QCD Phase Transitions at finite temperature and densities within FRG approach

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666. WE-Heraeus-Seminar on "From Correlation Functions to QCD Phenomenology"
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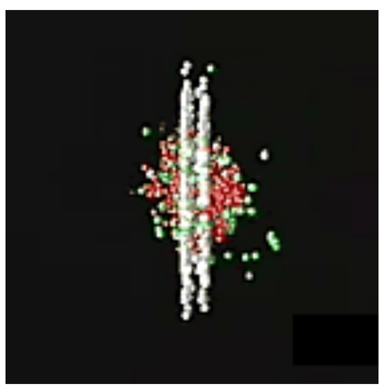
Collaborators:
fQCD collaboration (J. Braun, L. Corell, A. Cyrol, WF, M. Leonhardt, M. Mitter, J.M. Pawlowski, M. Pospiech, F. Rennecke, N. Strodthoff, N. Wink), Bernd-Jochen Schaefer, Rui Wen, Chuang Huang, Ke-Xin Sun et al.

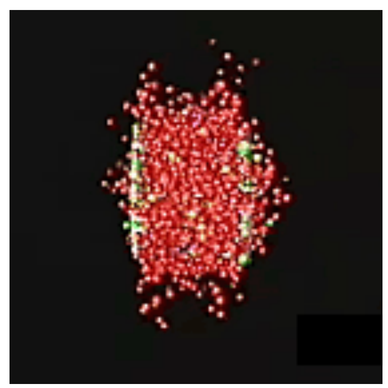
Outline

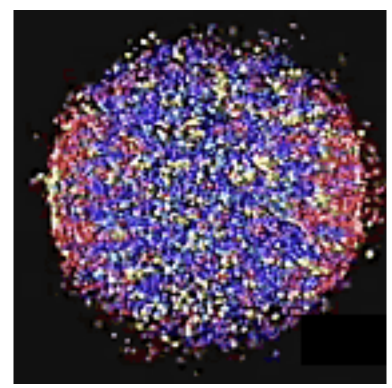
- Introduction
- Baryon number fluctuations, probability distribution in low energy effective models
- Quantum fluctuations of gluons and QCD phase transition
- Summary and outlook

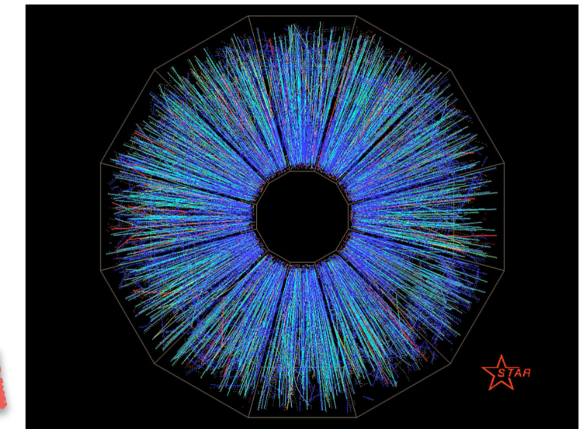
Heavy-ion collision

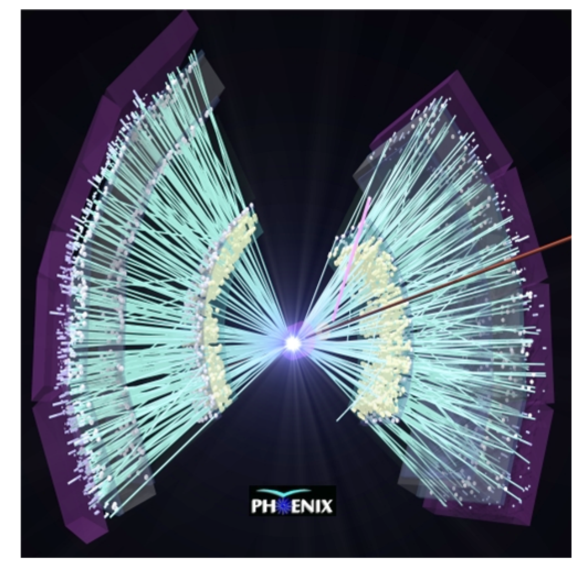

Ions about to collide

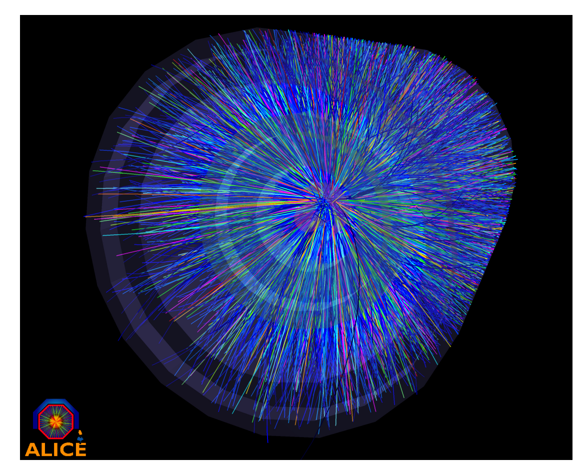

Ion collision


Plasma formation


Freeze out

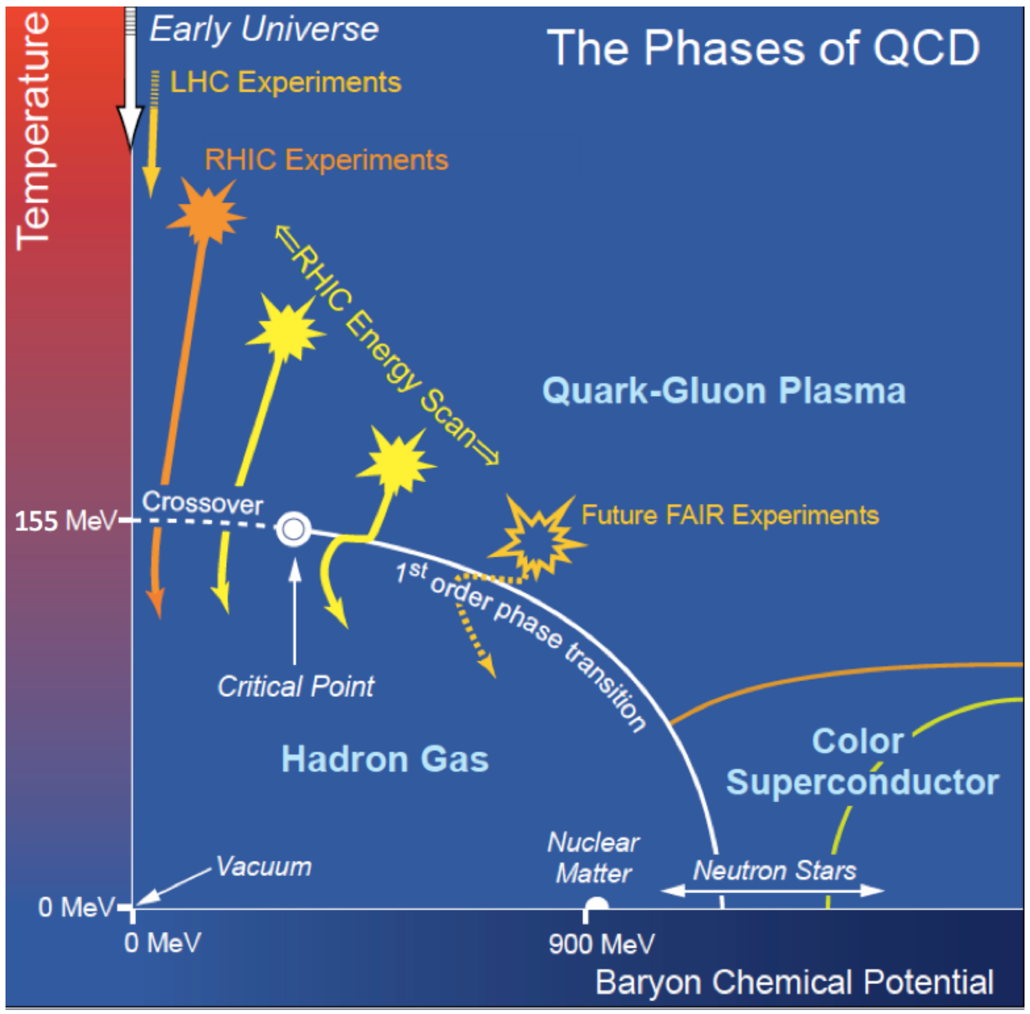

STAR


PHENIX


ALICE

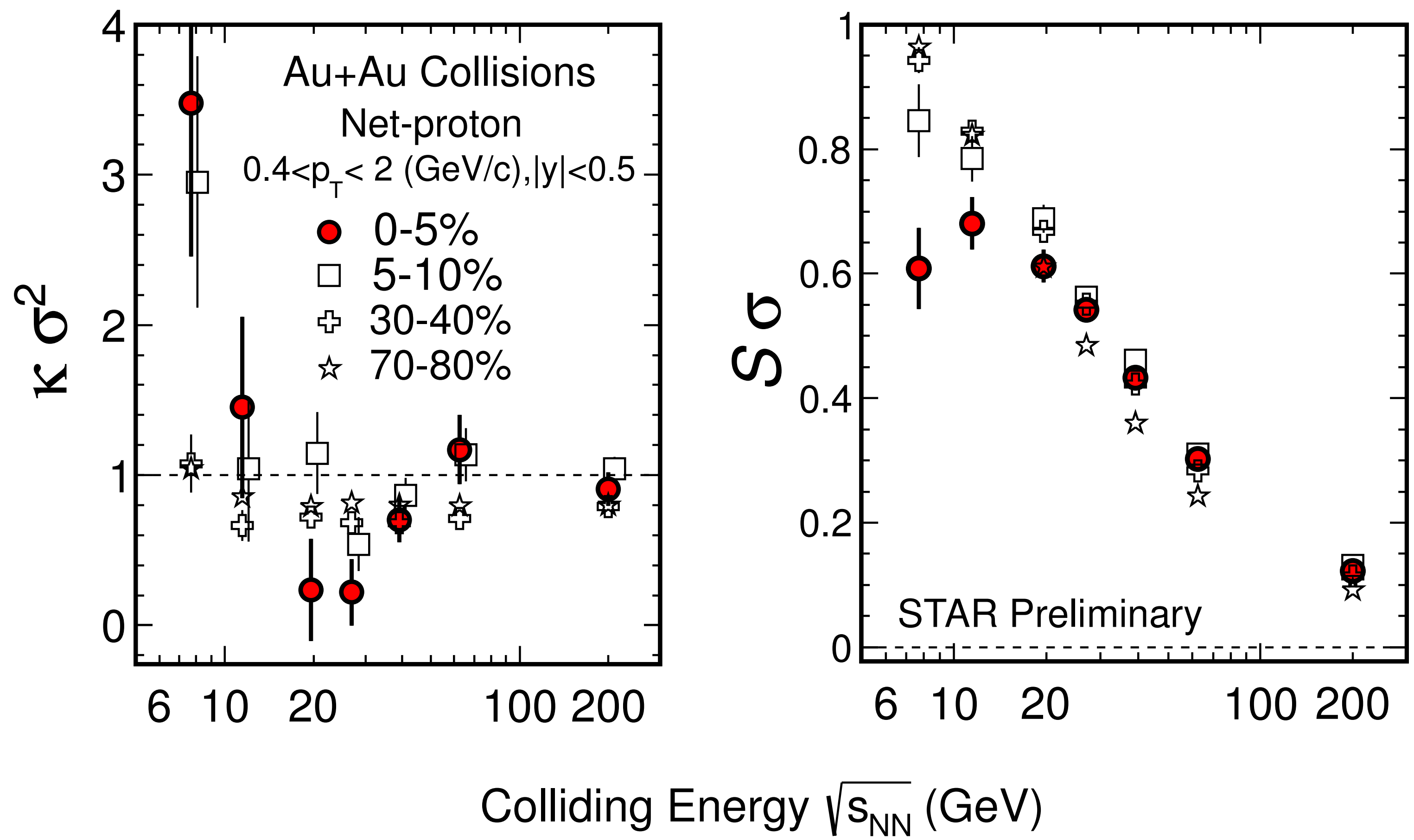
What we see

QCD Phase Structure



The Hot QCD White Paper (2015)

RHIC:

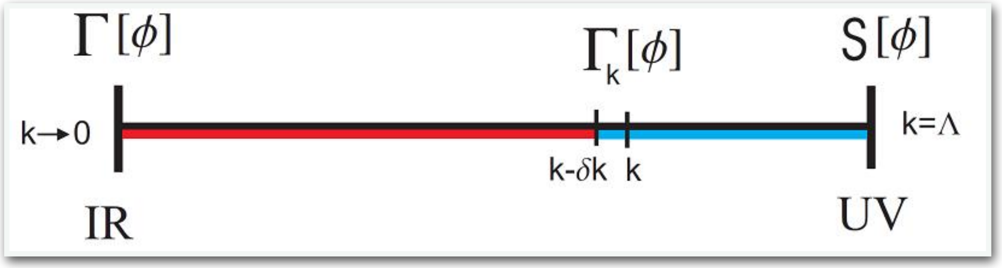


X.Luo (STAR), PoS CPOD2014, 019 (2014)

Quantum Fluctuations with FRG

Quantum Fluctuations with FRG

FRG



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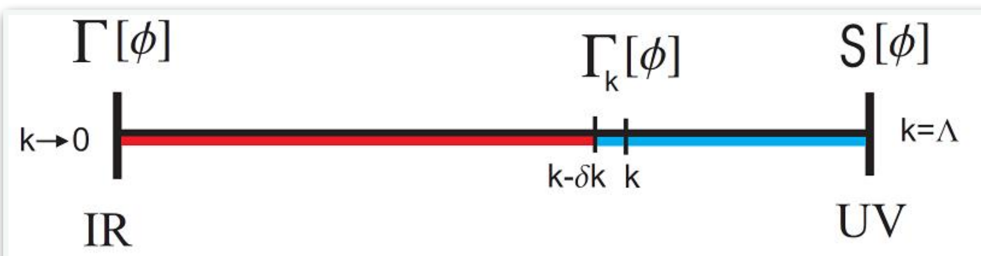

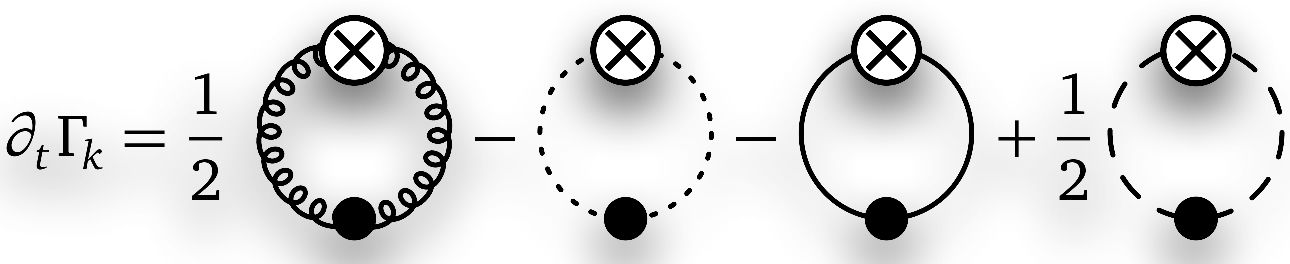

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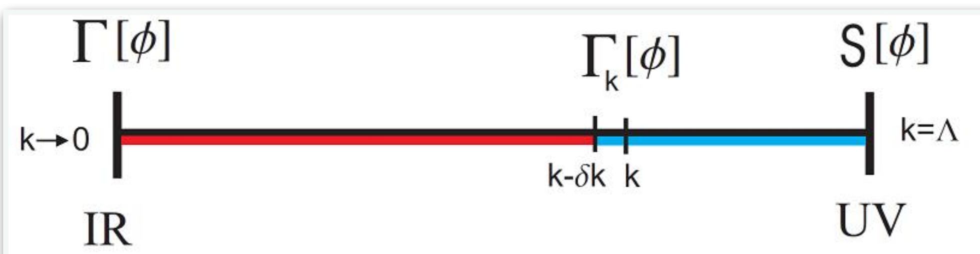

