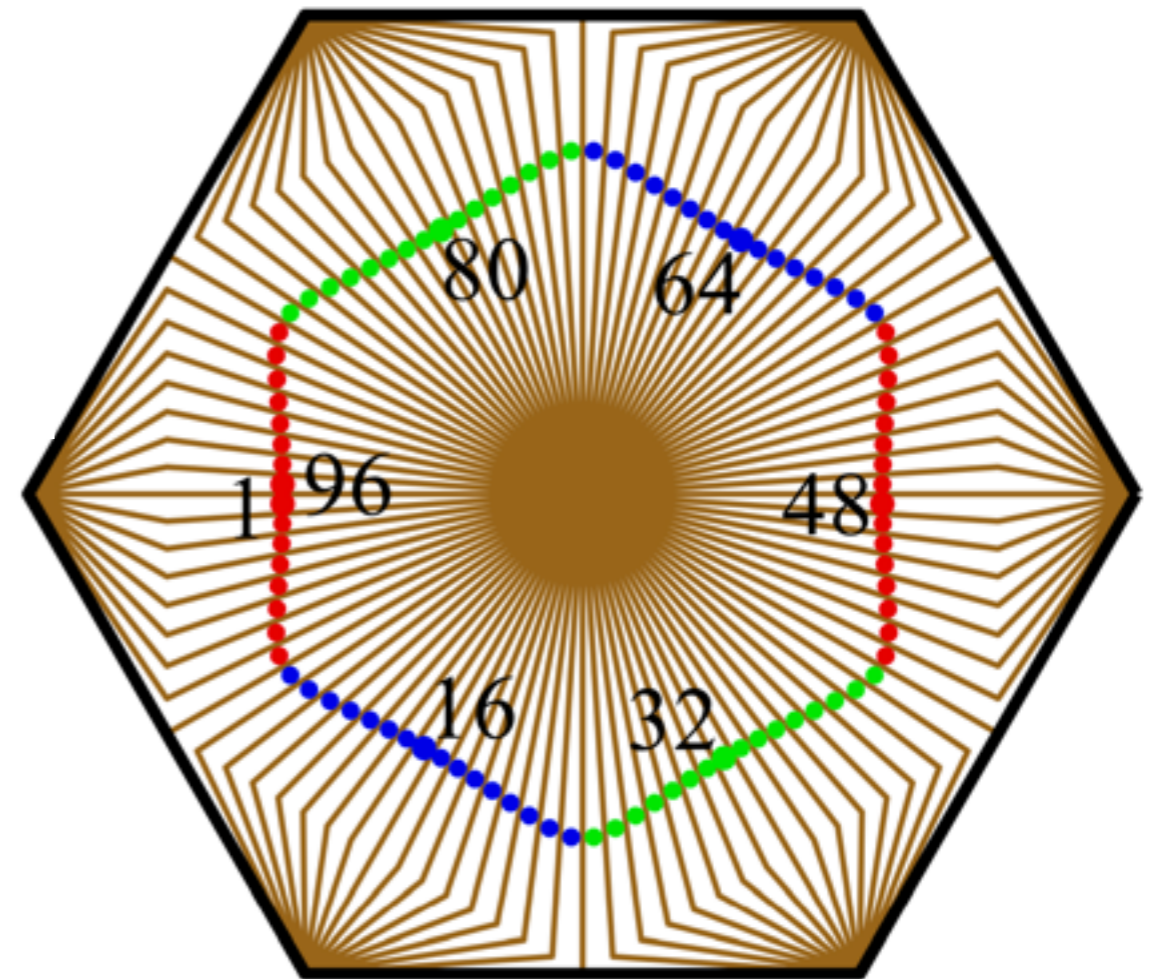
Specific heat: Cava group, PRB 2008



# Hexagonal d-wave superconductivity

$C_{6v}$	$E$	$C_2$	$2C_3$	$2C_6$	$3\sigma_v$	$3\sigma_d$
$A_1$	1	1	1	1	1	1
$A_2$	1	1	1	1	-1	-1
$B_1$	1	-1	1	-1	1	-1
$B_2$	1	-1	1	-1	-1	1
$E_1$	2	-2	-1	1	0	0
$E_2$	2	2	-1	-1	0	0

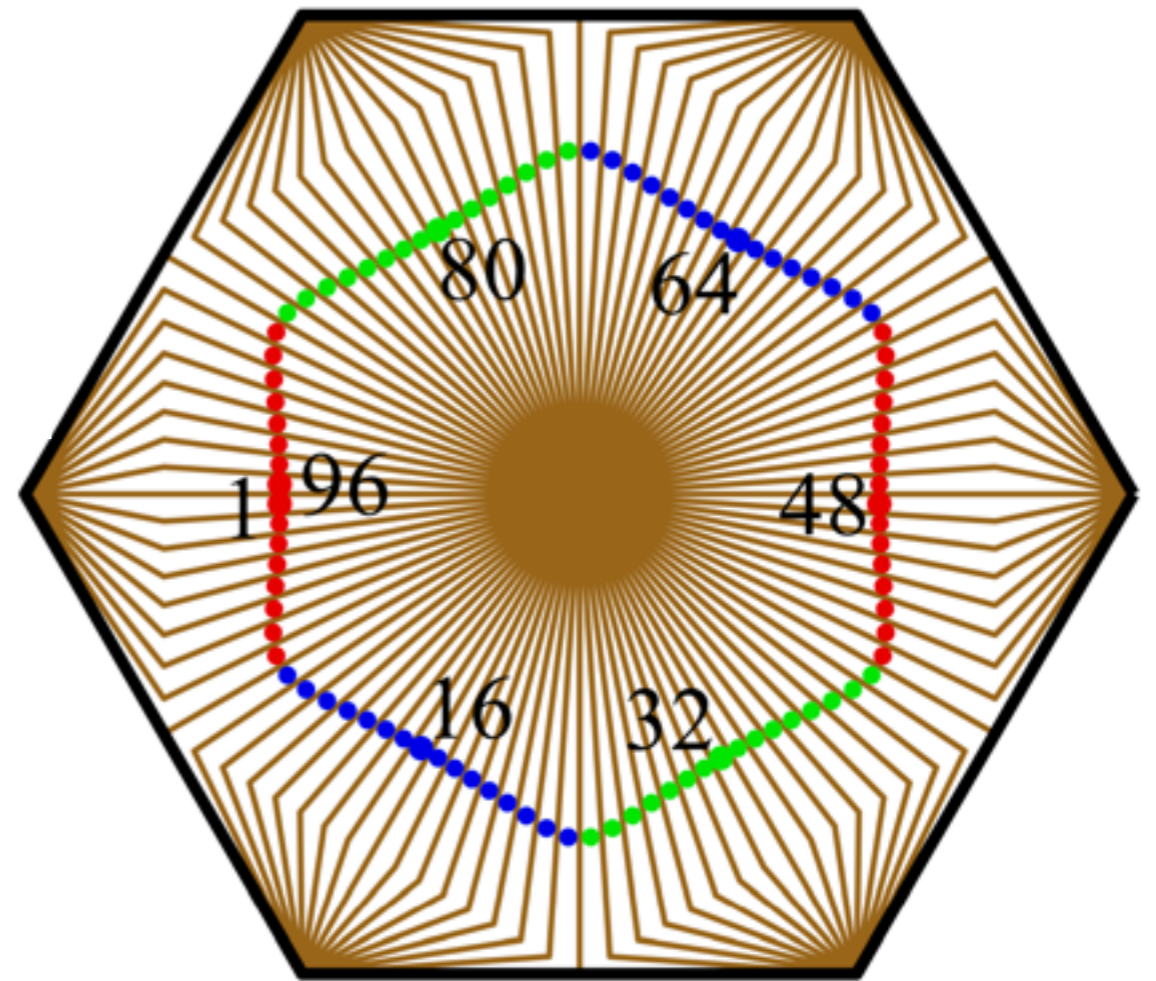
$$d_{x^2-y^2}/d_{xy}$$



# Hexagonal d-wave superconductivity

$C_{6v}$	$E$	$C_2$	$2C_3$	$2C_6$	$3\sigma_v$	$3\sigma_d$
$A_1$	1	1	1	1	1	1
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$E_1$	2	-2	-1	1	0	0
$E_2$	2	2	-1	-1	0	0

