Electromagnetic transition form factors of baryons in the space-like momentum region

Reinhard Alkofer[a],
Christian Fischer[b] and Helios Sanchis Alepuz[a]

[a] Institute of Physics, University of Graz
[b] Institute of Theoretical Physics, University of Gießen

From correlation functions to QCD phenomenology
April 3-6, 2018, Physikzentrum Bad Honnef, Germany

- *H. Sanchis Alepuz, C.S. Fischer, RA, Eur.Phys.J.* **A54** *(2018) 41 [arXiv:1707.08463].*

Review:

- *G. Eichmann, H. Sanchis Alepuz, R. Williams, C. S. Fischer, RA, Prog. Part. Nucl. Phys.* ***91*** *(2016) 1 [arXiv:1606.09602].*

From correlation functions to QCD phenomenology

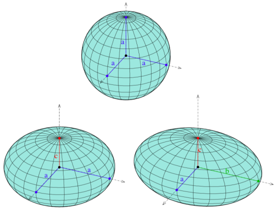
QCD correlation functions:
Input into hadron phenomenology via **QCD bound state eqs.**.

- Bethe-Salpeter equations for **mesons**
  form factors, decays, reactions, ...
- covariant Faddeev equations for **baryons**
  form factors, Compton scattering, meson production, ...

Why transition form factors?
E.g., sensitivity to shape!

Deformed quantum object:
Superposition of all orientations

Detect deviation from sphericity:
Transition between deformed states



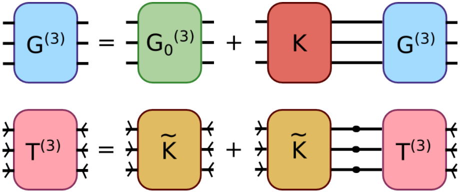
Baryons non-spherical? Why?

Outline

Relativistic Three-Fermion Bound State Equations

cf. talks by Eichmann, Roberts, Williams . . .

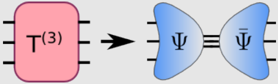
Dyson-Schwinger eq. for 6-point fct. $\Longrightarrow$ 3-body bound state eq.:



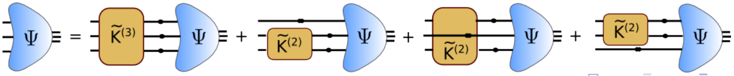
BOUND STATE:

Pole in $G^{(3)}$
or (equiv.) for $P^2 = -M_B^2$
Pole in $T^{(3)}$

bound state amplitudes:

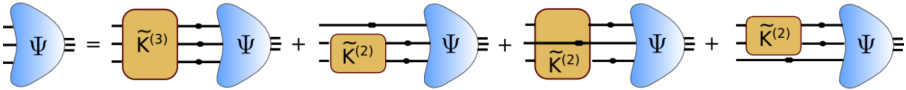


covariant 3-body bound state eq. (*cf.*, Bethe-Salpeter for 2-body BS)**:**



Relativistic three-fermion bound state equations

3-body bound state eq.:



NB: With 3-particle-irreducible interactions $\tilde{K}^{(3)}$ neglected:
Poincaré-covariant Faddeev equation.

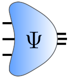
Elements needed for bound state equation:

- Tensor structures (color, flavor, Lorentz / Dirac) of the BS ampl.
- Full quark propagators for *complex* arguments
- Interaction kernels $K_{2,3}$

Needed for coupling to e.m. current:

- Full quark-photon vertex

Structure of Baryonic Bound State Amplitudes



$\sim \langle 0|q_\alpha q_\beta q_\gamma|B_\mathcal{I}\rangle \propto \Psi_{\alpha\beta\delta\mathcal{I}}$ (with multi-indices $\alpha = \{x, D, c, f, \ldots\}$)

and $\mathcal{I}$ baryon (multi-)index $\Longrightarrow$ baryon quantum numbers

C. Carimalo, J. Math. Phys. **34** (1993) 4930.

Comparison to mesonic BS amplitudes $\langle 0|q_\alpha \bar{q}_\beta|M_\mathcal{I}\rangle \propto \Phi_{\alpha\beta\mathcal{I}}$:

- scalar and pseudoscalar mesons: 4 tensor structures each
- vector and axialvector mesons: 12 tensor struct. each, 8 transv.
- tensor and higher spin mesons: 8 transverse struct. each

which are functions of two Lorentz-invariant variables.