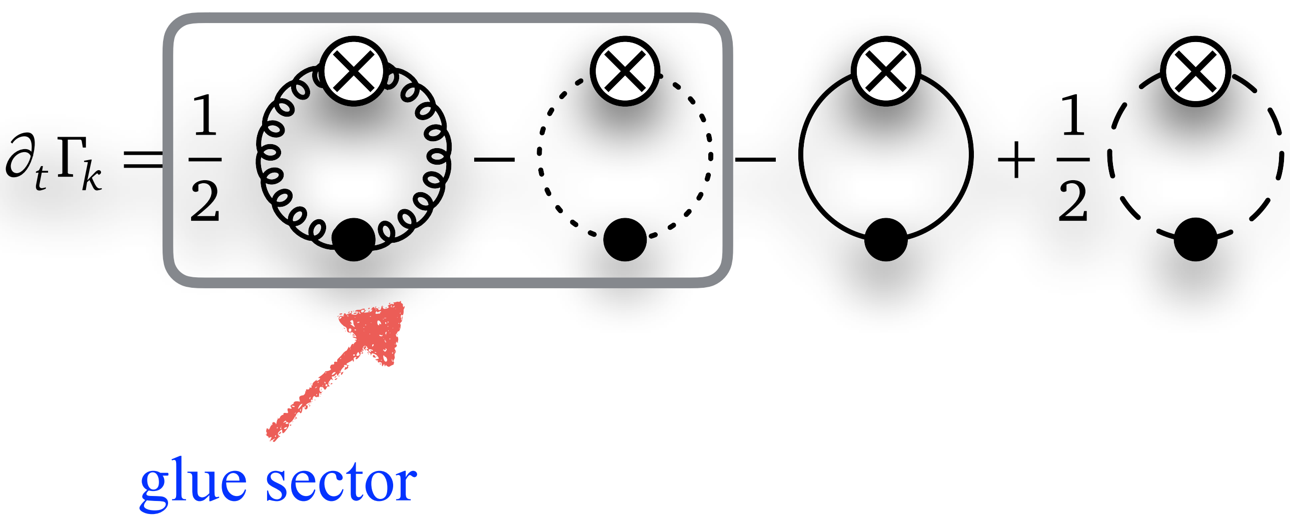

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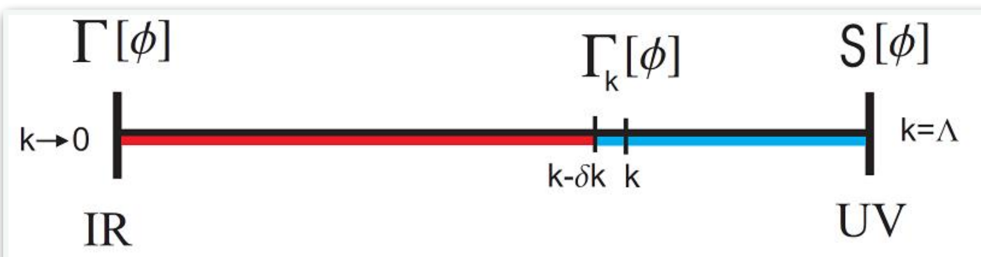

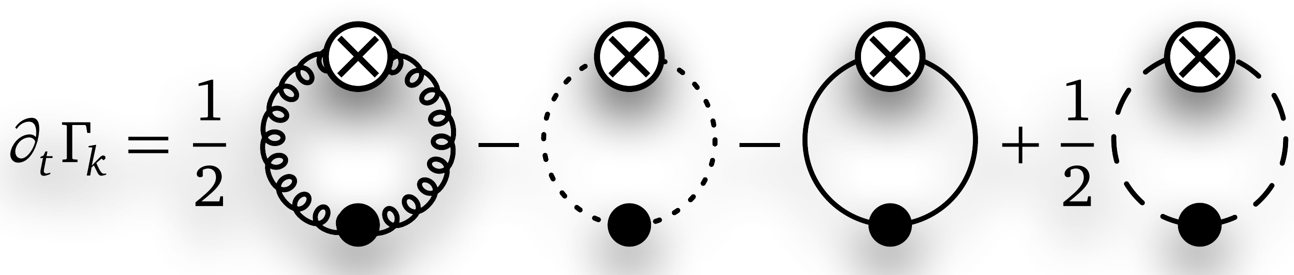

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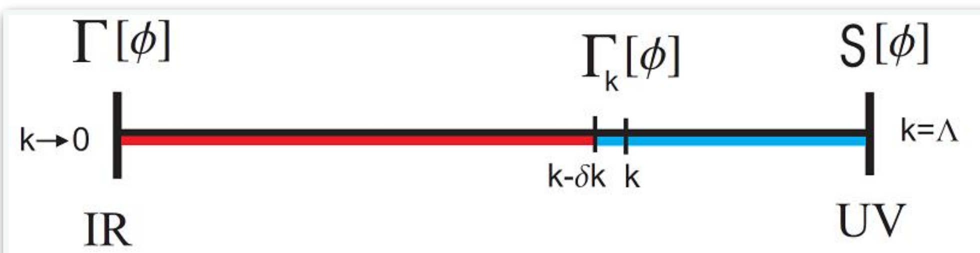
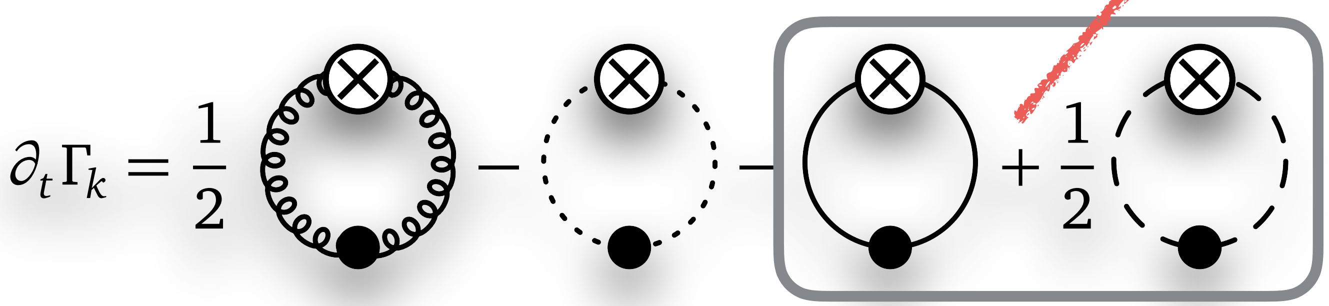




Matter part, effective models

2 flavor

$$\Gamma_k = \int_x \Big\{ Z_{q,k}\bar{q}(\gamma_\mu\partial_\mu - \gamma_0\mu)q + \frac{1}{2}Z_{\phi,k}(\partial_\mu\phi)^2$$

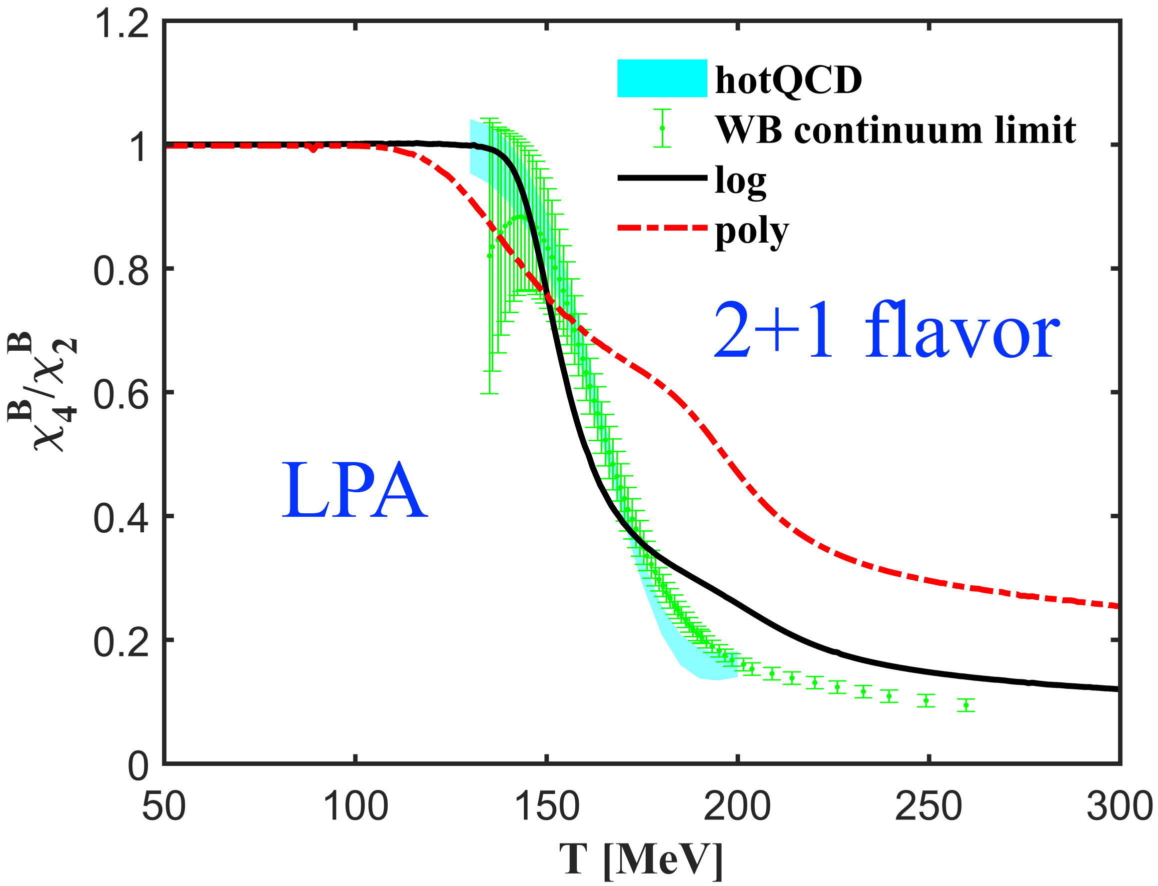
$$+ h_k\,\bar{q}\left(T^0\sigma + i\gamma_5\vec{T}\cdot\vec{\pi}\right)q + V_k(\rho) - c\sigma\Big\} + \cdots$$

2+1 flavor

$$\Gamma_k = \int_x \Big\{ \bar{q}\big[\gamma_\mu\partial_\mu - \gamma_0(\mu + igA_0)\big]q + V_{\mathrm{glue}}(L,\bar{L})$$

$$+ \mathrm{tr}(\partial_\mu\Sigma\cdot\partial_\mu\Sigma^\dagger) + i\bar{q}h_k\cdot\Sigma_5 q + \tilde{U}_k(\Sigma,\Sigma^\dagger)\Big\},$$

Baryon Number Fluctuations in 2+1 Flavor



Rui Wen, Chuang Huang, WF, in preparation

2+1 flavor PQM:

$$\Gamma_k = \int_x \Big\{ \bar{q}[\gamma_\mu \partial_\mu - \gamma_0(\mu + igA_0)]q + V_{\text{glue}}(L, \bar{L})$$

$$+ \text{tr}(\partial_\mu \Sigma \cdot \partial_\mu \Sigma^\dagger) + i\bar{q}h_k \cdot \Sigma_5 q + \tilde{U}_k(\Sigma, \Sigma^\dagger) \Big\},$$

scalar and pseudoscalar mesons in the octet and singlet:

$$\Sigma = T_a(\sigma_a + i\pi_a), \quad a = 0, 1, ...8$$

$$\Sigma_5 = T_a(\sigma_a + i\gamma_5 \pi_a),$$

the effective potential

$$\tilde{U}_k(\Sigma, \Sigma^\dagger) = U_k(\rho_1, \tilde{\rho}_2) - c_k \xi - j_L \sigma_L - j_S \sigma_S,$$

`t Hooft determinant

$$\xi = \det(\Sigma) + \det(\Sigma^\dagger),$$

$$\rho_1 = \text{tr}(\Sigma \cdot \Sigma^\dagger),$$

$$\tilde{\rho}_2 = \text{tr}\Big(\Sigma \cdot \Sigma^\dagger - \frac{1}{2}\rho_1\Big)^2,$$

also see talk by Fabian

Net Baryon Number Probability Distribution

Net Baryon Number Probability Distribution

The probability distribution:

canonical partition function

$$P(N;T,V,\mu) = \frac{Z(T,V,N)}{\mathcal{Z}(T,V,\mu)} \exp\left(\frac{\mu N}{T}\right),$$

grand canonical partition function

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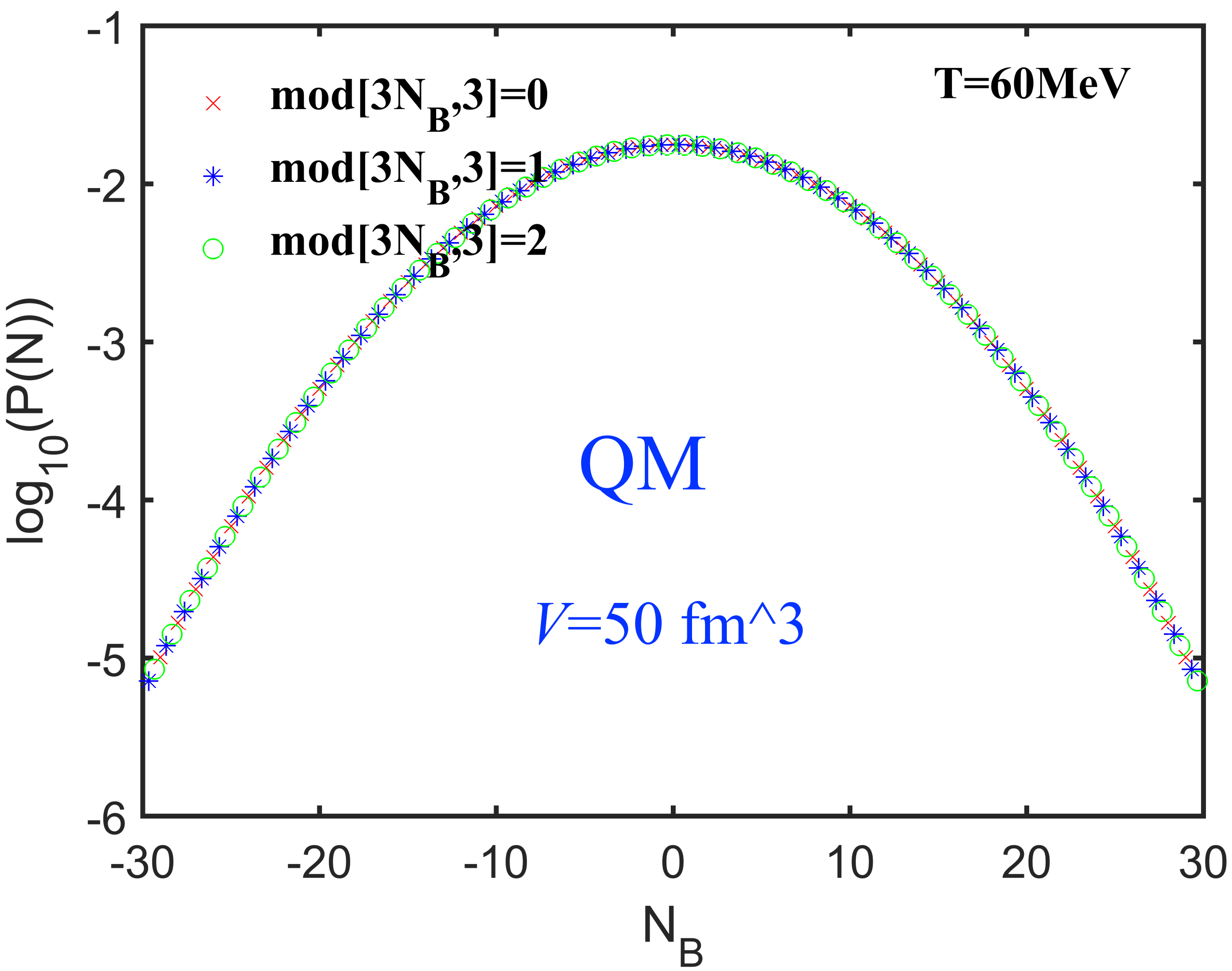