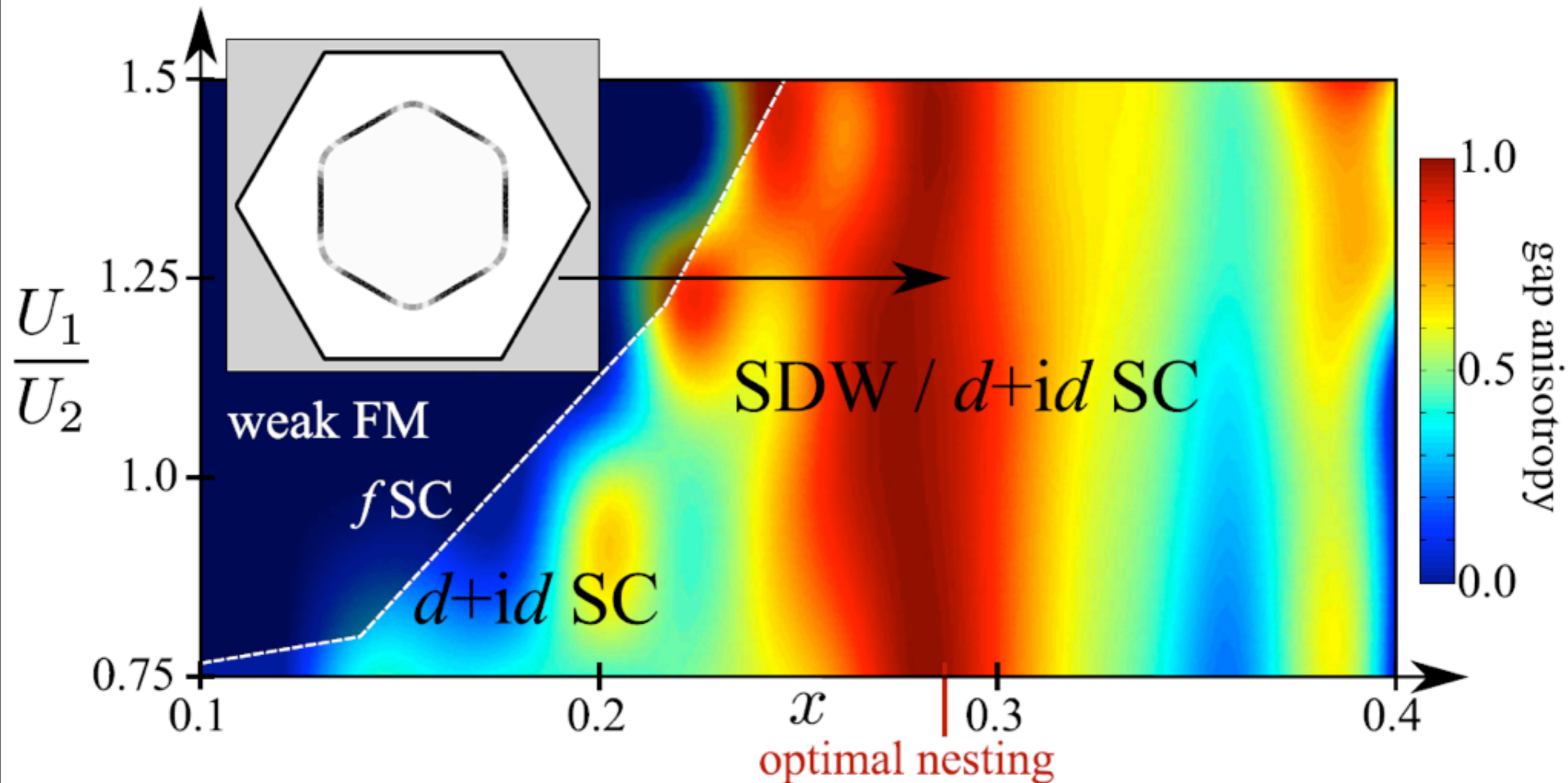
$$d_{x^2-y^2}/d_{xy}$$



The d-wave hexagonal point group representation is **two-dimensional**, implying a degeneracy at the instability level.

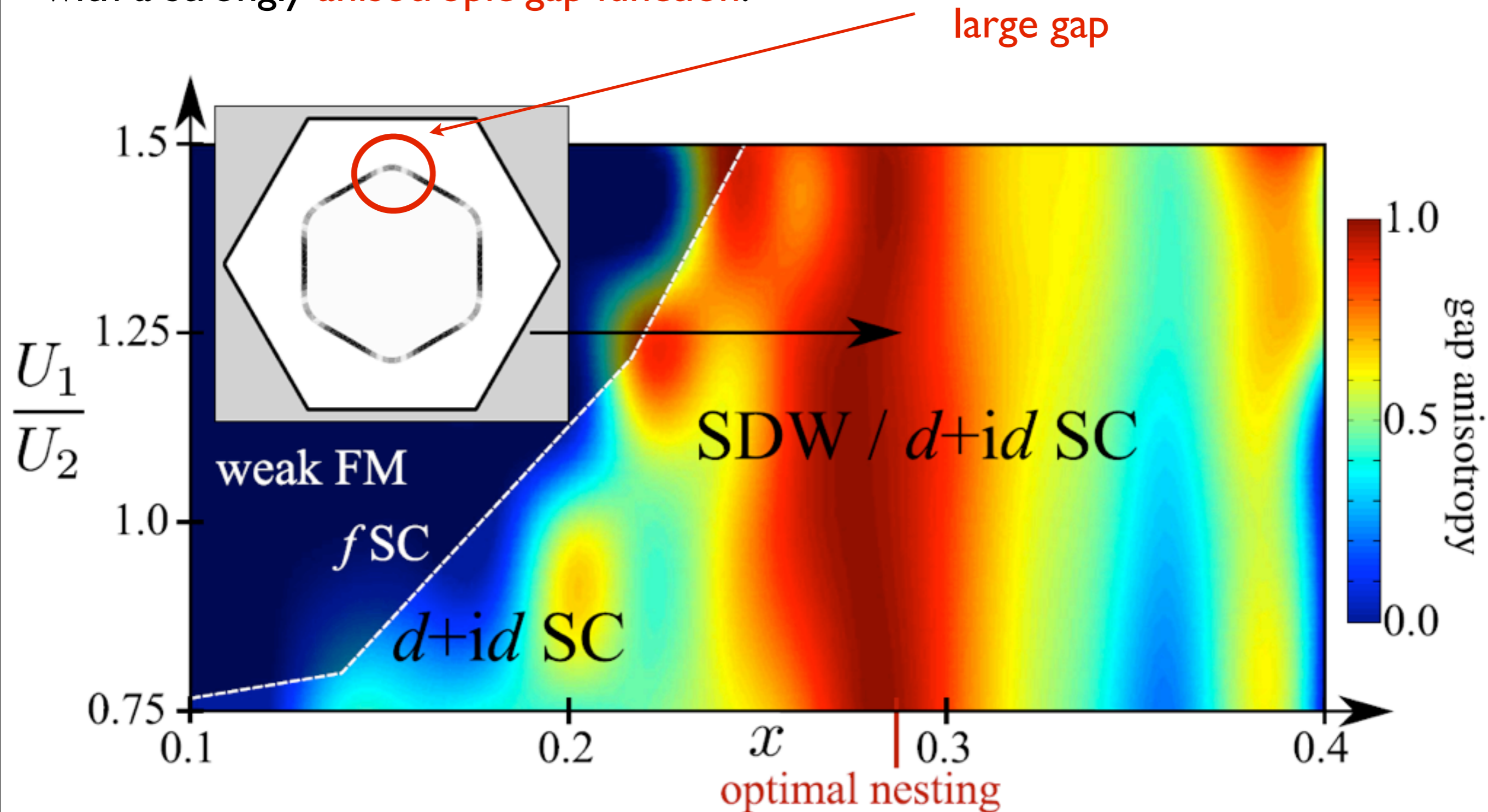
# FRG analysis: anisotropic d+id phase

The multi-orbital Hubbard model at  $x \sim 0.3$  yields a d+id-wave superconductor with a strongly **anisotropic gap function**.