C. H. Llewellyn-Smith, Annals Phys. **53** (1969) 521.

Structure of Baryonic Bound State Amplitudes

Facts about the decomposition:

- Independent of any truncation of the bound state equation.
- Only Poincaré covariance and parity invariance exploited.
- It includes all possible internal spin and orbital angular momenta.
- For positive-parity, positive-energy (particle) baryons it consists of

spin-$\frac{1}{2}$ particle: 64 elements

	# elements
s-wave	8
p-wave	36
d-wave	20

G. Eichmann et al., PRL 104 (2010) 201601

spin-$\frac{3}{2}$ particle: 128 elements

s-wave	4
p-wave	36
d-wave	60
f-wave	28

H. Sanchis Alepuz et al. PRD 84 (2011) 096003

Note: Four-spinor nature of baryon amplitudes, in used Dirac basis, e.g., upper components s-wave, lower components p-waves!

lower components ↔ antiparticles

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lower components $\leftrightarrow$ antiparticles

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H. Sanchis Alepuz et al. PRD 84 (2011) 096003

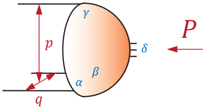
Each tensor structure is multiplied by a function of five Lorentz-invariant variables!

Structure of Baryonic Bound State Amplitudes

s	l	T_{ij}	
1/2	0	$\mathbb{1}\otimes\mathbb{1}$	**s waves (8)**
1/2	0	$\gamma^\mu_T\otimes\gamma^\mu_T$	
1/2	1	$\mathbb{1}\otimes\frac{1}{2}[\,\not{p},\not{q}\,]$	**p waves (36)**
1/2	1	$\mathbb{1}\otimes\not{p}$	
1/2	1	$\mathbb{1}\otimes\not{q}$	
1/2	1	$\gamma^\mu_T\otimes\gamma^\mu_T\,\frac{1}{2}[\,\not{p},\not{q}\,]$	
1/2	1	$\gamma^\mu_T\otimes\gamma^\mu_T\,\not{p}$	
1/2	1	$\gamma^\mu_T\otimes\gamma^\mu_T\,\not{q}$	
3/2	1	$3\,(\not{p}\otimes\not{q}-\not{q}\otimes\not{p})-\gamma^\mu_T\otimes\gamma^\mu_T\,[\,\not{p},\not{q}\,]$	
3/2	1	$3\,\not{p}\otimes\mathbb{1}-\gamma^\mu_T\otimes\gamma^\mu_T\,\not{p}$	
3/2	1	$3\,\not{q}\otimes\mathbb{1}-\gamma^\mu_T\otimes\gamma^\mu_T\,\not{q}$	
3/2	2	$3\,\not{p}\otimes\not{p}-\gamma^\mu_T\otimes\gamma^\mu_T$	**d waves (20)**
3/2	2	$\not{p}\otimes\not{p}+2\,\not{q}\otimes\not{q}-\gamma^\mu_T\otimes\gamma^\mu_T$	
3/2	2	$\not{p}\otimes\not{q}+\not{q}\otimes\not{p}$	
3/2	2	$\not{q}\otimes[\,\not{q},\not{p}\,]-\frac{1}{2}\gamma^\mu_T\otimes[\,\gamma^\mu_T,\not{p}\,]$	
3/2	2	$\not{p}\otimes[\,\not{p},\not{q}\,]-\frac{1}{2}\gamma^\mu_T\otimes[\,\gamma^\mu_T,\not{q}\,]$	

$$\chi(x_1,x_2,x_3)=\langle 0|\,T\,\psi(x_1)\,\psi(x_2)\,\psi(x_3)\,|N\rangle$$

Momentum space:
Jacobi coordinates p, q, P
⇒ 5 Lorentz invariants
⇒ 64 Dirac basis elements



$$\chi(p,q,P)=\sum_k \; f_k(p^2,\,q^2,\,p\cdot q,\,p\cdot P,\,q\cdot P) \quad \textbf{Momentum}$$
$$\tau^k_{\alpha\beta\gamma\delta}(p,q,P) \quad \textbf{Dirac} \quad \otimes\ \textbf{Flavor}\ \otimes\ \textbf{Color}$$