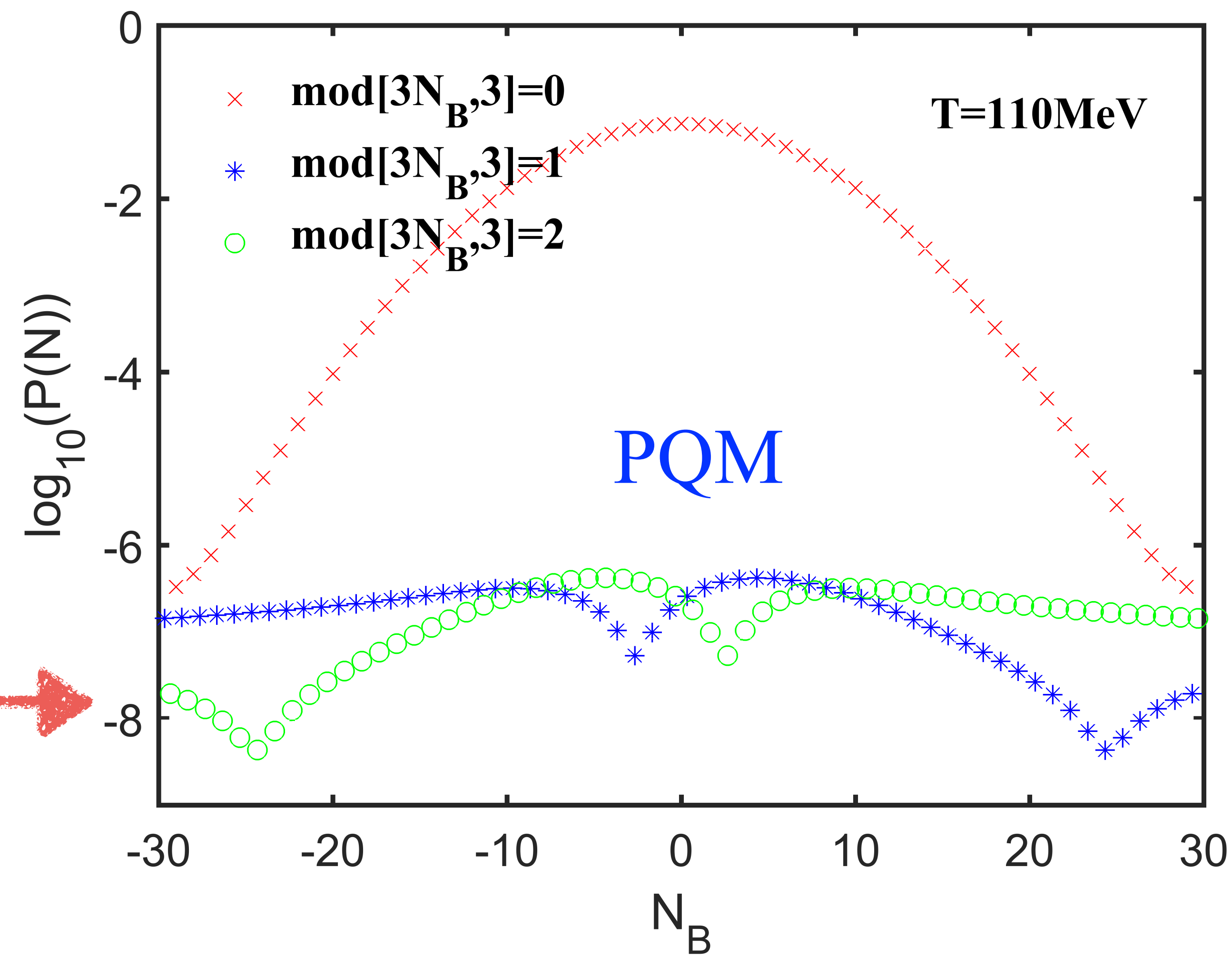

Ke-xin Sun, Rui Wen, WF, in preparation

Net Baryon Number Probability Distribution

Net Baryon Number Probability Distribution

Why do we need to calculate
the probability distribution?

Net Baryon Number Probability Distribution

Why do we need to calculate the probability distribution?

- Sensitive to different dynamics, especially the glue dynamics.
- Input to transport simulations in heavy ion collisions
- Non-critical effects are easily involved, such as the detector acceptability cut, volume fluctuations etc.
- Related investigations are in progress.

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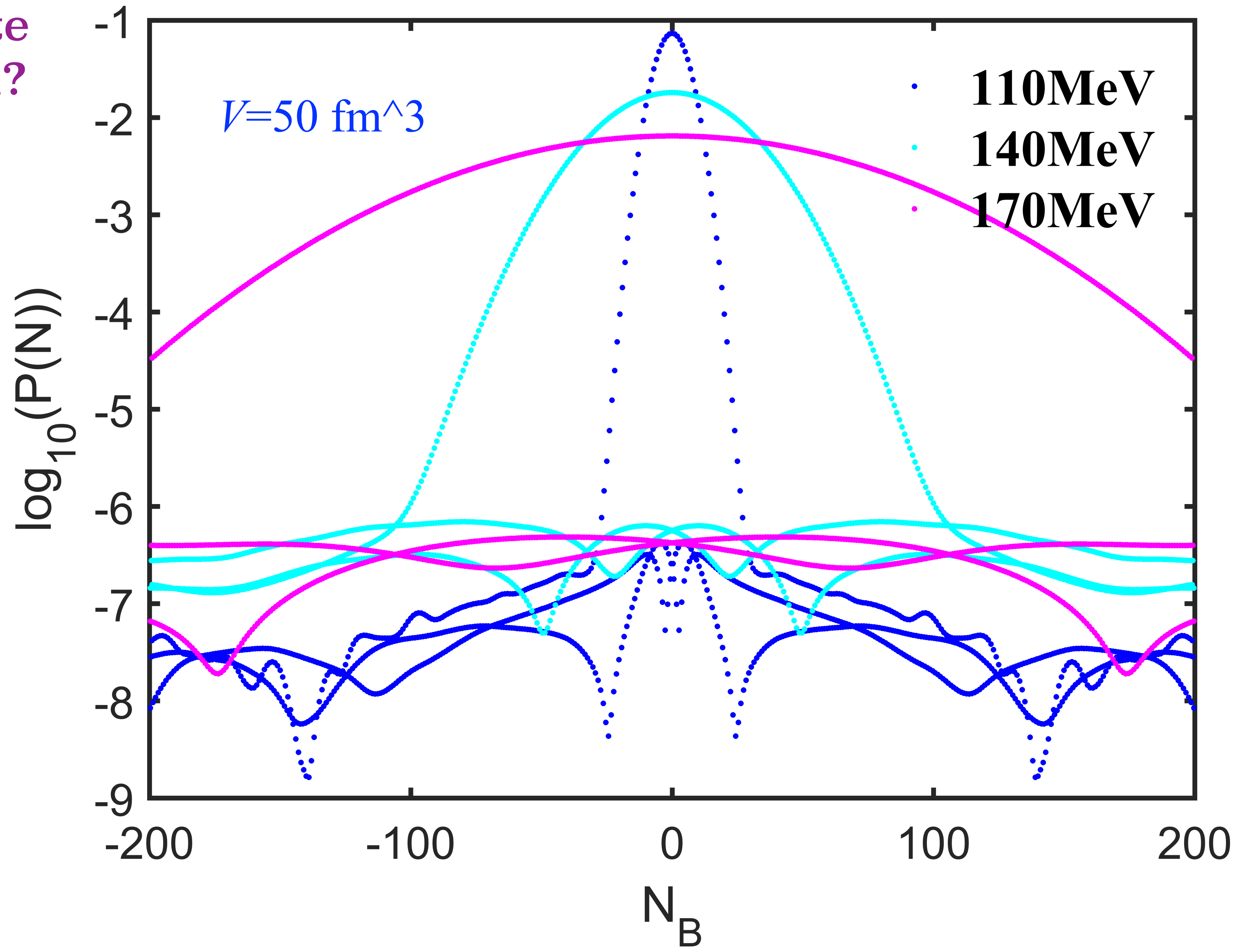
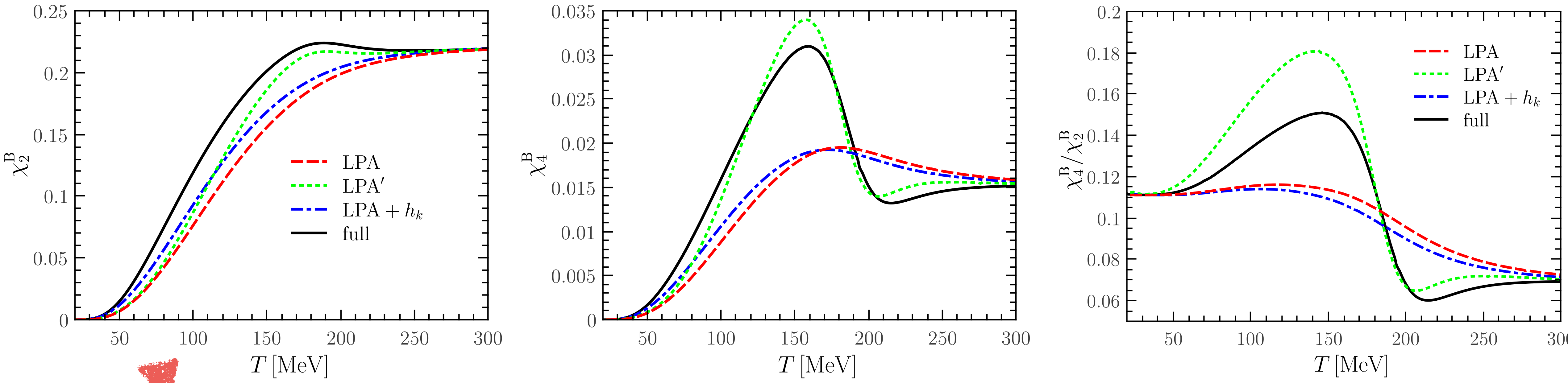
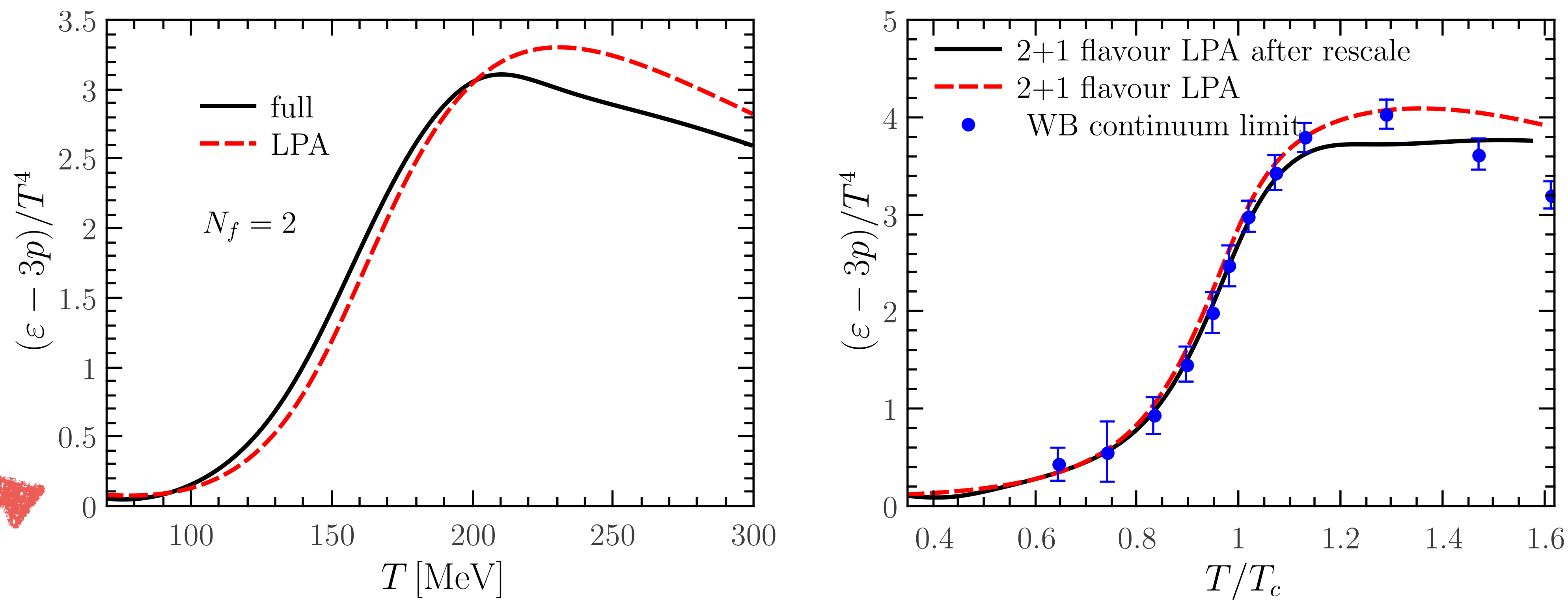