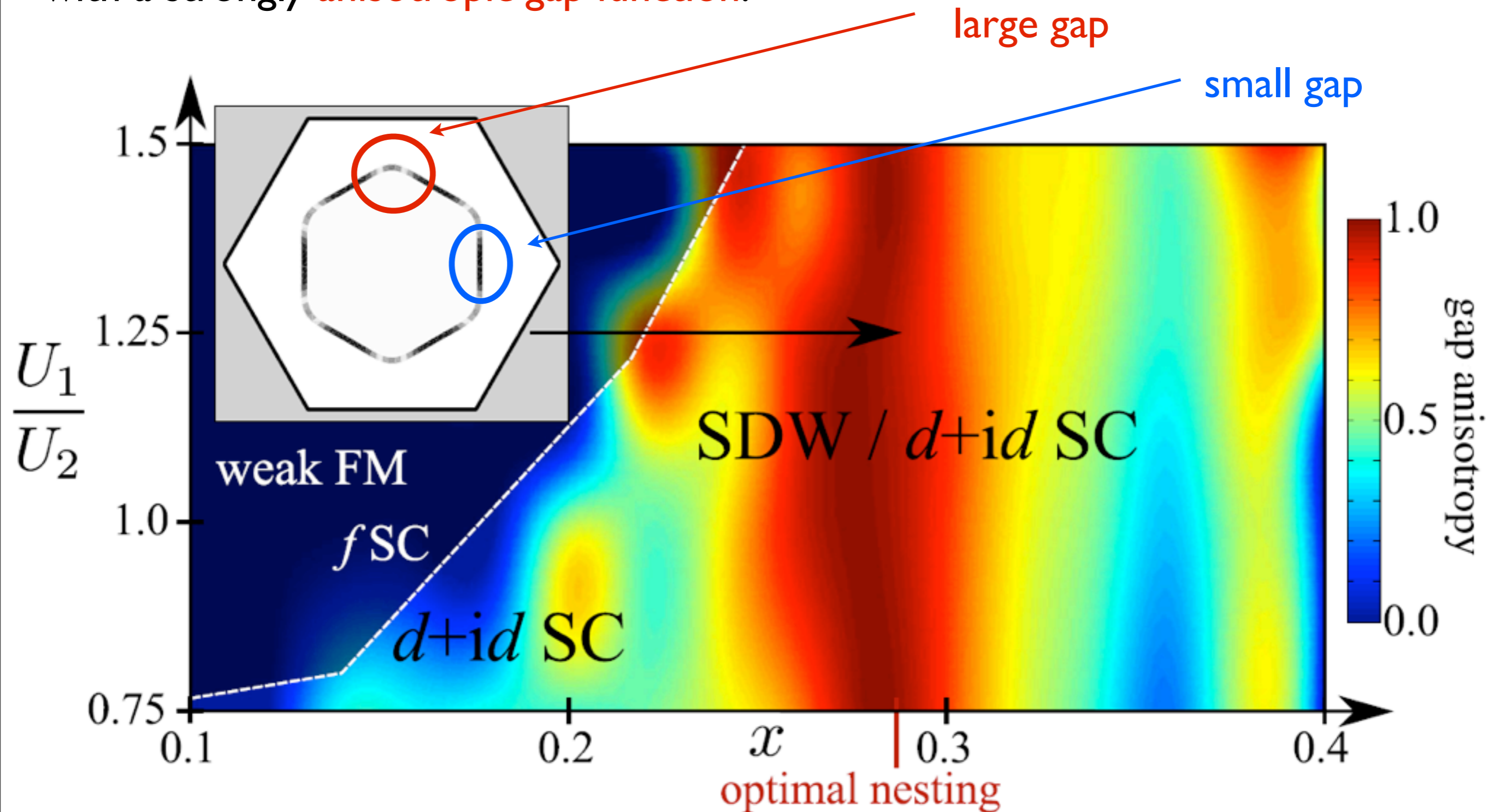
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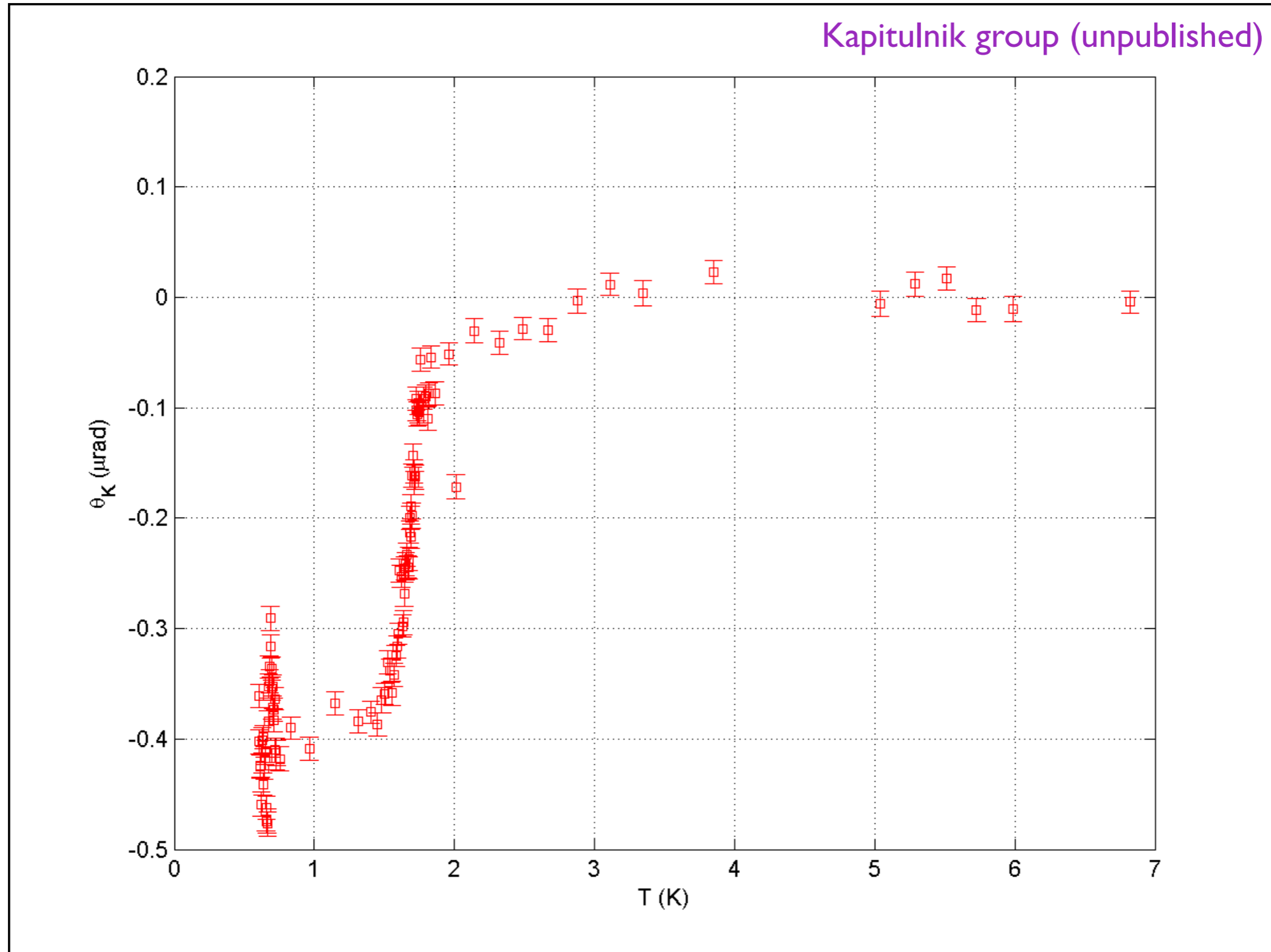