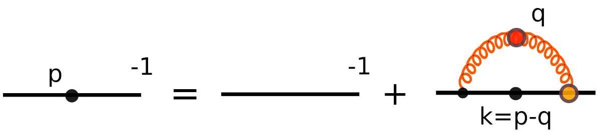
Complete, orthogonal **Dirac tensor basis**
(partial-wave decomposition in nucleon rest frame):
Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)

$$T_{ij}\,(\Lambda_\pm\gamma_5 C\otimes\Lambda_+)$$
$$(\gamma_5\otimes\gamma_5)\,T_{ij}\,(\Lambda_\pm\gamma_5 C\otimes\Lambda_+)$$

$$(A\otimes B)_{\alpha\beta\gamma\delta}=A_{\alpha\beta}\,B_{\gamma\delta}$$

Quark Propagator and Rainbow Truncation

Dyson-Schwinger eq. for Quark Propagator:



$$S^{-1}(p) = Z_2 S_0^{-1} + g^2 Z_{1f} \int \frac{d^4k}{(2\pi)^4} \gamma^\mu S(k) \Gamma^\nu(k,p;q) D_{\mu\nu}(q)$$

Rainbow truncation

Projection onto tree-level tensor γ_μ, restrict momentum dependence

$$Z_{1f}\frac{g^2}{4\pi} D_{\mu\nu}(q)\Gamma_\nu(k,p;q) \to \begin{cases} Z_{1f}\frac{g^2}{4\pi} T_{\mu\nu}(q) \frac{Z(q^2)}{q^2}\left(Z_{1f} + \Lambda(q^2)\right)\gamma_\nu \\ =: Z_2^2 T_{\mu\nu}(q)\frac{\alpha_{eff}(q^2)}{q^2}\gamma_\nu \end{cases}$$

Interaction Kernels and Rainbow-Ladder Truncation

- Truncation of the quark-gluon vertex in the quark DSE.
- The BSE interaction kernel must be truncated accordingly.
- **Physical requirement: Chiral symmetry**
  axial WT id. relates quark DSE and bound-state eq. kernel.

Ladder truncation

$q\bar{q}$ kernel compatible with rainbow truncation and axial WT id.:

$$K^{q\bar{q}} = 4\pi Z_2^2 \frac{\alpha_{eff}(q^2)}{q^2} T_{\mu\nu}(q)\gamma^\mu \otimes \gamma^\nu$$

Together constitute the DSE/BSE **Rainbow-Ladder truncation**.

Note: the truncation can and should be systematically improved!

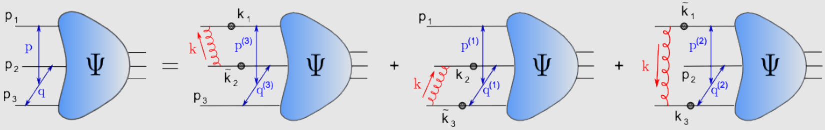
Interaction Kernels and Rainbow-Ladder Truncation

Rainbow-Ladder truncated three-body BSE:

The three-body irreducible kernel $K^{(3)}$ is neglected
(Faddeev approximation).

- Quark-quark interaction $K^{(2)}$: **same as quark-antiquark truncated kernel.** (!Different color factor!)

Rainbow-Ladder truncated **covariant Faddeev equation**



Interaction Kernels and Rainbow-Ladder Truncation

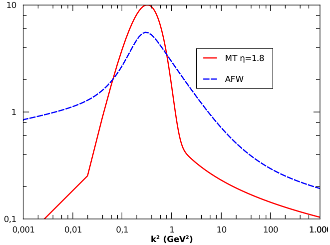
Models for effective interaction:

Maris-Tandy model (Maris & Tandy PRC60 1999)

$$\alpha(k^2) = \alpha_{IR}(k^2; \Lambda, \eta) + \alpha_{UV}(k^2)$$

- Purely phenomenological model.
- Λ fitted to f_π.
- Ground-state pseudoscalar properties *almost* insensitive to η around 1.8

Describes very succesfully hadron properties.



DSE motivated model (R.A.,C.S. Fischer,R. Williams EPJ A38 2008)

$$\alpha(k^2; \Lambda_S, \Lambda_B, \Lambda_{IR}, \Lambda_{YM})$$

- DSE-based in the deep IR.
- Designed to give correct masses of π, ρ and η' *($U_A(1)$ anomaly!)*.
- 4 energy scales! Fitted to π, K and η'.