Ke-xin Sun Rui Wen, WF, in preparation

Quantum Fluctuations beyond LPA



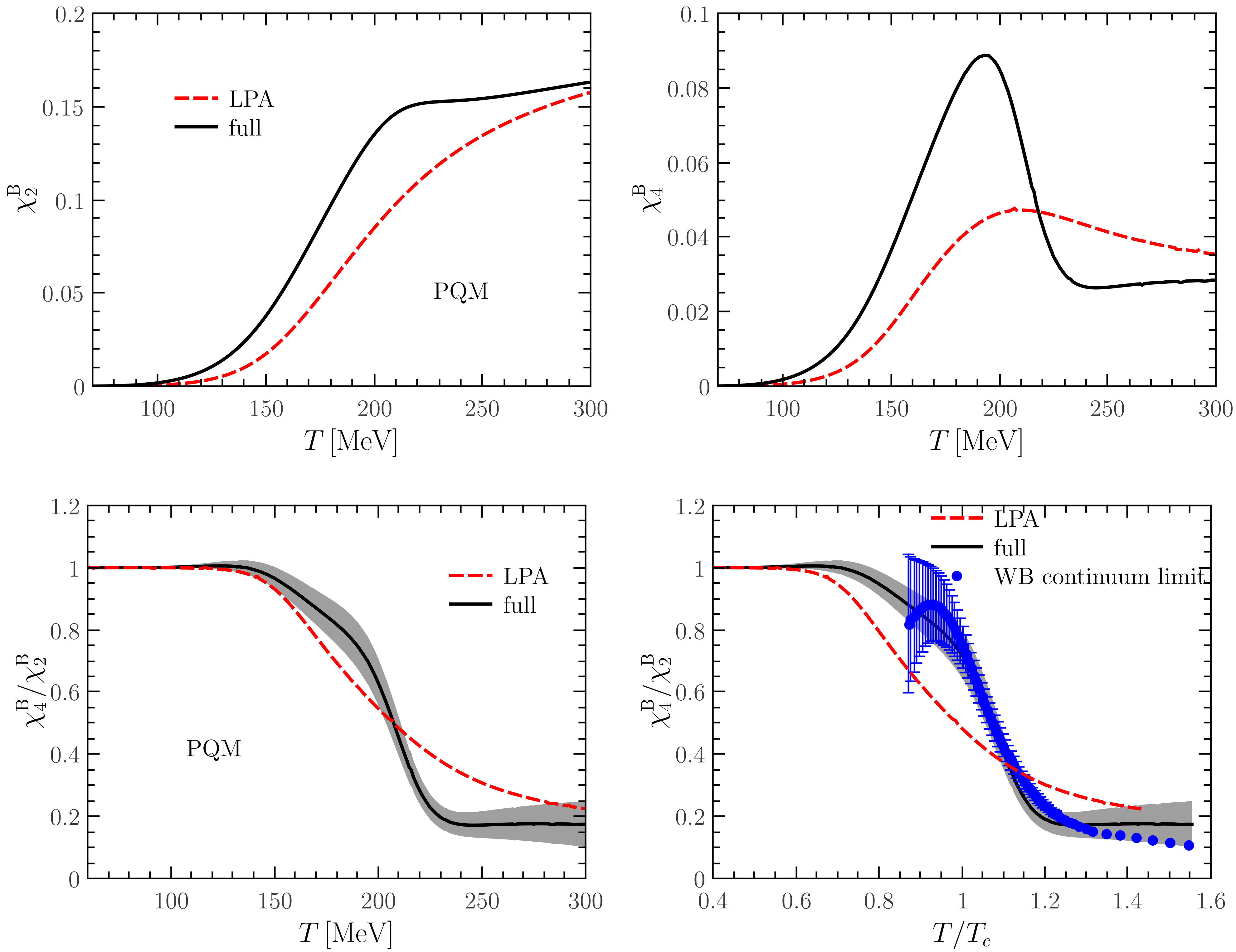
QM



PQM

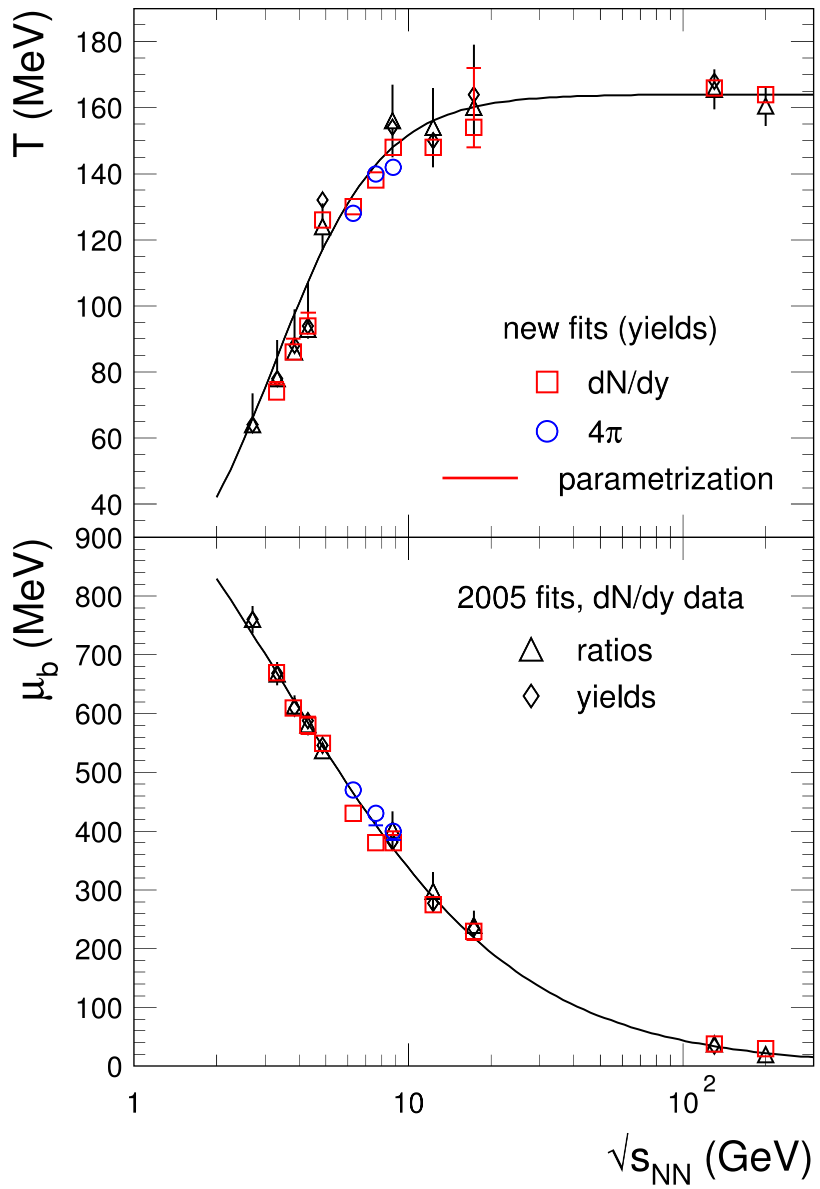
WF, J.M. Pawlowski, PRD 92 (2015) 116006

Baryon number fluctuations beyond LPA



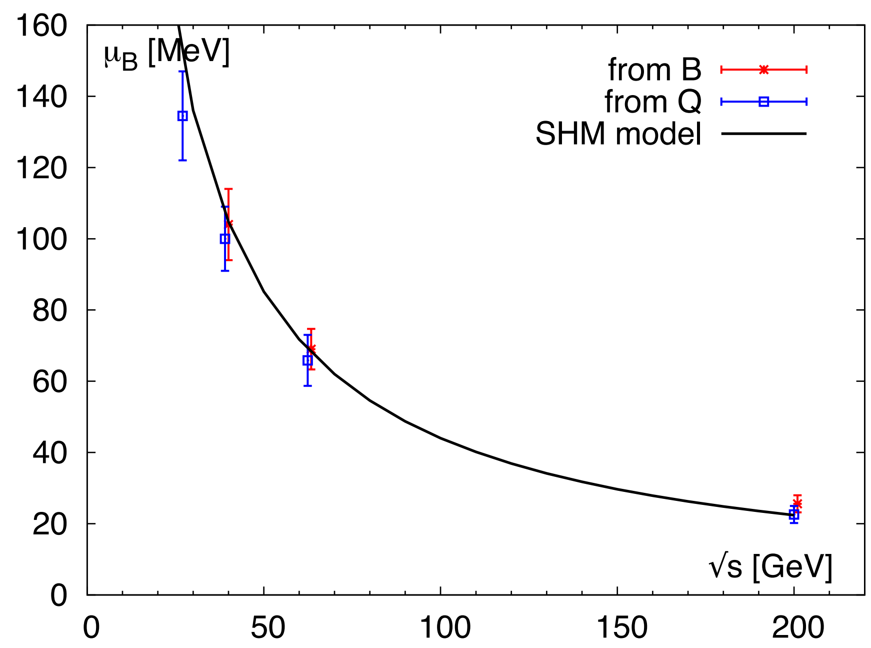
WF, J.M. Pawlowski, PRD 92 (2015) 116006

Freeze-out line



Freeze-out temperature and chemical potential obtained from the Statistical Hadronization Model

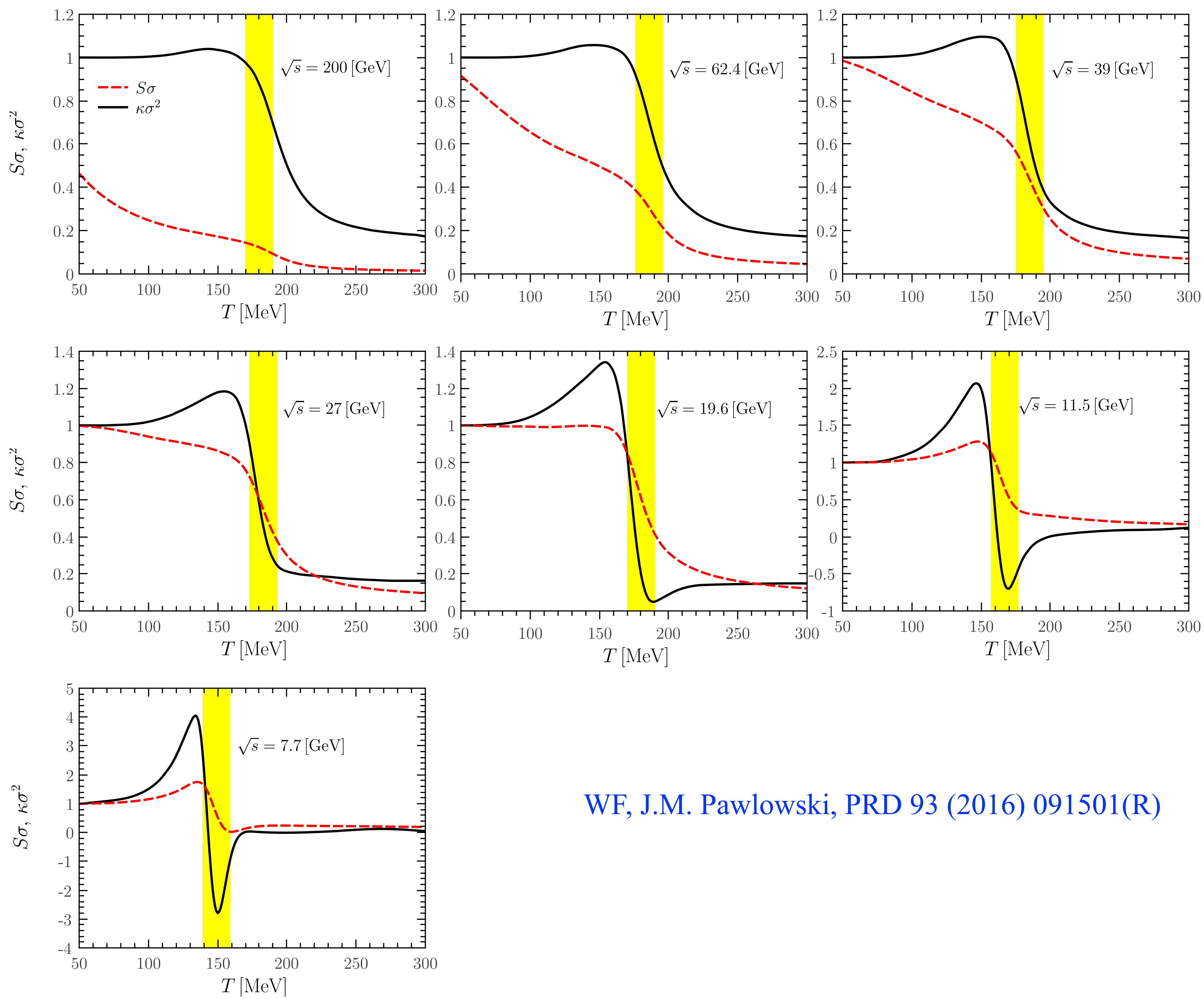
A. Andronic, P. Braun-Munzinger, J. Stachel, Phys.Lett. B673 (2009) 142



Freeze-out chemical potential obtained from lattice simulations

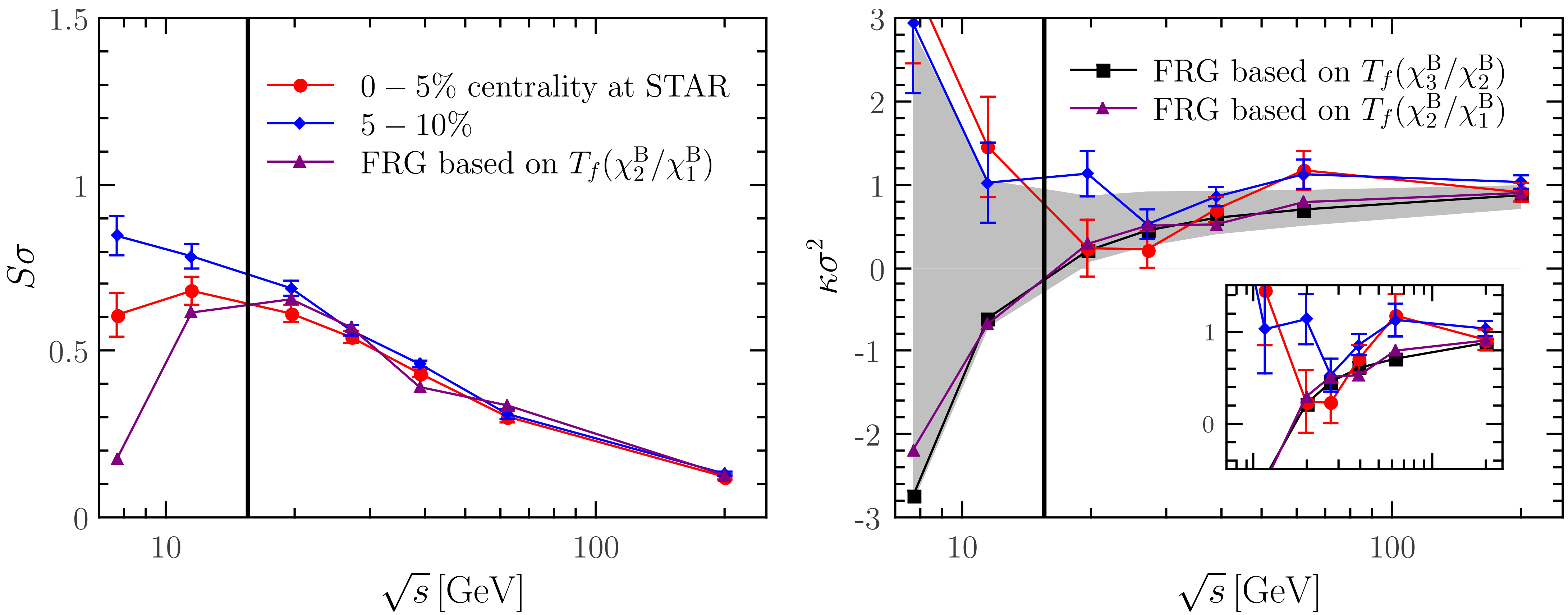
S. Borsanyi *et al.*, Phys.Rev.Lett. 113 (2014) 052301

Correlating the skewness and kurtosis of baryon number distributions



WF, J.M. Pawlowski, PRD 93 (2016) 091501(R)

Comparison with experimental measurements



WF, J.M. Pawlowski, PRD 93 (2016) 091501(R)

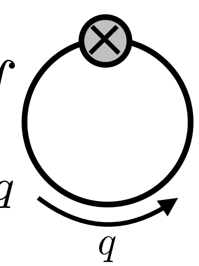
Silver Blaze Property and the Frequency Dependence

Frequency-dependent quark anomalous dimension:

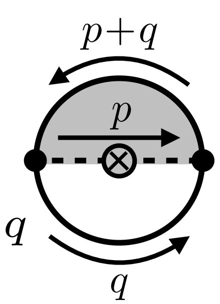
$$\eta_{q,k}(p) = \frac{1}{Z_{q,k}(p)} \frac{1}{4N_c N_f} \frac{\partial^2}{\partial|\vec{p}|^2} \mathrm{Tr}\Big(i\vec{\gamma}\cdot\vec{p}\,\partial_t \tilde{\Gamma}^{(2)}_{q\bar{q},k}(p) \Big)$$

Insert this into the flow of effective potential, and perform the two-loop summation

$$\partial_t V_k^q = -\int_q$$



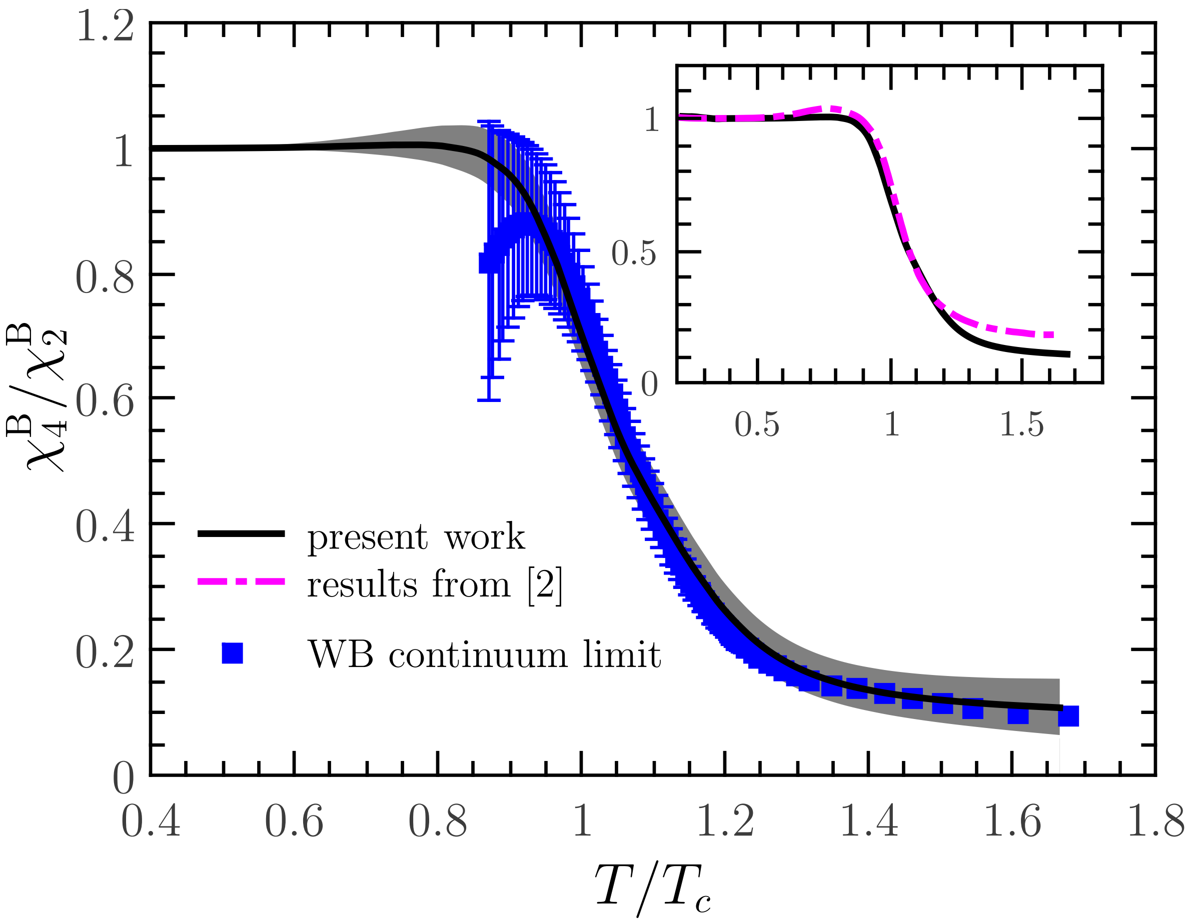
$$\approx -\iint_{p,q}$$

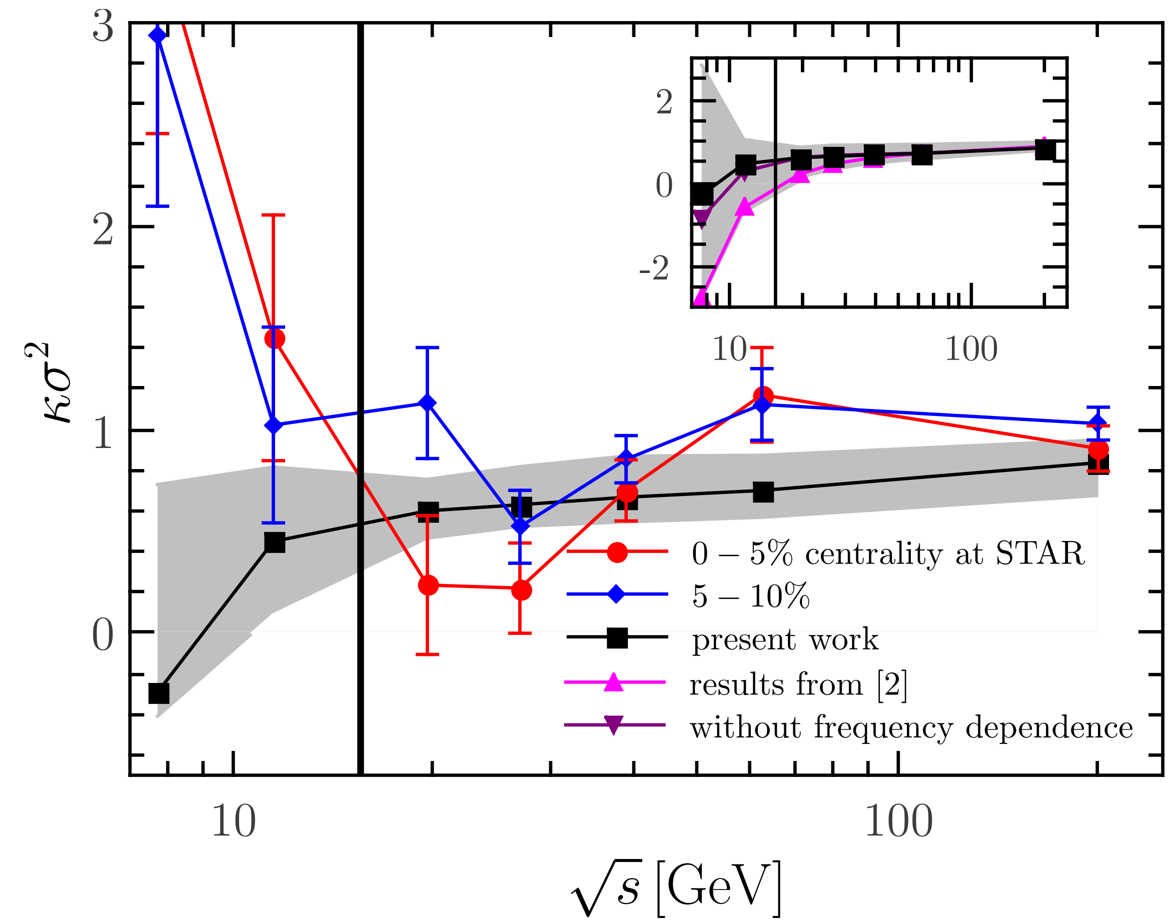


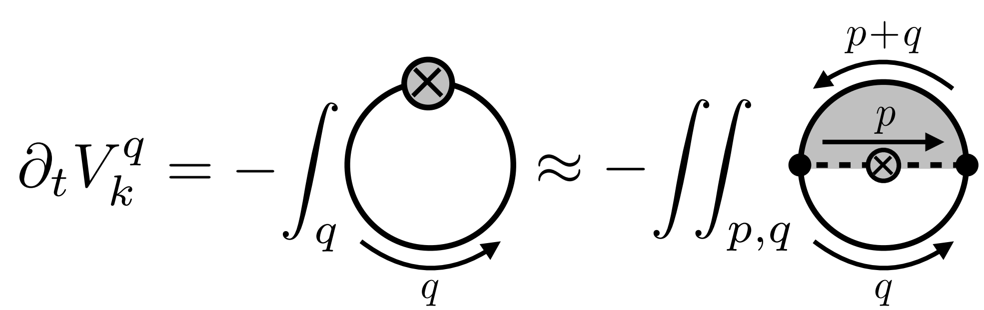
One obtains

$$\begin{aligned}\partial_t V_k(\rho) = \frac{k^4}{360\pi^2}\Big\{ & 12(5-\eta_{\phi,k})\big[(N_f^2-1)\mathcal{B}_{(1)}(\bar{m}^2_{\pi,k}) \\ & + \mathcal{B}_{(1)}(\bar{m}^2_{\sigma,k})\big] - 5N_c\Big(48N_f\mathcal{F}_{(1)}(\bar{m}^2_{F,k}) \\ & + \frac{1}{2\pi^2}(-4+\eta_{\phi,k})\bar{h}_k^2\Big[\mathcal{F}\mathcal{F}\mathcal{B}_{(1,1,2)}(\bar{m}^2_{F,k},\bar{m}^2_{\sigma,k}) \\ & + (N_f^2-1)\mathcal{F}\mathcal{F}\mathcal{B}_{(1,1,2)}(\bar{m}^2_{F,k},\bar{m}^2_{\pi,k})\Big]\Big)\Big\},\end{aligned}$$

Two-loop Results







WF, J.M. Pawlowski, F. Rennecke, Bernd-Jochen Schaefer, PRD 94 (2016) 116020

From effective models to QCD

Summary on effective model including mesonic fluctuations

- Quantitative agreement with lattice results below ~1.2 Tc
- No bump on the baryon number kurtosis
- Discrepancy observed at large temperature or density because of the UV cutoff effect
- UV cutoff should be pushed up higher, and glue quantum fluctuations should be included

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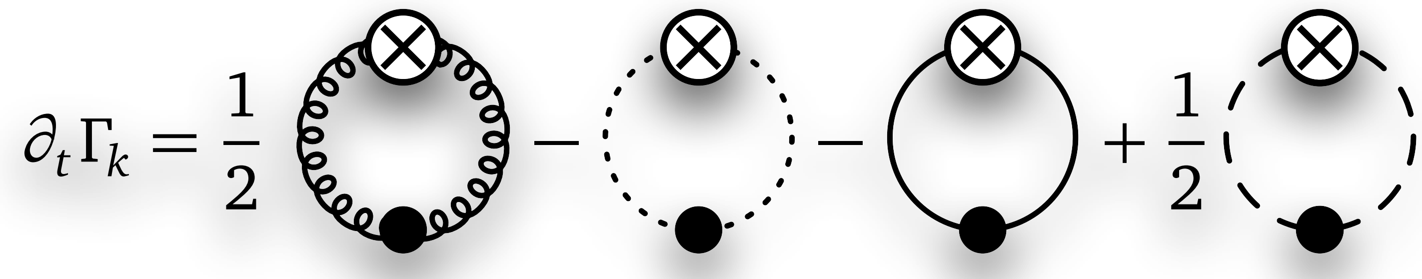

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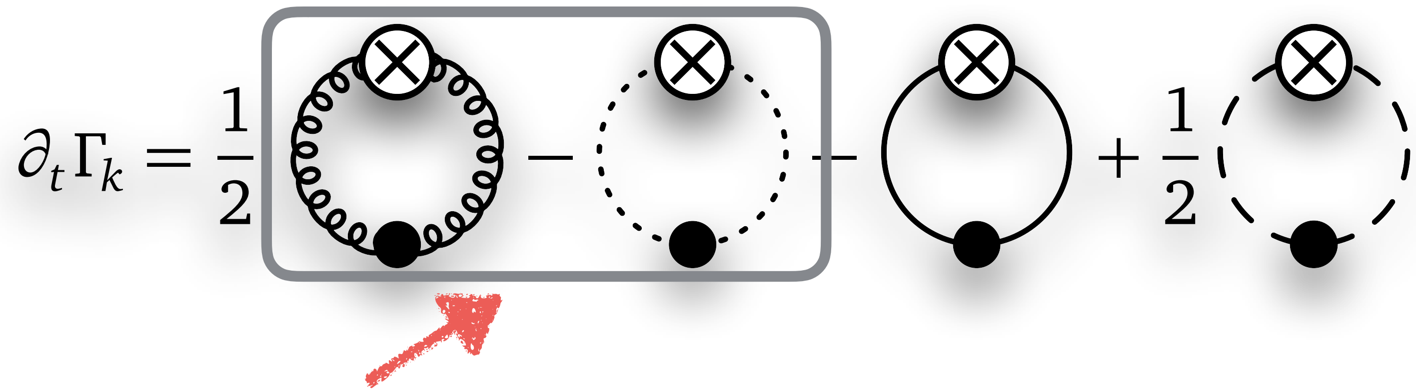

Come back!

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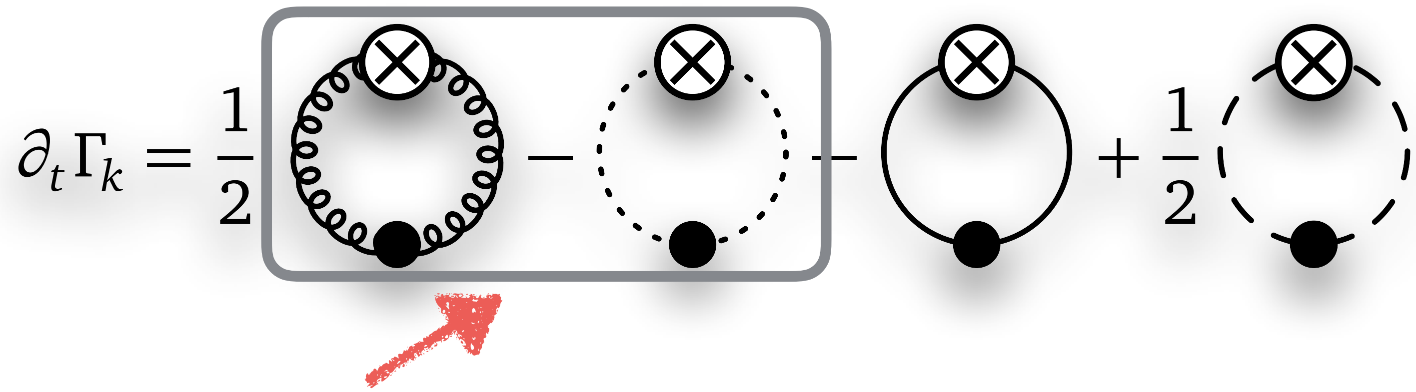

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Effective model:

$$\Gamma_k = \int_x \Big\{ Z_{q,k}\bar{q}(\gamma_\mu \partial_\mu - \gamma_0 \mu) q + \frac{1}{2} Z_{\phi,k}(\partial_\mu \phi)^2$$