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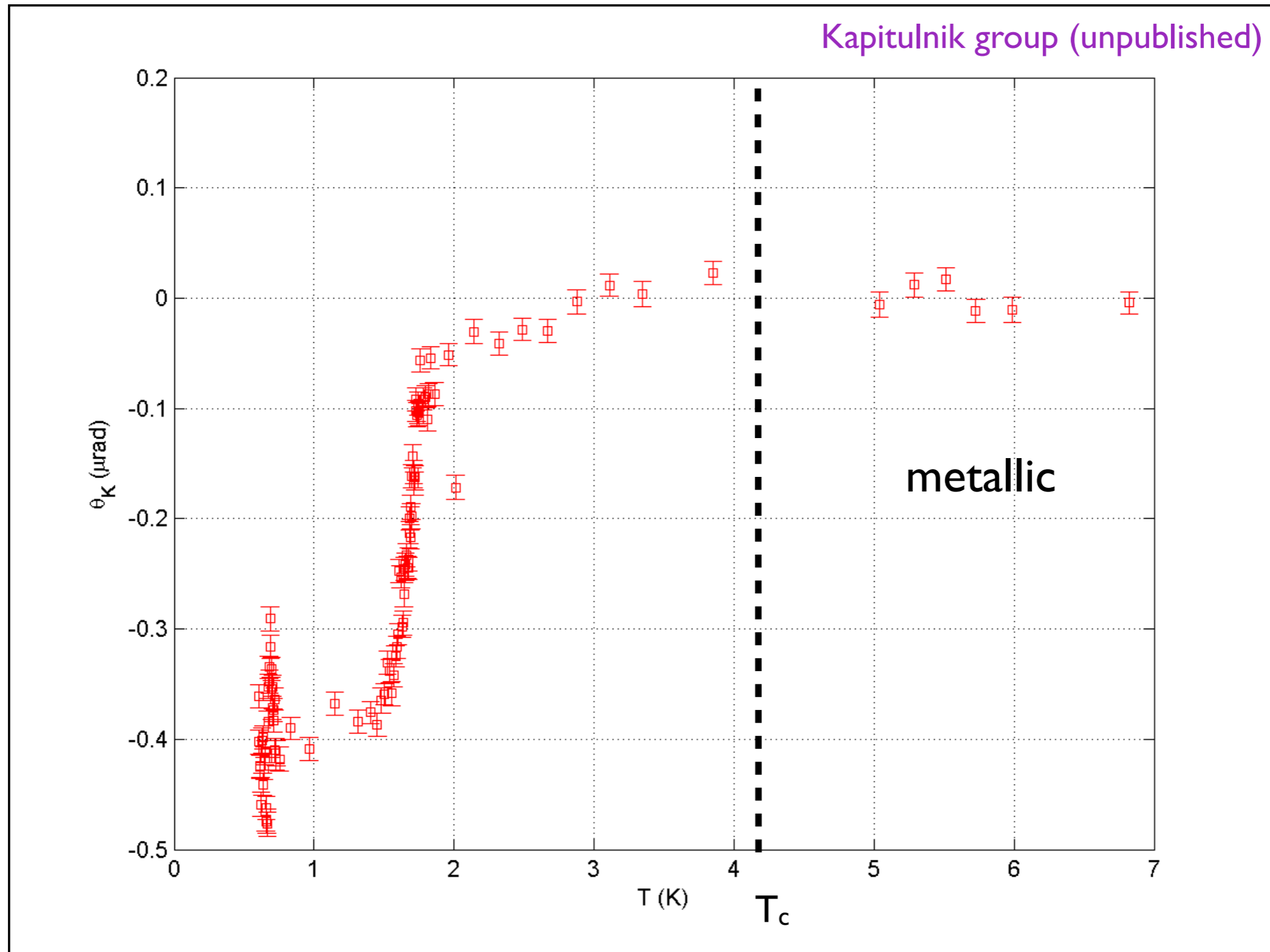