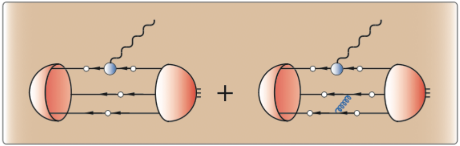
Interaction Kernels and Rainbow-Ladder Truncation

Note: The resulting qq-interaction is chirality-conserving, flavour-blind and current-quark mass independent.

Beyond Rainbow-Ladder

- "Corrections beyond-RL" refers to corrections to the effective coupling but also to additional structures beyond vector-vector interaction.
- They can induce a different momentum dependence of the interaction.
- They can also **induce a quark-mass and quark-flavour dependence of the interaction**
- Question: how important are beyond-RL effects?

Electromagnetic current in the three-body approach:



by "gauging of equations"

M. Oettel, M. Pichowsky and L. von Smekal, Eur. Phys. J. A **8** (2000) 251;
A. N. Kvinikhidze and B. Blankleider, Phys. Rev. C **60** (1999) 044003.

Impulse appr. + Coupling to spectator q + Coupling to 2-q kernel (not present in RL appr.) + Coupling to 3-q kernel (not present in Faddeev appr.)

Additional Input: Quark-Photon Vertex

cf. talk by Sternbeck

Coupling of E.M. Current and Quark-Photon Vertex

cf. talk by Sternbeck

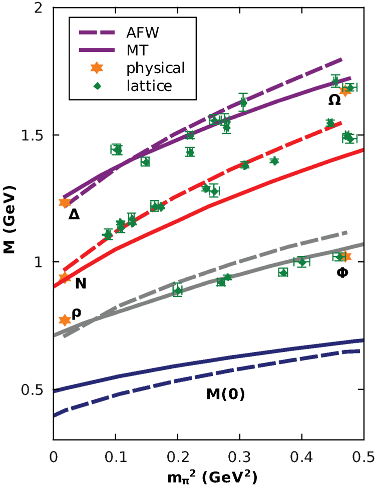
Quark-Photon Vertex:

- Vector WT id. determines vertex up to purely transverse parts: "Gauge" part (Ball-Chiu vertex) completely specified by dressed quark propagator.
- Can be straightforwardly calculated in Rainbow-Ladder appr.:
  - important for renormalizibility (Curtis-Pennington term),
  - anomalous magnetic moment,
  - contains ρ meson pole!

The latter property is important to obtain the correct physics!

All elements specified to calculate baryon amplitudes and properties:
Use computer with sufficient RAM ($\sim$ tens of GB) and run for a few hours . . .

Some Selected Earlier Results



PoS QNP2012 (2012) 112

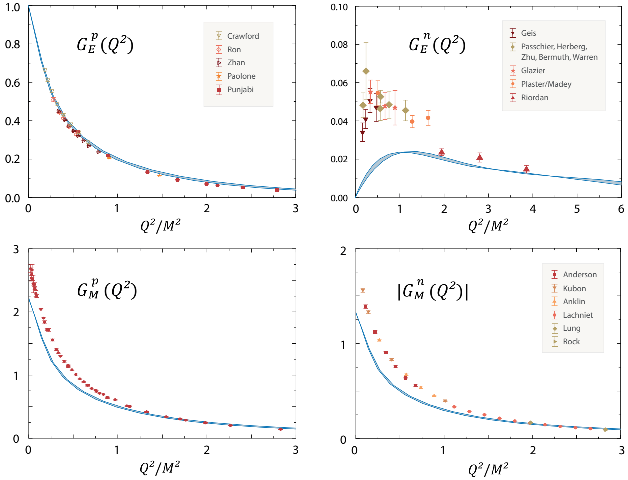
- Both models designed to reproduce correctly DχSB and pion properties within RL. **They capture beyond-RL effects at this quark-mass.**
- This behaviour extends to other light states (ρ, N, Δ), one gets a good description.
- Both interactions similar at intermediate momentum region: **$\sim$ 0.5 − 1 GeV is the relevant momentum region for DχSB & ground-state hadron props.**
- Slight differences at larger current masses, however, **qualitative model indep.**