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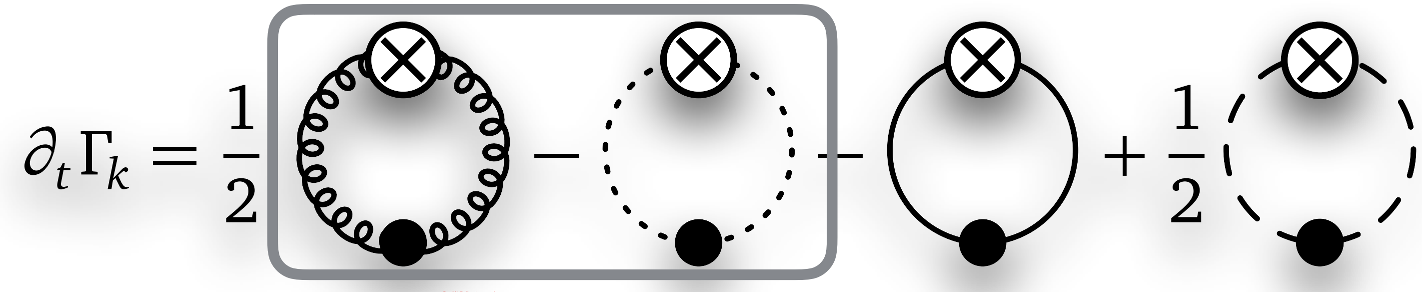

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Rebosonized QCD:

$$\Gamma_k = \int_x \Big\{ \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + Z_{c,k}(\partial_\mu\bar{c}^a)D^{ab}_\mu c^b + \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2$$
$$+ Z_{q,k}\bar{q}(\gamma_\mu D_\mu)q - \lambda_{q,k}\left[(\bar{q}\,T^0 q)^2 - (\bar{q}\,\gamma_5\vec{T}q)^2\right]$$
$$+ h_k\left[\bar{q}(i\gamma_5\vec{T}\vec{\pi} + T^0\sigma)q\right] + \frac{1}{2}Z_{\phi,k}(\partial_\mu\phi)^2$$
$$+ V_k(\rho) - c\sigma\Big\} + \Delta\Gamma_{\text{glue}}$$

2 flavor

Flow Equations

Wetterich equation:

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}\mathrm{STr}\Big\{\partial_t R_k \big(\Gamma_k^{(2)}[\Phi] + R_k\big)^{-1}\Big\}$$

$$= \frac{1}{2}\mathrm{STr}\Big\{\tilde{\partial}_t \ln\big(\Gamma_k^{(2)}[\Phi] + R_k\big)\Big\}$$

with $t = \ln(k/\Lambda)$ and

$$\big(\Gamma_k^{(2)}[\Phi]\big)_{ij} := \frac{\overrightarrow{\delta}}{\delta \Phi_i}\Gamma_k[\Phi]\frac{\overleftarrow{\delta}}{\delta \Phi_j} \qquad \Gamma_k^{(2)} + R_k = \mathcal{P} + \mathcal{F}$$

Vertex expansion:

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\{\tilde{\partial}_t \ln(\mathcal{P}+\mathcal{F})\} = \frac{1}{2}\mathrm{STr}\tilde{\partial}_t \ln \mathcal{P}$$

$$+ \frac{1}{2}\mathrm{STr}\tilde{\partial}_t\Big(\frac{1}{\mathcal{P}}\mathcal{F}\Big) - \frac{1}{4}\mathrm{STr}\tilde{\partial}_t\Big(\frac{1}{\mathcal{P}}\mathcal{F}\Big)^2 + \cdots$$

3d optimized regulator:

$$R_{\mathrm{F},k}(q) = Z_{q,k} i\vec{q}\cdot\vec{\gamma}\, r_F(\frac{\vec{q}^2}{k^2}), \quad \text{with} \quad r_F(x) = (\frac{1}{\sqrt{x}} - 1)\Theta(1-x)$$

$$R_{\mathrm{B},k}(q) = Z_{\phi,k}\vec{q}^2\, r_B(\frac{\vec{q}^2}{k^2}), \quad \text{with} \quad r_B(x) = (\frac{1}{x} - 1)\Theta(1-x)$$

Feynman Diagrams

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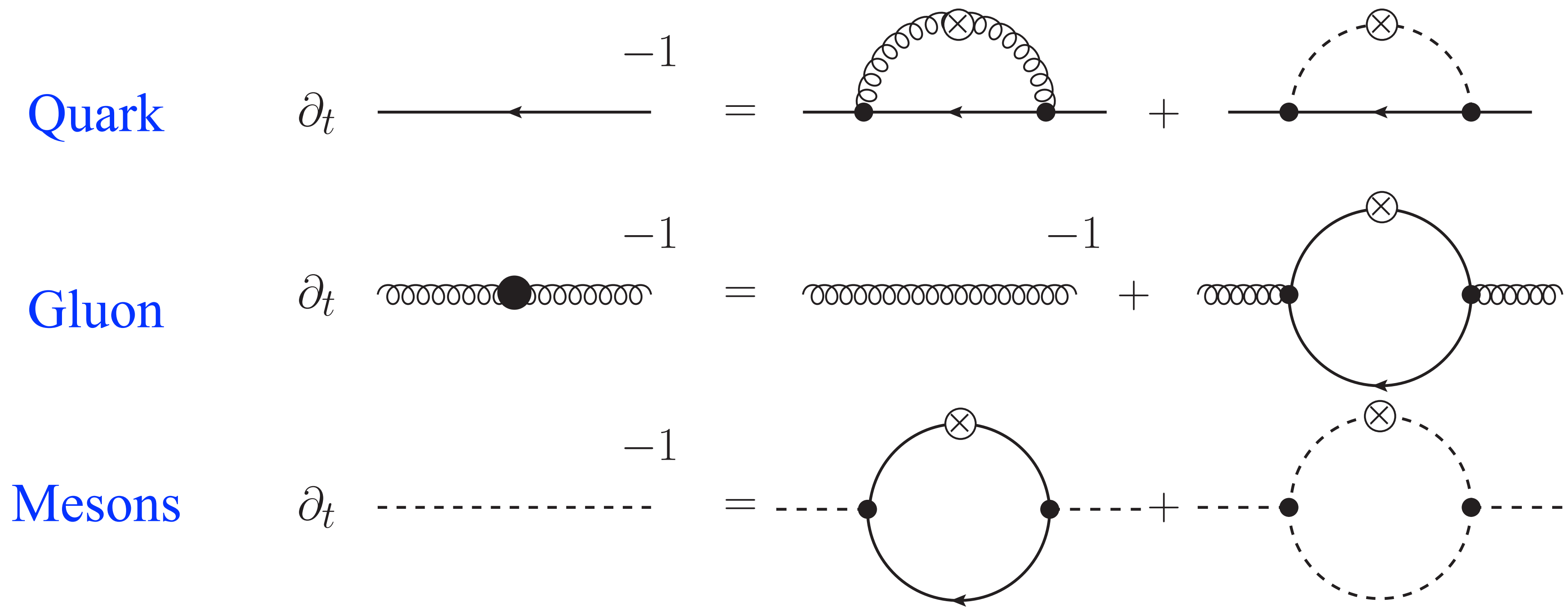

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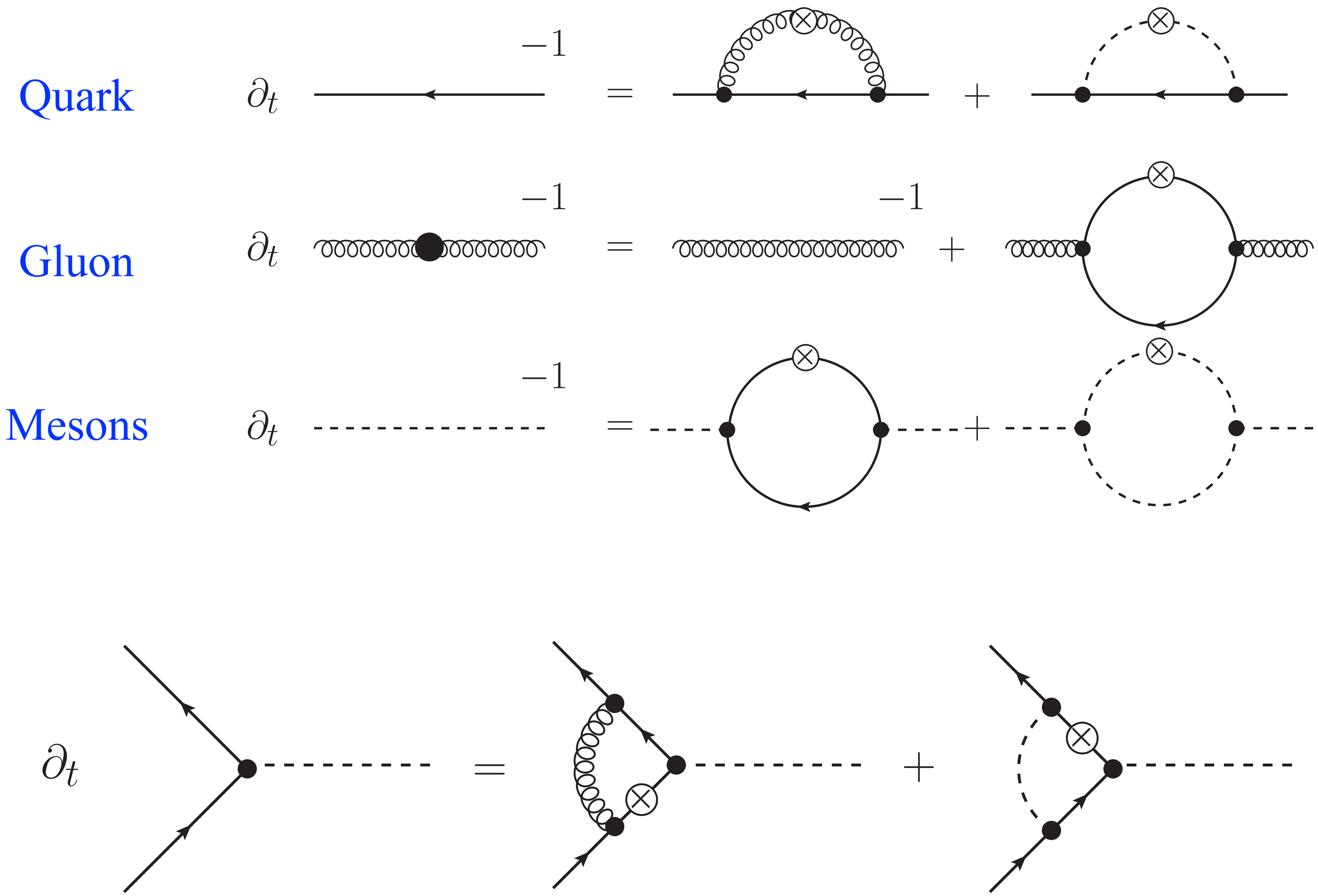
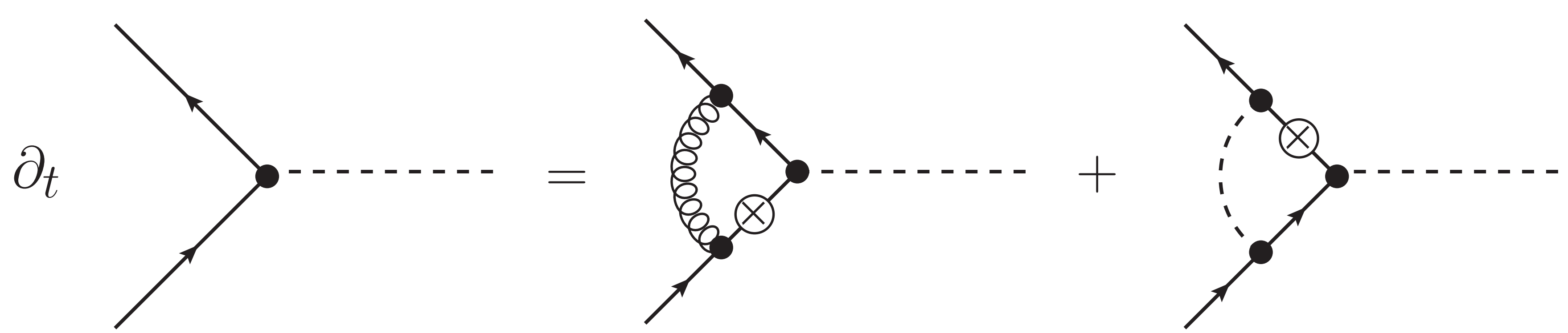


Yukawa coupling

Feynman Diagrams



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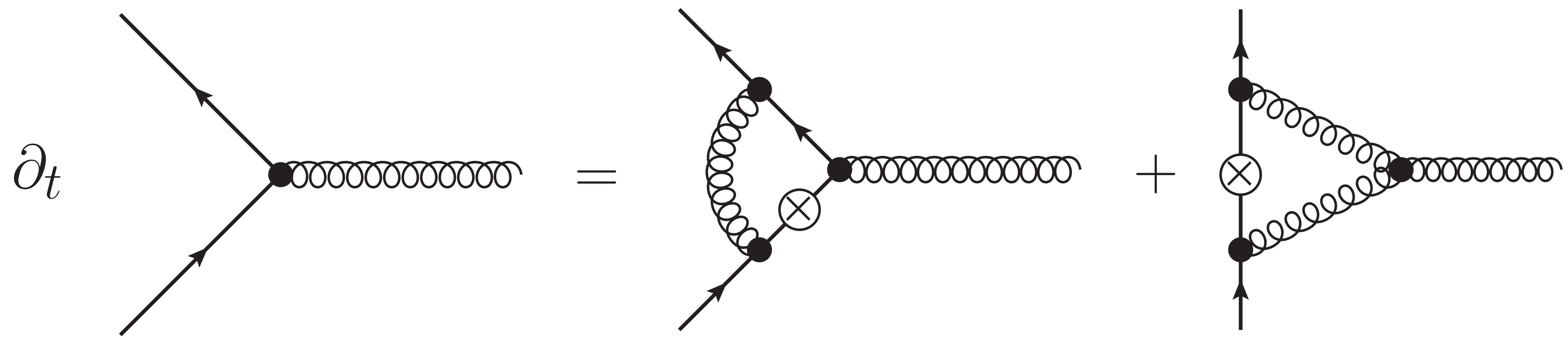
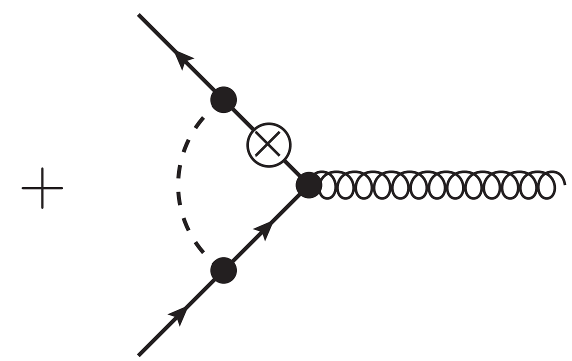
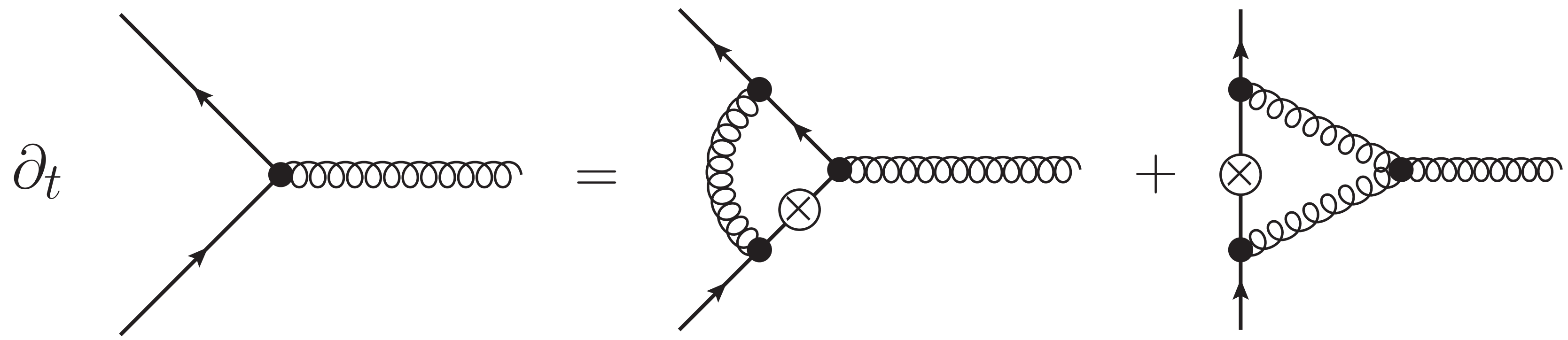