# T violation probed by Kerr angle

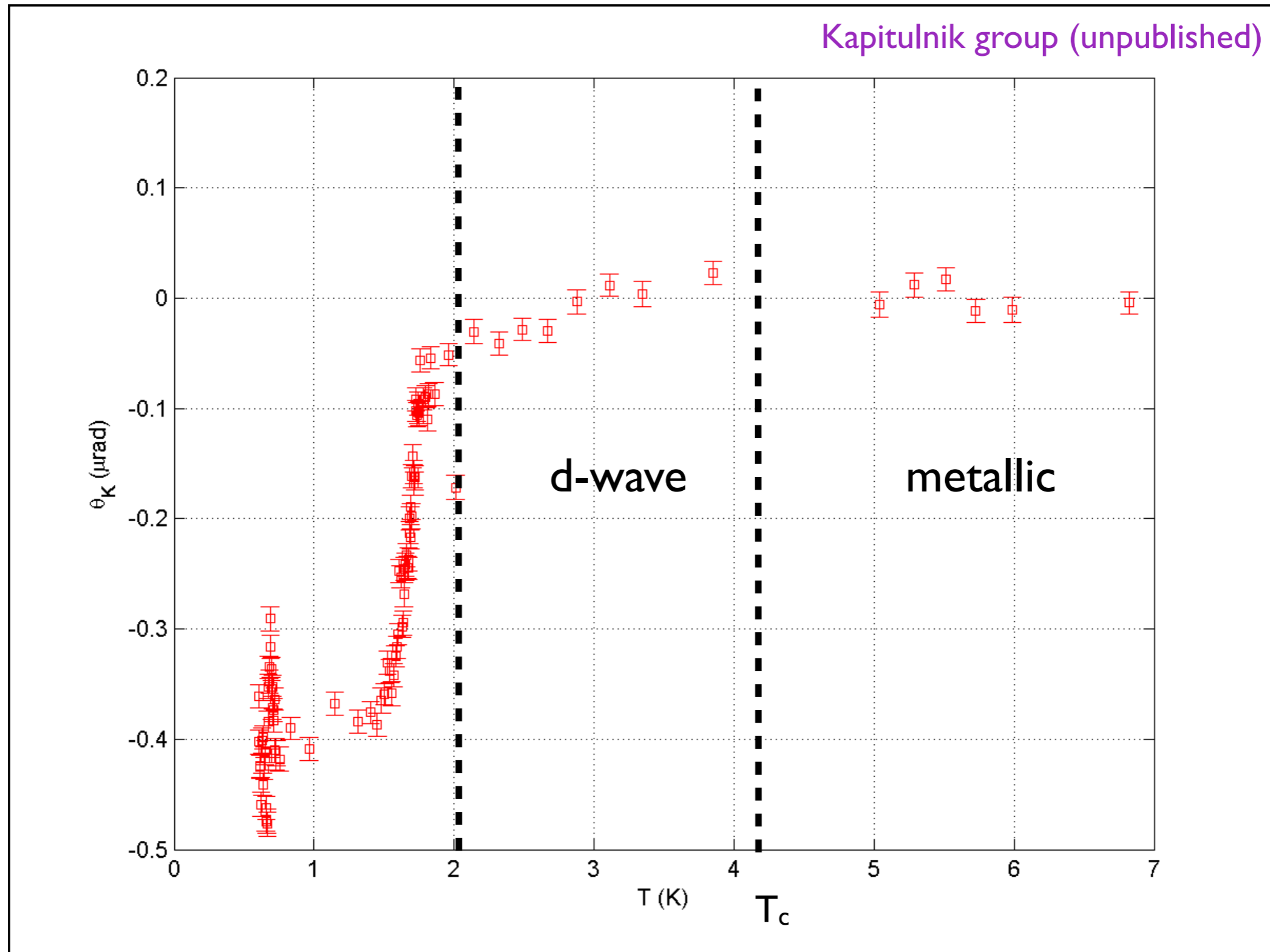




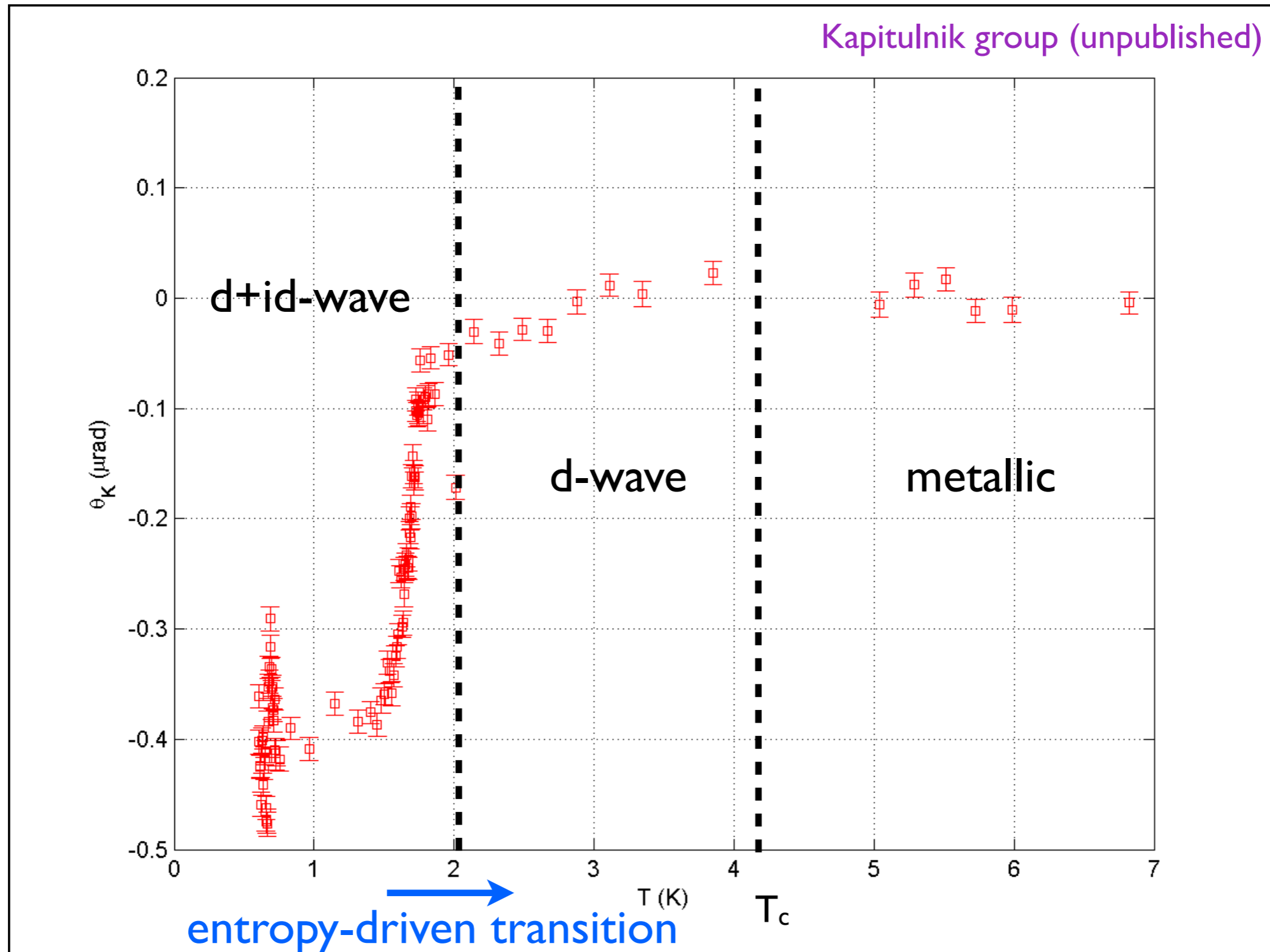
# T violation probed by Kerr angle



# T violation probed by Kerr angle



# T violation probed by Kerr angle



Future

# SrPtAs - a Weyl superconductor?

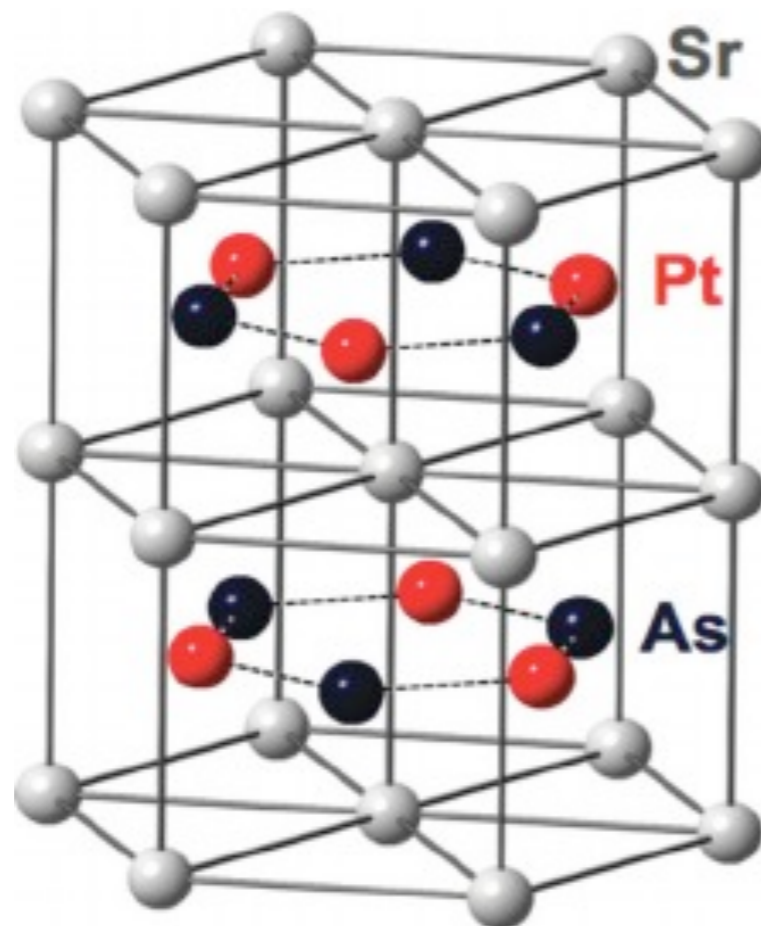
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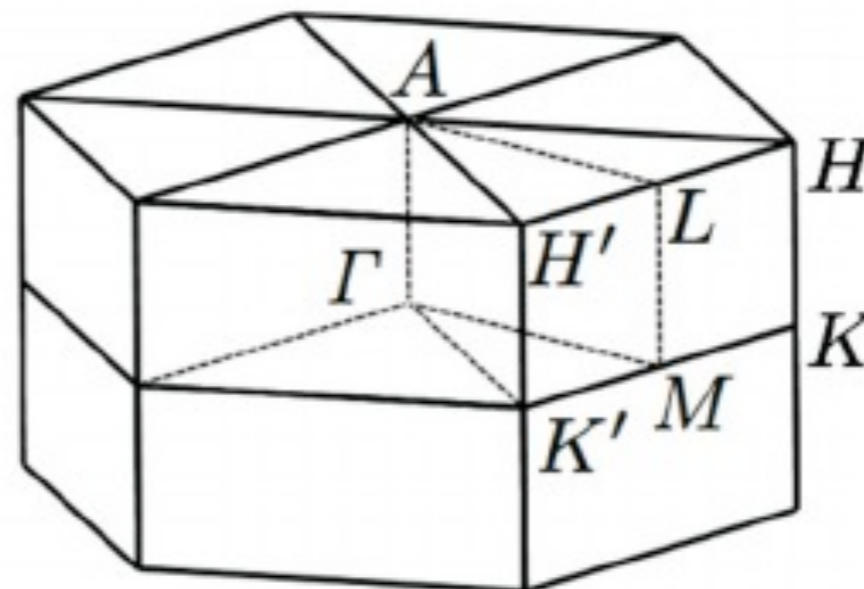