Nucleon electromagnetic form factors

Nucleon em. FFs
vs. momentum transfer
Eichmann, PRD 84 (2011)

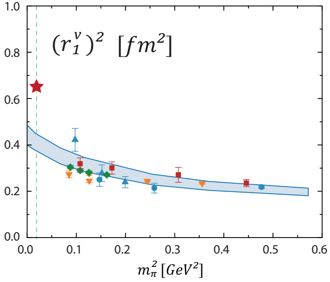
- Good agreement with recent **data** at large Q^2
- Good agreement with **lattice** at large quark masses
- **Missing pion cloud** below ~2 GeV^2, in chiral region

~ **nucleon quark core** without pion effects



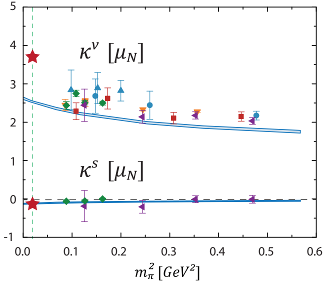
Nucleon electromagnetic form factors

Nucleon charge radii:
isovector (p-n) Dirac (F1) radius



- **Pion-cloud effects** missing in chiral region (⇒ divergence!), agreement with lattice at larger quark masses.

Nucleon magnetic moments:
isovector (p-n), isoscalar (p+n)



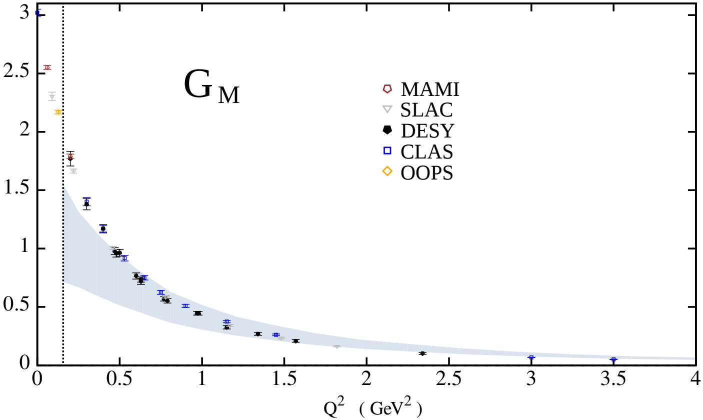
Eichmann, PRD 84 (2011)

- **But:** pion-cloud **cancels** in κ^s ⇔ **quark core**

Exp: $\kappa^s = -0.12$
Calc: $\kappa^s = -0.12(1)$ **!!**

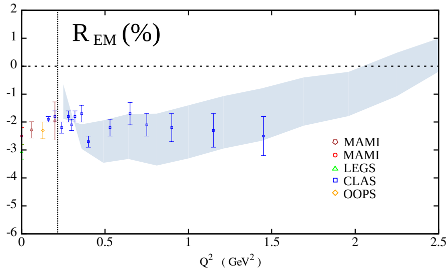
$\Delta \rightarrow N\gamma$ Electromagnetic Transition Form Factors

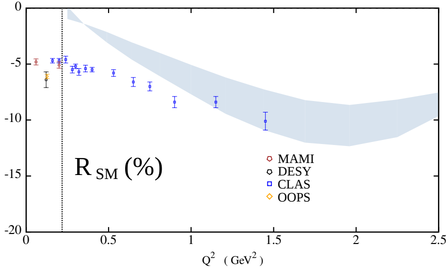
Sanchis Alepuz, Fischer, RA, Eur.Phys.J. **A54** (2018) 41 [arXiv:1707.08463].



Magnetic f.f.: Large Q^2 good, at small Q^2 missing pion cloud effects?!

$\Delta \rightarrow N\gamma$ Electromagnetic Transition Form Factors





$$R_{EM} = -\frac{G_E^*}{G_M^*}$$

Deformations of N and Δ!

- Non-rel. quark model:

  sub-lead. d-wave?

- Relativistically: Four-spinors

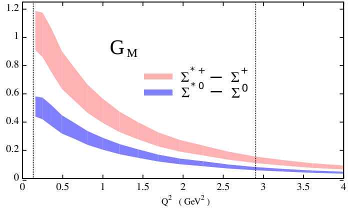
  with **lower components**!