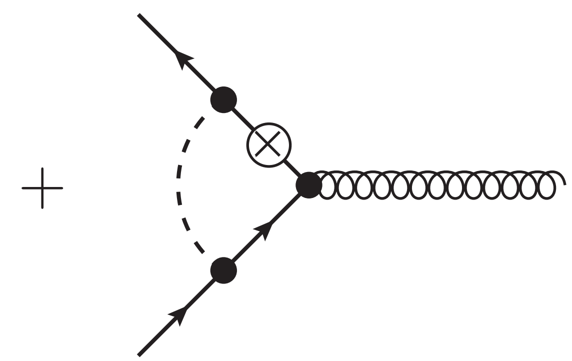
Quark gluon vertex

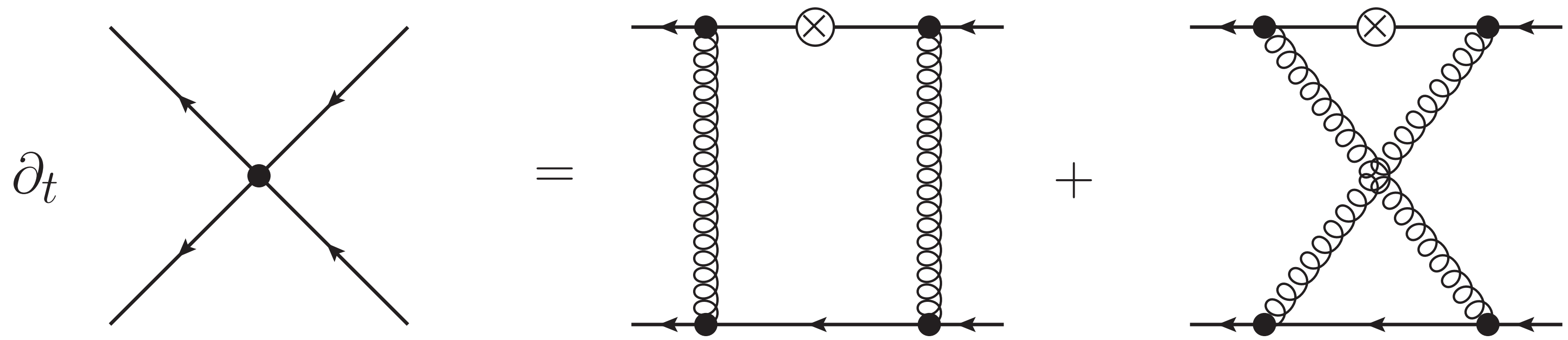


Feynman Diagrams

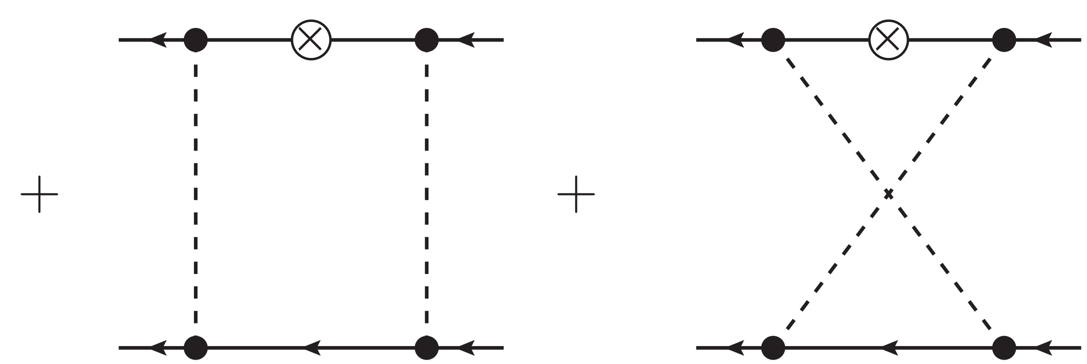


Quark gluon vertex

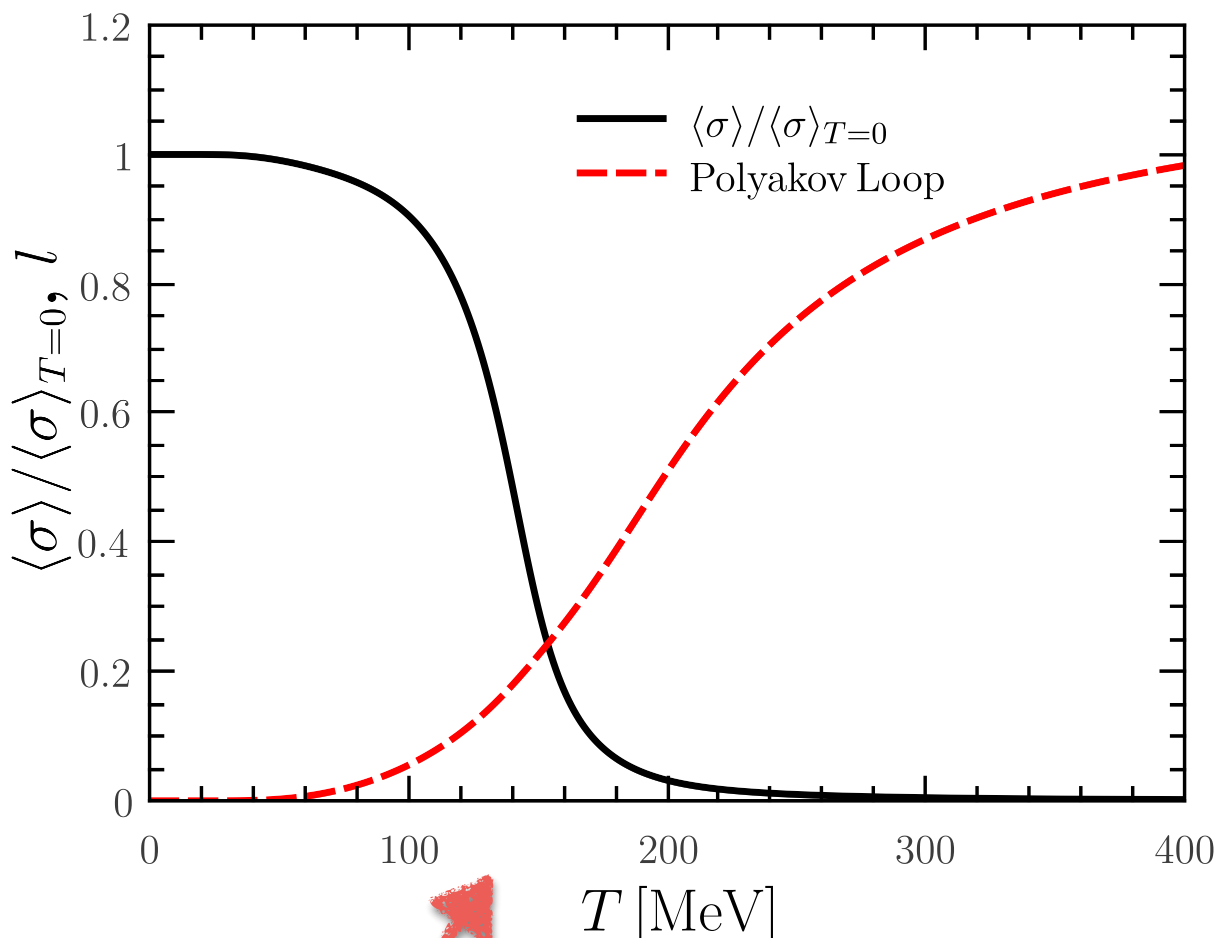


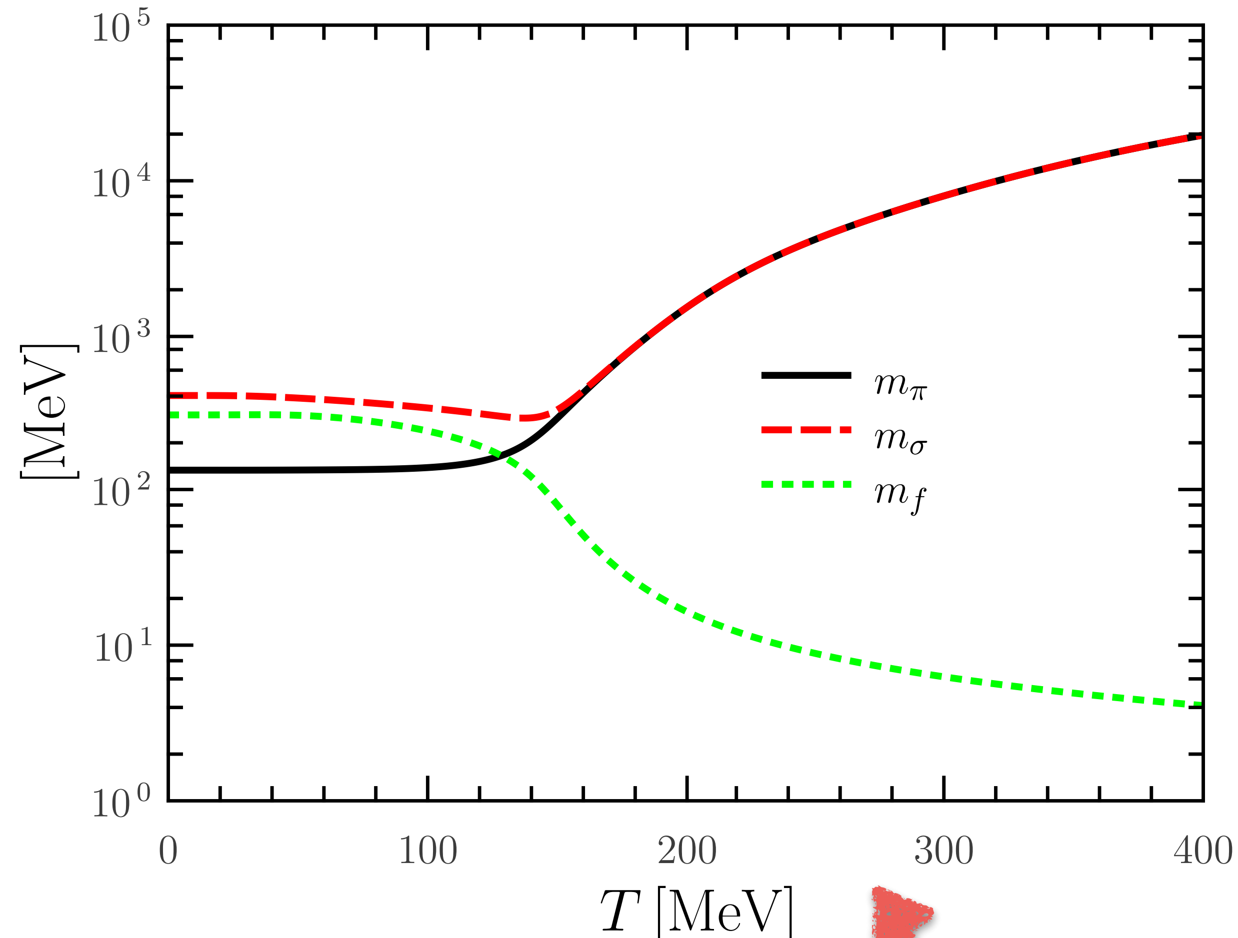


Four-fermion coupling



QCD Phase Transition with T





Order parameter

Meson and quark mass

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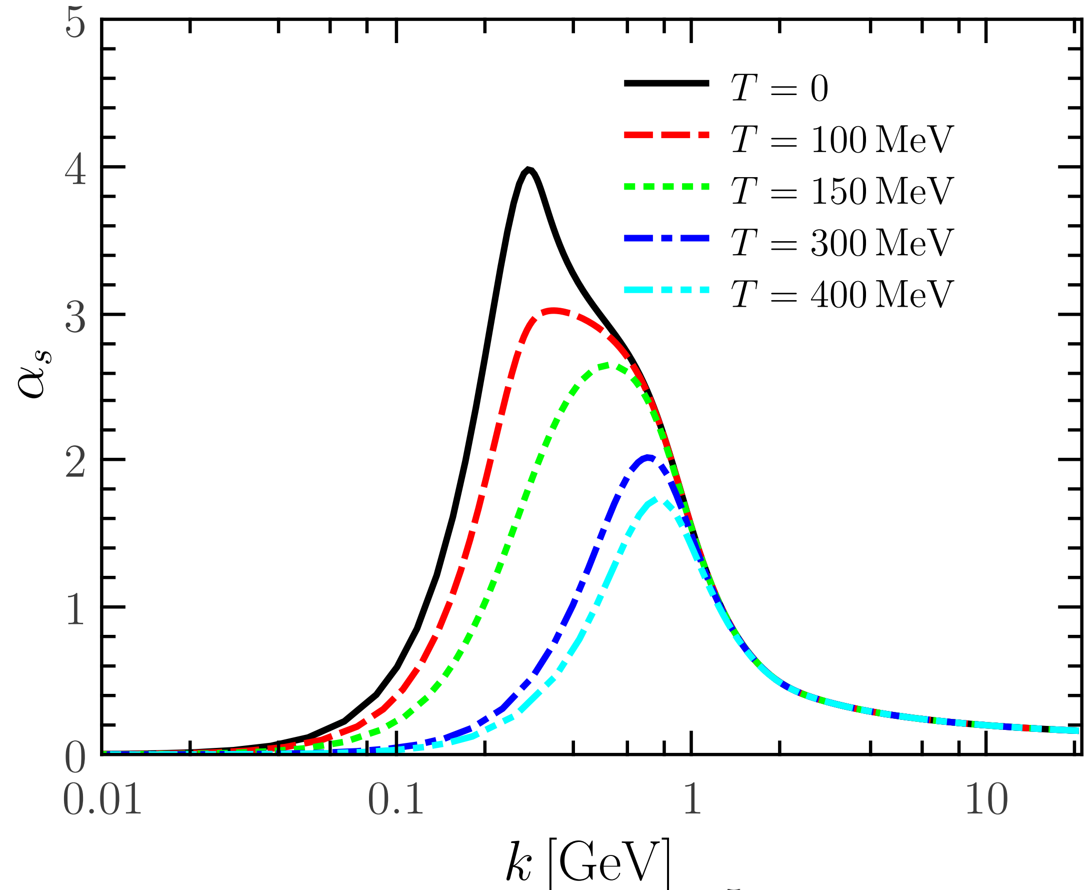
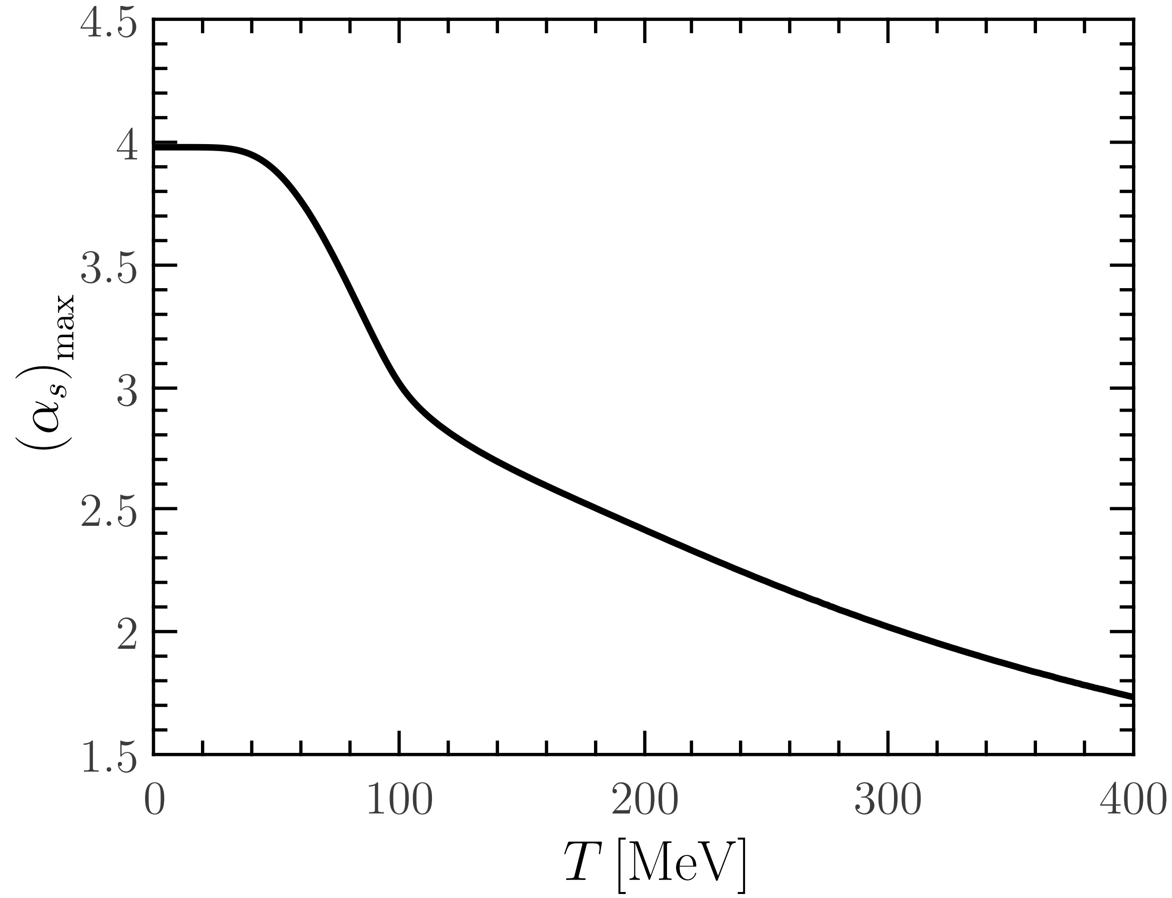
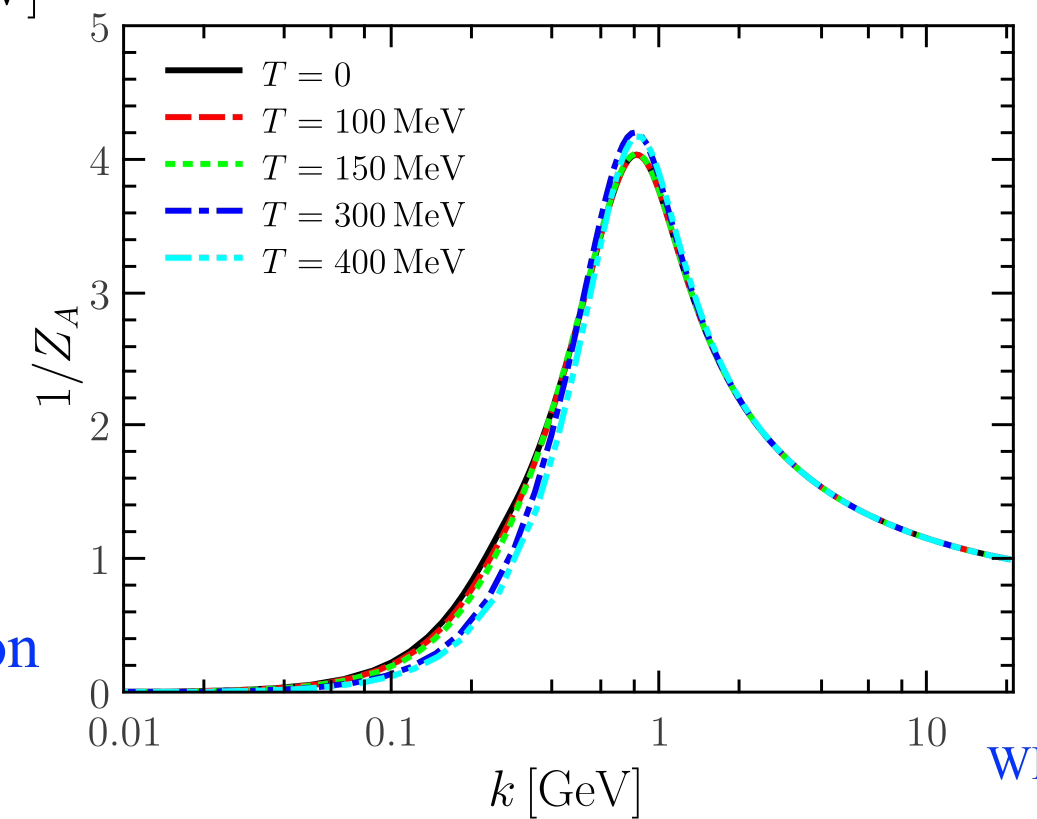