## Chiral *d*-wave superconductivity in SrPtAs

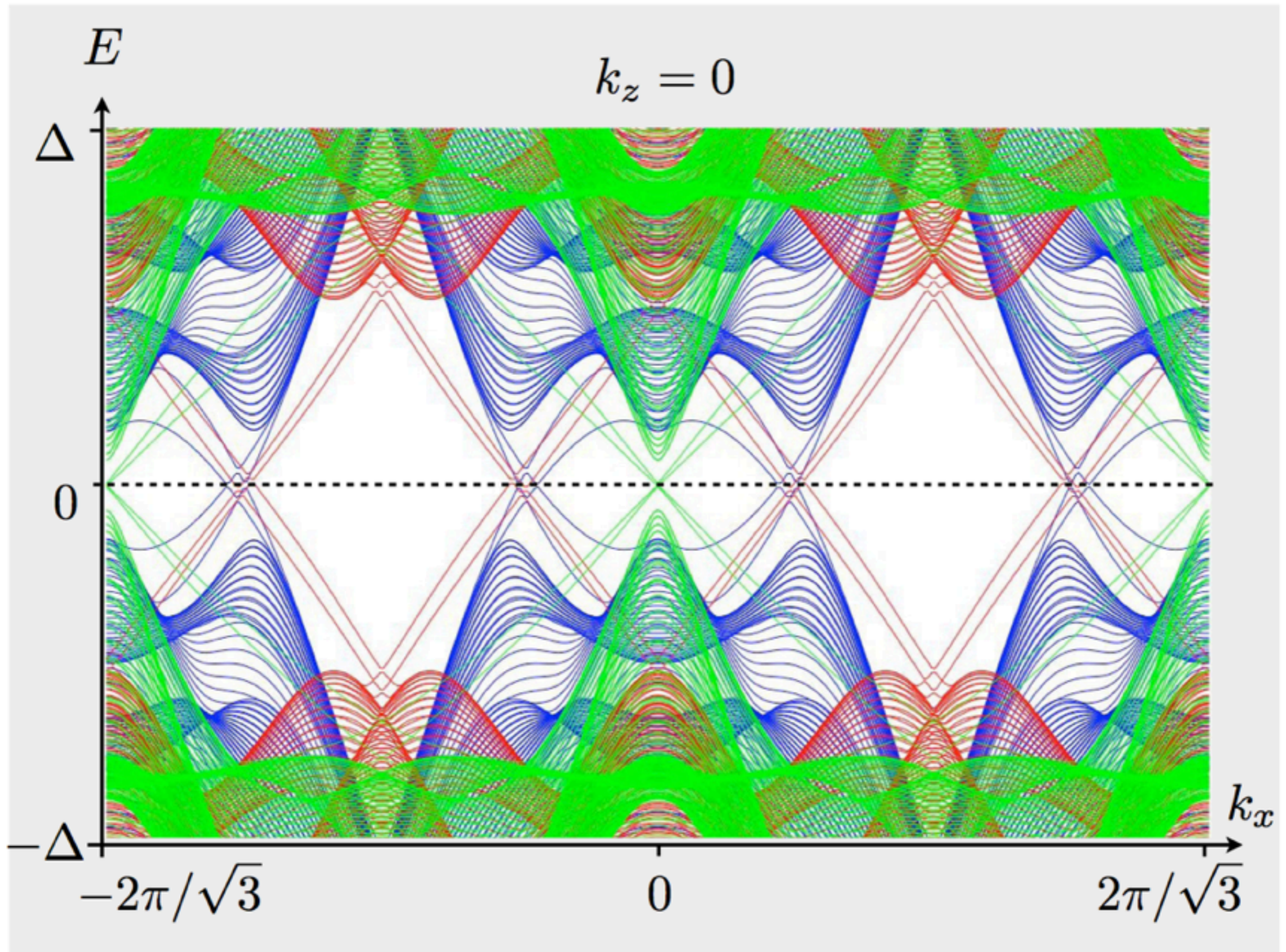
Mark H. Fischer,<sup>1,2</sup> Titus Neupert,<sup>3,4</sup> Christian Platt,<sup>5</sup> Andreas P. Schnyder,<sup>6</sup> Werner Hanke,<sup>5</sup> Jun Goryo,<sup>7</sup>  
Ronny Thomale,<sup>5</sup> and Manfred Sigrist<sup>4</sup>



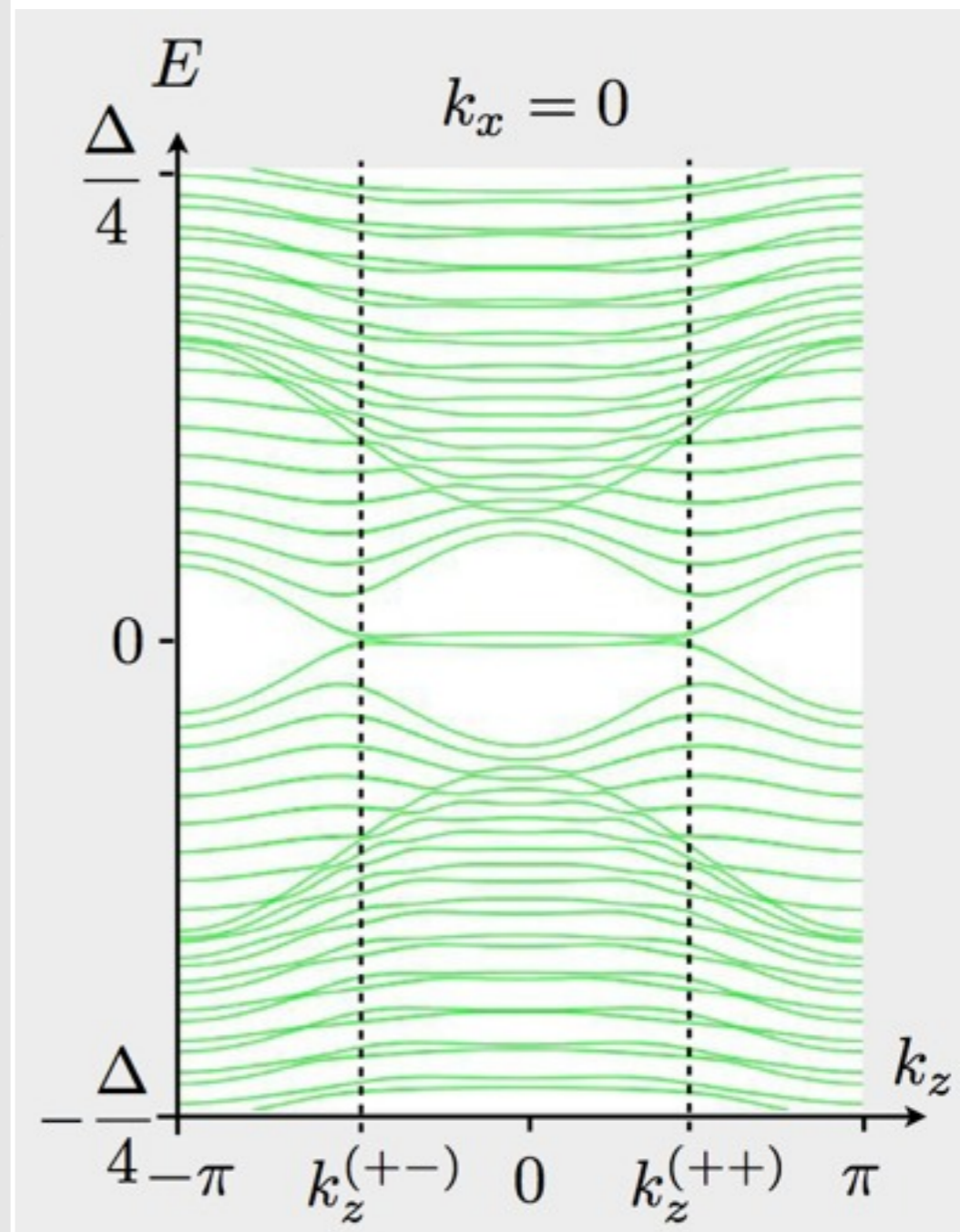
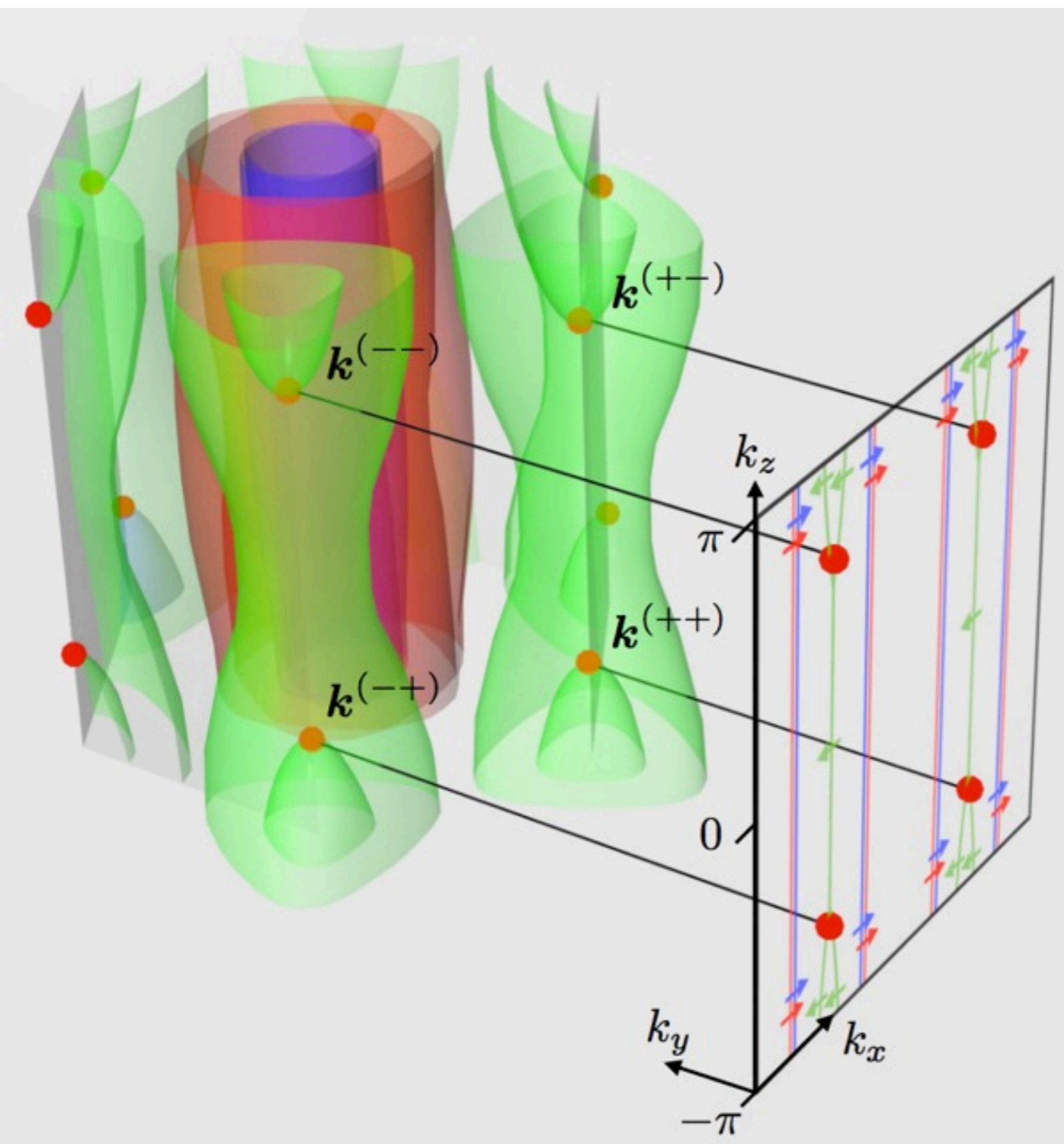
(a)