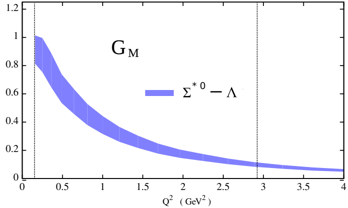
  Leading order: p-wave!

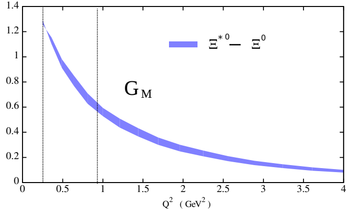
- Inherent to the approach!

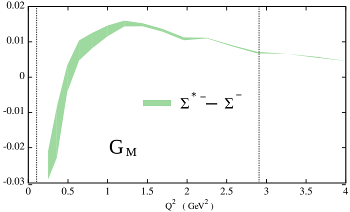
$$R_{SM} = -\frac{M_N^2}{M_\Delta^2}\sqrt{\lambda_+\lambda_-}\frac{G_C^*}{G_M^*}$$

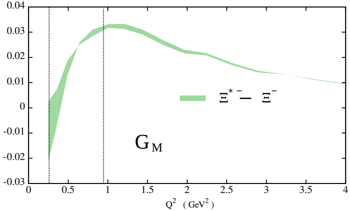
Magnetic Hyperon-Octet Transition Form Factors











Tiny isospin, small SU(3) breaking!

Magnetic Hyperon-Octet Transition Form Factors

transition	ΔN	$\Sigma^{*\,+}\Sigma^{+}$	$\Sigma^{*\,0}\Sigma^{0}$	$\Sigma^{*\,0}\Lambda$
$G_M(0)(\eta = 2.0)$	2.0	1.1	0.5	1.0
$G_M(0)(\eta = 1.6)$	0.8	1.3	0.6	1.1
exp.	3.04 (11)	4.10 (57)		3.35 (57)

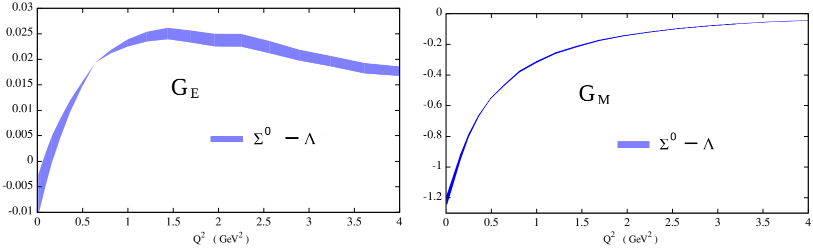
transition	$\Xi^{*\,0}\Xi^{*\,0}$	$\Sigma^{*\,-}\Sigma^{-}$	$\Xi^{*\,-}\Xi^{*\,-}$
$G_M(0)(\eta = 2.0)$	1.8	-0.05	-0.07
$G_M(0)(\eta = 1.6)$	1.5	-0.04	-0.02
exp.		$<$0.8	

Extrapolated result for $G_M(0)$ for quark-core calculation compared to estimates of experimental values.

$\Sigma^0 - \Lambda$ transition

- Only octet-octet transition
- PANDA (FAIR): also time-like transition f.f.
- Considerable theoretical interest
- Related to low-energy constants,

  Our results: $\left.\frac{dG_E}{dQ^2}\right|_{Q^2=0} = 0.053..0.073\,, \qquad \left.\frac{dG_M}{dQ^2}\right|_{Q^2=0} = 1.93..1.75\,.$



Expt.: $|\mu_{\Sigma^0\Lambda}| = (1.61 \pm 0.08)\ \mu_N$

Summary and Outlook

Hadrons from QCD bound state equations:

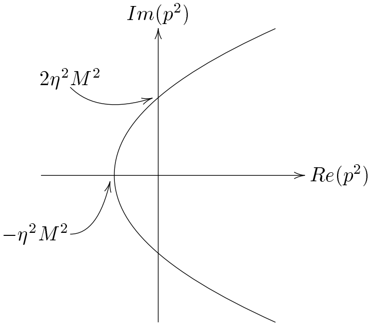
- QCD bound state equations:
  Unified approach to mesons and baryons feasible!
- In rainbow-ladder appr. meson observables
  and octet / decuplet masses and (e.m., axial, . . .) form factors;
  incl. N, Δ & hyperon transition form factors, . . .
- **Lower spinor components $\Longrightarrow$ non-spherical baryons!**

NB: Calculation of bound state amplitudes in boosted frames have started recently.
[H. Sanchis-Alepuz, in preparation]

- Even in ground state form factors beyond rainbow-ladder effects at small Q^2! Likely hadronic (pionic) effects!
- Systematic approach:
  Include knowledge on quark-gluon and on quark-photon vertices.