Running of the Strong Coupling





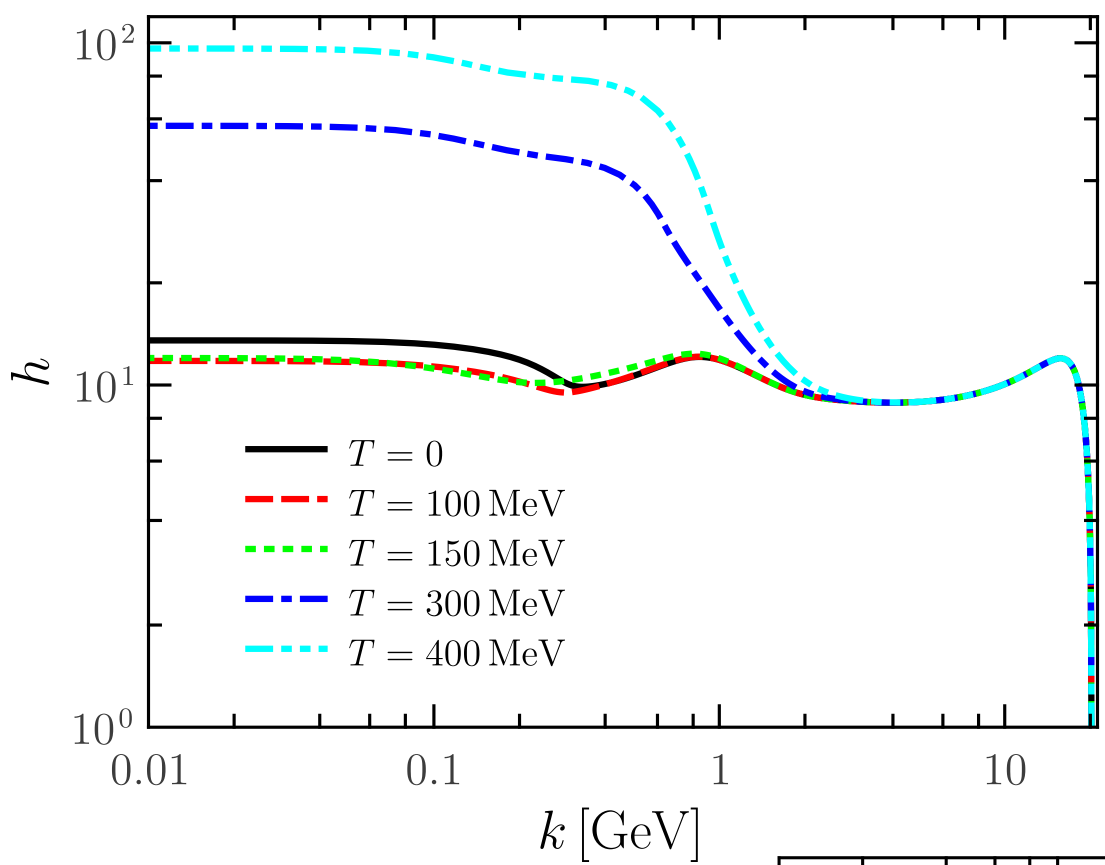


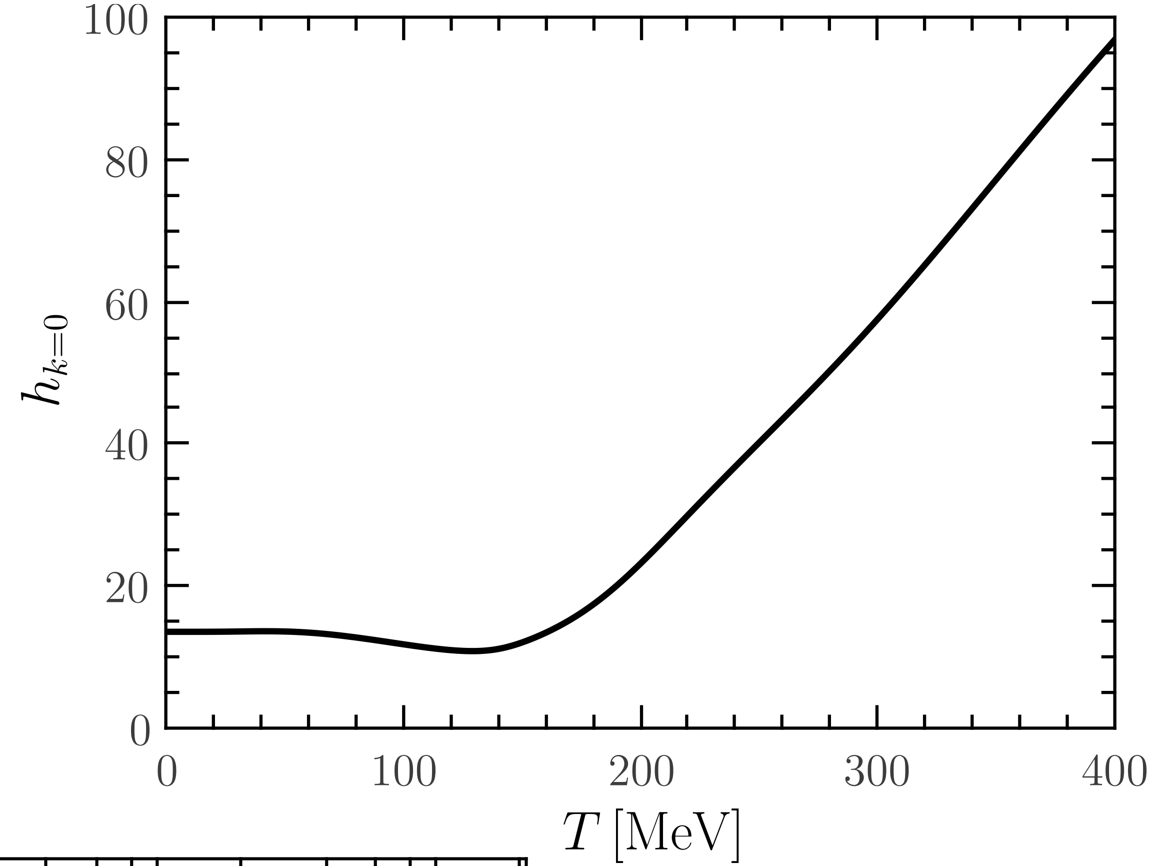
Gluon dressing function

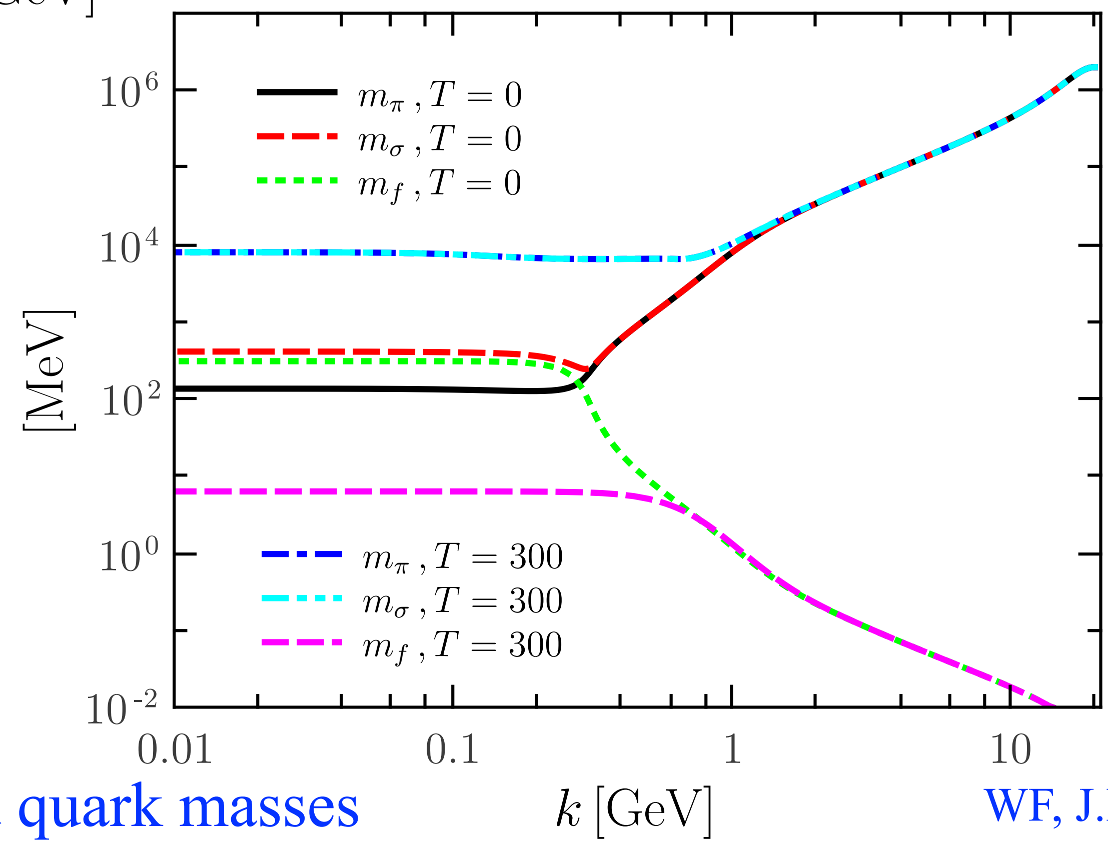
Note only the quark loop for the gluon are included for the moment, thermal effects of gluon loops are in progress.

WF, J.M. Pawlowski, F. Rennecke, in preparation

Yukawa coupling and Masses







Meson and quark masses

WF, J.M. Pawlowski, F. Rennecke, in preparation

Summary and outlook

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Thank you very much for your attentions!