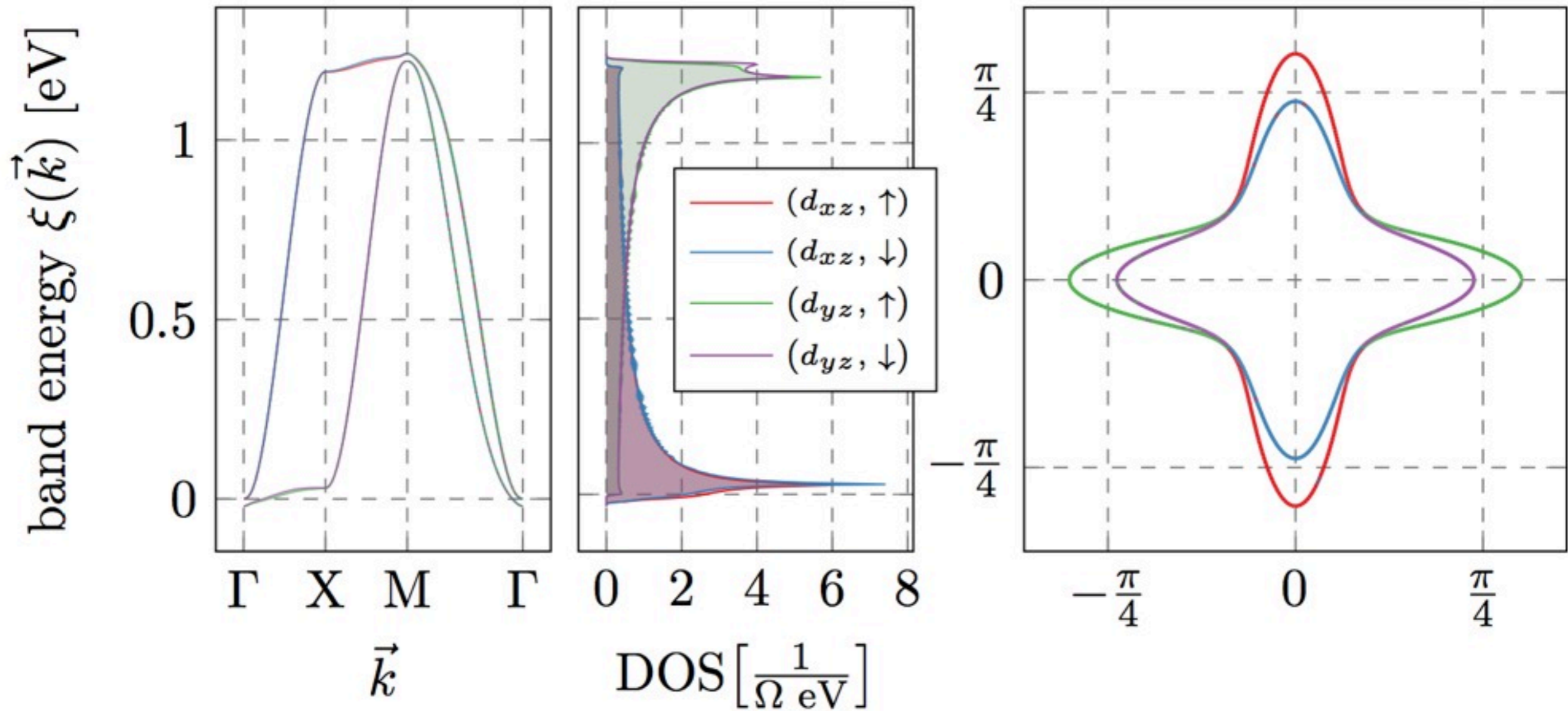
# Bogoliubov spectrum



# SrPtAs: First prediction of a Weyl superconductor

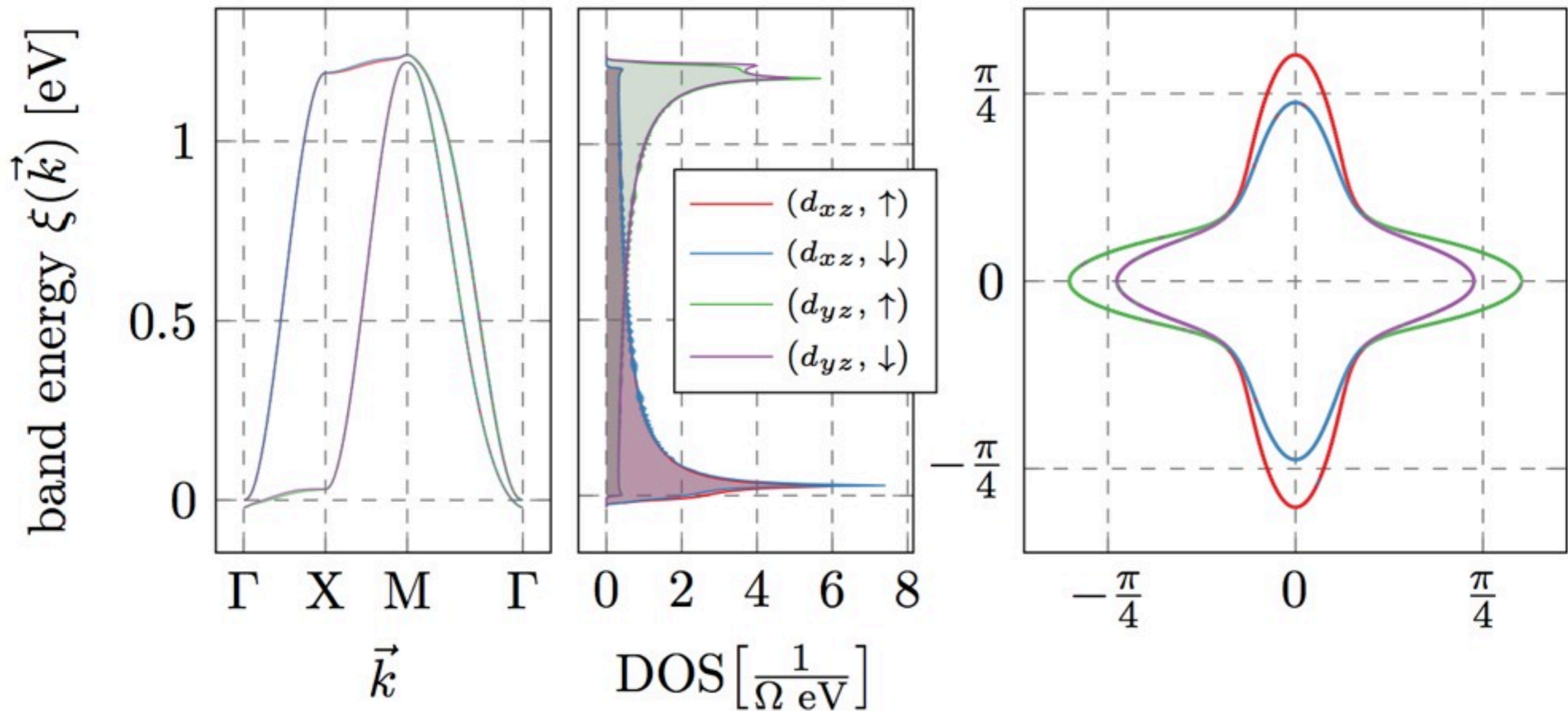


# LAO/STO - tight binding setup



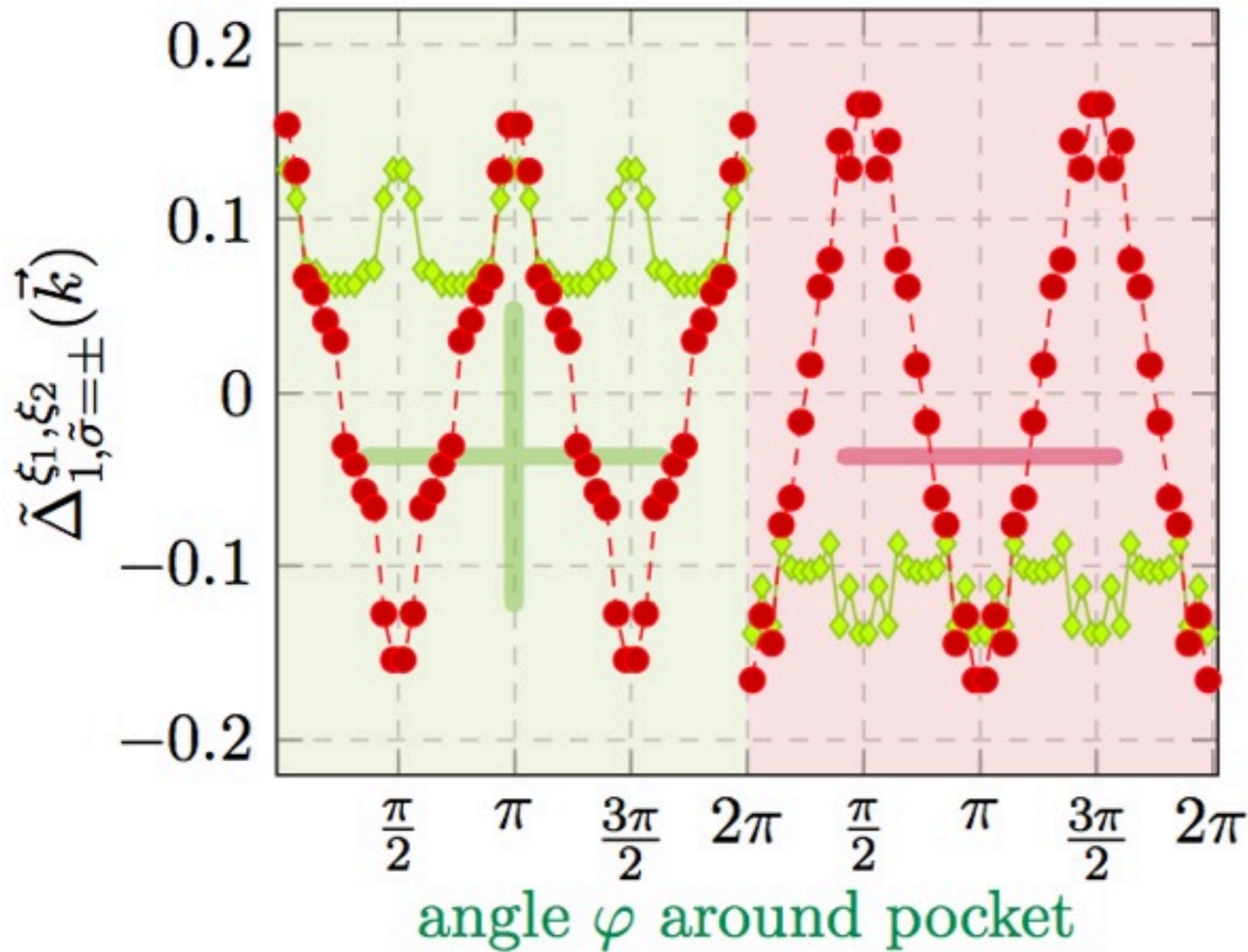