Quark Propagator and Rainbow Truncation

All (non-perturbative) approaches to QFT employ Euclidean momenta:
Connection to the world of real particles requires analytic continuation!



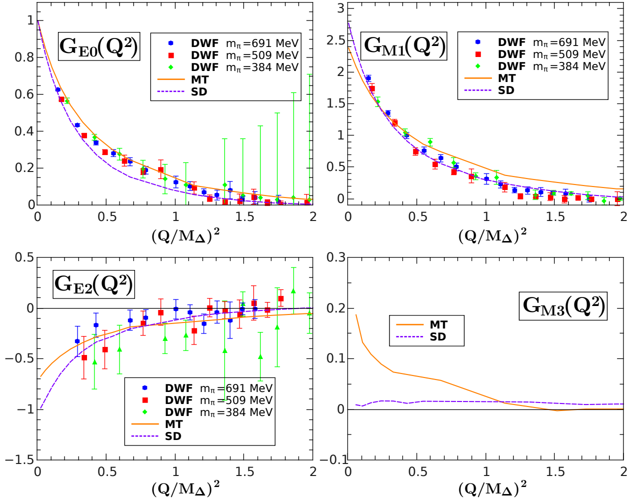
In bound state eqs.:

- Knowledge of the quark propagator inside parabolic region required.
- Parabola limited by nearest quark singularities: $M < 2mq(3mq)$ for mesons (baryons)
- ground states unaffected by singularities.

- Lattice: Values for real $p^2 \geq 0$ only.
- Dyson-Schwinger / ERG eqs.: complex p^2 accessible.

Δ electromagnetic form factors

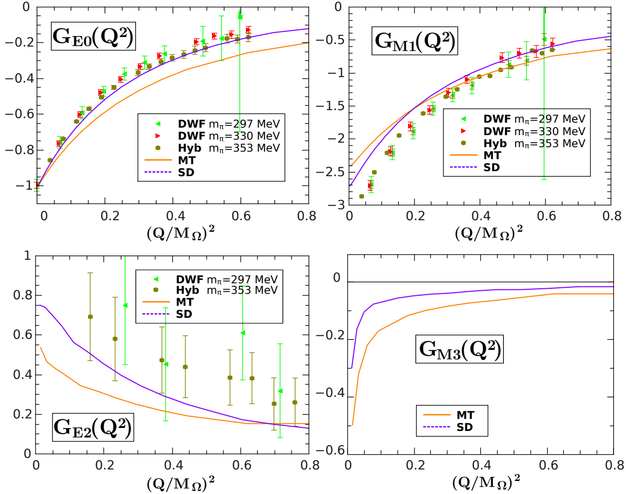
H. Sanchis-Alepuz *et al.*, Phys. Rev. D **87** (2013) 095015 [arXiv:1302.6048 [hep-ph]].



- G_{E2} and G_{M3}: Deviation from sphericity!
- Important: Difference to quark-diquark model in G_{E2} and G_{M3}.
- Large G_{E2} for small Q^2!
- “Small” G_{M3}?

Ω electromagnetic form factors

H. Sanchis-Alepuz *et al.*, Phys. Rev. D **87** (2013) 095015 [arXiv:1302.6048 [hep-ph]].



- Again deviation from sphericity!
- Only weak quark mass dependence!

Electromagnetic Transition Form Factors

[H. Sanchis Alepuz, C. S. Fischer, RA, arXiv:1707.08463.]

Technical complications: Limits on accessible Q^2 range due to

- Poles of quark propagator.
- Analytic continuation of Chebychev expansion.

Here: Maris-Tandy model interaction (bands indicate parameter dep.)

Transition f.f. more involved: Initial & final states have different masses!

NB: Calculation of bound state amplitudes directly in boosted frames have started recently [H. Sanchis-Alepuz, R. Williams, in preparation].