# LAO/STO - tight binding setup

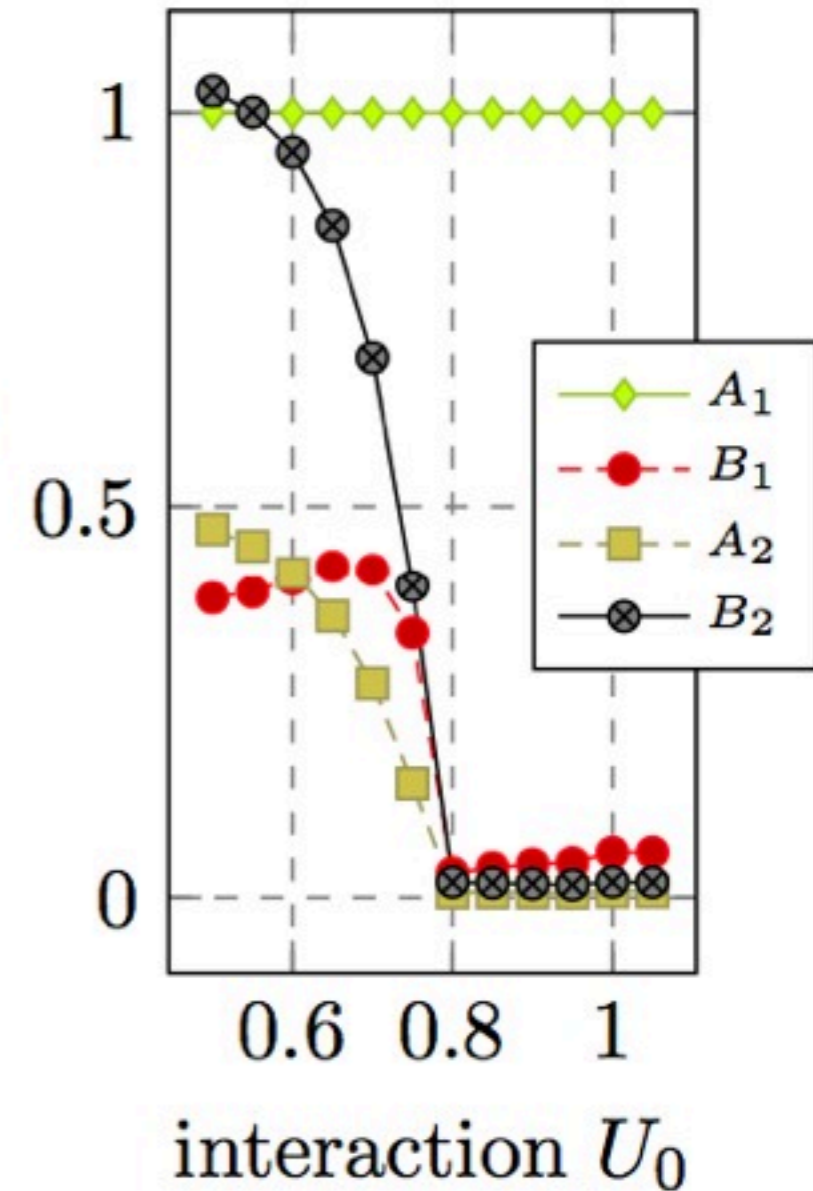


New methodological step: Spin-orbit FRG with complete **double group implementation**.

# LAO/STO - s-wave vs. d-wave



rel. eigenval.  $\frac{\xi^n}{\xi_{A_1}}$  Cooper ch.



# Research team

